Time value of money-Concepts and Calculations

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Lecture – 07
Comparison of all compounding methods

Welcome to the lecture series on Time value of money-Concept and Calculations. The present lecture is never takes to Comparison of all compounding methods.

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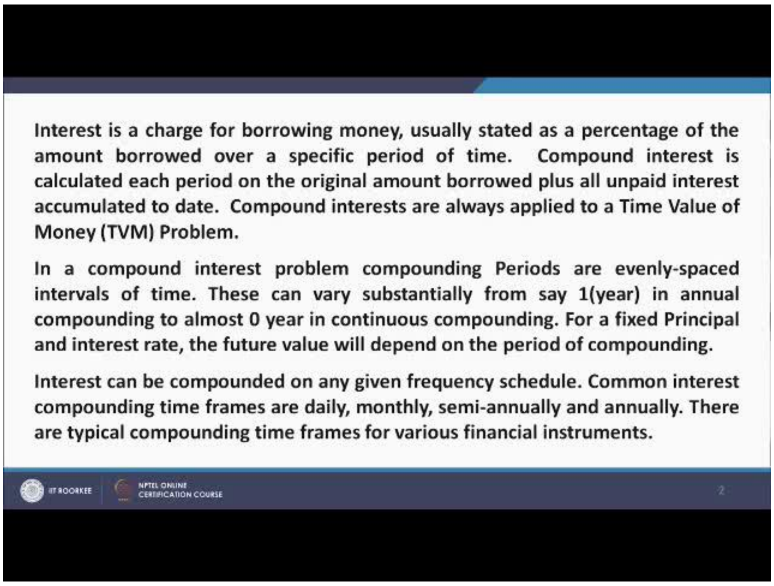

An interest is a charge for borrowing money, usually stated as a percentage of the amount borrowed over a specific period of time. Compounding interest is calculated each period on the original amount borrowed plus all unpaid interest accumulated to date. Compound interests are always applied to a Time Value of Money problem.

In a compound interest problem compounding periods are evenly-spaced intervals of time. These can vary substantially from say 1 year in annual compounding to almost 0 year in continuous compounding. For a fixed principle and interest rate the future value will depend on the period of compounding. Interest can be compounded on any given frequency schedule. Common interest compounding time frames are daily, monthly, semi-annually and annually. There are typical compounding time frames for various financial instruments.

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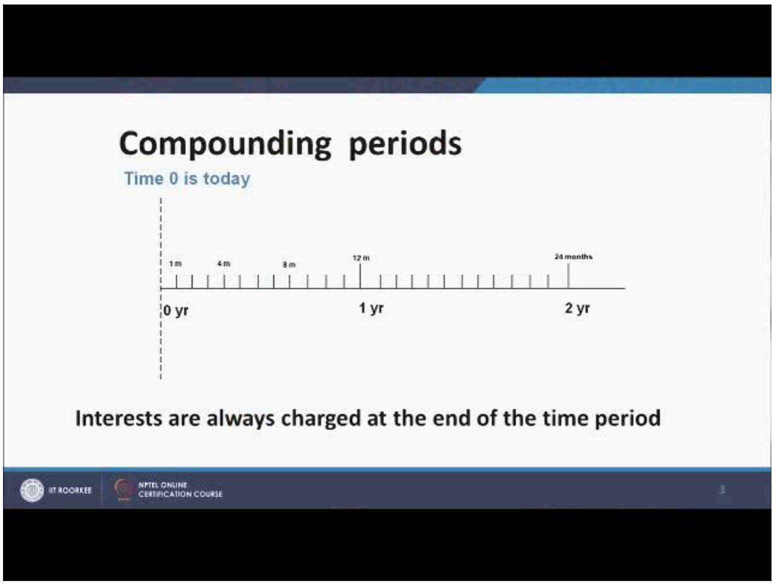

Now, different compounding periods as soon here can be applied. Compounding can be for a year or per year compounding, per month compounding, per day compounding, per 3 month compounding, etcetera, and etcetera. It should be noted that interest are always charged at the end of the time period.

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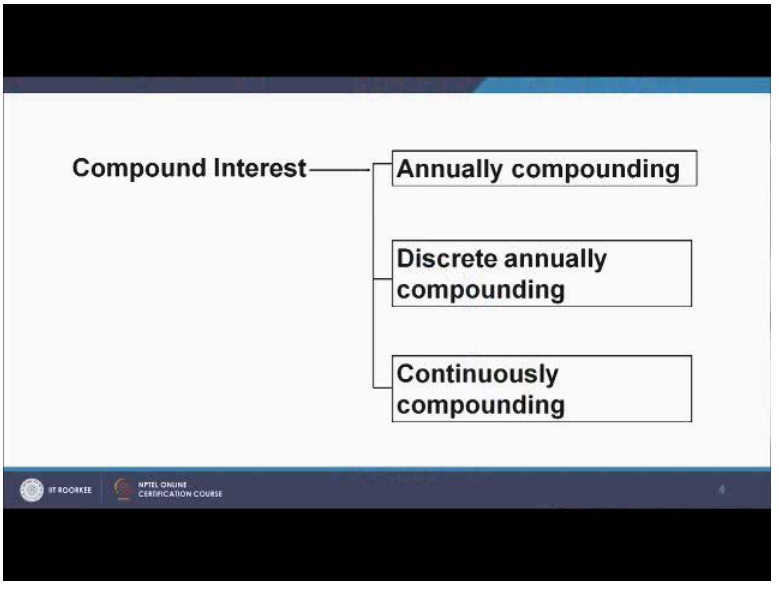

If you see the different type of compounding the compound interest can be divided in three parts; annually compounding, discrete annually compounding, and continuously compounding.

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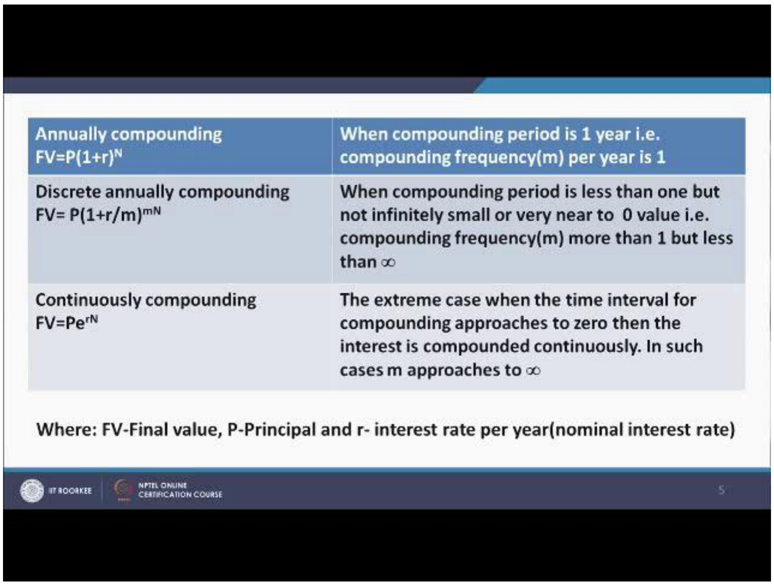

Annually compounding $FV=P(1+r)^N$	When compounding period is 1 year i.e. compounding frequency(m) per year is 1
Discrete annually compounding $FV= P(1+r/m)^{mN}$	When compounding period is less than one but not infinitely small or very near to 0 value i.e. compounding frequency(m) more than 1 but less than ∞
Continuously compounding $FV=Pe^{rN}$	The extreme case when the time interval for compounding approaches to zero then the interest is compounded continuously. In such cases m approaches to ∞

Where: FV-Final value, P-Principal and r- interest rate per year(nominal interest rate)

Now annually compounding is given by FV is equal to P 1 plus r to the power N, when compounding period is 1 year that is compounding frequency m per year is 1. Discrete annually compounding is given by FV is equal to P 1 plus r divided by m and whole to the power mN, when compounding period is less than 1, but not infinitely small or very near to 0 value.

That is compounding frequency m is more than 1, but less than infinite. Continuously compounding the FV is equal to Pe to the power rN, this is the extreme case when the time interval for compounding approaches to 0 and the interest is compounded continuously in such case m approaches to infinite.

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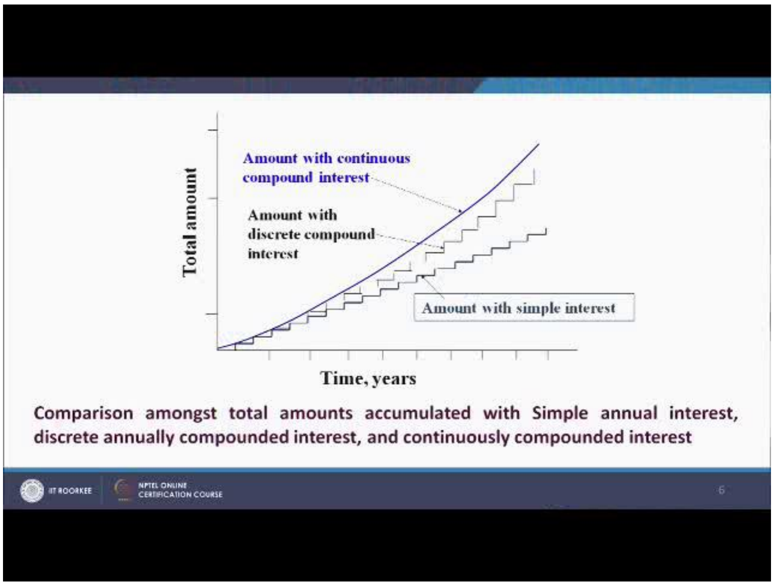

Now, if you see how the different compounding methods produce total amount after certain time. Then we find that s value for simple interest is the lowest one and s value for the continuous compounding interest is the highest one and in between is amount with discrete compounding interest.

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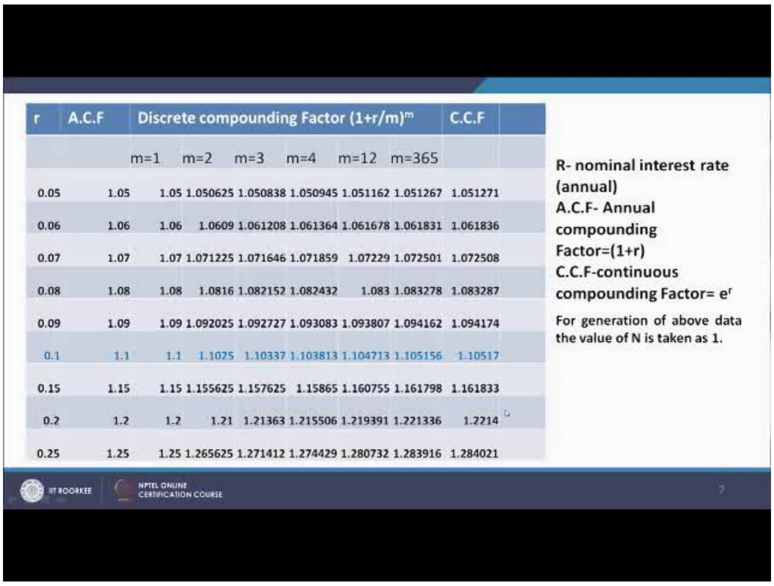

r	A.C.F	Discrete compounding Factor $(1+r/m)^m$						C.C.F
		m=1	m=2	m=3	m=4	m=12	m=365	
0.05	1.05	1.05	1.050625	1.050838	1.050945	1.051162	1.051267	1.051271
0.06	1.06	1.06	1.0609	1.061208	1.061364	1.061678	1.061831	1.061836
0.07	1.07	1.07	1.071225	1.071646	1.071859	1.07229	1.072501	1.072508
0.08	1.08	1.08	1.0816	1.082152	1.082432	1.083	1.083278	1.083287
0.09	1.09	1.09	1.092025	1.092727	1.093083	1.093807	1.094162	1.094174
0.1	1.1	1.1	1.1025	1.10337	1.103813	1.104713	1.105156	1.10517
0.15	1.15	1.15	1.155625	1.157625	1.15865	1.160755	1.161798	1.161833
0.2	1.2	1.2	1.21	1.21363	1.215506	1.219391	1.221336	1.2214
0.25	1.25	1.25	1.265625	1.271412	1.274429	1.280732	1.283916	1.284021

Now, if we try to compare the annual compounding factor 1 plus r continuous compounding factor e to the power r. And discrete compounding factor 1 plus r divided by m to the power m for in for different values of r then this table can be created.

In this table we will see that when compounding becomes high that is m is equal to 365 the C.C.F which continuous compounding factor e to the power r is almost equal to the discrete compounding factor and there is little change between these two at least up to four digit they are equal.

Now, if you float the percent raise in interest factor in comparison to annual compounding factor.

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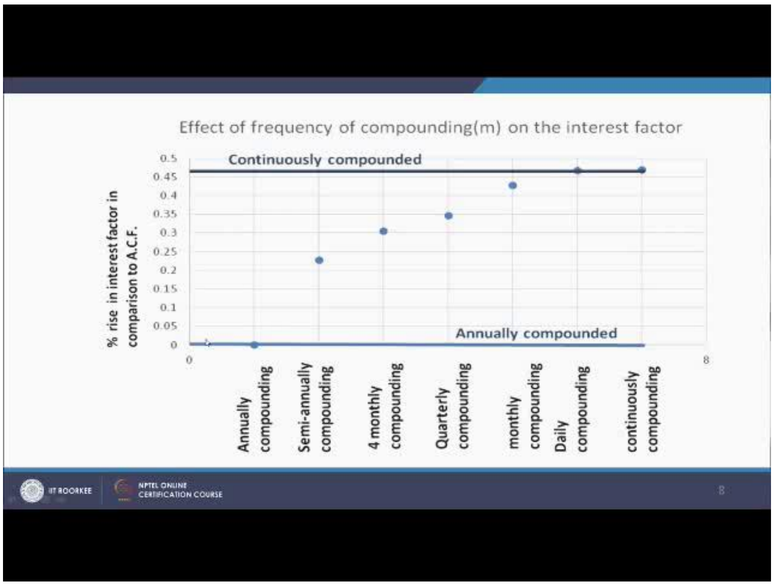

So, it is the annual compounding factor, this line is annual compounding factor and this is continuously compounding factor. And when the frequency of compounding increases when it is annually compounding this falls on the annually compounded line, semi-annually it fix up, then four monthly compounding, then quarterly compounding, then monthly compounding, then daily compounding.

So, at the daily compounding and continuously compounding it almost touches the continuously compounded factor. That means, when we go for daily compounding we almost touch the continuously compounding or compounded factor.

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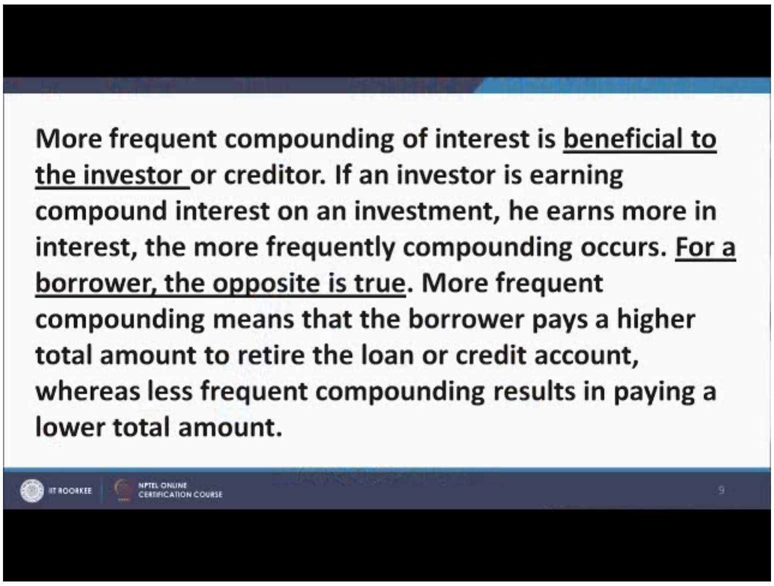

More frequent compounding of interest is beneficial to the investor or creditor. If an investor is earning compound interest on an investment, he earns more in interest, the more frequently compounding occurs. For a borrower the opposite is true. More frequent compounding means that the borrower pays a higher total amount to retire the loan or credit account, whereas less frequent compounding results in paying a lower total amount.

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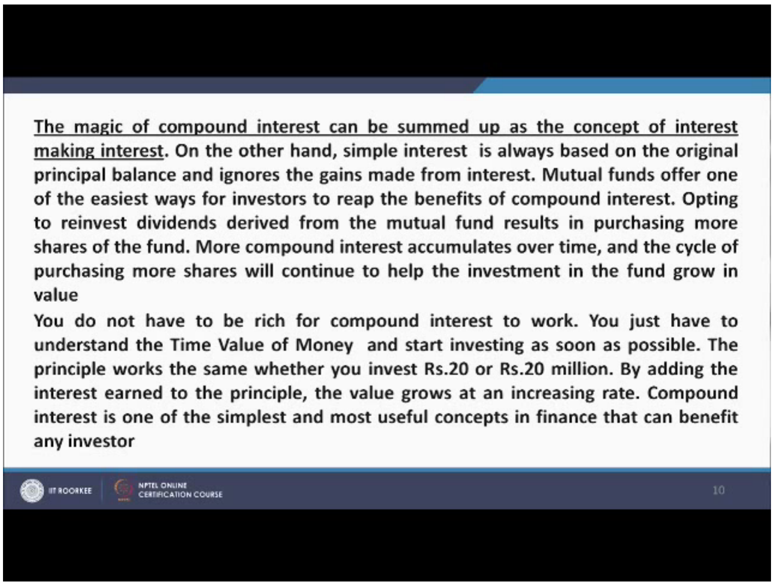

The magic of compound interest can be summed up as the concept of interest making interest. On the other hand simple interest is always based on the original principle balance and ignores the gains made from interest. Mutual funds offer one of the easiest ways for investors to reef the benefits of compound interest. Opting to reinvest dividends derived from the mutual fund results in purchasing more shares of the fund. More compound interest accumulates over time and the cycle of purchasing more shares will continue to help the investment in the fund grow in value.

You do not have to be rich for compound interest to work. You just have to understand the time value of money and start investing as soon as possible. The principle works the same way whether you invest Rupees 20 or 20 million. By adding the interest earned to the principle, the value grows at an increasing rate. Compound interest is one of the simplest and most useful concepts in finance that can benefit any investor.

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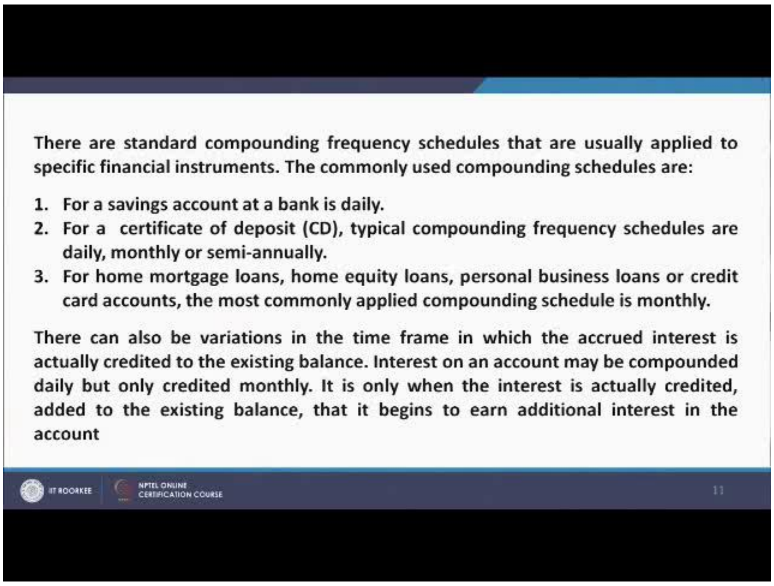

There are standard compounding frequencies schedules that are usually applied to specific financial instruments. The commonly used compounding schedules are; for a saving account at a bank. It is daily for a certificate of deposit CD; typical compounding frequency schedules are daily, monthly, or semi-annually. For home mortgage loans, home, equity loans, personal business loans or credit card accounts the most commonly applied compounding schedule is monthly.

There can also be variations in the time frame in which the accrued interest is actually credited to the existing balance. Interest on an account may be compounded daily but only credited monthly. It is only when the interest is actually credited added to the existing balance that it begins to earn additional interest in the account.

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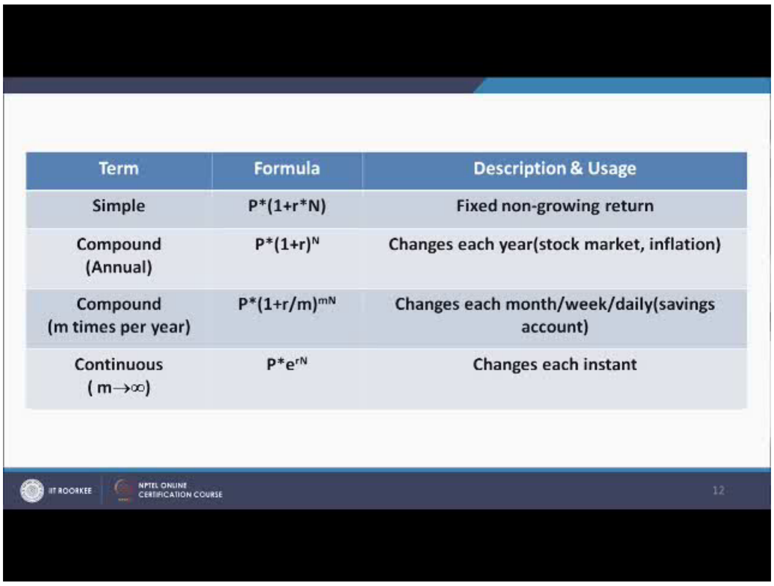

Term	Formula	Description & Usage
Simple	P*(1+r*N)	Fixed non-growing return
Compound (Annual)	$P*(1+r)^{N}$	Changes each year(stock market, inflation)
Compound (m times per year)	$P*(1+r/m)^{mN}$	Changes each month/week/daily(savings account)
Continuous ($m\rightarrow\infty$)	$P*e^{rN}$	Changes each instant

Now, you see for simple interest the formula is P into 1 plus r into N, this gives you the FV value, fixed non-growing return we call it. The compound interest when it is annual it is P into 1 plus r to the power N changes each year stock market and inflation N use this compounded m times per year the formula is P into 1 plus r divided by m whole to the power mN changes each month week daily and this is used in savings account and continuous m tends to infinite. So, s is equal to P into e to the power rN changes each instant.

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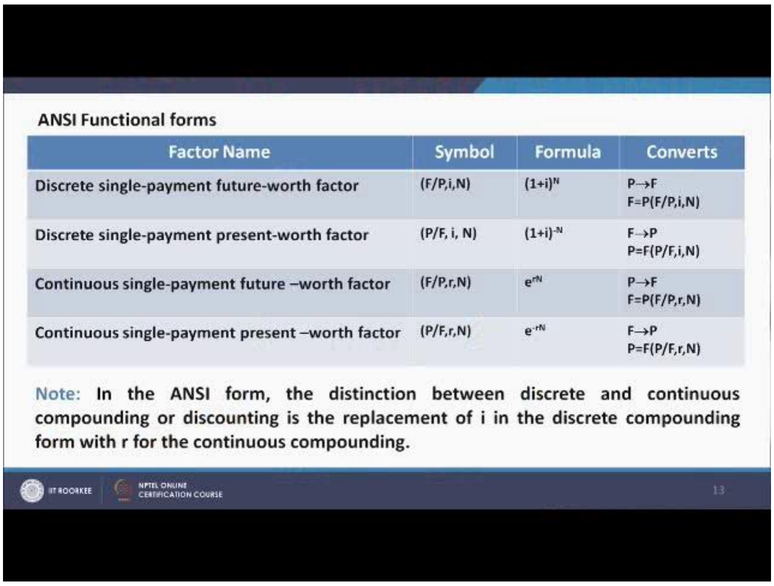
ANSI Functional forms

Factor Name	Symbol	Formula	Converts
Discrete single-payment future-worth factor	(F/P,i,N)	$(1+i)^N$	P→F F=P(F/P,i,N)
Discrete single-payment present-worth factor	(P/F, i, N)	$(1+i)^{-N}$	F→P P=F(P/F,i,N)
Continuous single-payment future –worth factor	(F/P,r,N)	e^{rN}	P→F F=P(F/P,r,N)
Continuous single-payment present –worth factor	(P/F,r,N)	e^{-rN}	F→P P=F(P/F,r,N)

Note: In the ANSI form, the distinction between discrete and continuous compounding or discounting is the replacement of i in the discrete compounding form with r for the continuous compounding.

Now if you see ANSI functional forms and different factors. Then discrete single payment future worth factor symbol is in brackets F by P comma i comma N brackets closed. The formula is 1 plus i to the power N this converts P to F and written as F is equal to P into de factor that is F by P comma i comma N. Discrete single payment present worth factor; the symbol is in brackets P oblique F comma i comma N this is 1 plus i whole to the power minus N, it converts F to P and is written as P is equal to F and in brackets P by F comma i comma N.

Continuous single payment future worth factor this is denoted by F by P comma r comma N and given as e to the power rN this converts P to F and is given as F is equal to P into that factor which is F by P comma r comma N. Continuous single payment present worth factor; this is P by F rN in brackets is given by e to the power minus rN and this converts F to P and written as P is equal to F into de factor that is F by P comma r comma N.

In the ANSI form the distinction between discrete and continuous compounding or discounting is the replacement of i in the discrete compounding form with r for the continuous compounding form.

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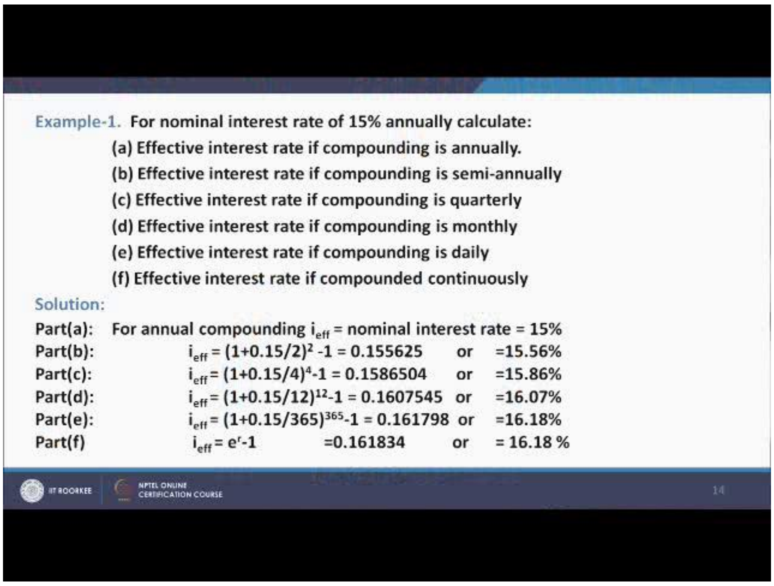

Now let us see some examples; for nominal interest rate of 15 percent annually calculate; part a, effective interest rate if compounding is annually; b, effective interest rate if compounding is semi-annually; c, effective interest rate if compounding is quarterly; d, effective interest rate if compounding is monthly; e, effective interest rate if compounding is daily; and F, effective interest rate if compound at continuously.

Solution; part a for annual compounding i effective is equal to nominal interest rate and thus i effective in this case is 15 percent. Now for part b when it is compounding semi-annually i effective is equal to 1 plus 0.15 by 2 to the power 2 minus 1 it comes out to be 0.155625 or 15.56. Now see here that i effective is more than the nominal interest rate of 15 percent. Now part c, where it is compounded quarterly, so i effective is equal to 1 plus 0.15 divided by 4 whole to the power 4 minus 1 comes out to be 0.1586504 or 15.86 percent.

Part d, where it is compounding monthly i effective is equal to one in brackets 1 plus 0.15 divided by 12 whole to the power 12 minus 1 it comes out to be 0.1607545 or 16.07 percent. Now part e, where compounding is daily this is in brackets i effective is equal to in brackets 1 plus 0.15 divided by 365 and whole to the power 365 minus 1 is 0.161798 or 16.18 percent. And if you see the compound at continuously i effective is e to the power r minus 1 comes out to be 0.161834 or 16.18 percent.

What we see here the i effective in the case of compounding daily and i effective in the case of compounded continuous base almost equal. And hence i daily is almost equal to the i effective.

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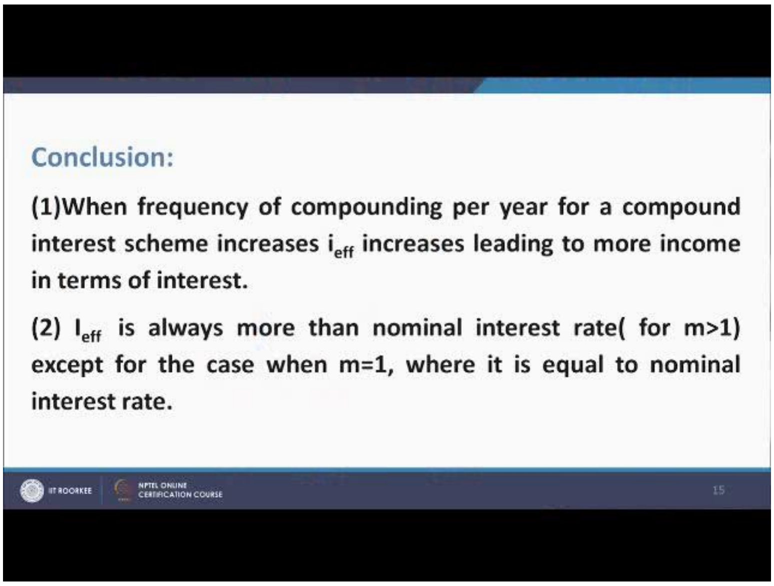

Conclusion: when frequency of compounding per year for a compounding interest scheme increases i effective increases leading to more income in terms of interest. Number 2; i effective is always more than nominal interest rate for m is greater than 1, except for the case when is equal to m is equal to 1 where it is equal to nominal interest rate.

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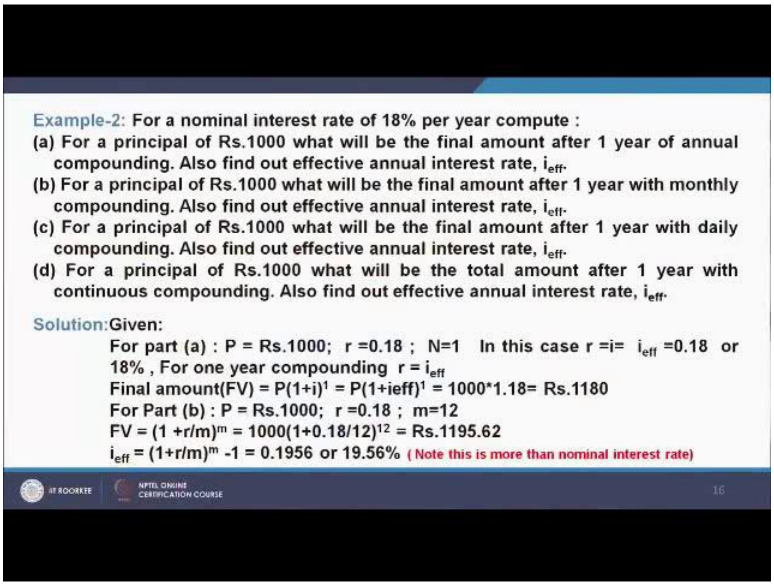

Now let us take another example. For a nominal interest rate of 18 percent per year compute for a principle of Rupees 1000 what will be the final amount after 1 year of annual compounding. Also find out effective annual interest rate. Part b, for a principle of 1000 what will be the final amount after 1 year with monthly compounding. Also find out the effective annual interest rate i effective.

C part, for a principle of Rupees 1000, what will be the final amount after 1 year with daily compounding. And also find out the effective annual interest rate i effective. And d, for a principle of Rupees 1000 what will be the total amount after 1 year with continuous compounding. And also find out the effective annual interest rate i effective.

Now for part a; P is equal to 1000, r is equal to 0.18, N is equal to 1. In this case r is equal to i is equal to i effective is equal to 0.18 or 18 percent. For 1 year compounding r is equal to equal to i effective. The final amount FV or the future value is equal to P 1 plus i to the power 1 is equal to P in the brackets 1 plus i effective is equal to 1000 into 1.18 is equal to 1180. That means, if i invest 1000 with r is equal to 18 percent for 1 year and i will get 1180.

Part b; P is equal to 1000, r is equal to 0.18, m is equal to 12 that means monthly compounding. Here FV is equal to P into 1 plus r divided by m whole to the power m is equal to 1000 1 plus 0.18 divided by 12 whole to the power 12 is comes out to be Rupees 1195.62. So, this amount has increased.

If you see that for 1 year compounding the final amount is 1180, but same 1 year compounding but for monthly compounding for 1 year the amount is 1195.62. Now, i effective is equal to 1 plus r divided by m to the power m minus 1 comes to be 19.56 percent. Now note this is more than the nominal interest rate.

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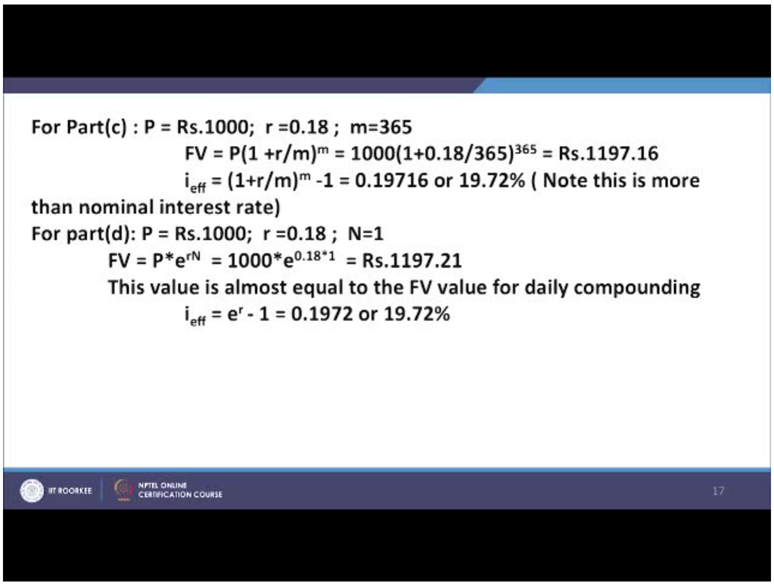

Part c; P is equal to 1000, r is equal to 0.18, m is equal to 365 that means daily compounding. So, FV value is equal to P into 1 plus r divided by m whole to the power m comes out to be 1197.16, and the i effective is 0.19716 or 19.72. Note this is more than the nominal interest rate. Now part d; P is equal to 1000, r is equal to 0.18, N is equal to 1. And for continuous compounding FV is equal to P into e to the power rN. So, this is 1000 into e to the power 0.18 into 1 comes out to be Rupees 1197.21.

So, if you see this only difference is between the daily compounding and continuous compounding is about 50.0; about 5 paisa. These values is almost equal to the FV value of daily compounding and i effective for continuous compounding is e to the power r minus 1 comes out to be 19.72 percent. And if you see the i effective of daily compounding which is also 19.72 percent.

Thank you.