Time value of money-Concepts and Calculations

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Lecture – 06
Continuous compounding

Welcome to the Lecture 6 on Time value of money-Concepts and Calculation. The present lecture is devoted to Continuous Compounding.

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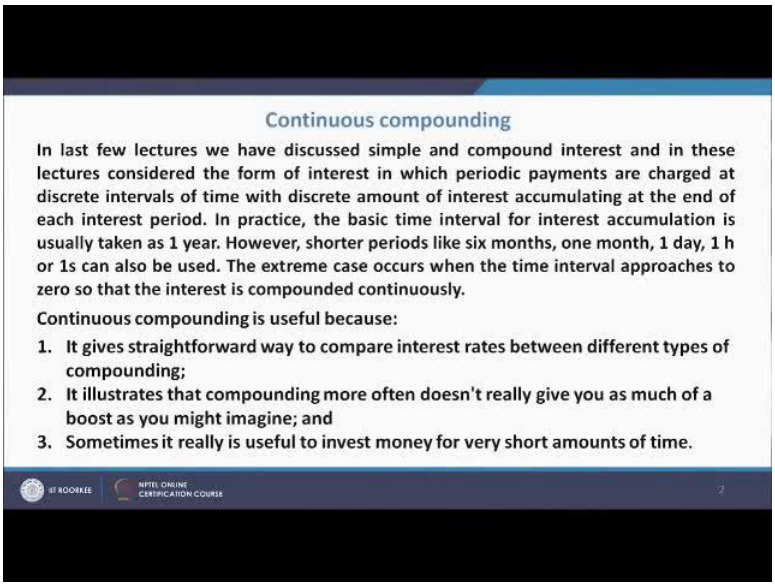

In last few lecture we have discussed simple and compound interest and in these lectures consider the form of interest in which periodic payments are charged at discrete intervals of time with discrete amount of interest accumulating as the end of each interest period. In practice the basic time interval for interest accumulation is usually taken as 1 year. However shorter periods like 6 months, 1 month, 1 day, 1 hour or 1 second can also be used. In the extreme case when the time interval approaches to 0. So, that the interest is compounded continuous is called Continuous Compounding.

Continuous compounding is useful because, it gives straight forward way to compare interest rates between different types of compounding. Second, it illustrates that compounding more often does not really give you as much of a boost as you might imagine. And third, sometimes it really is useful to invest money for very short amount of time.

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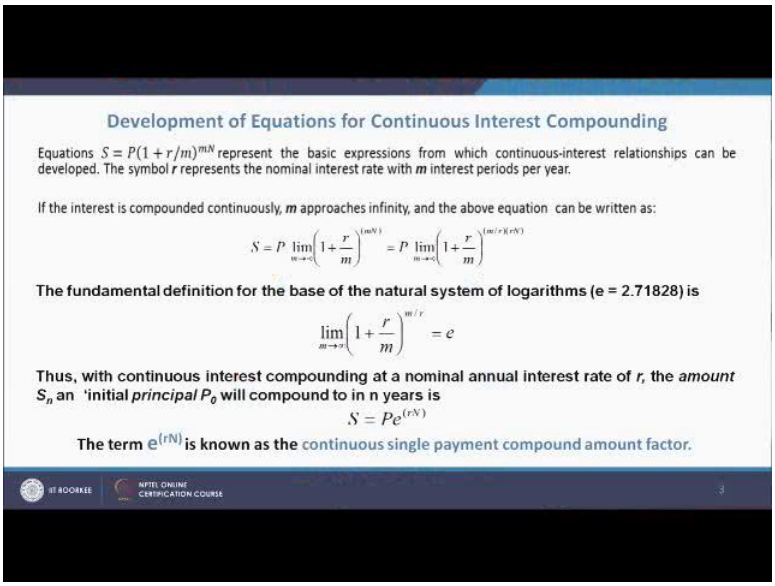

Now, development of equation for continuous compounding the basic equations which we know that S is equal to P 1 plus r divided m to the power m N which is use for discreet compounding represents the basic equation for which continuous interest relationships can be derived. The symbol r represents the nominal interest rate with m interest period per year. If the interest is compounded continuously m approaches infinity and the above equation can be rewritten has S equal to P limit m tends to infinity in bracket 1 plus r by m whole to the power m N is equal to P limit m tends to 0 in the bracket 1 plus r by m whole to the power m by r into r into N.

The fundamental definition for the base of the natural system of logarithms that is e is equal to 2.71828 is limit m tends to infinity 1 plus r by m to the power m by r is equal to e. Thus with continuous interest compounding at nominal interest rate of r the amount S N for initial principal P 0 compounded for m years is equal to S equal to P e to the power rN. The term e to the power rN is known as the continuous single payment compound amount factor.

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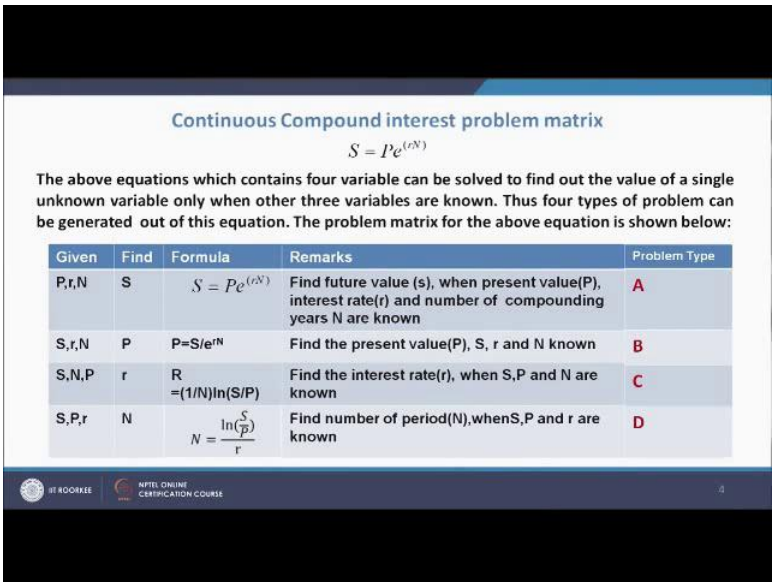

Now, let us see at what type of problems can be created for this continuous compound interest. The equation is S is equal to P e to the power rN. The above equation which contains four variables that is S P r and N can be solve to find out the value of single unknown variable only when other three variables are known. Thus 4 type of problem can be generated out of this equation. The problem matrix of the above equation is shown below.

Now if P r and N are given we can find out the value of S, where S is equal to P into e to the power rN. In this case we have to find out the value of S that is future value when present value P interest rate r and number of compounding years N r known. This type of problem will be called problem type A.

The second type of problem will be given S r and N, we have to calculate the value of p. So, the formula we should be used for this purpose is P is equal to S divided by e to the power rN. Find the present value P provided S r and N are known. Such type of problem will be called problem type B.

The third type problem S N and P will be given and r as to be found out. For this problem r is equal to 1 by N ln S by P. Find the interest rate r where S P and N are known. Such type of problem is called problem type C.

The forth and the last type of problem is S P and r are given N has to be found out. An N is equal to ln S by P whole divided by r. In this case number of periods N has to be found out when S P and r are known. This type of problem will be called problem type D.

Let us start with different problems and it is solution.

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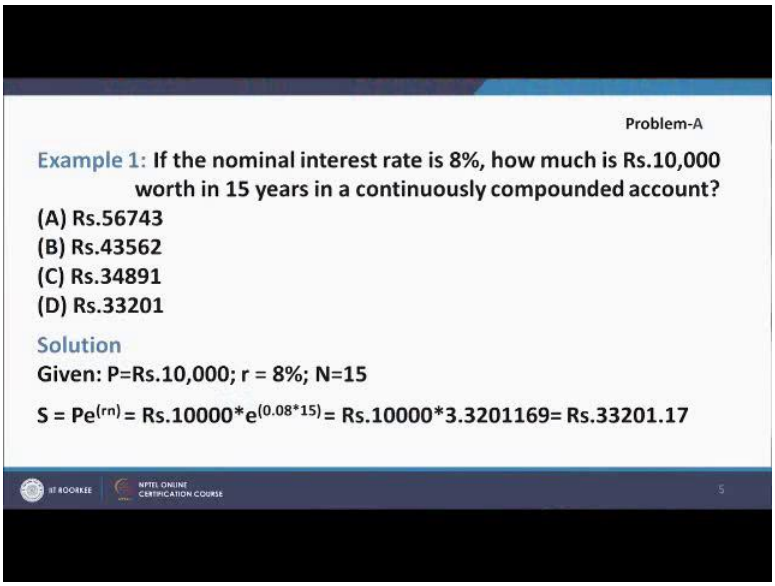

Now problem number A or the problem type A for example 1; if the nominal interest rate is 8, percent how much is Rupees 10000 towards in 15 years in a continuous compounded account? There are four answers, the solution is P is given as 10000, nominal interest rate is given as 8 percent, and N is given has 15 years. So, S as to be found out S is P into e to the power rN.

So, S is equal to 10000 into e to the power 0.08 into 15. This comes out to be 10000 into 3.3201169 comes out to be Rupees 33201.17. That means if we invest 10000 we can nominal interest rate of 8 percent for 15 years it will provide you Rupees 33201.17 if the compounding is continuous.

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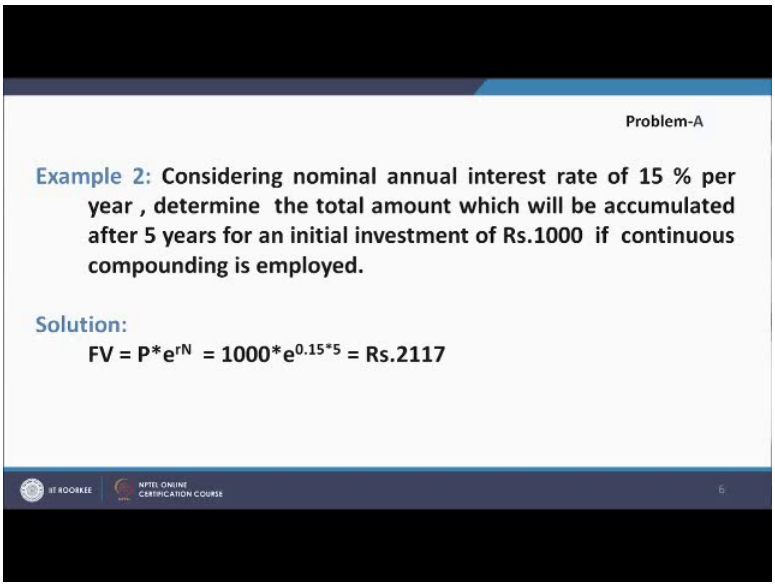

Again problem type A; example 2. Consider nominal annual interest rate of 15 percent per year; determine the total amount which will be accumulated after 5 years for an initial investment of 1000 if continuous compounding is employed. Solution; we know what is the value of P that is 1000, we know the value of interest rate which is nominal interest rate it is 15 percent, we know the value of N which is 5 years. So, we have to find out S or FV and FV is equal to P into e to the power rN is equal to 1000 into e to the power 0.15 the value of r into 5 which is the value of N.

And this gives us Rupees 2117. That is if 1000 Rupees invested today for 5 year with a nominal annual interest rate of 15 percent and the compounding is continuous it will convert into Rupees 2117.

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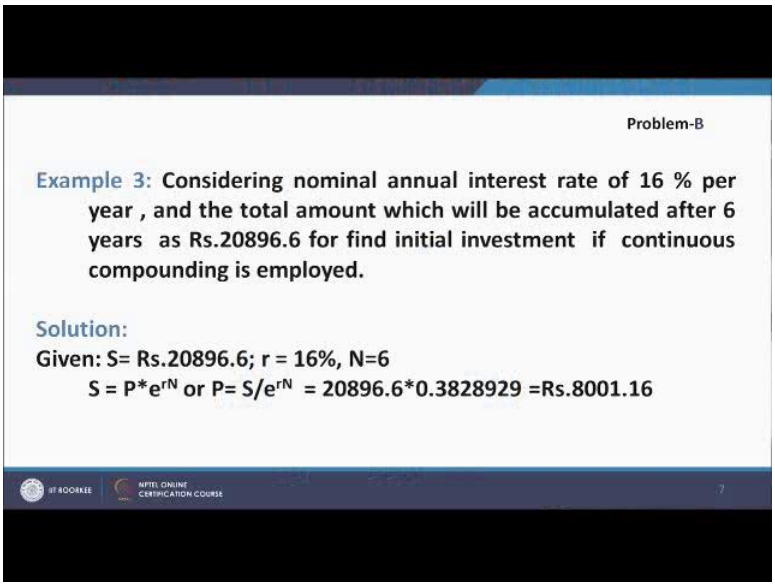

Problem type B example 3, considering nominal annual interest rate of 16 percent per, year and the total amount which will be accumulated after 6 year as 20896.6 for this find initial investment if continuous compounding is employed. Solution S is given or FV is given has 20896.6, r is given has 16 percent, N is 6. So, S equal to P into e to the power rN or P is equal to S divided e to the power rN.

So, this is equal to 20896.6 into 0.3828929 and this value 0.3828929 is equal to 1 by e to the power rN. So, the multiplication gives Rupees 8001.16. That means, if I invest 8001.16 for 6 year with nominal annual interest rate of 16 percent it will give me 20896.6 Rupees.

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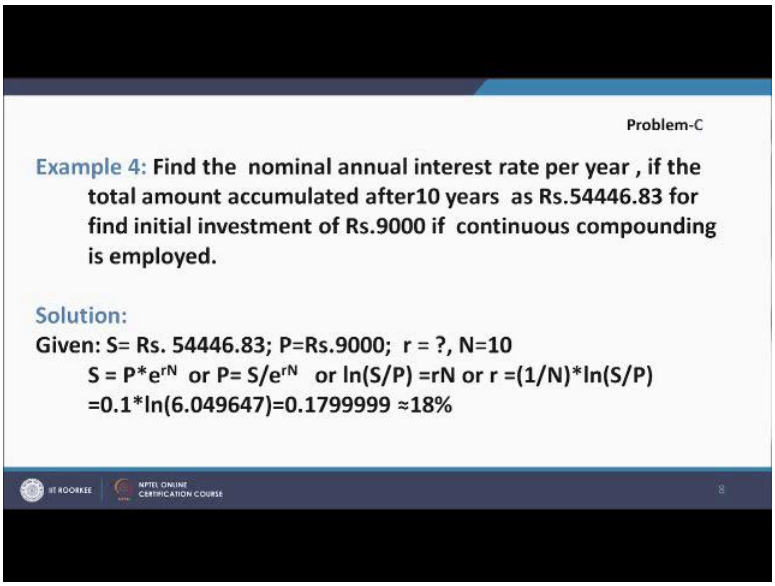

Now problem C, this is given in example 4. Find the nominal annual interest rate per year if the total amount accumulated after 10 years as 54446.83 for this we find the initial investment of Rupees 9000 if continuous compounding is employed. So, if we analyze this problem we find that S is given 54446.83, P is given as Rupees 9000, N is given as 10 years, but what is the value of r if value of r is not there. So, we have to find it out so again we use the same equation S is equal to P into e to the power rN or P is equal to S divided by e to the power rN or we can write down S by P is equal to e to the power rN.

And if we take log both the side becomes ln S by P is equal to rN or r is equal to 1 by N into ln S by P. If we substitute the value of the N S and P N into this, so this is equal to 0.1 into ln 6.049647 and this is equal to 0.174444 and that is approximately 18 percent. That means, 9000 will be converted into 54446.83 Rupees in 10 years if nominal interest rate r is equal to 18 percent.

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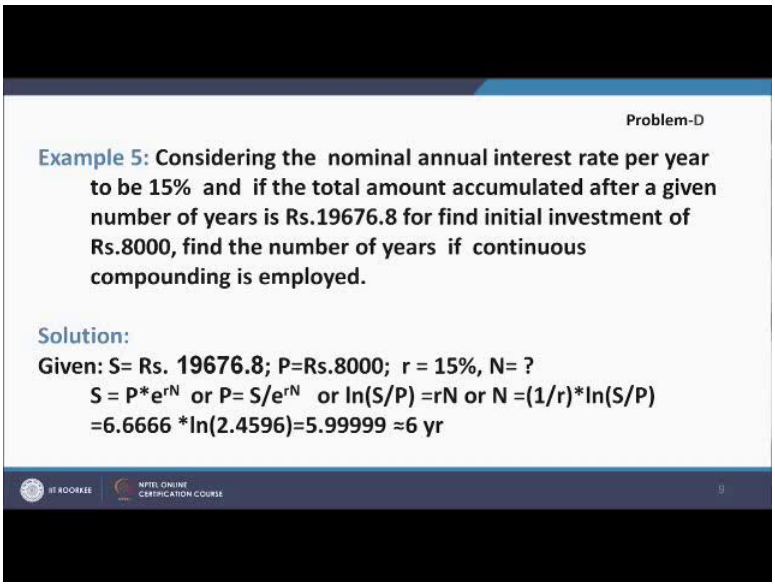

Now problem type D for example 5; considering the nominal annual interest rate per year to be 15 percent and if the total amount accumulated after given number of S is Rupees 19676.8. Find initial investment of 8000; find the number of years if continuous compounding is employed. Solution; given S is equal to Rupees 19676.8, P is 8000, r is equal to 15 percent and what is the value of N.

So, again we use the same equation S is equal to P into e to the power rN or P is equal to S by e to the power rN or we can write down e to the power rN is equal to S by P if we take log both the sides ln S by P is equal to rN or N is equal to 1 by r into ln S by P. This is equal to 6.6666 into ln 2.4596 and we will multiply these two it comes out to be 5.99999 which is equivalent to 6 years. That means, the Rupees 8000 invested with nominal interest rate of 15 percent we will convert into Rupees 19676.8 in 6 years if continuous compounding is employed.

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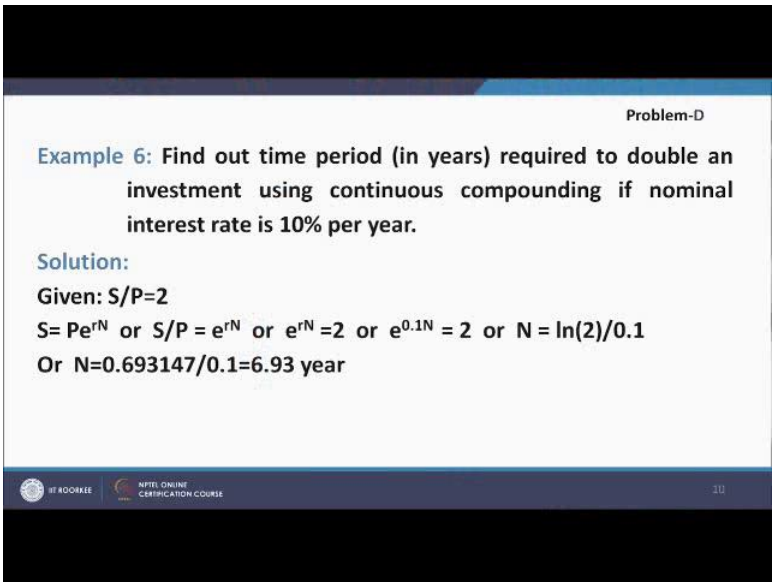

Now, again problem D this is example 6. Find out time period in years required to double an investment within continuous compounding if nominal interest rate is 10 percent per year.

Solution; here S by P is equal to 2 and we know that for continuous compounding S is equal to P e to the power rN or S by P e to the power rN or e to the power rN is equal to 2, or e to the power 0.1 N when we put the value of r as 0.1 because it is 10 percent then it becomes e to the power 0.1 N is equal to 2 and if we take log of both the sides then N is equal to ln 2 divided by 0.1 or N is equal to 0.693147 which is the natural logarithmic value of two divided by 0.1 is equal to 6.93 years.

So, a value will double in 6.93 years if nominal interest rate is 10 percent and continuous compounding is used.

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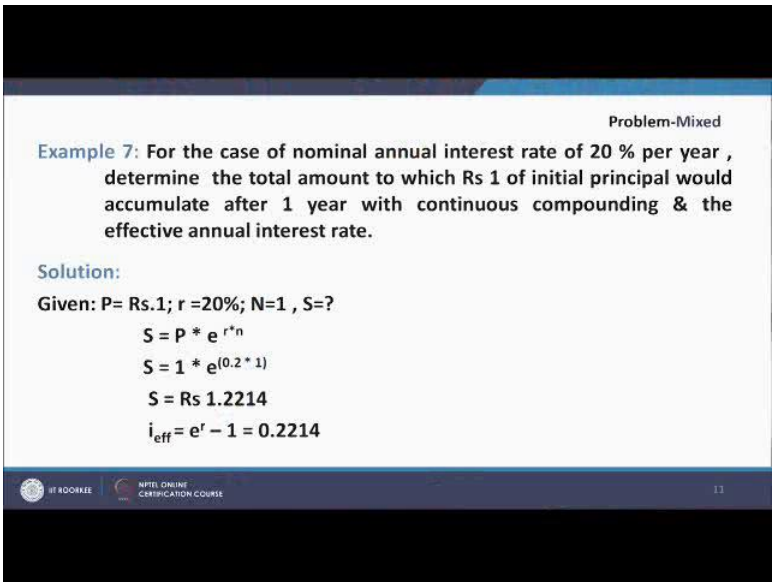

Let us take a mix problem for this we considering example 7. For the case of nominal annual interest rate of 20 percent per year determine the total amount to which Rupees 1 of initial principal would accumulate after 1 year with continuous compounding and the effective annual interest rate. What are the things which are given P is equal to Rupees 1, r is equal to 20 percent, N is equal to 1, and S is what, that means we have to find out S when the value of P r and N are known.

So, again we use the formula for continuous compounding S is equal to P into e to the power rN or S equal to 1 e to the power 0.2 into one because r is 0.2 and N is equal to 1. So, S is equal to Rupees 1.2214. Now I effective is equal to e to the power r minus 1 and this comes out to be 0.2214. That means, if I invest Rupees 1 I can get Rupees 1.2214 in 1 year if nominal interest rate is 20 percent and continuous compounding is used. And that is why the i effective is 22.14 percent and in fraction it is 0.2214.

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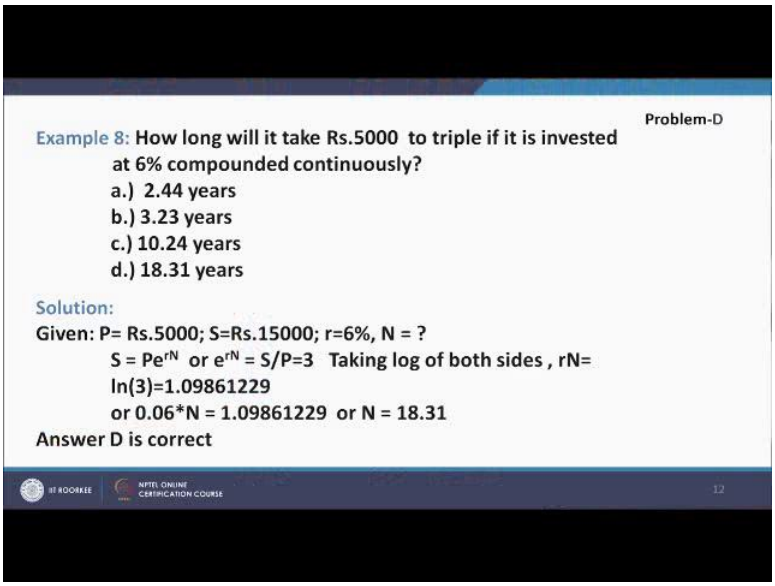

Now problem type D, example 8. How long will it take Rupees 5000 to triple if it is invested at 6 percent compounded continuously? So, we have 4 options and let us take the solution. What have given P is equal to Rupees 5000, S is equal to Rupees 15000 because it is going to triple 5000 into 3; 15000, r is 6 percent and what is the value of N.

So, we use again the formula for continuous compounding S is equal to P e to the power rN or e to the power rN equal to S by P is equal to 3, taking log both sides, rN is equal to log 3 is equal to 1.09861229. So, 0.06 which is the value of r into N is equal to 1.09861229, so N is equal to 18.31. So, your answer d is correct.

So, it tells that a 5000 Rupees input can be triple to 15000 Rupees if nominal interest rate is 6 percent in 18.31 years if continuously compounded.

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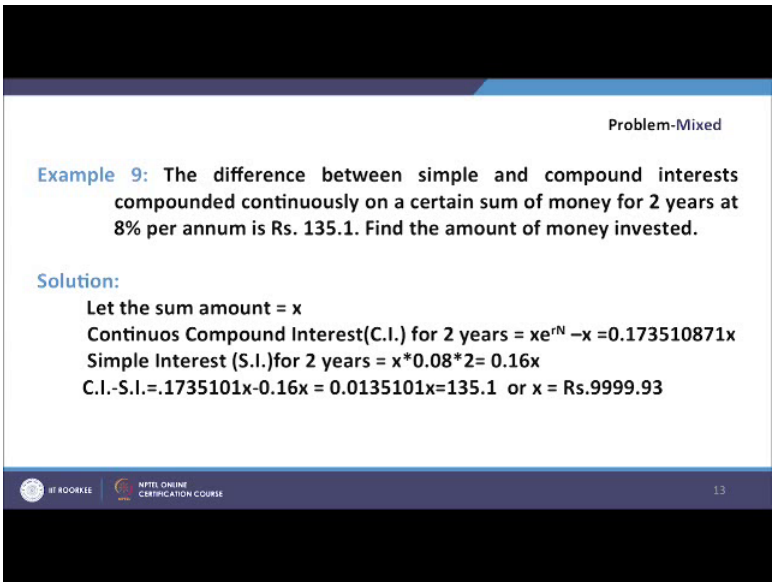

Again a mix problem, example 9; the difference between simple and compound interest compounded continuously on a certain some of money for 2 years at 8 percent per annum is Rupees 135.1. Find the amount of the money invested. Let the sum of money is x. So, continuous compound interest for 2 years will be x into e to the power rN minus x which comes out to be 0.173510871x. Now simple interest for 2 years is x into 0.08 into to 2 which is 0.16x.

So, continuous interest compounding interest minus simple interest is equal to 0.1735101 x minus 0.16x and this is equal to 0.0135101x is equal to 135.1, so x is equal to 9999.93.

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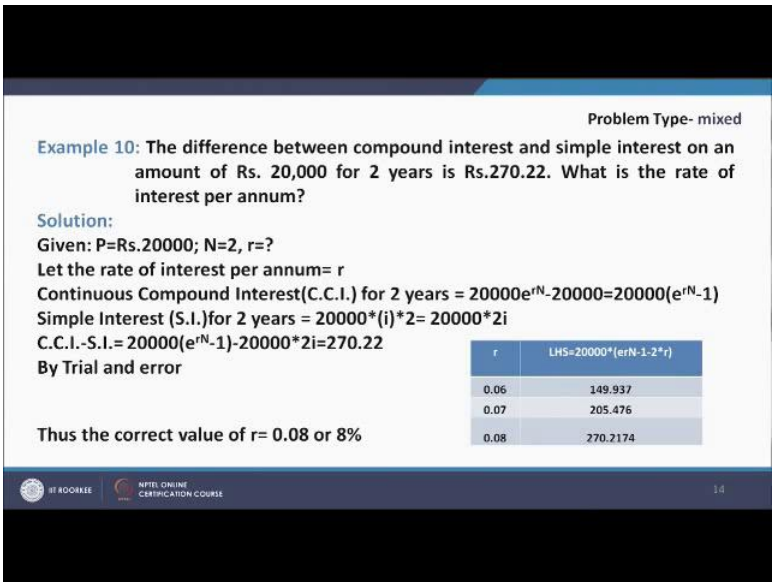

Now again problem type mixed, example 10. The difference between compound interest and simple interest on an amount of Rupees 20000 for 2 years is Rupees 270.22. What is the rate of interest per annum? Given that P is equal to Rupees 20000, N is equal to 2 and what is the value of r. So, we have to find out r, when P and N are given. Let the rate of interest per annum is r. So, continuous compound interest C.C.I for 2 years is 2000 into e to the power rN minus 2000 which comes are to be 20000 in brackets e to the power rN minus 1.

Simple interest S.I for 2 years is 20000 into i into 2, so it is 20000 2i. So, C.C.I minus S.I that is continuous compound interest manners simple interest is equal 20000 in bracket e to the power rN minus 1 into 20000 into 2 into i is called to 270.22. By trial and error if we put r is equal to 0.06 the left hand side becomes 149.937, if we take r equal to 0.07 the left hand side which is equal to 20000 into e to the power rN minus 1 minus 2 r becomes205.476 and you we take r is equal to 0.08 it is 270.2174. So, right hand side is 270.22 and at the value of r becomes 0.08 it is 270.2174 and hence the correct value of r is 0.08 that is 8 percent.

Thank you.