

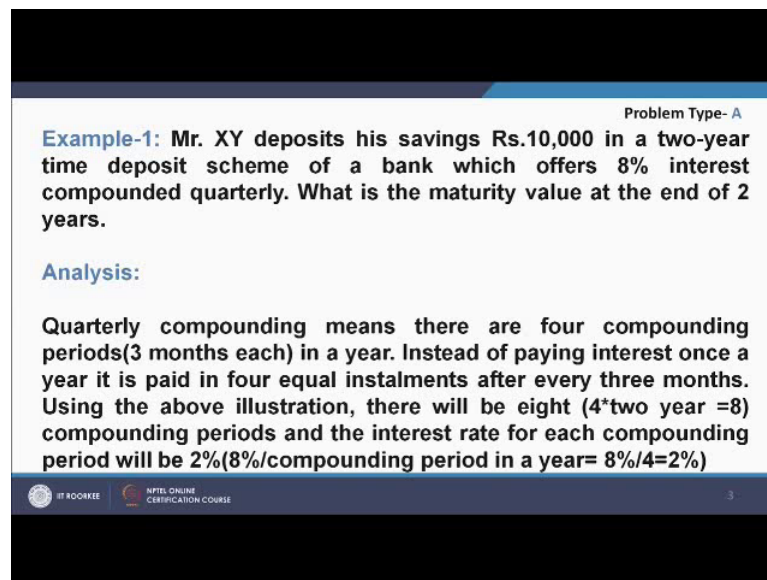
Time value of money-Concepts and Calculations
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Lecture – 05
Discrete annually compounding- 1 & 2

Welcome to the lecture series Time value of money-Concepts and Calculations. In this lecture will deal Discrete annually compounding. Discrete compounding refers to the method by which interest is calculated and added to the principle at specific interval of time. For example, interest may be compounded daily, weekly, monthly, or even yearly. The frequency at which interest is compounded has its slight effect on an inventor's effective annual yield.

With greater frequency of compounding that is has holding periods become smaller and smaller. The effective rate gradually increases, but in small amounts. Discrete compounding relates to measurable holding periods and a finite number of holding periods.

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Problem Type- A

Example-1: Mr. XY deposits his savings Rs.10,000 in a two-year time deposit scheme of a bank which offers 8% interest compounded quarterly. What is the maturity value at the end of 2 years.

Analysis:

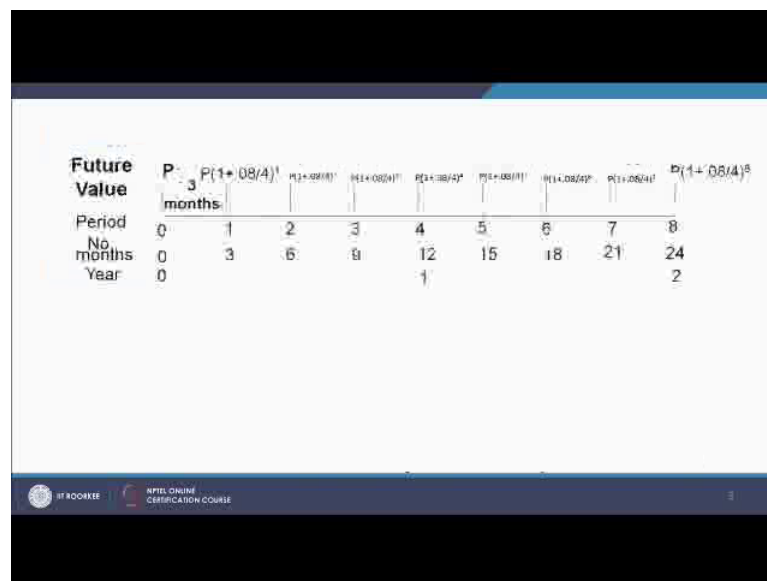
Quarterly compounding means there are four compounding periods(3 months each) in a year. Instead of paying interest once a year it is paid in four equal instalments after every three months. Using the above illustration, there will be eight ($4 \times \text{two year} = 8$) compounding periods and the interest rate for each compounding period will be 2% ($8\% / \text{compounding period in a year} = 8\% / 4 = 2\%$)

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For example Mr. XY deposits his saving Rupees 10,000 in a 2 year time deposit scheme of a bank which offers 8 percent interests compounded quarterly. What is the maturity value at the end of second year?

Now, let us analyze this problem. Quarterly compounding means there are four compounding periods 3 month each in a year. Instead of paying interest once in a year it is paid in four equal installments after every 3 months. Using the above illustration there will be eight 4 into 2 year that is equal to 8 compounding periods and the interest rate for each compounding period will be 2 percent which we calculated as 8 percent divided by compounding period in a year is equal to 8 percent divide by 4 is equal to 2 percent.

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Now, if you see this. If I see here periods it starts with 0 then 1 2 3 4 5 6 7 and 8, so eight compounding periods. And if I see month wise the first compounding period hence after 3 months, second after 6 months, third after 9 months, fourth after 12 months and fifth after 15 month, six after 18 month, seventh after 21 month, and eighth after 24 month. And year wise if we see its starting year is 0 here is the first year and here is the second year. Now if we calculate what will be the future value, the distance of between one period is 3 months the 0 to 1 is 3 month and so we are investing P amount at t equal to 0. And this amount grows to P into 1 plus 0.8 divided by 4 to the power 1 after the end of the first year. After the end of the second year it is P into 1 plus 0.8 divided by 4 square.

And in similar manner at the end of eighth period this is P into 1 plus 0.8 divided by 4 to the power 8. So, this is shown in the time line.

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Solution from first principle

Example-1: Mr. XY deposits his savings Rs.10,000 in a two-year time deposit scheme of a bank which offers 8% per annum compounded quarterly. What is the maturity value at the end of 2 years

Solution:

Interest rate per period = $8/4=2\%=0.02$; Years(N)=2, Principal= Rs.10,000

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So, Mr. XY deposits his saving of 10,000 in a 2 year time period scheme when if the bank which offers 8 percent annual compounded quarterly what is the matured value at the end second year, so we solve this. So, the interest period is 8 divided by 4 is equal to 2 percent. So, interest rate per period is 2 percent which is 0.02 and we have 2 years and principle is 10,000. Now if we calculate from the first principle what will be the result?

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Period end (months)	Beginning amount, Rs	Interest rate per period	Amount of interest, Rs.	Beginning principal	Period ending principal
0	10000	0.02	0	10000	10000
1(3 months)	10000	0.02	200	10000	10200
2(6 months)	10200	0.02	204	10200	10404
3(9 months)	10404	0.02	208.08	10404	10612.08
4(12 months)	10612.08	0.02	212.2416	10612.08	10824.32
5(15 months)	10824.32	0.02	216.4864	10824.32	11040.81
6(18 months)	11040.81	0.02	220.8162	11040.81	11261.62
7(21 months)	11261.62	0.02	225.2325	11261.62	11486.86
8(24 months)	11486.86	0.02	229.7371	11486.86	11716.59

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So, here a period end is 0 at 0 periods we invest 10,000 which is the principal amount. Interest per period is 0.02, amount of interest earn is 0, and beginning principal is

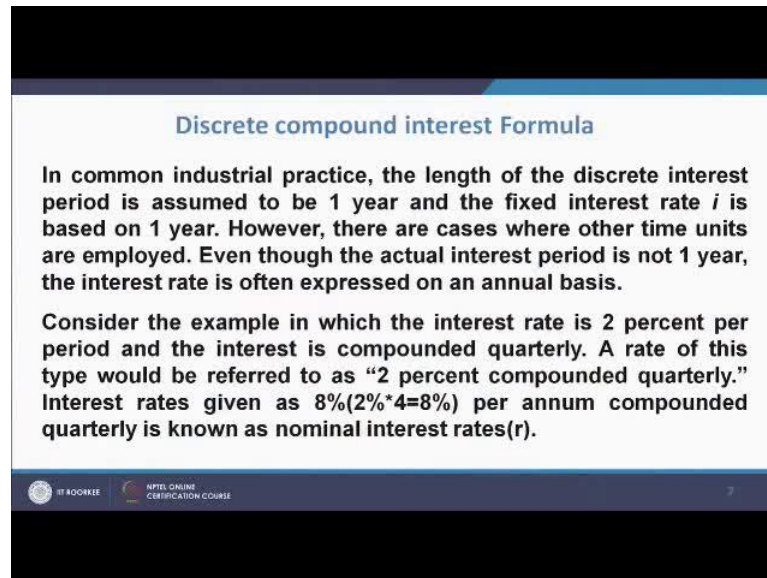
10,000, and period ending principal is 10,000. Because at t equal to 0 obviously no interest will be earned, but at the end of that period this is first period after 3 months we have beginning amount one 10,000 interest per period is 0.02. So, interest earned is 10,000 into 0.02 comes out to be 10,000 plus 200.

So, period ending principal becomes 10,000 200 now and this period ending principal will become the period start principal for second period and again the 0.02 we earn 204 interest. So, my beginning principal is 10,200 and ending principal is 10404. This will work as a starting value for the starting up the period 3. So, beginning amount of period three is 10404 interest per period is 0.02 interest earned is 208.08. Now this beginning principal was 10404 so the ending principal is 10612.08. Similarly, for starting of the fourth period this value which is 10612.08 becomes the principal amount here and interest rate is 0.02, so interest earned is 212.24.

Now the final value becomes 10,824.32 again this becomes the principal of the start of the fifth period, and the value grows to 11040.81 at the end of the fifth period. So, it becomes the starting value at the start of the sixth period and at the end of these six periods this becomes 11261.62. At the start of the seventh period this is 11261.62 and the end of the seventh period it is 11486.86. So, at the start of the eighth period this is the principal amount 11486.86. The interest per period is 0.02, so the final value to become 11716.59.

So, this is the amount which the bank will offer at the end of second year. So, what calculation method we showed just now is from first principles.

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Discrete compound interest Formula

In common industrial practice, the length of the discrete interest period is assumed to be 1 year and the fixed interest rate i is based on 1 year. However, there are cases where other time units are employed. Even though the actual interest period is not 1 year, the interest rate is often expressed on an annual basis.

Consider the example in which the interest rate is 2 percent per period and the interest is compounded quarterly. A rate of this type would be referred to as "2 percent compounded quarterly." Interest rates given as 8%(2%*4=8%) per annum compounded quarterly is known as nominal interest rates(r).

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Now let see what is discrete compound interest formula; in common industrial practice the length of the discrete interest period is assume to be 1 year, and the fixed interest rate i is based on 1 year. However, there are cases were other time units are employed. Even though the actual interest period is not 1 year the interest rate is often expressed on annual basis this will always see that 8 percent compounded quarterly that 8 percent is 8 percent annually. And in such cases basically we express nominal interest rate r . And nominal interest rates r is expressed always annually.

Consider the example in which interest rate is 2 percent per period and the interest is compounded quarterly. So, a rate of this type would be refereed as 2 percent compounded quarterly. Now i P r that is interest rate per period is equal to the nominal interest rate divided by the number of periods. So, interest rate is given as 8 percent per annum compounded quarterly that 8 percent comes from 2 percent into 4 which it becomes 8 percent. So, in such cases will find that in variably normal interest rates are is given which reads like 8 percent per annum compounded quarterly.

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The “r” is nominal rate compounded “m” times per year. The interest rate is r/m and the amount (S_n or FV) at the end of one year or N years as the case may be is:

$$S_n = P_0 \left[1 + \frac{r}{m} \right]^m \quad S_n \text{ For one year} \quad \dots \text{Eq.1}$$
$$S_n = P_0 \left[1 + \frac{r}{m} \right]^{mN} \quad S_n \text{ For N years} \quad \dots \text{Eq.2}$$

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The r is nominal rate compounded N times per year the interest rate is r by m and the amount S_n or FV at the end of 1 year or N years has the case may be is given by S_n is equal to P_0 , S_n is the final value of the sum P_0 , P_0 is the principal amount or present value in brackets 1 plus r by m brackets close to the power m. This is when I am calculating the S_n value which is the final value for 1 year, and this is equation 1. S_n equal to P_0 1 plus r 2 divided by m whole to the power m N is the value of S_n which is the final value for N years when it is compounded m times per year.

Now take a problem we are solving it with equations; first we are seen in example one which is Mr. X deposits saving 10,000 in 2 years time deposit scheme of a bank which offers 8 percent per annum compounded quarterly. What is the maturity value at the end of 2 years? This problem has solved earlier using very fundamental concept here will solve the same problem using formula which we saw just now.

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Solution of the problem using formula Eq.2 **Problem Type- A**

Example-1: Mr. XY deposits his savings **Rs.10,000** in a two-year time deposit scheme of a bank which offers 8% per annum compounded quarterly. What is the maturity value at the end of 2 years

Solution:
Given $r = 8\%$; $P = \text{Rs.}10,000$; $N = 2$; $m = 4$ (quarterly compounded)

$$S_n = P_0 \left[1 + \frac{r}{m} \right]^{mN} \quad S_n \text{ For } N \text{ years} \quad \dots \text{Eq.2}$$
$$S_n \text{ or } FV = 10000 \left[1 + \frac{0.08}{4} \right]^{4 \times 2} = \text{Rs.}11716.59$$

It should be noted that the result (Rs.11716.59) obtained for the above problem using first principle and that of Eq.2 are same and hence it can be concluded that Eq.2 is correct

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So, solution is r is 8 percent which is nominal interest rate P is Rupees 10,000 N is 2 and m is 4 because it is quarterly compounded. So, if we use this formula S_n is equal to $P_0 \left[1 + \frac{r}{m} \right]^{mN}$ done S_n or FV is equal to 10,000 $1 + \frac{0.08}{4}$ which is the nominal interest rate and m is 4 year because quarterly compounding to the power 4 into 2 which comes out to be 11716.59. It should be noted that the result Rupees 11716.59 often from the above problem using first principle and that of the equation 2 are the same. And enhance it can be concluded that equation 2 is correct.



Now, let us see what type of problems can be created for the discrete compound interest.

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Discrete Compound interest problem matrix

$$S = P \left[1 + \frac{r}{m} \right]^{mN} \quad \dots \text{Eq.2}$$

The above equations which contains five variables(S,P,r,m,N) can be solved to find out the value of a single unknown variable only when other four variables are known. Thus five types of problem can be generated out of these equations. The problem matrix for above equation is shown below:






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And we call it is a problem matrix. Now that the problem matrix can be generated from the equation 2 is S equal to P into 1 plus r by m to the power m N. Now if we analyze the unknowns and variables so we find that it is one equation enhance one unknown can be solved using this one equation. Further, the ever the equation which contains five variables, and what are those five variables S, P, r, m, and N can be solved to find of the value of a single unknown variable only when other four variables are known. Thus five types of problem can be generated out of this equation. The problem matrix for the above equation is shown in the next slide. This is our problem matrix.

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Given	Find	Formula	Remarks
P,r,m,N	S	$S=P(1+r/m)^{mN}$	Find future value (s), when present value(P), interest rate(r),frequency of compounding per year (m) and number of years N of compounding are known(Problem-A)
S,r,m,N	P	$P=S/(1+r/m)^{mN}$	Find the present value(P), when S,R,m and N known(Problem-B)
S,N,P,m	r	$R=m[(S/P)^{1/mN}-1]$	Find the interest rate(r), S,N,P and m known(P-C)
S,P,m,r	N	$N = \frac{\ln(S/P)}{m \ln(1+r)}$	Find number of years (N),when S,P,m and r known (Problem-D)
S,P,r,N	m	$(S/P)^{1/mN}-1=r/m$	Determine m –the frequency of compounding per year when S,P,N,r are known. The value of "m" can be determined using trial and error method (Problem-E)

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The first type of problem which I call problem A; in the problem A, P, r, m and N are given and we have to find out the value of S, and the formula used will be $S = P \left(1 + \frac{r}{m}\right)^{mN}$. And what is type of problem this is, that future value S has to be found out when present value P, interest rate r, frequency of compounding per year m and number of years N of compounding are known. If we come across type of problem will call it a problem A.

Now, for the second type of problem; the given is S, r, m, and N and we have to find out what is the value of P. So, P is equal to $S \div \left(1 + \frac{r}{m}\right)^{mN}$. In this problem we have to find out the present value of P were S, r, m and N are known and we will call this type of problem; problem B.

Now if we see the third type of problem. In the third type of problem S, N, P and m are given and we have to find out r which is the nominal interest rate. And we can calculate these by the formula $r = m \left[\left(\frac{S}{P}\right)^{\frac{1}{mN}} - 1\right]$. In this case we have to find out the interest rate r where S, N, P and m are known. And such type of problems will be called problem C.

The fourth type of problem is when S, P, m and r are given and we have to find out N. So, we will use the formula $N = \frac{\ln(S/P)}{\ln\left(1 + \frac{r}{m}\right)}$. In this case we have to find out the number of years N when S P m and r unknown. Such type of problem will be called problem D.

Now the fifth type of problem is where S, P, r, and N are given and we have to find out the value of m. The formula here will be $\left(\frac{S}{P}\right)^{\frac{1}{mN}} = 1 + \frac{r}{m}$. Here we have to determine the value of m the frequency of compounding per year when S, P, N, and r are known. The value of m can be determined using trial and error method, because we find that in the left end side there is m and in the right end side there is m so we have to use trial and error method to solve this problem and to calculate m. And such type of will be designated has problem E.

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Conclusion:

1. Under an annual interest r with annual compounding, money in a bank account will grow with a multiplier $(1+r)$ each year
2. If compounding occurs m times per year, the multiplier for each compounding (at the end of period) will be $(1+r/m)$
3. To find the final value of amount compounding m times per year for N years following equation should be used:

$$S_n \text{ or } FV = P_0 \left[1 + \frac{r}{m} \right]^{mN}$$

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What conclusion we derive out of this? The conclusion number 1; under an annual interest r the annual compounding, money in a bank account will grow with a multiplier $1 + r$ each year. Number 2; if compounding occurs m times per year the multiplier for each compounding at the end of the period will be $1 + r$ by m . And third conclusion is to find out the final value of amount compounding m times per year for N years following equation should be used S_n and FV is equal to P_0 in brackets $1 + r$ divided by m whole to the power m into N .

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4. To convert annual compounding to discrete compounding change r to r/m and N to $m \cdot N$ as shown below:

$$S_n \text{ or } FV = (1+r)^N \rightarrow S_n \text{ or } FV = (1+r/m)^{m \cdot N}$$

Simple compounding	discrete compounding
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Fourth conclusion; to convert involves compounding to discrete compounding change r to r by m and N to m into N has shown below. So, the first formula which is simple compounding or annually compounding is FV is equal to 1 plus r to the power N , but if I go for discrete compounding the FV will be 1 plus r divided by m and whole power m into N . So, what I am doing r is changed to r by m and m is changed to m into N .

Now let us see, what is the value of m ; that is compounding frequency when different time periods are decided? In a discrete compounding problem the value of m may vary widely depending on frequency of compounding.

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In a discrete compounding problem the value of "m" may vary widely depending of frequency of compounding

Value of m	Compounding frequency Per year	Value of m	Compounding frequency per year
m=1	Yearly compounding	m=365	Daily compounding
m=2	Semi-annually compounding	m=8736	Hourly compounding
m=4	Quarterly compounding	m=524160	Compounding per minute
m=12	Monthly compounding		

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So, m is equal to 1 when yearly compounding is used. When m is equal to 2 semi-annually compounding is used. When m is equal to 4 quarterly compounding is used. When m is equal to 12 monthly compounding is used or other way we can say for monthly compounding m is equal to 12 or quarterly compounding m is equal to 4.

For daily compounding m assumes a value of 365. For hourly compounding m assumes a value of 8736. And for compounding per minute the aim as a value of 524160.

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Variation of discrete compounding factor $(1+r/m)^m$ on different values of m and nominal interest rate r

Value of m	r = 5%	r = 8%	r = 10%	r = 20%
1 (annually)	1.05	1.08	1.1	1.2
2(semi-annually)	1.050625	1.0816	1.1025	1.21
4 (quarterly)	1.050945	1.082432	1.103813	1.215506
12(monthly)	1.051162	1.083	1.104713	1.219391
365 (daily)	1.051267	1.083278	1.105156	1.221336
8760 (hourly)	1.051271	1.083287	1.10517	1.2214
524160(minutely)	1.051271	1.083287	1.105171	1.221403

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Now, see the effect of discrete compounding factor that is $1 + r/m$ to the power m . On different values of m and nominal interest rates. Now suppose r is 5 which is nominal interest rate per year and for this value r equal to 5 percent if I am doing annually compounding then my discrete compound factor which is given by $1 + r/m$ to the power m becomes 1.05. For semi-annually when m value is 2 it is 1.050625. When it is quarterly that is m equals to 4 it is 1.0509454. And what is monthly 12 m is equal to 12 it is 1.051162. When it is daily that m is 365 it is 1.051267. And when it is hourly 8760 it is 1.051271. And what is minutely that means each minute it is compounding an m is equal to 524160 this is 1.051271.

What indication it gives? That when frequency is increasing the factor is not changing much. For example, when m is equal to 8760 and m is equal to 524160. The value of discrete compounding factor which is $1 + r/m$ to the power m becomes almost same up to say 6 digits. So, what we find that after certain amount of frequency this factor does not take much. The similar conclusions can be found out for r equal to 8 percent; r is equal to 10 percent, and r equal to 20 percent.

We see that for hourly and minutely compounding the values are almost same for all nominal interest values whether it is 5 percent, whether it is 8 percent, whether it is 10 percent or whether it is 20 percent. Let us take problems now.

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The slide contains the following text:

Problem Type- A

Example-2 : If Rs.10000 were loaned for a total time of 1 year at nominal interest rate of 18% per year compounded monthly then find the future value of amount after 1 year.

Solution:

For the present problem:
P= Rs.10,000
r=0.18 N=1
m=12 (12 months in a year)
Future value = ?

Future value= $P*(1+r/m)^{mN} = 10000(1+0.18/12)^{12*1} = \text{Rs.11956.18}$

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Problem type A is example 2. If Rupees 10,000 were loaned for a total time of 1 year. At nominal interest rate of 18 percent per year compounded monthly then find the future value of amount after 1 year. Now let us analyze this problem. In this problem P is given that is Rupees 10,000, r nominal interest rate is given 0.18 that is 18 percent, N is equal to 1 and m is equal to 12 has the compounding is monthly and there are 12 months in a year. And what is demanded is the future value.

So I can use my future value equation, future value is equal to P into 1 plus r by m to the power m N; that is 10,000 into 1 plus 0.18 divided by 12 in brackets to the power 12 into 1 which becomes Rupees 11956.18. That means, if I invest Rupees 1000 with 18 percent interest rate compounded monthly then it will convert into 11956.18. However, you will find that this value will be always more if compounding is used for yearly.

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Problem Type- Mixed

Example-3: For the case of nominal annual interest rate of 20 % per year, determine

- (a) The total amount to which Rs 1 of initial principal would accumulate after 1 year with annual compounding and the effective interest rate.
- (b) The total amount to which Rs 1 of initial principal would accumulate after 1 year with monthly compounding and the effective interest rate.
- (c) The total amount to which Rs 1 of initial principal would accumulate after 1 year with daily compounding and the effective annual interest rate.
- (d) The total amount to which Rs.1 of initial principal would accumulate after 1 year with hourly compounding and the effective annual interest rate.

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Now, let us take a mix problem which is example 3. For the case of nominal annual interest rate of 20 percent per year determine. Part a; the total amount to which Rupees 1 of initial principal would accumulate after 1 year with annual compounding and the effective interest rate. Part b; the total amount to which Rupees 1 of initial principal will accumulate after 1 year with monthly compounding and the effective interest rate. Part c; the total amount to which Rupees 1 of initial principal would accumulate after 1 year with daily compounding and the effective annual interest rate. Number d; the total amount to which Rupees 1 of initial principal would accumulate after 1 year with hourly compounding and the effective annual interest rate.

Now, here if you see what we doing that the time period is been changed in part a time period is 1 year, in part b it is the compounding is monthly, in c it is daily and in d it is hourly. And then we are also asking to calculate the effective interest rate which will give us a feeling that how this effective interest rate increases when frequency of compounding per year is increase, but it at aesthetically remains constant.

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Ans: (a) $P_0 = \text{Principal} = \text{Rs } 1$; $N = 1$; $r = i = i_{\text{eff}} = 0.2$ or 20% because compounding is annual
 $S_n = P_0 (1 + i_{\text{eff}})^N = 1 * (1 + 0.2)^1 = \text{Rs. } 1.2$

(b) For part (b) $P_0 = 1$; $r = 0.20$; $m = 12$, $N = 1$ $S_n = P_0 \left[1 + \frac{r}{m} \right]^m$
 $S_n = P_0 * (1 + (r/m))^m = \text{Rs } 1.2194$

(c) For part (c) $P_0 = 1$; $r = 0.20$; $N = 1$, $m = 365$
 $S_n = P * [1 + (r/m)]^m = 1 * [1 + (0.20/365)]^{365} = \text{Rs } 1.2213$
 $i_{\text{eff}} = [(1 + (r/m))^m] - 1 = 0.2213$

(d) For part (d) $P_0 = \text{Rs. } 1$; $r = 0.20$, $N = 1$; $m = 8760$
 $i_{\text{eff}} = [(1 + (r/m))^m] - 1 = 0.2213999$
 $S_n = P * [1 + (r/m)]^m = 1.2213999$

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Now for answer part a; P_0 is the principal which is Rupees 1, N is equal to 1 r is equal to i and i effective is 0.2 or 20 percent, because for annual compounding r is equal to i and i is equal to i effective and that is why r is equal to i is equal to i effective is equal to 0.2 or 20 percent. But S_n that is the final value is $P_0 1$ plus i effective to the power N is equal to 1 into 0.02 to the power 1 is equal to 1.2. Now, this S_n can be found out in different way I can replace i is equal to r is equal to i effective value for annual compounding, but part b the things will change.

For part b; P_0 is equal to 1, r is equal to 0.20, m is equal to 12 and N is equal to 1. So, I have use this formula S_n is equal to P_0 in brackets 1 plus r divided by m to the power m . So, S_n is equal to P_0 into 1 plus r divided by m in the whole bracket to the power m is equal to 1.2194. So, what we saw that if it is annually compounding the S_n value is only 1.2, when it is monthly compounding the value rises to 1.2194. So, slight rise in the value. So, any guy who is doing this will get more money.

Now in part c; P_0 is equal to 1, r equal to 0.20, N equal to 1 and m is equal to 365. Here compounding is daily, so S_n is equal to P into 1 plus r by m whole to the power m comes out to be Rupees 1.2213. Now this is 1.2213 is more than Rupees 12194; and hence, when I increase the frequency of compounding by compounding it daily, then my money S_n value that is final value increases slightly. And i effective is equal to this is formula of i effective 1 plus r by m whole to the power m minus 1 so comes out to be

0.2213. That means that I effective have also increase then the r value if frequency of compounding is increasing.

For part d; P 0 is equal to 1, r 0 is equal to 0.20, N equal to 1 and m is 8760 because it is hourly compounding and i is equal to 0.2213999. So, here we see that up to four digits it is matching with the daily compounding. So, i effective have not increase much when the frequency has gone beyond daily to hourly and S n is equal to 0.2213999.

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Problem Type- A

Example-4: If Rs. 1000 were loaned for a total time of 1 year at nominal interest rate of 24% per year compounded monthly then find the find future amount after 1 year.

or

If Rs. 1000 were loaned for a total time of 1 year at a monthly interest rate of 2% then find the find future amount after 1 year.

Solution:

$$S_n = P_0 \left[1 + \frac{r}{m} \right]^m$$

r = 2x12 = 24% per year
m = 12 months
S_n = 1000 [1 + (0.24/12)]¹² = 1268.5

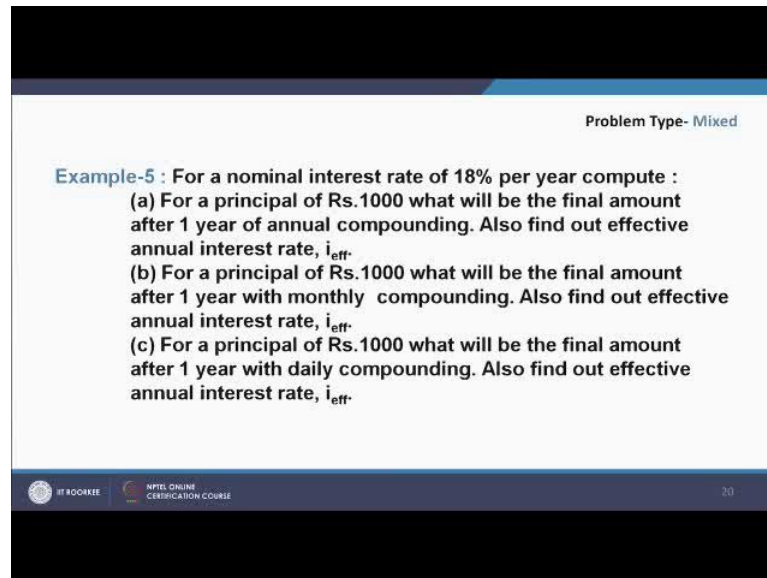
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Now takes the problem type A, this is example 4. If Rupees 1000 were loaned for a total time of 1 year at nominal interest rate of 24 percent per year compounded monthly. Then find the future value amount after 1 year. Or if 1000 were loan for a total time of 1 year at a monthly interest rate of 2 percent then find the future value of amount after 1 year.

Now, the second part of the problem it is monthly interest rate is given and in the first part of the problem 24 percent per year compounded monthly is a nominal interest rate is given. So, we should not confuse here. The nominal interest rate is i P r that is per period interest rate into number of period that is 12, so r it comes out to be 24 percent per year. S n is equal to P 0 1 plus r divided by m 2 to the power m because N here is 1, so m into 1 so that is why we have written m because the compounding is for 1 year, but compounding monthly.

So, m is equal to 12, S_n is equal to 1000, $1 + 0.24$ divided by 12 whole to the power 12 is equal to 1268.5. That means, if I invest Rupees 1000 for 1 year with a nominal interest rate of 24 percent then after 1 year it will convert into 1268.5 when it is compounded monthly. And these values will be more than the value of S_n if it is only compounded yearly.

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The slide is titled "Problem Type- Mixed" in the top right corner. The main text reads: "Example-5 : For a nominal interest rate of 18% per year compute : (a) For a principal of Rs.1000 what will be the final amount after 1 year of annual compounding. Also find out effective annual interest rate, i_{eff} . (b) For a principal of Rs.1000 what will be the final amount after 1 year with monthly compounding. Also find out effective annual interest rate, i_{eff} . (c) For a principal of Rs.1000 what will be the final amount after 1 year with daily compounding. Also find out effective annual interest rate, i_{eff} ." The slide footer includes the IIT Roorkee logo, "NPTEL ONLINE CERTIFICATION COURSE", and the number "20".

Let us take a mix type of a problem which is example 5. For a nominal interest rate of 18 percent per year compute for a principal of 1000 what will be the final amount after 1 year of annual compounding. Also find the effective annual interest rate. Part b; for a principal of 1000 what will be the final amount after 1 year with monthly compounding. And also effective annual interest rate. Part c; for the same principle of 1000, find out the final amount after 1 year when daily compounding is used. And also find out the effective interest rate.

Here also we are finding the final value when frequency of compounding is changing and as a same time is also finding out the effective annual interest rate which gives you a feel that your final value will increase with number of compound when number of the frequency of compounding is increasing.

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Solution:
Given:
For part (a) : $P = \text{Rs.}1000$; $r = 0.18$; $N = 1$
In this case $r = i = i_{\text{eff}} = 0.18$ or 18% , For one year compounding $r = i_{\text{eff}}$
Final amount(FV) = $P(1+i)^1 = P(1+i_{\text{eff}})^1 = 1000 \times 1.18 = \text{Rs.}1180$
For Part (b) : $P = \text{Rs.}1000$; $r = 0.18$; $m = 12$
 $FV = (1 + r/m)^m = 1000(1 + 0.18/12)^{12} = \text{Rs.}1195.62$
 $i_{\text{eff}} = (1 + r/m)^m - 1 = 0.1956$ or 19.56% (Note this is more than nominal interest rate)
For Part(c) : $P = \text{Rs.}1000$; $r = 0.18$; $m = 365$
 $FV = (1 + r/m)^m = 1000(1 + 0.18/365)^{365} = \text{Rs.}1197.16$
 $i_{\text{eff}} = (1 + r/m)^m - 1 = 0.19716$ or 19.72% (Note this is more than nominal interest rate)

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Now the solution is for part a; P is equal to Rupees 1000, r is equal to 0.18, N is equal to 1. In this case r is equal to i i effective is equal to 0.18 because for yearly compounding r is equal to i effective is equal to i. So, final amount FV is equal to P into 1 plus i to the power of 1 or P equal to 1 plus i effective to the power 1 is comes out to be 1180.

For part b; P is equal to 1000, r is equal to 0.18, m is equal to 12. So, FV is equal to 1 plus r by divided by m whole to the power m. This is 1000 1 plus 0.18 divided by 12 whole to the power 12 comes out to be 1195.62. Here we see very clearly that when compounding is yearly my final value is 1180.

And when compounding is monthly my final value is 1195.62 that means, it has increased when my compounding period has increase. So, i effective will also increase because effective is a parameter this shows whether be compounded value will be more or less. So, i effective is equal to 1 plus r by m to the power m minus 1 is comes out to be 19.56 percent. Now, this percent is more than the r value of 18 percent nominal interest rate. It shows clearly that if I am increasing the frequency of compounding the every value will increase.

Now for part c; P is equal to Rupees 1000, r is equal to 0.18, and m is equal to 365. So, now in frequency has increased to daily free compounding. So, FV is equal to 1 plus r by m to the power m which comes out to be Rupees 1197.16. So, we see that further view

FV value has increase and so the i effective will also increase and i effective have become 19.72.

Now if we see here the monthly compounding and daily compounding the i effective value has not changed much only of the tune of say point 0.2 percent also. But you see from yearly compounding to monthly compounding it has changed by 1.56 percent. So, when we increase number that is compounding frequency the difference had increased i effective becomes lower and lower and assumes at an aesthetic value.

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Problem Type- Mixed

Example-6: If Mr.XX pays 5.12% nominal interest compounded quarterly, what is the effective annual interest rate?

(A) 5.219%
(B) 7.250%
(C) 9.893%
(D) 12.552%

Solution
 $i_{\text{eff}} = (1+r/m)^m - 1 = (1+0.0512/4)^4 - 1 = 0.052191 = 5.2191\%$

Answer is A

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Let us take another example which is example 6. If Mr. XX pays 5.12 to nominal interest compounded quarterly, what is the effective annual interest rate? So, the problem here is very clear if somebody is clever, he can pick up the result very quickly. The nominal interest rate is 5.12 and compounded quarterly. So, the effective interest rate will be slightly more than 5.12 and that we will observe here. So, results are a 5.219 percent, b 7.250 percent, c 9.893 percent, and then d is 12.556 percent.

Now solution i effective is the formula of i effective $1 + r$ by m whole to the power m minus 1 comes out to be 5.2191 percent. So, we see that this is little bit higher than the nominal interest rate. So, any answer which is very close to nominal interest rate or it is not very different than the nominal interest will be the correct answer.

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Problem Type- C

Example-7: Ten years ago Rs.10,000 was deposited in a bank account, and today it is worth Rs.22595 The bank pays interest semi-annually. What was the nominal annual interest rate paid on this account?

(A) 2.93%
(B) 4.45%
(C) 5.02%
(D) 8.32%

Solution:
P=Rs.10,000
S= Rs.22595
m=2
N=10
Or $(1+r/2) = (2.2595)^{0.05} = 1.041599$ or $r=2*(1.041599-1)=0.083198$ or 8.32%
Answer is D

$$S = P \left[1 + \frac{r}{m} \right]^{mN} \text{ or } 22595 = 10000 \left[1 + \frac{r}{2} \right]^{2 \times 10}$$

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Now let us see the problem type C, example 7. Ten years ago Rupees 10,000 was deposited in a bank account, and today it is worth 22595. The bank pays interest semi-annually. What was the nominal annual interest rate paid on this account? So, what is available what is demanded is P is given 10,000, S is given 22595, m is given 2 because it is semi-annually that means, in a year 2 times N is equal to 10. So, what is r? So, here we see that the formula S is equal to P 1 plus r divided by m to the power m N. So, here 22595 is equal to 10,000 1 plus r by 2 whole to the power 2 into 10. So, we have to find out the value of r. So, if we solve this equation then it is 1 plus r by 2 is equal to 2.2595 to the power 0.05, and r comes out to be by solving comes out to be 8.32 percent. So obviously, your answer d is correct.

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Problem Type- B

Example-8: Five years ago a certain sum of money was deposited in a bank account, and today it is worth Rs.10775. The bank pays 6% nominal interest compounded quarterly. What was the amount of money deposited ?

(A) 7563.3
(B) 8996.25
(C) 8000.12
(D) 10000

Solution: $P = ?$, $S = 10775$; $m = 4$, $N = 5$, $r = 6\%$
 $10775 = P(1 + 0.06/4)^{4 \times 5} = P \times 1.346855$
Or $P = \text{Rs. } 8000.12$

$$S = P \left[1 + \frac{r}{m} \right]^{mN}$$

Answer is C

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Now, let us take another problem which is problem type B example number 8. Five years ago a certain sum of money was deposited in a bank account, and today it is worth 10775. The bank pays 6 percent nominal interest compounded quarterly. What was the amount of money deposited? Solution; yes that is FV is given as 10775, N is given as 5 because it is quarterly, so quarterly compounded m is 4, number of years N is given 5, r is given 6 percent. Now question is what is the value of P?

So, we have this equation S is equal to P in brackets $1 + \frac{r}{m}$ whole to the power m into n . So, if we put these values into this 10775 is equal to P into $1 + 0.06$ divided by 4 to the power 4 into 5 . So, it comes out to be P into 1.346855 or P is equal to 8000.12 . So, if I spend 8000 Rupees it will grow to 10775 at the interest rate nominal interest rate is 6 percent and if the compounding is quarterly.



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Problem Type- D

Example-9: Some years ago Rs.7000 was deposited in a bank account, and today it is worth Rs.11259. The bank pays 8% nominal interest compounded quarterly. For how many years the money was invested ?
(A) 2.3333
(B) 4.999
(C) 5.999
(D) 10.231

Solution:
Given; P=Rs.7000, S=11259, r=8%, m=4, N=?
 $11259=7000(1+0.08/4)^{4N}$ or $4N = \ln(11259/7000)/\ln(1.02)$
 $4N = 0.475257/0.01980=23.9997$
Or $N = 23.99969/4=5.9999$ yr
Answer is C

$$S = P \left[1 + \frac{r}{m} \right]^{mN}$$

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Now, let us the problem type D and example number 9. Some years ago Rupees 7000 was deposited in a bank account, and today it is worth 11259. The bank pays 8 percent nominal interest compounded quarterly. For how many years the money was invested?

Now, if we analyze this problem the given is P is equal to 7000, S is equal to 11259, r is equal to 8 percent, m is equal to 4 and what is the value of N m is N is unknown, so four variables known four variables unknown. So, we can solve because we have one equation. So, this is $11259 = 7000(1 + 0.08/4)^{4N}$ So $4N = \ln(11259/7000)/\ln(1.02)$. So, if I take the log $4N = \ln(11259/7000)/\ln(1.02)$. So, if we solve this then $4N = 23.9997$ and N is equal to $23.99969/4 = 5.9999$ and which is approximately equal to 6 years.

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Problem Type- E

Example-10: Five years ago Rs.9000 was deposited in a bank account, and today it is worth Rs.14091.13. The bank pays 9% nominal interest compounded for an unknown frequency per year. Find the value of the frequency of compounding per year ?
(A)2 (C) 4
(B)3 (D) 12

$$S = P \left[1 + \frac{r}{m} \right]^{mN}$$

Solution:
Given; P=Rs.9000, S=14091.13, r =9%, m= ?, N=5
 $14091.13=9000(1+0.09/m)^{m \times 5}$ or $(1+0.09/m)^{5m} = 14091.13/9000 = 1.565681$
Let m=4 then LHS = 1.5605 RHS=1.565681
 m=10 then LHS= 1.565158 RHS =1.565681
 m=12 Then LHS=1.565681 RHS =1.565681 As at m=12 LHS=RHS
the assumed value is correct
Answer is D

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Now problem type E, example 10. Five years ago Rupees 9000 was deposited in a bank account, and today it is worth 14091.13. The bank pays 9 percent nominal interest compounded for an unknown frequency per year. Find the value of the frequency of compounding per year.

Now the same equation S equal to P in brackets 1 plus r by m to the power m N. Now the solution what is given and what is required let us see P is given 9000, S is given 14091.13, r is given 9 percent, m is not given, and N is given 5. So, P, S, r and N is available and we have to find out what is the value of N. Now if you remember in the first part of this lecture I have told you that then we find out m it is trial and error crosses, because left hand side contents same and right hand side also contents same. So, you have to go through a trial and error crosses. So, we can right down using this equation 14091.13 equal to 9000 in brackets 1 plus 0.09 divided by m to the power m into 5.

So, we converted into two parts that is 1 plus 0.09 by m to the power 5 m is equal to 1.565981. So, let m is equal to 4 then LHS is equal to 1.5605. And RHS is equal to 1.565981. When m is equal to 10 LHS is equal to 1.565158, and RHS is equal to 1.565981. And for m equal to 12 LHS becomes equal to 1.565981 because RHS remains constant so whenever changing m LHS is changing. Now for m equal to 12 the LHS has

become equal to RHS that is right hand side LHS is left hand side becomes is equal to right hand side. So, as at m equal to 12 LHS equal to RHS the assumed value is correct.

Thank you.