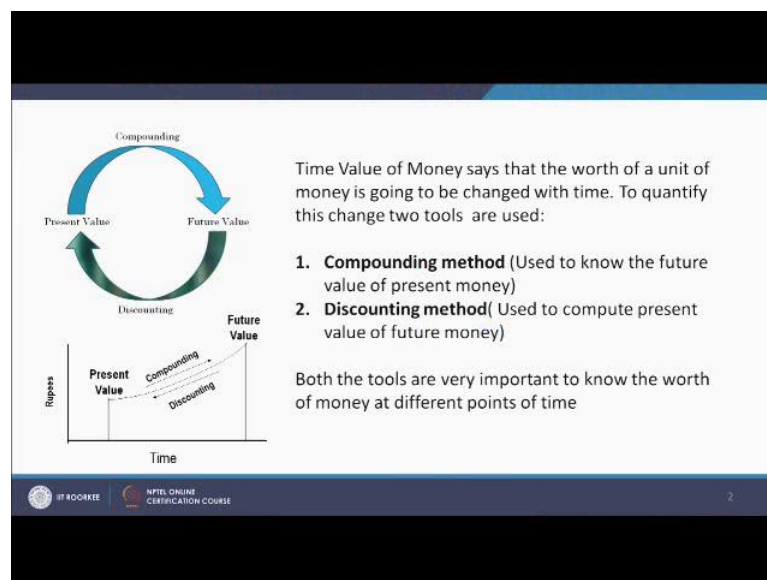


**Time value of money-Concepts and Calculations**  
**Prof. Bikash Mohanty**  
**Department of Chemical Engineering**  
**Indian Institute of Technology, Roorkee**

**Lecture – 04**  
**Compounding Techniques- 1&2**

Welcome to the lecture series on Time value of money-Concepts and Calculation. This lecture is devoted to Compounding and that too compounding annually. Time value of money says that, the worth of a unit of money is going to be changed with time.

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To quantify this change two tools are used; the first tool is compounding methods use to know the future value of present money and discounting method use to compute present value of future money. Both the tools are very important to know the worth of money at different point of time. We start with the present value and using compounding reach to the future value, and we start with the future value and reach to present value using discounting method has shown in the figure.

Let us see the comparison between compounding and discounting methods based on different basis.

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Comparison between Compounding and discounting		
Basis for comparison	Compounding	Discounting
Meaning	The method used to determine the future value of present investment is known as Compounding.	The method used to determine the present value of future cash flows is known as Discounting.
Concept	If some money is invested today, what will be the amount one gets at a future date.	What should be the amount we need to invest today, to get a specific amount in future.

Now if we see the basis is meaning then compounding is the method used to determine the future value of present investment is known as Compounding. And what is discounting the method used to determine the present value of future cash flows is known as Discounting. Now if we say the basis is concept then for compounding if some money is invested today what will be the amount one gets at a future date. And the same concept for discounting is what should be the amount we need to invest today to get a specific amount in future.

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Basis for compariso	Compounding	Discounting
Use of	Compound interest rate.	Discount rate
Known	Present Value	Future Value
Factor	Future Value Factor or Compounding Factor	Present Value Factor or Discounting Factor
Formula	$FV = PV (1 + r)^N$	$PV = FV / (1 + r)^N$

What is the factor which is used for compounding? For compounding future value factor or compounding factor is used, and for discounting present value factor or discounting factor is used. And if you see the formula for compounding FV is equal to PV in brackets  $1 + r$  to the power N. And for discounting PV is equal to FV divided by in brackets  $1 + r$  to the power N.

## Definition of Compounding

Compounding refers to the process of earning interest on both the principal amount, as well as accrued interest by reinvesting the entire amount in order to generate more interest.

Compounding is the method used in finding out the future value of the present investment. The future value can be computed by applying the compound interest formula :

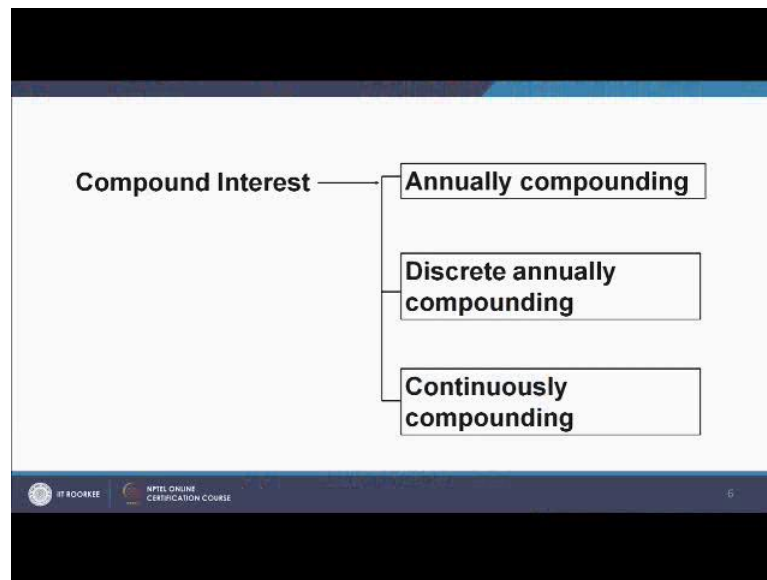
**Money makes money, and the money that money makes makes more money**

-Benjamin Franklin

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Compounding is the method used in finding out the future value of the present investment. The future value can be computed by applying the compound interest formula which we have given in the last slide. As per the Benjamin Franklin, ‘money makes money, and the money that money makes more money’.

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Now the compound interest can be divided into three parts; the first is annually compounding, the second is discrete annually compounding, and the third is continuously compounding. In these three methods what is changing is the time period.

If the time period is 1 year then it is annually compounding. If the time period is less than 1 year say every month then it is discrete annually compounding. And if the time period is very small then we call it a continuously compounding.

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Derivation of formula for annually compounding			
The compound amount due after any discrete number of interest periods (in present case year) can be determined as follows:			
Period	Principal at start of period	Interest earned during the period ( $i$ = interest rate per year)	Compound amount $S$ at the end of period
1	$P$	$Pi$	$P+Pi=P(1+i)$
2	$P(1+i)$	$P(1+i)i$	$P(1+i)+P(1+i)i = P(1+i)^2$
3	$P(1+i)^2$	$P(1+i)^2i$	$P(1+i)^2+P(1+i)^2i = P(1+i)^3$
$N$	$P(1+i)^{N-1}$	$P(1+i)^{N-1}i$	$P(1+i)^{N-1}+P(1+i)^{N-1}i = P(1+i)^N$

In this lecture we will deal with annually compounding only. Now let us see the derivation of formula for annually compounding. The compound amount due after any discrete number of interest periods in this present case a year can be determined as follows. Let us take the period 1 that means, 1 year. The principle at the start of the period is a  $P$  then interest earned during that period is  $P$  into  $i$ , if  $i$  is the interest rate per year then the compounded amount  $S$  that is which includes the principle as well as interest earned is  $P$  plus  $Pi$  which is equal to  $P$  into brackets  $1$  plus  $i$ .

Now for the second year the principle is  $P$  into  $1$  plus  $i$ , because compound interest the interest is also charged on principle as well as interest earned. So, at the start of second year the principle is  $P$  in brackets  $1$  plus  $i$ . Now the interest earned in the second year will be  $P$  in brackets  $1$  plus  $i$  into  $i$ . This is the interest which will be earned in second year. So, the compound amount will be now  $P$  in brackets  $1$  plus  $i$  plus  $P$  in brackets  $1$  plus  $i$  into  $i$ . And if I take  $P$   $1$  plus  $i$  common then it becomes  $P$   $1$  plus  $i$  into  $1$  plus  $i$  which becomes  $P$  into  $1$  plus  $i$  whole square. So, end of the second year the compound amount will be  $P$   $1$  plus  $i$  in brackets whole square.

At the start of the third year the  $P$   $1$  plus  $i$  whole square will now work as the principle, and the interest at the end of the third year will be  $P$   $1$  plus  $i$  square into  $i$ . So, the compounded amount will be  $P$   $1$  plus  $i$  whole square plus  $P$  into  $1$  plus  $i$  whole square into  $i$ , and if I take common then it becomes  $P$   $1$  plus  $i$  whole cube. Similarly, if I can write extend this logic to the  $n$ th period then the start of the  $n$ th period the principle will be  $P$   $1$  plus  $i$  to the power  $N$  minus  $1$ , because in the third period start of the third period the principle is  $P$   $1$  plus  $i$  whole square which I can write down as  $P$   $1$  plus  $i$  to the power  $3$  minus  $1$ .

So, on this same logic for  $n$ th period I can write down the principle will be  $P$   $1$  plus  $i$  to the power  $N$  minus  $1$ . Then interest earned on this principle will be  $P$   $1$  plus  $i$  to the power  $N$  minus  $1$  into  $i$ . And the compounded amount will be  $P$   $1$  plus  $i$  to the power  $N$  minus  $1$  plus  $P$   $1$  plus  $i$  to the power  $N$  minus  $1$  into  $i$ . And if I solved this it becomes equal to  $P$   $1$  plus  $i$  to the power  $n$ .

Now, this gives us the formula for annually compounding.

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Therefore, if the total amount of principal plus compounded interest due after "N" interest periods is designated as S,

$$\text{then } S = P(1+i)^N$$

The term  $(1+i)^N$  is commonly referred to as the **compound value interest factor (CVIF)**;

$$\text{CVIF} = (1+i)^N$$

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Therefore, if the total amount of principles plus compounded interest after N interest period is designated as S, then S is equal to P 1 plus i to the power N. Now for all compounding problems which is compounding annually we will use this formula which is S is equal to P 1 plus i to the power N. The term 1 plus i to the power N is commonly referred to as the compound value interest factor CVIF. So, I can write down CVIF is equal to 1 plus i to the power N.

Now, let us see how many type of problems can be created on annual compound interest.

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**Annual Compound interest problem matrix**

$$S = P(1+i)^N \quad \dots \text{Eq.1}$$

The above equations which contains four variable(S,P,i,N) can be solved to find out the value of a single unknown variable only when other three variables are known. Thus four types of problem can be generated out of these equations. The problem matrix for these equations are shown in the next slide:

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So, I have written my equation  $S$  is equal to  $P(1+i)^N$ . Now this equation can solve only one unknown, because I have only one equation and using one equation I can solve only one unknown. The above question which contains four variable;  $S$ ,  $P$ ,  $i$ ,  $N$ , can be solved to find out the value of a single unknown variable only when other three variables are known.

That means, in these equations if I feed three known I can find out the fourth unknown variable. Thus four type of can be generated out of these equations. So, what I will see now that four type of problems can be created using this equations. The problem matrix for these equations is shown in the next slide. This is a problem matrix.

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Given	Find	Formula	Remarks	Problem Type
$P, i, N$	$S$	$S = P(1+i)^N$	Find future value (s), when present value(P), interest rate(i) and number of period N are known	A
$S, i, N$	$P$	$P = S/(1+i)^N$	Find the present value(P), when future value(S), interest rate i and number of periods known	B
$S, N, P$	$i$	$i = (S/P)^{1/N} - 1$	Find the interest rate(i), when future value(S), present value(P) and number of periods N known	C
$S, P, i$	$N$	$N = \frac{\ln(\frac{S}{P})}{\ln(1+i)}$	Find number of period(N), when future value(S), present value(P) and interest rate(i) know	D

So, the first type of problems which I call problem type A in which  $P$  will be given,  $i$  will be given, and  $N$  will be given, and what has to be found out the  $S$  has to be found out. So, the equation which will be used for this purpose is  $S$  is equal to  $P(1+i)^N$ . So, the problem is find future value  $S$  when present value  $P$  interest rate  $i$  and number of period  $N$  are known. So, when I find such types of problem I call it a problem A.

The second type of problem will be in which  $S$  will be given,  $i$  will be given, and  $N$  will be given, and we have to find out  $P$ . So, the equation used in this problem will be  $P$  is equal to  $S$  divided by  $(1+i)^N$ . And the problem is to find the present

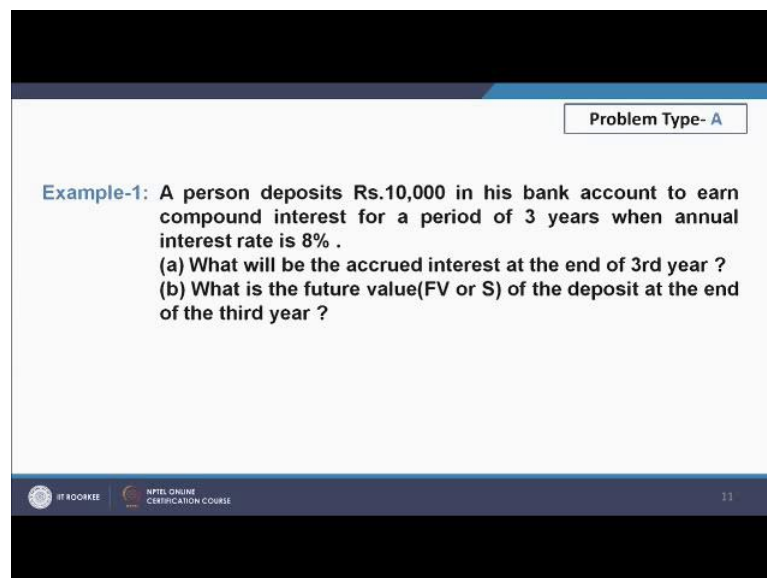
value  $P$  when future value  $S$  interest rate  $i$  and number of periods known. Such type of problems will be called problem type B.

Now, the third type of problem which I will call it problem type C. In this  $S$  will be supplied,  $N$  will be supplied, and  $P$  will be supplied, and one has to calculate the value of  $i$ . And the equation will be  $i$  is equal to  $\ln(S/P)$  divided by  $N$  minus 1. In this discussion find the interest rate  $i$  when future value  $S$ , present value  $P$  and the no of periods  $N$  is known. And will call this type of problems problem type C.

Now in the last problem type which will called D;  $S$  is given,  $P$  is given,  $i$  is given, and we have to find out  $N$ . So, equation will be  $N$  is equal to  $\ln(S/P)$  divided by  $\ln(1 + i)$  in brackets. So, here one has to find the number of periods when future value  $S$ , present value  $P$ , and interest rate  $i$  are known.

Now let us take problems based on this matrix. So, first we take problem type A.

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Problem Type- A

**Example-1:** A person deposits Rs.10,000 in his bank account to earn compound interest for a period of 3 years when annual interest rate is 8% .

(a) What will be the accrued interest at the end of 3rd year ?

(b) What is the future value(FV or S) of the deposit at the end of the third year ?

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Example 1; a person deposits Rupees 10,000 in his bank accounts to earn compound interest for a period of 3 years when annual interest rate is 8 percent. What will be the accrued interest at the end of third year part a. And part b, what is the future FV or  $S$  of the deposit at the end of the third year.

So, if you analyze the problem matrix then we find that it demands accrued interest and final value enhance false in the category of problem type A.



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**Solution:**

(a)  $P$  = Principal amount ( invested at time=0)  
 $i$  = interest rate per year  
 $N$  = No. of years  
 $P$  = Rs.10,000/- ;  $i$  = 8% ;  $N$  = 3  
Interest earned after period , $N$   
$$= P \cdot i/100 + P(1+i/100)^1(i/100) + P(1+i/100)^2(i/100)$$
$$= 10000 \cdot 0.08 + 10000(1.08)^1 \cdot 0.08 + 10000(1.08)^2(0.08)$$
$$= 800 + 864 + 933.12 = 2597.12$$

(b) Future value(FV) = Principal amount( $P$ )+accrued interest  
$$= 10,000 + 2597.12 = \text{Rs. } 12597.12$$
  
or Future value after 3 years =  $P(1+i/100)^N = 10000(1.08)^3 = \text{Rs. } 12597.12$

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Now let us see the solution. In the A part  $P$  is the principle amount because it is invested at time equal to 0 in the time line,  $i$  equal to interest rate per year,  $N$  is number of years. So in this problem  $P$  is equal to Rupees 10,000,  $i$  is equal to 8 percent, and  $N$  is equal to 3. And what is demand at here, that interest earned after period  $N$ . Now this problem can be solved in many a ways, but I am solving it using the first principle.

So, my 1st year interest will be  $P$  into  $i$  divided by 100. It should be noted always that  $i$  is always given in percentage like in this case 8 percent, but when it will be used in the formula it will be used as a ratio. So,  $i$  will be converted into 8 divided by a 100, so 0.08. So the 1st years interest will be  $P$  into  $i$  divided by 100, and the second years interest will be  $P$  1 plus  $i$  divided by 100 in brackets to the power 1 into  $i$  divided by 100. And for the third year's interest will be  $P$  1 plus  $i$  divided 100 to the power 2 into  $i$  by 100.

This has been taken from the derivation of the formula for annually compound interest. So, when we put values this is 10,000 into 0.08 plus 10,000 into 1.08 to the power 1 into 0.08 plus 10,000 into 1.08 to the power 2 into 0.08; because 1 plus  $i$  divided by 100 gives 1.08. So, when we solved it we find that interest is 2597.12. Now if we calculate the future value, future value is principle amount  $P$  plus accrued interest in 3 years. So, this is 10,000 plus 2597.12 is equal to Rupees 12597.12.

Now the same can be directly calculated by the using the formula which we have learned,  $S$  is equal to  $P$  into 1 plus  $i$  to the power  $N$ . So, if we use that formula then it is

10,000 into 1.08 to the power 3 is equal to 12597.12. And the part a from this data can be back calculated that means, the interest earned in 3 years will be equal to Rupees 12597.12 minus 10,000 which will come out to be 2597.12.

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**Problem Type- A**

**Example-2:** Will you prefer to get Rs.5000 today or the same amount after 5 years ?  
The annual interest rate is 8%.

**Solution:** After five years the principal amount of Rs.5000 will grow to :

If compound interest compounding annually is considered:

Future value=  $5000(1+8/100)^5$  = Rs.7346.64/-

As the future value (after 5 years) of Rs.5000/- is far more than present value of Rs.5000/-, one should prefer to get Rs.5000/- today than the same after 5 years. After getting the money today one should deposit this in the bank and get Rs.7346.64 after 5 years.

Note: Future value(FV) is the value at some future time of the principal amount( present amount of money) evaluated at a given interest rate

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Let us take another problem, and the problem is also type A. It is example 2; the question is will you prefer to get 5000 today or the same amount after 5 years. The annual interest rate is 8 percent. This is the time value problem, clearly tells that if somebody offers you 5000 Rupees today or offers you the same 5000 Rupees after 5 years and gives you alternative that you can select one of this, which one you should select. Let us say analyze this problem. After 5 years the principle amount of Rupees 5000 will grow to if compound interest compounding annually is considered then future value of this 5000 will be 5000 in the brackets 1 plus 8 divided by 100 to the power 5.

This 5000 in 5 years will convert into 7346.64 Rupees. As the future value of this 5000 after 5 years is far more than the present value of 5000 we should prefer to get 5000 today than the same after 5 years. After getting the money today we should deposit this in the bank and get Rupees 7346.64 after 5 years which will be around 2346.64 Rupees more than the 5000 Rupees. Note, future value FV is the value at some future time of the principle amount evaluated at a given interest rate

Another problem we take, the problem type is A.

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**Problem Type- A**

**Example-3:** For a principal amount of Rs.1000/- deposited in a bank what will be the final value of the money at the end of 1st year, 10<sup>th</sup> year and 50<sup>th</sup> year if:

(a) Annual interest rate is 10% and simple interest is charged.  
(b) Annual interest rate is 10% and compound interest compounding annually is considered.

**Solution:**

(a) (i) Simple interest at the end of 1st year=  $1000 \times (10/100) \times 1 = \text{Rs.}100$   
Final value =  $1000 + 100 = \text{Rs.}1100$   
(ii) Simple interest at the end of 10<sup>th</sup> year=  $1000 \times (10/100) \times 10 = \text{Rs.}1000$   
Final value =  $1000 + 1000 = \text{Rs.}2000$   
(iii) Simple interest at the end of 50<sup>th</sup> year=  $1000 \times (10/100) \times 50 = \text{Rs.}5000$   
Final value =  $1000 + 5000 = \text{Rs.}6000$

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For principle amount of 1000 deposited in a bank what will be final value of money at the end of 1st year, 10th year and 15th year, if annual interest rate is 10 percent and simple interest is charged. Annual interest rate 10 percent and compound interest compounding annually is considered. This problem gives or permits us to find out if I invest my money based on simple interest or invest the same money based on compound interest what will be the difference.

Let us see the solution a part; that means simple interest. Simple interest at the end of the 1st year will be 1000 into 10 divided by 100 into 1 which is 1 year will be 100 Rupees. So, the final value will be 1000 plus 100 is equal to Rupees 1100. That means if I use the simple interest of 10 percent then my final value will be 1100. That means, from the principle of 1000 two it will grow to 1100 after 1 year. At the same way the simple interest at the end of 10th year is equal to 1000 10 by 100 into 10 which comes out to be 1000. So, the final value is 1000 plus the interest 1000 comes out to be 2000. That means, if I invest 1000 today at the rate of 10 percent and simple interest is used then at the end of 10th year I will get a value of Rupees 2000 only.

In the similar way if I calculate what I will get at the end of 15th year then the interest will be 1000 into 10 by 100 into 50 will be 5000. And my final value will be 1000 my principle value and my interest 5000 which will be 6000. So, what we conclude here that after 1 year we get 1100, after 10 years we get 2000 and after 15th year we get 6000. So,

the money will grow like this, but if we see the same how it is growing when compound interest is charged will find a large change.

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(b) (i) Final value at the end of 1st year when compound interest is charged  $=1000(1+10/100)^1 = \text{Rs.}1100$

(ii) Final value at the end of 10<sup>th</sup> year when compound interest is charged  $=1000(1+10/100)^{10} = \text{Rs.}2593.72$

(iii) Final value at the end of 50<sup>th</sup> year when compound interest is charged  $=1000(1+10/100)^{50} = \text{Rs.}1,17,390.85$

**Note: Pl. Note the huge difference in the future value when simple and compound interest is used.**

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Now, in the part b we are using the compound interest. So, final value at the 1st year when compound interest is charged is 1000 in the brackets 1 plus 10 divided by 100 to the power 1 is 1100. Final value at the 10th year is 1000 into 1 plus 10 by 100 to the power 10 comes out to be 2593.72. So, what we see here the final value is the same for the compound interest as well as the simple interest when the it is 1st year. In the second year the difference is less only a 593.72 Rupees.

But if we see the 15th year then the final value is 1000 into 1 plus 10 by 100 to the power 50 which comes out to be 1,17,390.85, whereas for the simple interest it was far less. So, what we conclude that we get more benefit if the money is invested for a longer in duration of time using compound interest.

Now, let us take another problem which is problem type B.



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**Problem Type- B**

**Example-4:** Presume that you are going to receive an amount of Rs.50,000 six years from now. How much loan should you take from a bank today, such that the money received (i.e. Rs.50,000) after five years from now will be exactly enough to pay off the loan plus interest at that time ? Assume the bank charges an annual interest rate of 10% ?

**Solution:**

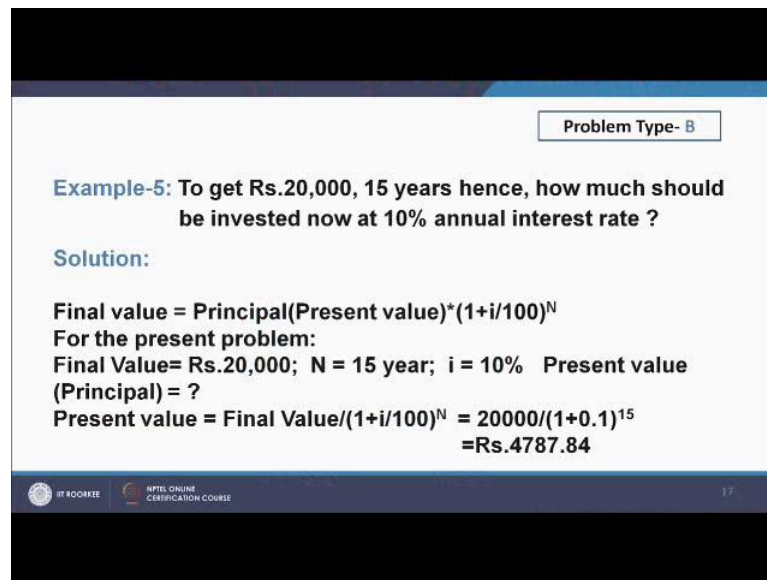
Future value after six years =  $P \cdot (1+i/100)^N$   
Future value = Rs.50,000 ;  $N=6$  ;  $i=10\%$  ;  $P=?$   
 $P$  is the present value of the money which will be borrowed from bank as loan  
 $P = \text{Future value} / (1+i/100)^6 = 50000 / (1+0.1)^6 = \text{Rs.}28223.69$

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Presume that you are going to receive an amount of Rupees 50,000 6 years from now. How much loan should you take from a bank today such that the money received after 5 years from now will be exactly enough to pay off the loan plus interest at that time. Assume the bank charges and annual interest rate of 10 percent.

Now, if you analyze this problem that our future value is 50,000, number of years is 6  $i$  is 10 percent, and the demand what should be my principle amount.  $P$  is the present value of the money which will be borrowed from the bank as loan. So,  $P$  is equal to future value divided 1 plus  $i$  by 100 to the power 6, so this is 50,000 divided by 1 plus 0.1 to the power 6 is comes out to the 28223.69. So, if we takes loan of 28223 today then after 6 years it will grow to 50,000 Rupees for an interest rate of 10 percent.

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The slide is titled "Problem Type- B" in a box at the top right. It contains the following text:

**Example-5:** To get Rs.20,000, 15 years hence, how much should be invested now at 10% annual interest rate ?

**Solution:**

**Final value = Principal(Present value)\*(1+i/100)<sup>N</sup>**

**For the present problem:**  
**Final Value= Rs.20,000; N = 15 year; i = 10% Present value (Principal) = ?**

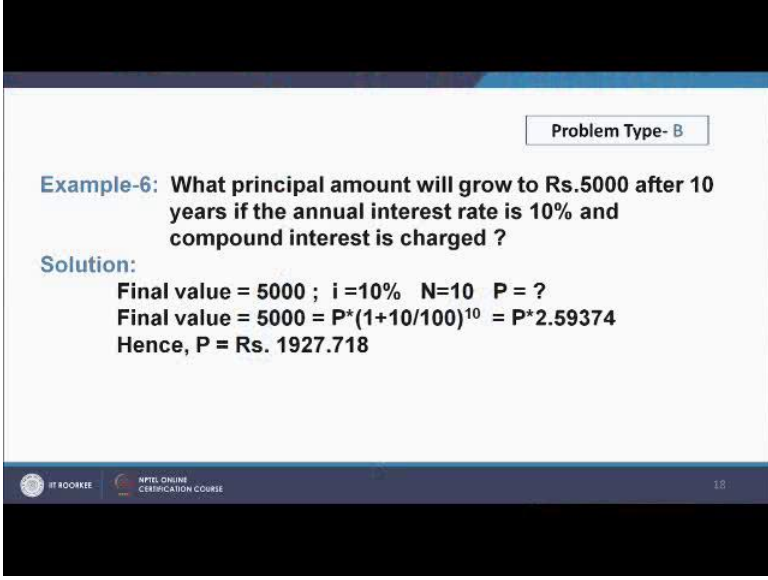
**Present value = Final Value/(1+i/100)<sup>N</sup> = 20000/(1+0.1)<sup>15</sup>**  
**=Rs.4787.84**

At the bottom of the slide, there are logos for "IIT ROORKEE" and "NPTEL ONLINE CERTIFICATION COURSE" on the left, and the number "17" on the right.

Now, another problem type B. To get Rupees 20,000 15 years hence how much should be invested now at 10 percent annually interest rate. It tells that how much money I should invest today at 10 percent annual interest rate so that after 15 years I get 20,000 Rupees. The solution of this problem is the final value is equal to principle which is present value into 1 plus i divided by 100 to the power N.

For the present problem final value is given as 20,000, N is 15 years, i is 10 percent and it ask what is the present value or the principle amount. So, we will use the same equation present value is equal to final value divided by 1 plus i divided 100 to the power N. I would say here that always i is expressed in terms of percentage, but when i is used in the equation it is used as a fraction and that is why 10 is divided by 100. So, it comes out to be 20,000 divided by 1 plus 0.1 to the power 15 which gives us Rupees 4787.84. That means, if I invest 4787.84 today at an annual interest rate of 10 percent then after 15 years I will get 20,000 Rupees.

(Refer Slide Time: 31:33)



The slide features a dark blue header and footer. The main content area is white. In the top right corner, there is a small box labeled 'Problem Type- B'. The text 'Example-6: What principal amount will grow to Rs.5000 after 10 years if the annual interest rate is 10% and compound interest is charged ?' is displayed in blue and black. Below this, the word 'Solution:' is in blue. The solution steps are listed in black: 'Final value = 5000 ; i =10% N=10 P = ?', 'Final value = 5000 =  $P \cdot (1 + 10/100)^{10} = P \cdot 2.59374$ ', and 'Hence, P = Rs. 1927.718'. The footer contains logos for 'IIT ROORKEE' and 'NPTEL ONLINE CERTIFICATION COURSE' on the left, and the number '18' on the right.

Problem Type- B

**Example-6:** What principal amount will grow to Rs.5000 after 10 years if the annual interest rate is 10% and compound interest is charged ?

**Solution:**

Final value = 5000 ; i =10% N=10 P = ?  
Final value = 5000 =  $P \cdot (1 + 10/100)^{10} = P \cdot 2.59374$   
Hence, P = Rs. 1927.718

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Again problem type B, what principle amount will grow to 5000 after 10 years if the annual interest rate is 10 percent and compound interest is charged. This is also a problem where principle is amount is amount unknown, final amount is known, number of years for which the money is invested is known, and interest rate known. So, it is a problem type B and for problem type B our formula is final value is equal to present value into 1 plus 10 by 100 to the power 10 or P is equal to final value divided by 1 plus 10 divided by 100 to the power 10.

So, from this equation we can very well calculate the value of P which come at to be 1927.718. That means, if today I spend 1927.718 Rupees at 10 percent compound interest annually then it will grow to 5000 Rupees after 10 years.


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Problem Type- C

**Example-7:** Assume the total cost of a college education will be Rs.60,000 when your child enters college in 12 years. You have Rs.10,000 to invest today. What annual rate of interest must you earn on your investment to cover the cost of your child's education ?

**Solution:**

Final value(FV)=60000 =  $10000(1 + 0.01*i)^{12}$   
or  $(1+0.01*i) = (60,000/10000)^{1/12} = (6)^{1/12} = 1.1610366$   
or  $i = (1.1610366-1)/0.01 = 16.104\%$

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Now, take a different problem which is problem type C. This is the example 7. I assume the total cost of a college education will be 60,000 Rupees, when your child enters college in 12 years. You have Rupees 10,000 to invest today. What annual rate of interest must you earn on your investment to cover the cost of your child's education? You have 10,000 in your hand you want 60,000 Rupees after 12 years what should be the interest rate which will grow this 10,000 Rupees to 60,000 Rupees after 12 years. So, our equation is final value is 60,000, present value is 10,000, interest rate is unknown and time is 12 years.

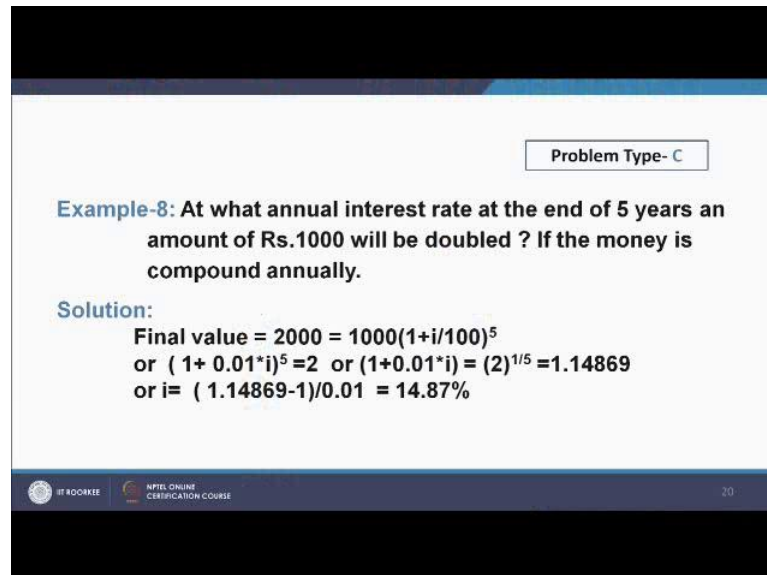
So, I can write down that final value is 60,000 is equal to present value 10,000 into 1 plus 0.01 into i to the power 12. Here I have used i divided by 100 and that 1 by 100 is giving this factor 0.01. That means, I am considering here i in percentage so whatever result will come out by solving this equations i it will be in percentage and not in fraction. Or I can write down 1 plus 0.1 into i is equal to 60,000 divided by 10,000 to the power 1 by 12 this is equal to 6 to the power 1 by 12 which comes out to be 1.1610366. So, 1 plus 0.01 i is equal to 1.1610366.

So, from here I can find out i which will be 1.1610366 minus 1 divided by 0.01 which comes to be 16.104 percent. That means, if the interest rate is 16.104 percent then only the Rupees 1000 which we you will invest today will grow to 60,000 Rupees in 12 years.



And if you get interest rate below this then it will not grow to 60,000 Rupees. So, the person we have to search a bank which will give him 16.104 percent interest rate.

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Problem Type- C

**Example-8:** At what annual interest rate at the end of 5 years an amount of Rs.1000 will be doubled ? If the money is compound annually.

**Solution:**

Final value = 2000 =  $1000(1+i/100)^5$   
or  $(1 + 0.01*i)^5 = 2$  or  $(1+0.01*i) = (2)^{1/5} = 1.14869$   
or  $i = (1.14869-1)/0.01 = 14.87\%$

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Take another problem of problem type C. This is example; at what annual interest rate at the end of 5 years an amount of Rupees 1000 will be doubled, if the money is compounded annually. Again we know the time its 5 years, P this is principle is 1000 this will double that means, FV which is final value is 2000, so we need here what is the annual interest rate. Again the same equation you can use final value is equal to present value into 1 plus i divided by 100 to the power N.

In this case the final value is equal to 2000 which is equal to 1000 into 1 plus i divided by 100 to the power 5. We can write down 1 plus 0.01 into i to the power 5 is equal to 2. Now this 0.01 factor which is multiplied by i is basically comes from 1 by 100, because I have used i divided by 100 so I can write this as 0.01 into i. So, 1 plus 0.01 into i to the power 5 is 2. Again I will emphasize that here I am using i as percentage directly so whatever result I will get after solving the i will be in percentage not has a fraction.


So, if you proceed 1 plus 0.01 i is equal to 2 to the power 1 by 5 which come out to be 1.14869 or i is equal to 1.14869 minus 1 and this is in brackets divided by 0.01 which comes out to be 14.87 percent. That means if 1000 it will be invested for 5 years it will be doubled in 5 years provided the annual interest rate is 14.87 percent.


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Problem Type- D

**Example-9:** How long(approximate years) it will take to double Rs.1000 at a annual interest rate of 10.41% per year ? If compound interest is paid.

**Solution:**  
Let after N years the money will be doubled  
Final value =2000 =  $1000(1+i/100)^N = 1000(1.1041)^N$   
or  $(1.1)^N = 2$   
or  $N = \ln(2)/\ln(1.1041)$   
=0.693147/0.09903  
=6.999 year  $\approx 7$  year

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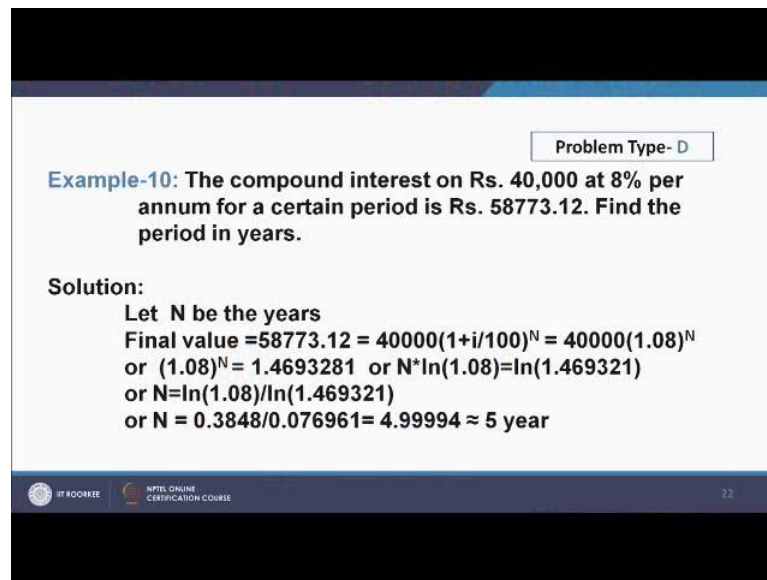
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Let us take another problem, the problem type is D. The example is 9; how long export it demands approximate years, it will take to double Rupees 1000 at an annual interest rate of 10.41 percent if the compound interest is paid. If you analyze these examples then it gives interest rate 10.41 it gives the value of P that is principle or present value to be 1000, it gives the final value because the money has to be doubled. So, the final value is 2000 which is 1000 into 2. And it requires the years that is N that for how many years this 1000 should be invested at 10.41 percent interest rate so that it doubles

Let us see the solution we consider that after 10 years the money will be doubled. So, the final value is 2000 is equal to 1000 1 plus i divided by 100 to the power N. So, is becomes 1000 into 1.1041 to the power [noise], and we have to find out this value of N. So, this becomes 1 plus 1 to the power N is equal to 2 or I should say that 1.1041 to the power N is equal to 2. So, if I take natural logarithm of both the sides then N into ln 1.1041 is equal to ln 2 or N will be equal to ln 2 divided by ln 1.1041.

If I take the natural logarithmic of 2 it is 0.6931478 and if I take the logarithmic of 1.1041 it is 0.09903. So when we divide it, it becomes 6.999 years which is approximately 7 years. That means, if I invest Rupees 1000 today at an interest rate of 10.41 per year then it will takes 7 years to double this money.

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Problem Type- D

**Example-10:** The compound interest on Rs. 40,000 at 8% per annum for a certain period is Rs. 58773.12. Find the period in years.

**Solution:**  
Let N be the years  
Final value = 58773.12 =  $40000(1+i/100)^N = 40000(1.08)^N$   
or  $(1.08)^N = 1.4693281$  or  $N \cdot \ln(1.08) = \ln(1.469321)$   
or  $N = \ln(1.08) / \ln(1.469321)$   
or  $N = 0.3848 / 0.076961 = 4.99994 \approx 5$  year

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Another problem we take which is problem type D. This is example 10; the compound interest on Rupees 40,000 at 8 percent per annum for a certain period is Rupees 58773.12. Find the period in years. Here the compound interest is given which is 8 percent per annum, P is given which is 40,000 and FV which is the final value is given 58773.12, and what is unknown is the N which is number of years. Let us take the solution. Again the similar way let N be the number of years which will grow the 40,000 Rupees to 58773.12 at the interest rate of 8 percent per annum. So, final value is 58773.12 which is equal to 40,000 into 1 plus i divided by 100 to the power N which comes out to be 40,000 into 1.08 to the power N. So, the 1.08 to the power N is equal to 58773.12 divided by 40,000 which comes out to be 1.469321.

Now the find out the value of N what we have to do, we have to take the log of both the sides. So, if I take the log of both the sides then it converts into N into log 1.08 is equal to ln 1.469321. Or N is equal to ln 1.08 divided by ln 1.469321. Now if I take this natural logarithm of these values 1.08 I find it to be 0.3848. Similarly, if I take the natural logarithm of 1.469321 it is 0.076961. So, once I divide 0.3848 with 0.076961 it comes out 4.9994 which is approximately equal to 5 years. That means if 40,000 Rupees is invested for 5 years at the rate of 8 percent per annum then it will convert into Rupees 58773.12.

Now, let us take some problems which are mixed in nature. With problem type A, problem type B, problem type C, problem type D we have covered the matrix which is possible matrix. And after we have covered the possible matrix then we see some problems which are of mix type. For that I have taken several problems and an example 11 is one such problems.

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Problem Type- mixed


**Example-11:** There is 90% increase in an amount in 9 years at simple interest. What will be the compound interest of Rs. 20,000 after 5 years at the same rate?

**Solution:**

Let the rate of interest per annum =  $i$   
 Let the principal amount be =  $P$   
 90% increase in amount  $P = 0.9P$   
 Interest received after 9 years =  $P \cdot (i/100) \cdot 9 = 0.9P$  or  $i = 90/9 = 10\%$

Compound interest after 5 years for Rs. 20,000 =  $20000 \cdot (1 + 10/100)^5 - 20000 = 32210.2 - 20000 = \text{Rs. } 12210.2$

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Example 11 says there is 90 percent increase in an amount in 9 years at simple interest. What will be the compound interest of Rupees 20,000 after 5 years at the same rate? So, is a mixture of simple interest and compound interest problems and the from the first statement I have to find out what is the interest rate and what is the simple interest rate per annum. And in the second parts of the questions I have to use that interest rate to compute the compound interest of 20,000 Rupees for 5 years. So, let us see the solution.

Let the rate of interest per annum is  $i$ , let the principle amount be  $P$ . So, 90 percent increase in amount of  $P$  is  $0.9 P$ . So, interest received after 9 years is equal to when we use it as simple interest  $P$  into  $i$  divided by 100 into 9 is equal to  $0.9 P$ , because  $P$  into  $1 i$  divided by 100 into 9 is the interest earned by the principle amount  $P$  in 9 years using simple interest. So,  $i$  comes out to be 90 divided by 9 as 10 percent. Once  $i$  is known, that is  $i$  of the simple interest is known same  $i$  interest rate has to be applied for the compound interest so the problem becomes simpler, now we know the value of  $i$ . Compound interest after 5 years for Rupees 20,000 will be now 20,000 into 1 plus 10

divided by 100 to the power 5 minus 20,000 which will be equal to 32210.2 minus 20,000 which come out to be 12210.2.

Lets taken an another problem, and this problem is mixed type.

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Problem Type- mixed

**Example-12:** The difference between compound interest and simple interest on an amount of Rs. 20,000 for 2 years is Rs. 162. What is the rate of interest per annum?

**Solution:**

Let the rate of interest per annum =  $i$



Compound Interest (C.I.) for 2 years =  $20000(1+i/100)^2 - 20000 = 20000((1+0.01i)^2 - 1)$

Simple Interest (S.I.) for 2 years =  $20000 \times (i/100) \times 2 = 200 \times 2i$

C.I.-S.I. =  $20000((1+0.01i)^2 - 1) - 20000 \times 0.02i = 162$  or  $20000[1 + 0.02i + 0.0001i^2 - 0.02i] = 162$

or  $20000(0.0001i^2) = 162$

or  $2.0(i^2) = 162$  or  $i^2 = 81$  or  $i = 9\%$

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Example 12; the difference between compound interest and simple interest on an amount of Rupees 20,000 for 2 years is Rupees 162. What is the rate of interest per annum? The solution is let the rate of the interest per annum is  $i$ , so the compound interest abbreviated as C I for 2 years will be 20,000 into 1 plus  $i$  divided by 100 square minus 20,000. 20,000 into 1 plus  $i$  divided by 100 square is the FV value that is final value. And 20,000 is the principle which is the present value so final value minus present value is the interest. So, this comes out to be 20,000 in brackets 1 plus 0.1 into  $i$  whole square minus 1.

This 0.01 factor has come from  $i$  divided by 100. So,  $i$  divided by 100 can be written as  $i$  into 1 by 100 and 1 by 100 will convert into 0.01, so  $i$  divided by 100 is 0.01 into  $i$ . It should be noted that  $i$  here is percent is not a fraction, so when we solve for  $i$  whatever we get the results will be in percentage. So, the compound interest now is 20,000 into in the brackets 1 plus 0.1 into  $i$  whole square minus 1. Similarly, the simple interest for 2 years is 20,000  $i$  divided by 100 into 2 which is 200 into 2  $i$ .

Now, the problem gives that compound interest minus simple interest is 162. So,  $C I$  minus  $S I$  is equal to 20,000 in the brackets  $1 + 0.1 i$  whole square minus  $1$  minus  $20,000$  into  $0.02 i$  is equal to 162. Now if we solve this then we find that  $20,000$  into  $0.001 i$  square is equal to 162 because while solving you will find that  $1$  plus minus  $1$  cancels  $0.02 i$  also cancels with minus  $0.02 i$  and hence left out is the  $20,000$  into  $0.001 i$  square is equal to 162. And hence  $2.0 i$  square is equals to 162 and  $i$  square is equal to 81 and  $i$  is equal to 9 percent. So, if  $i$  is taken to be 9 percent then the condition which is given in the example two will be satisfied.

Again we take another example which is a problem type mixed it is example 13.



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Problem Type- mixed

**Example-13 :** The difference between simple and compound interests compounded annually on a certain sum of money for 2 years at 8% per annum is Rs. 6.4. Find the amount of money invested.

**Solution:**

Let the sum amount =  $x$   
 Compound Interest(C.I.) for 2 years =  $x(1+8/100)^2 - x$   
 $= 0.1664x$   
 Simple Interest (S.I.) for 2 years =  $x \cdot 0.08 \cdot 2 = 0.16x$   
 $C.I. - S.I. = 0.1664x - 0.16x$   
 $= 0.0064x = 6.4$   
 or  $x = \text{Rs.} 1000$

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The difference between simple and compound interest compounded annually on a certain sum of money for 2 years at 8 percent annum is Rupees 6.4. Find the amount of money invested.

Now, here amount of money is required. So, if you see the solution we presume that let the some of the amount is  $x$  then compound interest  $C I$  for 2 is equal to  $x$  into  $1$  plus  $8$  divided by  $100$  whole square minus  $x$  which comes out to be  $0.1664 x$ . Similarly, if we calculated the simple interest for 2 years it is  $x$  into  $0.08$  into  $2$  which comes out to be  $0.16 x$ . So, compound interest minus simple interest is equal  $0.1664 x$  minus  $0.16 x$  which is equal to  $0.0064 x$  is equal to  $6.4$ . And hence  $x$  is equal to  $1000$ .

Thank you.