Time value of money-Concepts and Calculations

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Lecture - 02
Interest Rates

Welcome to the lecture series on Time value of money-Concepts and Calculations.

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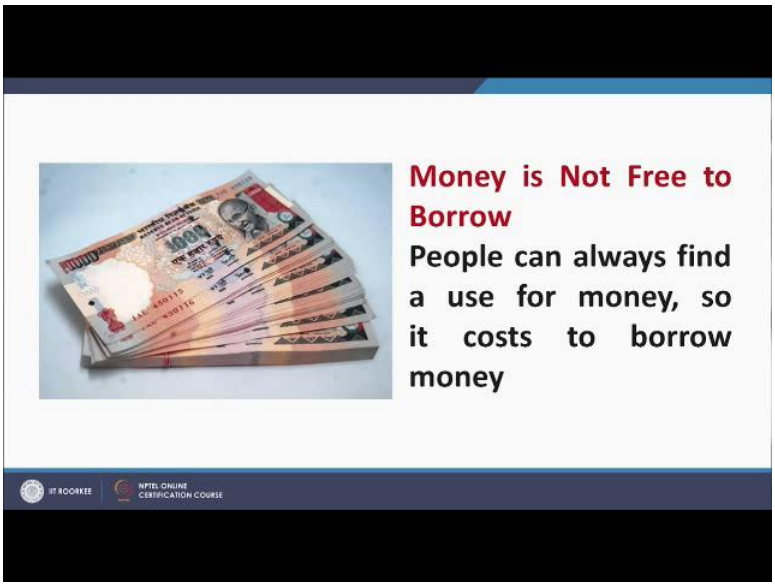

We all know that money is not free to borrow; people can always find a use of for money, so it costs to borrow money. This lecture is devoted to interest, and will see in this lecture different type of interest rates and then how they are correlated.

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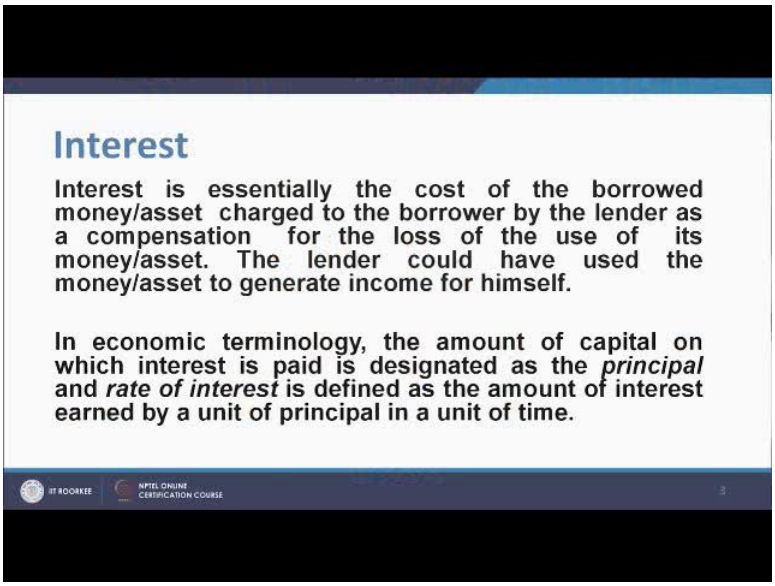

To define interest, interest is essentially the cost of the borrowed money or asset charged to the borrower by the lender as a compensation for the loss of the use of its money or asset. The lender could have used the money or asset to generate income for himself.

In economic terminology, the amount of capital on which interest is paid is designated as the principal and rate of interest is defined as the amount of interest earned by a unit of principle in a unit of time.

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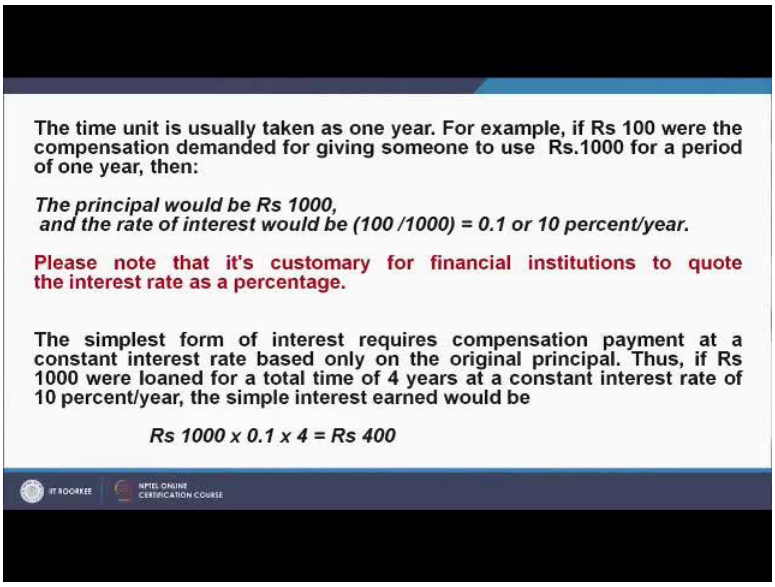

The time unit is usually taken as one year. For example, if rupees 100 were the compensation demanded for giving someone to use rupees 1,000 for a period of one year then, the principle would be 1,000, the rate of interest would be 100 divided by 1,000, which is equal to 0.1 or 10 percent per year.

Please note that it is customary for financial institutions to quote the interest rate as a percentage. The simplest form of interest requires compensation payment at a constant interest rate based on the original principal. Thus if rupees 1,000 per loan for a total time of 4 years at a constant interest rate of 10 percent per year, the simple interest earned will be rupees 1,000 into 0.1 into 4 which will come out to be rupees 400. Interest rates let us see that how many interest rates are prevalent.

The term interest rate is one of the most commonly used phrases in the consumer finance and fixed income investments and interest rate is the rate at which interest is paid by a borrower as a compensation for the use of money that he borrows from the lender.

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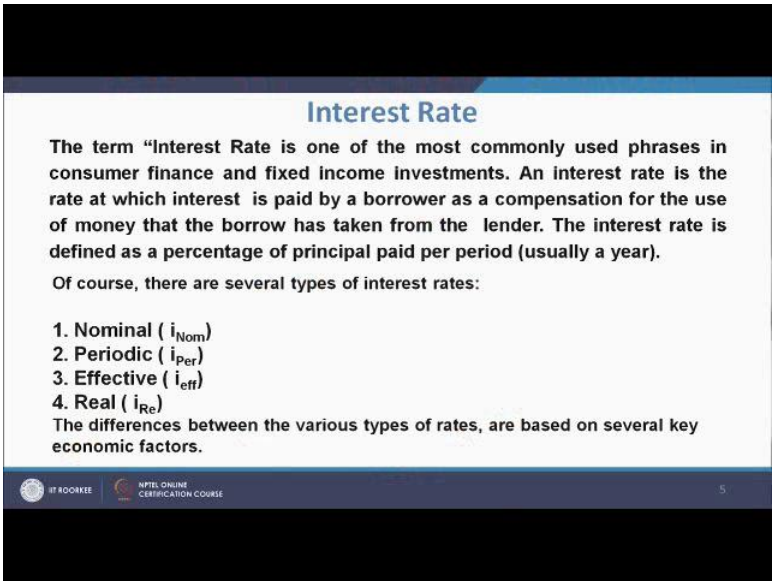

Interest rate is defined as a percentage of principal paid per period which is usually a year. Of course, there are several types of interest rates. The first one is called nominal, the second is periodic, and the third is effective and fourth is real. The differences between the various types of rates are based on several key economic factors and let us now see one by one these interest rates.

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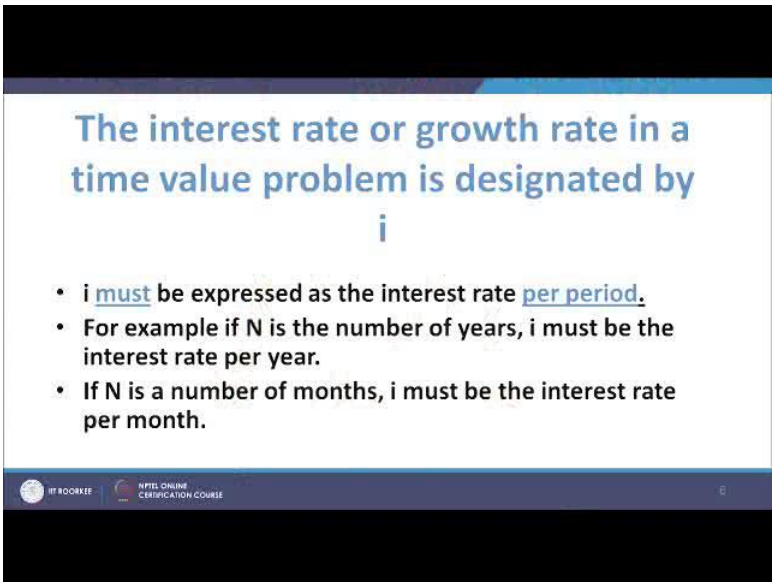

The interest rate or growth rate in a time value problem is designated by small i. What are the conditions? i must be expressed as the interest rate per period. Second one for example, if N is the number of years, i must be the interest rate per year and if N is the number of months, i must be expressed as interest rate per month. Here we see that how interest rate affects the total accumulated wealth.

Now if I am investing some principle at t equal to 0, it will grow to different values of some at the end of one year for different interest rates. So, if interest rate is 5 percent then, I find that the value S N that is the final value is less.

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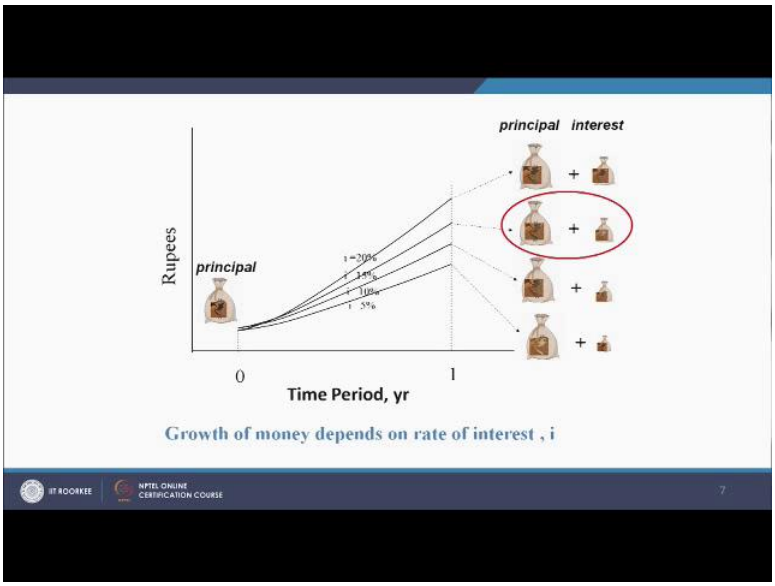

And if the interest is 20 percent, then the final value is more, but however, if when we analyze the principals remain same, but the interest changes with the interest rate. Lower is the interest rate, less interest you will get and higher is the interest rate, more interest you will get and thus the final value becomes more. Nominal interest rate, the nominal interest rate which is shown by r, small r is conceptually the simplest type of the interest rate. It is simply the stated interest rate for a given bond or loan.

This type of interest rate is referred to as the coupon rate or fixed income investment. As it is the interest rate, guaranteed by the user that was traditionally stamped on the coupons that were redeemed by the shareholders. The nominal interest rate is inessential, the actually monetary price that borrowers pay to lenders to use their money. These are stated in contracts and quoted by banks and brokers periods per year that is m must be governed.

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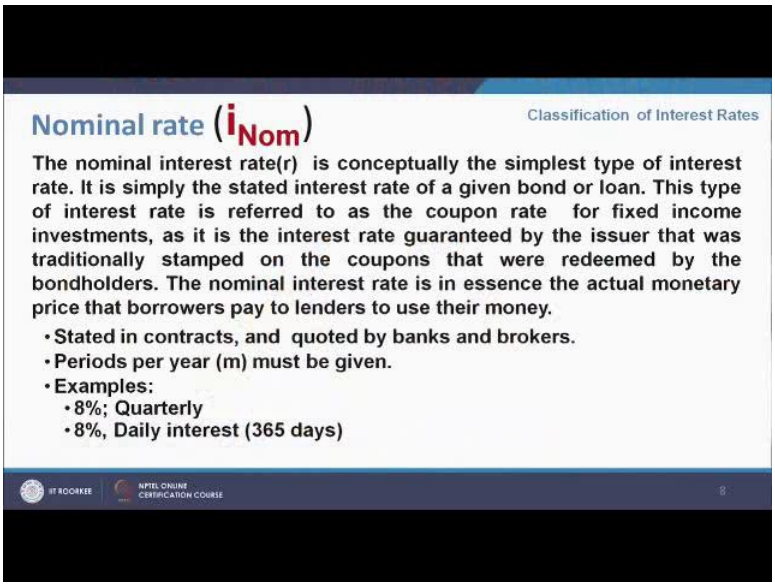

For example 8 percent quarterly, 8 percent interest daily that is 364 days. In common industrial practice the interest period is taken to be one year and interest rate is also defined per year basis.

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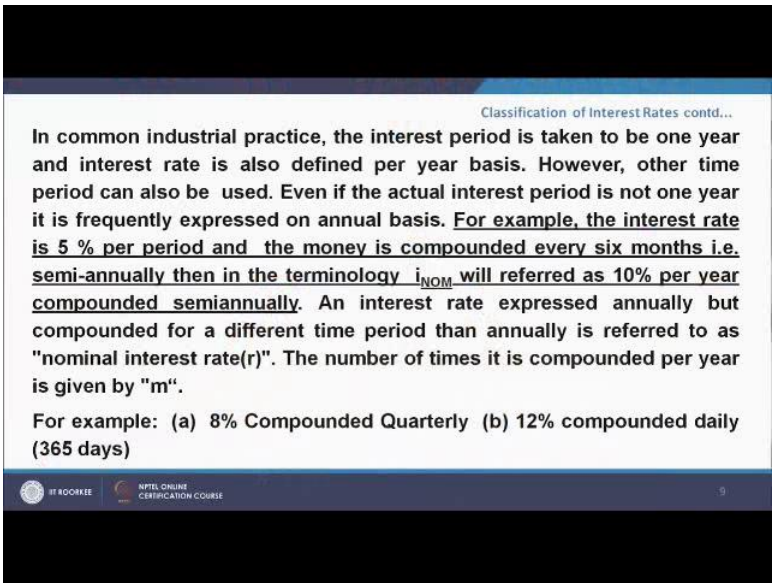

However other time period can also be used. Even if the actual interest period is not one year, it is frequently expressed on N 1 basis. For example, the interest rate is 5 percent per period and the money is compounded every 6 months that is semiannually, then the terminology i nominal will be referred as 10 percent per year compounded semiannually.

So, this is a method to write, we will write i nominal as 10 percent per year compounded semiannually. And interest rate expressed annually, but compounded for different time period than annually is referred to as nominal interest rate r. The number of time it is compounded per year is given by small m. For example, 8 percent compounded quarterly and 12 percent compounded daily.

So, if I want to find out the interest rate, actual interest rate then it will be 12 divided by 365 percent. Another type of interest rate is periodical rate, which is shown by i period, i period is equal to i nominal divided by small m. Where, m is the number of compounding periods per year.

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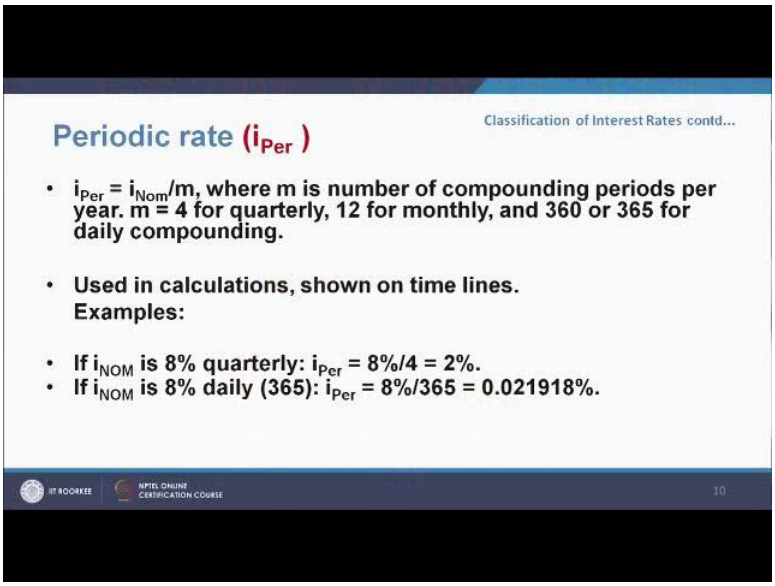

For quarterly m is equal to 4, m is equal to 12 for monthly and m is equal to 360 or 365 for daily compounding. These are used in calculations shown on time line, example is if i nominal is 8 percent quarterly, then i period will be 8 percent divided by 4 which comes out to be 2 percent; if i nominal is 8 percent daily, then i per.

That is periodic will be 8 percent divided by 365, which comes out to be 0.021917 percent. The third interest rate is effective annual rate, which is shown by i effective. The i effective is the annual rate which causes present value to grow to the same final value has under multi period compounding.

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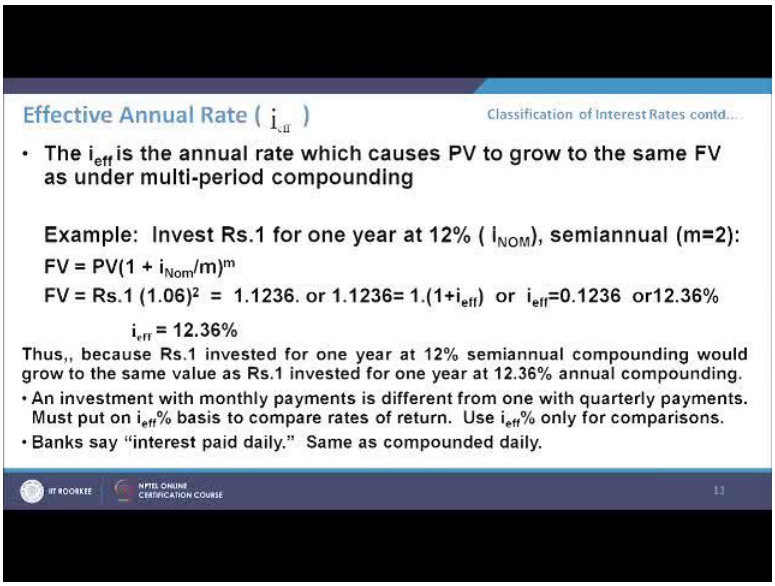

For example, invest rupees 1 for one year at 12 percent i nominal semiannually and when I say semiannually, that is 2 periods per year. So, m is equal 2, then FV is equal to PV into 1 plus i nominal divided by m and whole to the power m. So, FV comes out to be 1 into 1.06 square, which comes out to be 1.236 and when we compare with this i effective, then it becomes 1.1236 equal to one this is value of PV into 1 plus i effective. So, i effective come out to be 0.1236 or 12.36 percent.

Thus, i effective becomes 12.36 percent because if I invest rupees 1 for one year at 12 percent semiannually compounding, it will grow to the same value as 1 rupee invested for one year at 12.36 percent annually compounding. And invest with monthly payment is different from one with quarterly payments must put i effective basis to compare rates of return. Use i effective only for a comparisons.

Another example we will be that, suppose there are two banks, one is giving you compounding monthly. Another is giving you compounding daily for the money which you have put into the bank. So, if we want to compare that who is giving more interest than we have to calculate i effective for these two banks. Obviously, the bank which is giving daily compounding, the i effective will be more.

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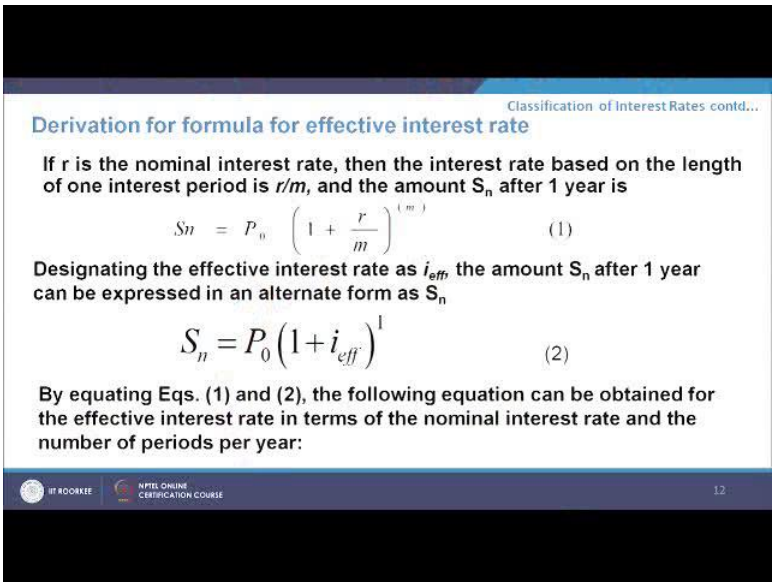

And thus we should invest money into that bank. Bank says interest paid daily same as compounded daily. Derivation of formula for effective interest rate if r is the nominal interest rate than the interest rate based on the length of one interest period is r divided by m and the amount S which is the final amount after one year can be written as S n is equal to P 0 in brackets 1 plus r divided by m brackets close to the power m. Designating the effective interest as i effective, the amount S n after one year can be expressed in the alternate form as, S n is equal to P 0 within brackets 1 plus i effective to the power 1.

Now, when we equate the S n values obtained from equation 1 and equation 2, then we can find out i effective for different compounding periods. By equation 1 and 2 the following equation can be obtained for the effective interest rate in terms of the nominal interest rate and the number of periods per year.

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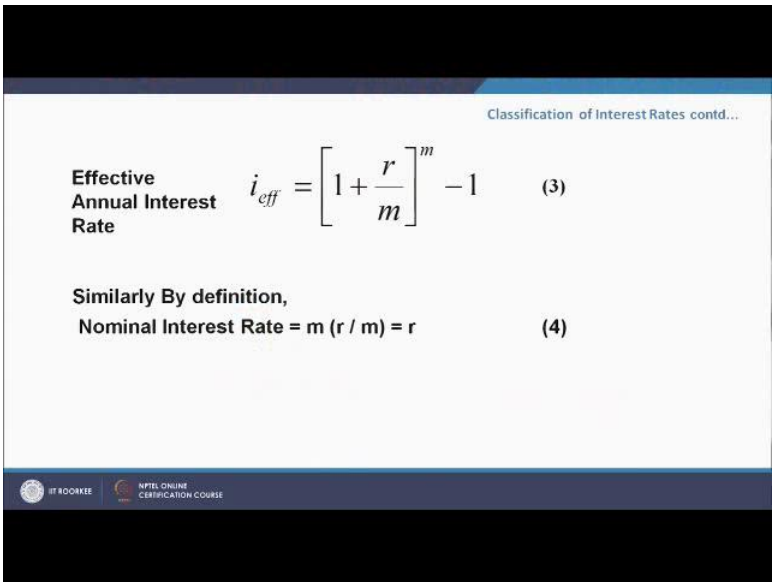

So, the final equation is given in equation number 3. So, effective annual interest rate i effective is equal to 1 plus r divided by m whole to the power m minus 1. By definition the nominal interest rate is equal to m into r by m which is equal to r and so in equation 4. Can be effective rate i effective ever be equal to the nominal rate? If this question is asked and the answer is yes, but only if n 1 compounding is used; that means, if annual compounding is used then i effective will be equal to i nominal, that is for m equal to 1. If m is greater than one, then i effective will always be greater than the nominal interest rate which is shown by r.

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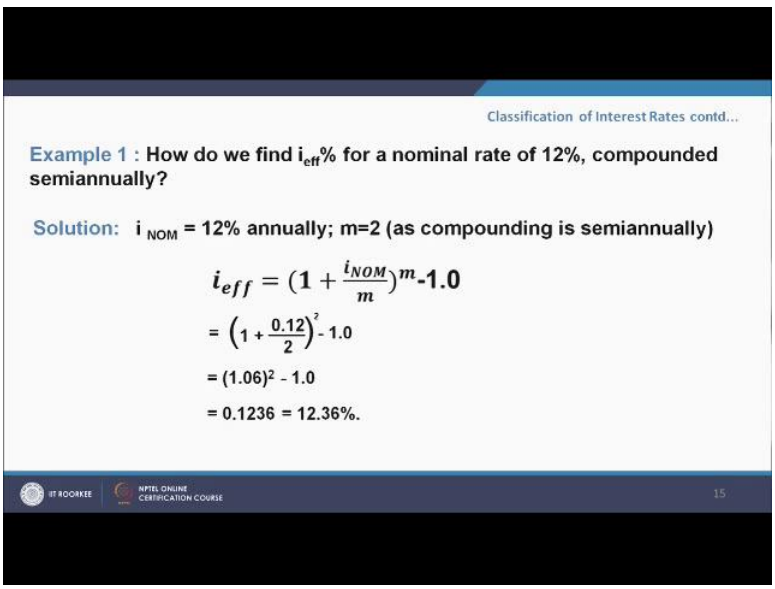

Let us take an example to demonstrate, what we want to tell you. How do we find i effective for a nominal rate of 12 percent compounded semiannually? What is given i nominal is 12 percent, which is always given annually and m is equal to 2 as compounding semiannually; that means, twice per year.

So, i effectively will be equal to 1 plus i nominal divided by m 2 whole to the power m minus 1.0 and this comes out to be 1 plus 0.12 divided by 2 because the i nominal is 12 percent that is why we have written 0.12 divided by 2 whole to the power 2 minus 1.0, this comes out to be 0.1236 which is 12.36 percent. Let us take example number 2. For nominal interest rate of 10 percent annually, calculate an effective interest rate.

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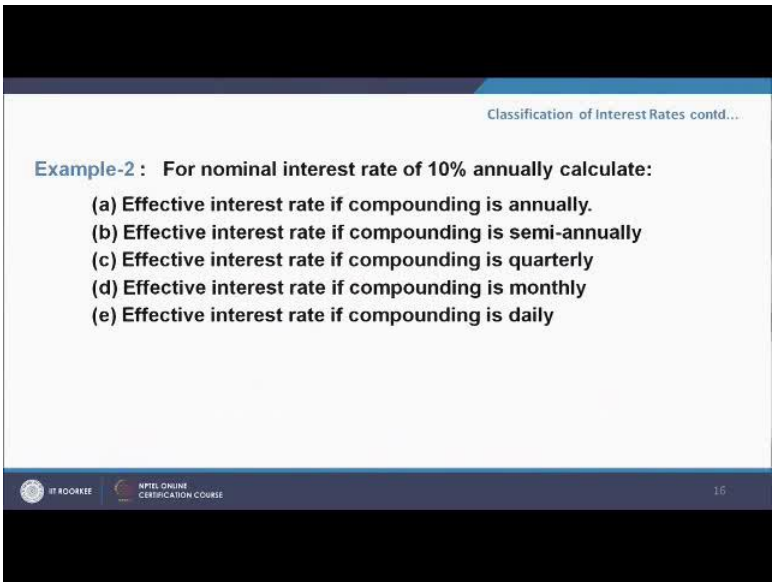

If compounding is annually, b effective interest rate if compounding semiannually, c effective interest rate if compounding is quarterly and d effective interest rate if compounding is monthly and e effective interest rate compounding is daily.

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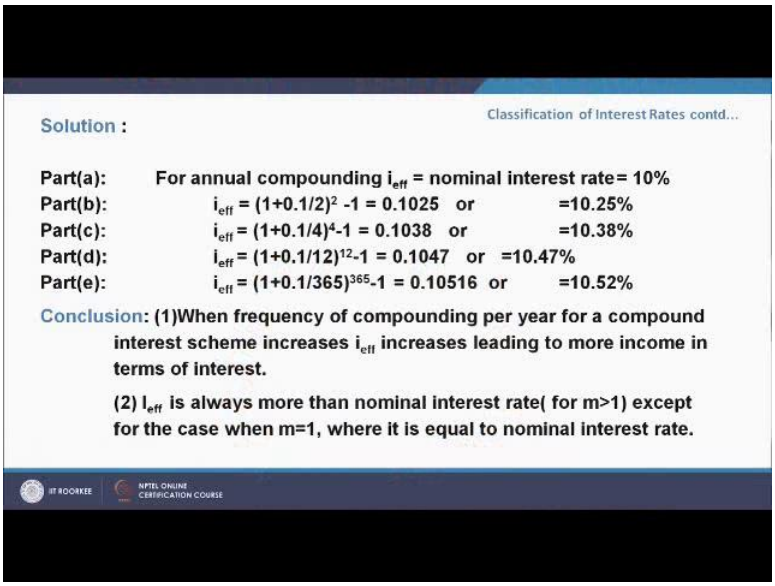

So, we take this same equation and put the value. Part a, for annually compounding we know that i effective is equal to nominal interest rate which is 10 percent. So, i effective is 10 percent for part a. For part b, i effective is equal to 1 plus 0.1 by 2 because m is equal to 2 year compounding semiannually to the power 2 minus 1, which comes out to be 0.1025 or 10.25 percent. Now when were going for quarterly m is equal to 4, the value comes out be 0.1038 or 10.38 percent. When were going for monthly m is equal to 12. So, i effective is equal to 1 plus 0.1 divided by 12 whole to the power 12 minus 1, which comes out to be 0.1047 that is, the value is 10.47 percent and when we are finding it for yearly then i effective is equal to 1 plus 0.1 divided by 365 whole to the power 365 minus 1 which comes out to be 0.10516 or 10.52 percent. What conclusion we make or of this example? The first, when frequency of compounding per year for a compound interest scheme increases, i effective increases leading to more income in terms of interest.

So, if the compounding period is shorter, then obviously the i effectively will be far far greater than the nominal interest rate. The second i effective is always more than nominal interest rate for m is greater than 1, except for the case when m equal to 1. In this case i effectively will be equal to nominal interest rate.

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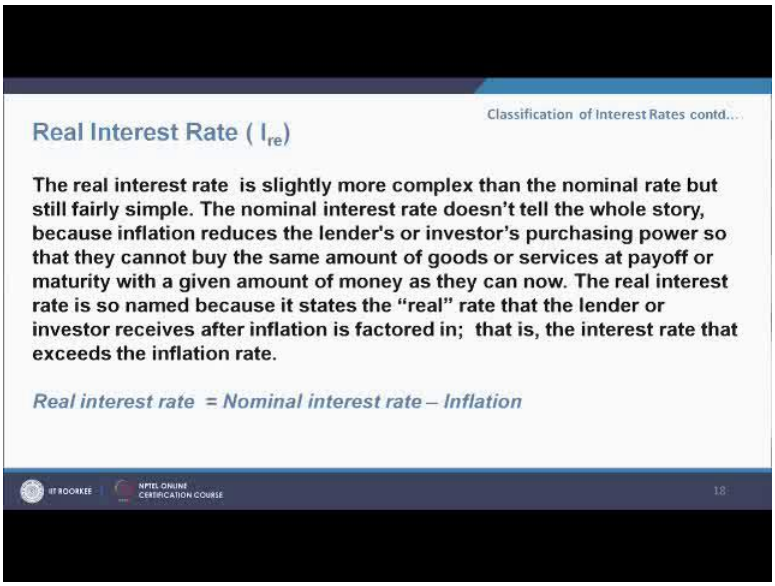

Real interest rate, the real interest state is slightly more complex than the nominal rate, but still fairly simple. The nominal interest rate does not tell the whole story because inflation reduces the lenders or investors purchasing power so that, they cannot buy the same amount of goods or services at payoff or maturity with a given amount of money as they can now.

The real interest rate is so named because it states the real rate at the lender or investor receives after inflation is factored in; that is, the interest rate that exceeds the inflation rate. So, really interest rate is equal to nominal interest rate minus inflation.

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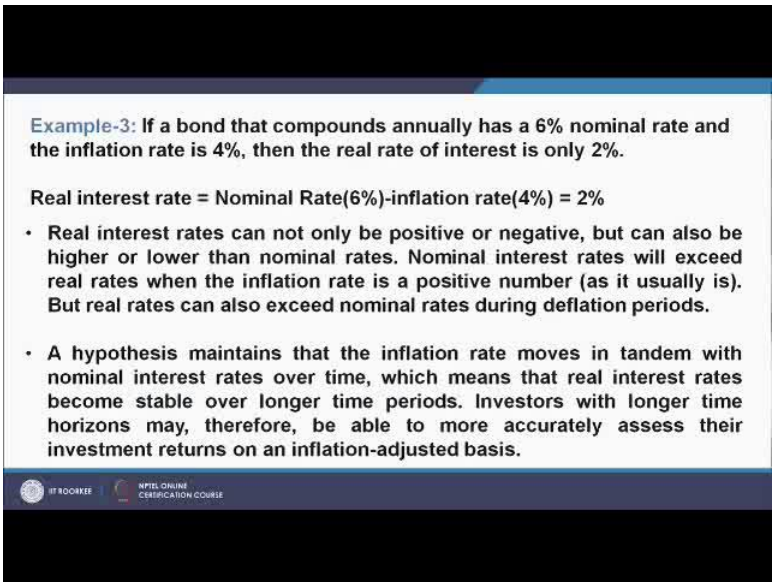

Let us take an example. If a bond that compounds annually has 6 percent nominal rate and the inflation is 4 percent, then the real rate if interest is 6 minus 4 equal to 2 percent.

So, we see here the real interest is nominal interest rate which is 6 percent minus inflation rate which is 4 percent comes out to be 2 percent. Real interest rate cannot only be positive or negative, but can also be higher or lower than nominal rates. Nominal interest rate will exceed real rates, when the inflation rate is a positive number as it usually is. But real rates can also exceed nominal rates during deflation periods. A hypothesis maintains that the inflation rate moves in tandem with nominal interest rate over time which means that, real interest rates become stable over long time periods. Investors with longer time horizon may therefore, be able to more accurately access their investment returns on an inflation adjusted basis.

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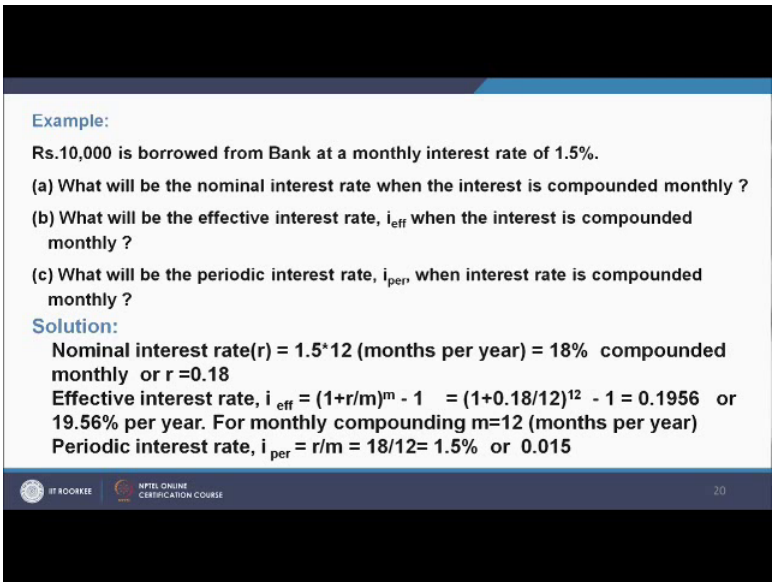

Let us take some examples rupees 10,000 are borrowed from a bank at a monthly interest rate of 1.5 percent. What will be the nominal interest rate when the interest is compounded monthly? What will be the effective interest rate when the interest is compounded monthly? And what will be the periodic interest rate when interest rate is compounded monthly? Solution; nominal interest rate are will be 1.5 into 12 because the bank is giving interest rate 1.5 percent monthly. So, I multiply with number of months. So, it comes out to be 18 percent compounded monthly or r is equal to 0.18.

Effective interest rate will be 1 plus r divided by m whole to the power m minus 1, which comes out to be 0.1956 or 19.56 percent per year. For monthly compounding m is equal to 12 because there are 12 months in a year. Periodic interest i period will be r by m which is 18 by 12 and is equal to 1.5 percent or 0.015.

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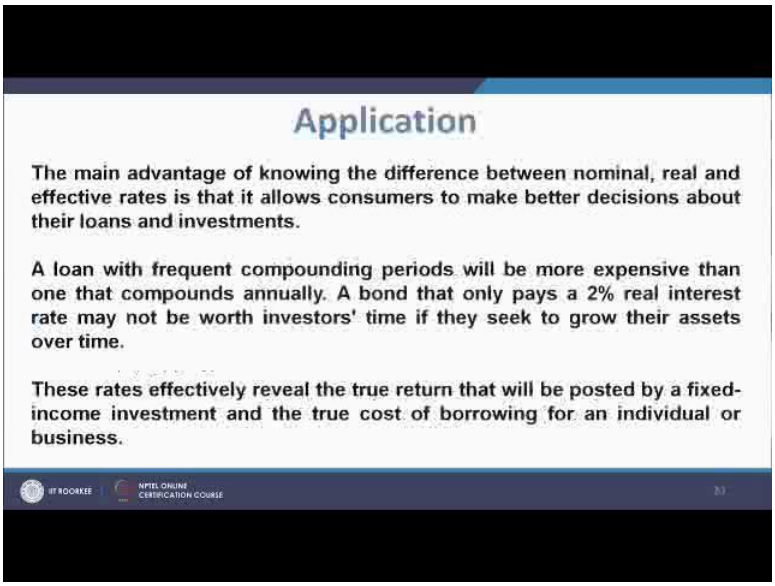

So, we see that the bank has shown an IPR, that is periodic interest as 1.5 percent. Now the application, the main advantage of knowing the difference between nominal, real and effective rates is that, it allows consumers to make better decisions about their loan and investments. A loan with frequent compounding periods will be more expensive than one that compounds annually. A bond that only pays a 2 percent real interest rate may not be worth investors' time if they seek to grow their assets over time. These rates effectively review the true return that will be posted by a fixed income investment and the true cost of borrowing for an individual or business.

Thank you.