Time value of money-Concepts and Calculations
Prof. Bikash Mohanty
Department of Chemical Engineering
Indian Institute of Technology, Roorkee

Lecture - 14
Valuation of Bonds- 1 and 2

Welcome to the lecture series on Time Value of money-Concepts and Calculations. In this lecture I will teach valuation of bonds part 1 and valuation of bond part 2.

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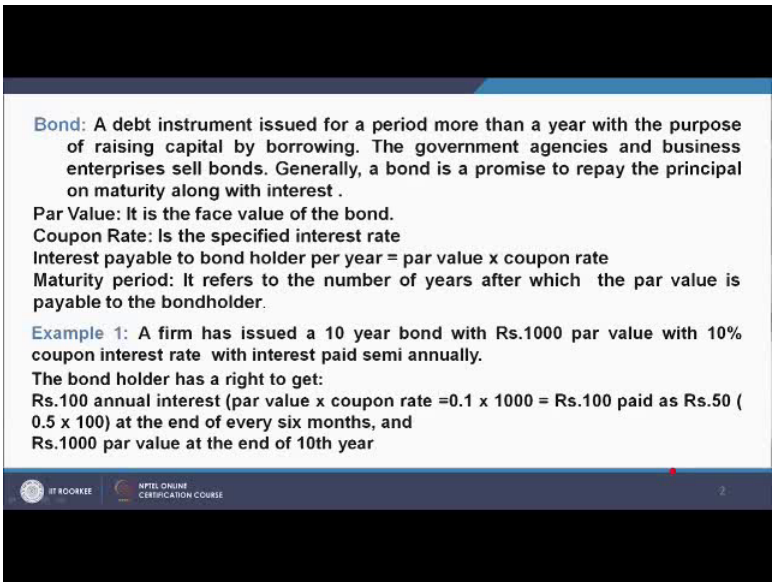

Bond a debt instrument issued for a period more than a year with the purpose of raising capital by borrowing. The government agencies and business enterprises sell bonds generally a bond is a promise to repay the principal on maturity along with interest.

Now, let us see something which is related to the bond. Par value it is the face value of the bond. Coupon rate is the specified interest rate, interest payable to bond holder per year is equal to par value into coupon rate. Maturity period it refers to the number of years after which the par value is payable to the bond holder.

Let us take an example. Example 1, a firm as issued a 10 year bond with 1,000 par values with 10 percent coupon interest rate, with interest paid semi-annually. The bond holder as a right to get rupees 100 and annual interest which comes out from par value into coupon rate, which is 0.1 into 1,000 is equal to 100 paid as rupees 50, 100 divided by 2

at the end of every 6 months and rupees 1,000 par value at the end of 10'th year.

Let us take an example. This is called example 2, a firm as issued a 10 year bond with 1,000 par values with 10 percent coupon rate as well as discounted; that means, discount rate and coupon rates are 10 percent with interest paid annually. Compute the value of the bond.

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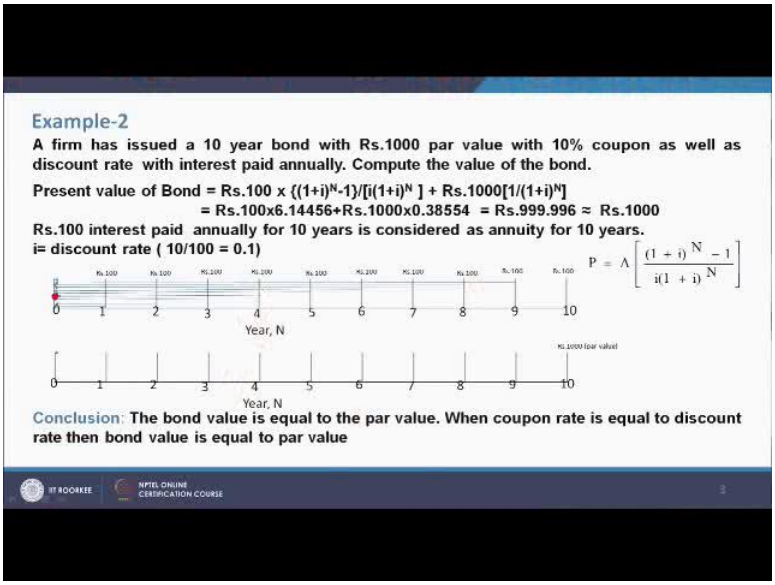

Now, if you analyze this, this bond is giving 100 rupees at the end of each year up to 10 years. So, this is shown in the time line at the end of the first year, you get 100 rupees and end of the second year, you get again 100 rupees so on so forth up to the end of 10'th year, you get 100 rupees. The present values of these 100 rupees can be calculated using this formula, which is being used to calculate the present value of annuity. This is P equal to A in brackets 1 plus i to the power N minus 1 divided by i 1 plus i to the power N. And then this line shows at the end of 10th year you will 1,000 rupees and the present value of these 1,000 rupees can be calculated by bringing these values to time line t equal to 0.

So, here the present value is calculated in two groups. This first group shows the present value of the annuity and second group is related to this figure; that is at the end of 10'th year, you will get 1,000. So, when you put the values this becomes 100 into 6.14456 plus 1,000 into 0.38554 which comes out to be rupees 999.996, which is almost equal to 1,000. Here the 100 rupees interest paid annually for 10 years is considered as annuity for 10 years and discount rate is 10 by 100 is 0.1 conclusions. The bond value is equal to

the par value when coupon rate is equal to discount rate, then bond value is equal to par value.

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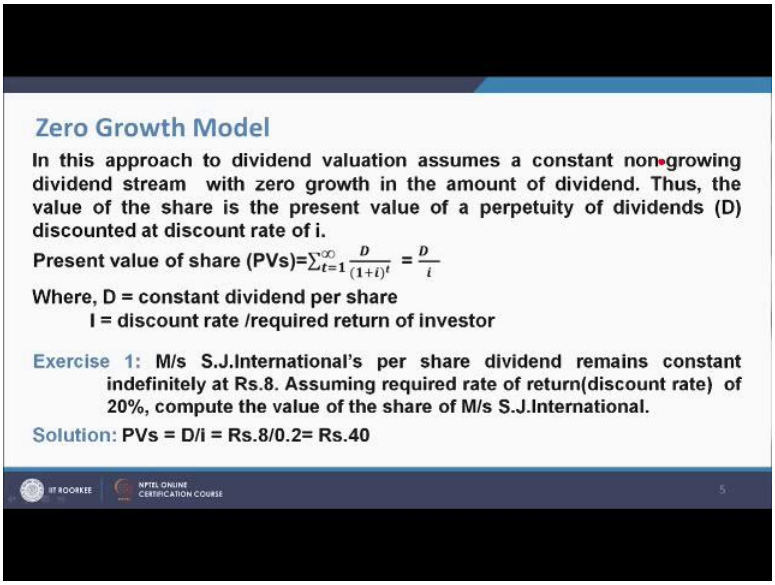

Now values of bond with different coupon and discount rate. The values of the bond are the present value of the contractual payment. It is issuer that is government or enterprises obliged to make from the beginning till maturity.

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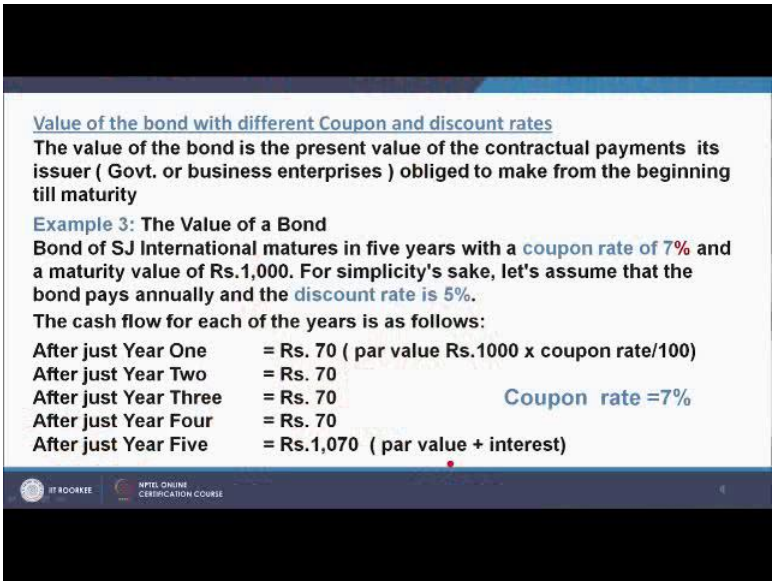

Now, to find out what will happen if the bond has got different coupon and discounted we have taken the example 3. The value of a bond of SJ International matures in 5 years

with a coupon rate of 7 percent and a maturity value of 1,000. For simplicity, let us assume that the bond pays annually and the discount rate is 5 percent. Here we see that coupon rate is 7 percent and discount rate is 5 percent and hence discount rate is different than coupon rate. In earlier questions, the coupon rate was equal to the discount rate. As the cash flow for each of the year is as follows; after just 1 year this 70 rupees will get, as the interest that is par value 1,000 into coupon rate divided by 100 coupon rate is 7 percent. So, this 0.07 into 1,000 is 70 rupees.

So, after just 2 years, 70 rupees, after the 3 years, 70 rupees, after the 4 years, 70 rupees and after the 5 year, this is 70 plus 1,000. So, it becomes 1,070. This is par value plus interest. Now thus the PV the present value of the cash flow is as follows.

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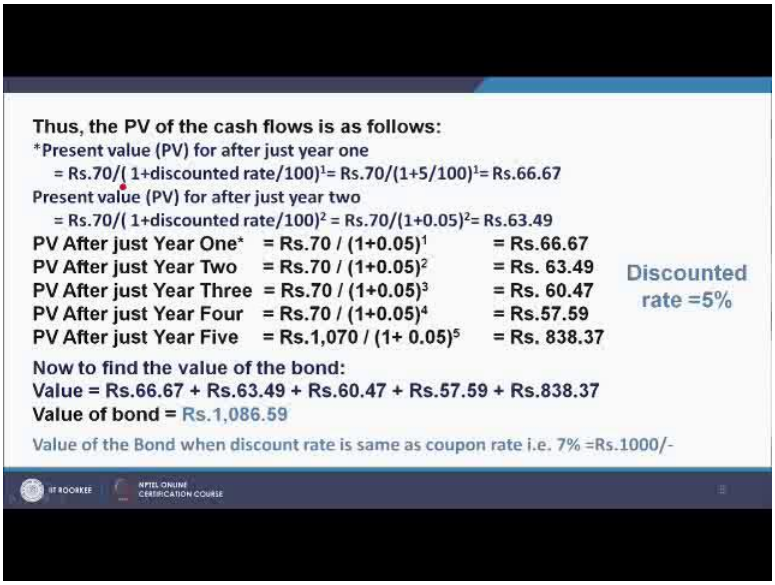

Present value for after just 1 year is rupees 70 divided by 1 plus discount rate divided by 100 to the power 1 is 70 divided by 1 plus 5 divided by 100, this becomes 66.67 rupees. Similarly, the present value after just 2 years is 70 divided by 1 plus 0.05 square which comes out to be 63.49. Similarly PV after just third year is 60.47, PV after just fourth year is it is 57.59, PV after 5 years is. This is 838.37. Here we will see that for discounting, we are using the discount rate. While finding out the interest, we are using coupon rate.

Now, to find out the value of the bond all this present values are added up. So, the bond value is 1086.59. The value of the bond when discount rate is same as the coupon rate

was 1,000, but in this case, it is different. We will illustrate the valuation bond with reference to basic bond valuation, yield to maturity and semi-annual interest and bond values.

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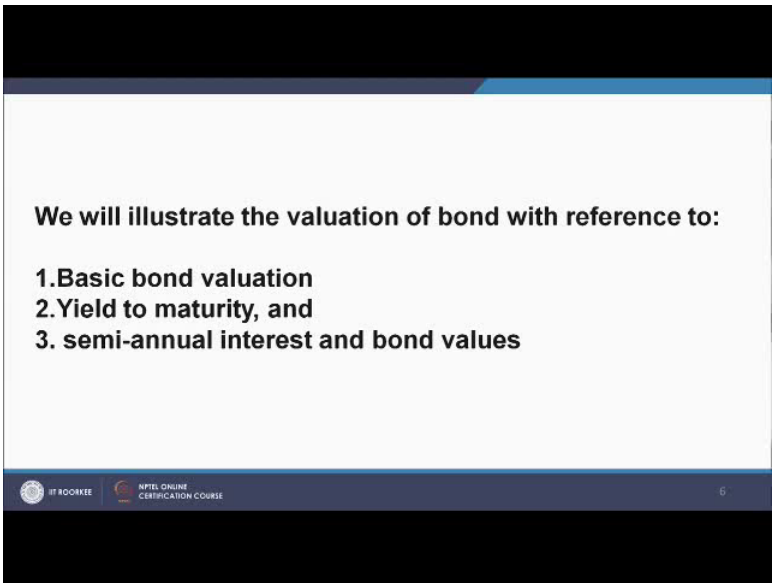

Now, let us take an example. Example 4, the bond of SJ International matures in 5 years with a coupon rate of 7 percent.

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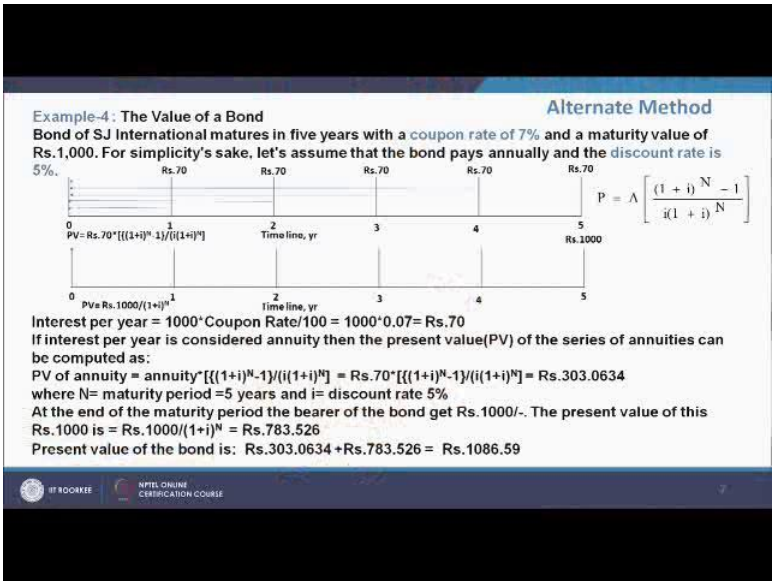

And the maturity value of 1,000. For simplicity, let us assume the bond pays annually and the discount rate is 5 percent. Now here, we are using formula to get these values.

So, every end of the every year you will be getting rupees 70, rupees 70, rupees 70, rupees 70 and rupees 70 up to 5th year and at the end of 5'th year rupees 1,000. So, interest for 1,000 into coupon rate divided by 100 is equal to 70 rupees. If interest per year is considered annually, annuity then the present value of the series of annuities can be calculated as PV of the annuity is annuity into this formula, which I had already told you in the last question and if you put just this, this comes out to be rupees 303.0634. Where N is the maturity period, 5 years and the i is discount rate is 5 percent.

At the end of the maturity period the bearer of the bond gets 1,000 rupees. So, if you want to find out the present value of 1,000 rupees this is equal to 1,000 divided by 1 plus i to the power N. Where N is 5, this comes out to be rupees 783.526 the present value of bond is therefore, this value 303.0634 plus this value 783.526 come out to be 1086.59.

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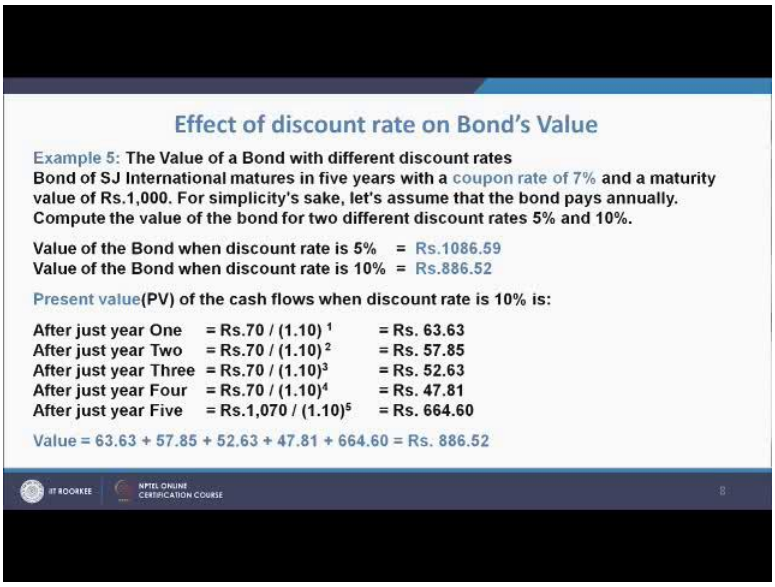

Now, you see the results are the same. Whether I can go from the first principal to find out the present value or I use the formula effect of discount rate on bonds value. So, what we saw here that we will find out the same problem we solved for two different discount rates, 5 percent and 10 percent and then we will find out the conclusion what the effect of discount rates on bonds value is. For this we have take an example 5; the value of a bond with different discount rates. Bond of SJ International matures in 5 years with a coupon rate of 7 percent and a maturity value of rupees 1,000. For simplicity, let us assume that the bond pays annually compute the value of the bond for two different discount rates 5

percent and 10 percent.

Now, this 5 percent values and 10 percent values has been taken with a purpose 5 percent is less than 7 percent and 10 percent is more than 7 percent. So, the value of the bond when discount rate is 5 percent is 1086.59 and the value of the bond when discount rate is 10 percent is 886.52 and the value of the bond, when coupon discount rate is 7 percent is 1,000. So, what we saw, that when the discount rate is 5 percent, the value is more than 1,000 and when the discount rate is 10 percent is more than 7 percent, the value is less than 1,000.

So, let us see here the present value of the cash flow when discount rate is 10 percent. So, this can be calculated from the first principal. The end of the first year interest 70 is when present value is calculated by dividing it with 1.10 to the power 1 here I am taking the interest discount rate to be 10 percent. This is 63.63. Similarly, these values can be converted to the present value. So, is comes out to be rupees 886.52 and this is for discount rate 10 percent. Now what conclusion we make, when discount rate is different than coupon rate. The value of the bond will differ from its par value. This is number 1 conclusion.

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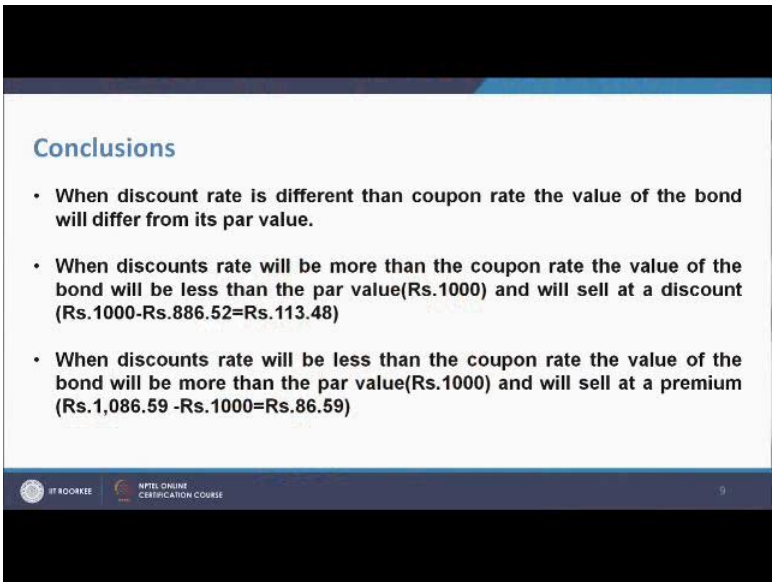

The number conclusion 2; when discount rate will be more than the coupon rate, the value of the bond will be less than the par value. Now par value was 1,000 and will sell at a discount I know, when the coupon rate was more that is 10 percent the value of the

bond was 886.52 which was less than 1,000. So, it will sell at a discount and discount will be rupees 1,000 minus rupees 886.52, which comes out to be rupees 113.48. When discount rate will be less than coupon rate, the value of the bond will be more than the par value and will sell at premium and this premium is equal to 1086.59 minus 1,000 which is the par value is rupees 86.59. So, when my discount rate was 5 percent, the bond will sell with the premium of 86.59.

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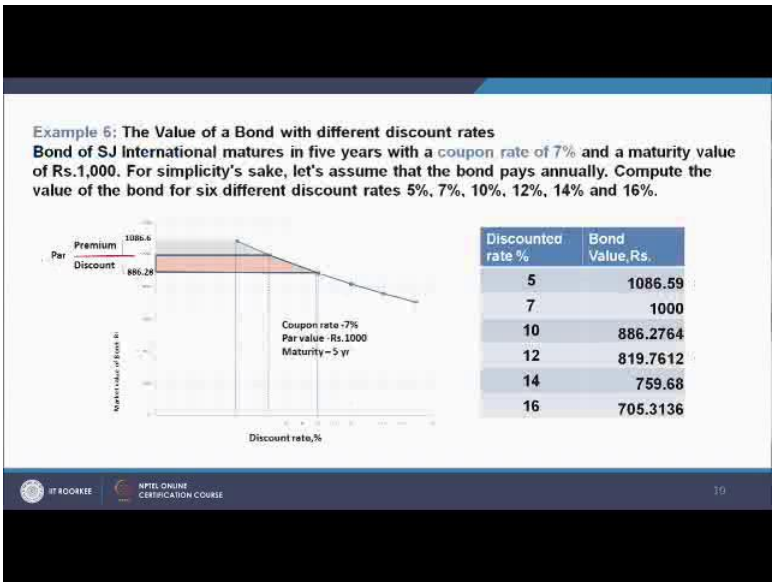

Now, let us take an example. In this example, we have calculated the value of the bond with different discount rates 5, 7, 10 percent, 12 percent, 14 percent and 16 percent. So, here we see when it is 5 percent, the value of the bond 1086.59. When it is 7 percent, it is 1,000. When it is 10 percent, it is 886.2764. When it is 12 percent, it is 819.7612. When it is 14 percent, it is 759.68 and when it is 16 percent, it is 705.3136.

So, when the discount rate is 7 percent, which is equal to coupon rate, the value of the bond is equal to the par value and the discount rate are more than 7 percent, the bond rate decreases. So, in this figure, it shows this is the value of the bond that is par value of the bond when discount rate is 7 percent and when discount rate is less than 7 percent, the value of the bond increases, but when the discount rate is more than the coupon rate the value of the bond decreases. So, this is discounting and this is premium. The green is premium, red is discounting

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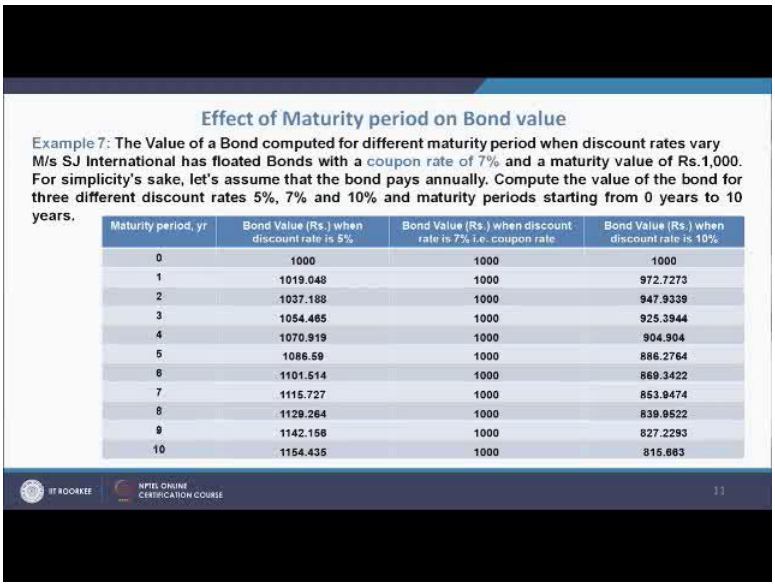

To show the effect of maturity period of bond value, we have takes an example 7. The value of a bond computed for different maturity period when discount rates vary. M/s SJ International has floated bond with a coupon rate of 7 percent and a maturity value of 1,000. For simplicity, let us assume that the bond pays annually. Compute the value of bond for three different discount rates; 5 percent, 7 percent and 10 percent and maturity periods starting from 0 years to 10 years and this have been, the results has been tabulated in this table.

Now, if see did maturity period is 0 then bond value is 1,000 for 5 percent discount rate 1,000 for 7 percent discount rate and 1,000 for 10 percent discount rate; that means that irrespective of the discount rate, if the maturity period is 0, the bond value remains 1,000. Now, if it is 1 year I find that the bond value for 5 percent discount rate is 1019.048 and for 7 percent it is 1,000 for 10 percent it is 972.7273; that means, bonds value will be more than 1,000, that is par value for even when the period, maturity period is 1 year and the bond value will be less than 1,000 when the discount rate is more than the coupon rate.

Now, here we see that when the bond discount rate is 7 percent which is equal to coupon rate for all time to come, that is all maturity periods, the bond value remains same that is 1,000; however, when the discount rate is 5 percent, the bond value increases with maturity period. For 1 year it was 1019.048 and for 10 years it is 1154.835; that means

the value is increasing with maturity period.

Let us take see the scenario for 10 percent discount rate. Here the value is decreasing and for the maturity period 1 year it was 972.7273 and for 10th year, this is 815.663. So, the value is decreasing, when I am increasing the maturity period.

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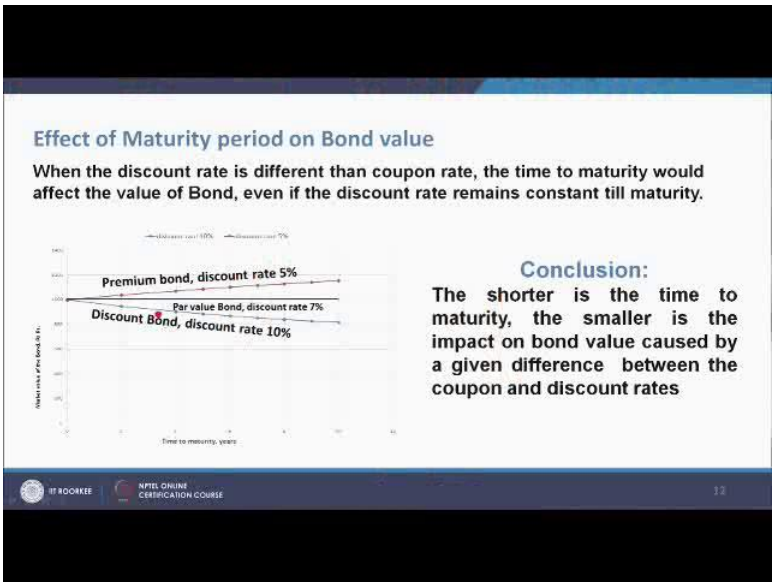

So, I get a plot like this. This will be a discount bond, this is the par value. So this will be obviously, a discounted bond this is a premium bond, when the discount rate is 5 percent. Now the changes like this; so when the discount rate is different than coupon rate the time to maturity would affect the value of the bond. Even if the discount rate remains constant till maturity and what is the conclusion? The shorter is the time to maturity, the smaller is the impact on bond value caused by a given difference between the coupon and discount rate.

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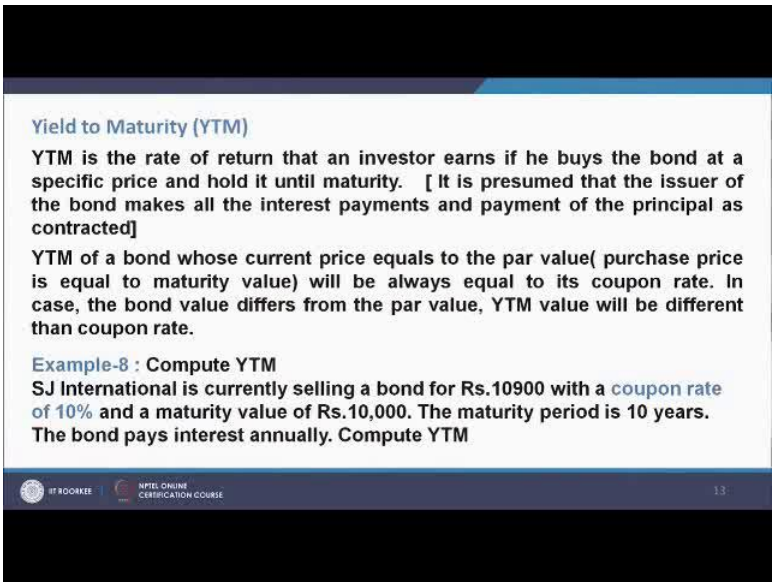

Now, the other method for finding out to evaluate the bond is yield to maturity. Which is called YTM? Yield to maturity, YTM is the rate of return that an investor earns if he buys the bond at a specific price and holds it until maturity it is presumed that the issuer of the bond makes all the interest payments and payments of the principal as contracted. YTM is a bond whose current price equals to par value, that is purchase price equals to maturity value, will be always equal to it is coupon rate.

In case, the bond value differs from the par value YTM value will be different than the coupon rate. To show this how to calculate YTM, let us take an example which is example number 8 to compute YTM. SJ International is currently selling a bond for rupees 1,900 with a coupon rate of 10 percent and a maturity value of 10,000. The maturity period is 10 years. The bond pays interest annually computes YTM. Now let us see the solution of this problem.

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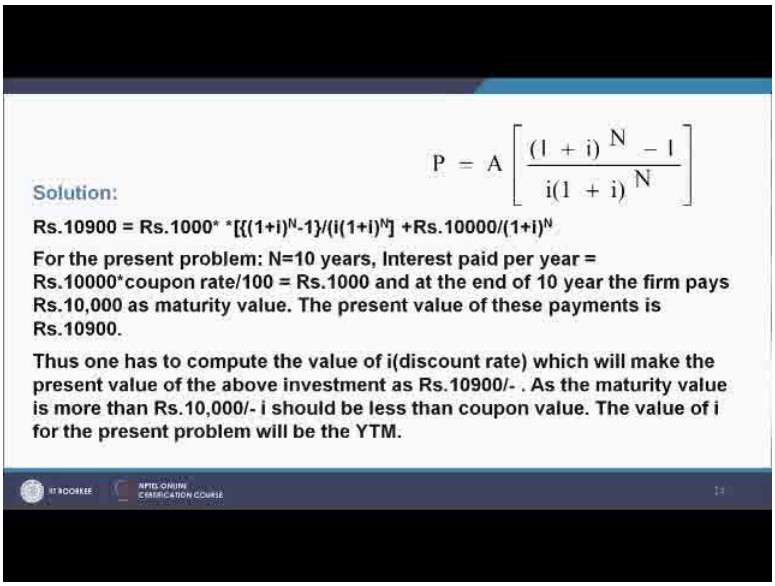

Now, for the present problem N is equal 10 years. Interest paid per year is 10,000 into coupon rate by 100 is equal to 1,000 and at the end of 10 years; the firm pays 10,000 as maturity value. Now if you see the interest that is 1,000. So, interest is paid for 10 years. So, interest is annuity and the present value of this annuity is this, where this A is the annuity 1,000 rupees and this formula I was used to find out the P, this is present value. And the end of 10 years one gets 10,000 and the present value of this 10,000 are 10,000 divided by 1 plus i to the power of N.

So, when we add the present values of these two, then this is equal to 10,900. So, the problem is that, what should be the value of i, which will give us this value and that value of i is required. Thus one has to compute the value of i discount rate, which will make the present value of the above investment as 10,900. As the maturity value is more than 10,000 i should be less than the coupon value. The value of i for the present problem will be the YTM.

Now, let us see the computation. As a first estimate, let us consider discount rate in this case YTM as 9 percent. The present value of this series above investment for 9 percent discount rate is rupees 10,641.77 which is less than 10,900.

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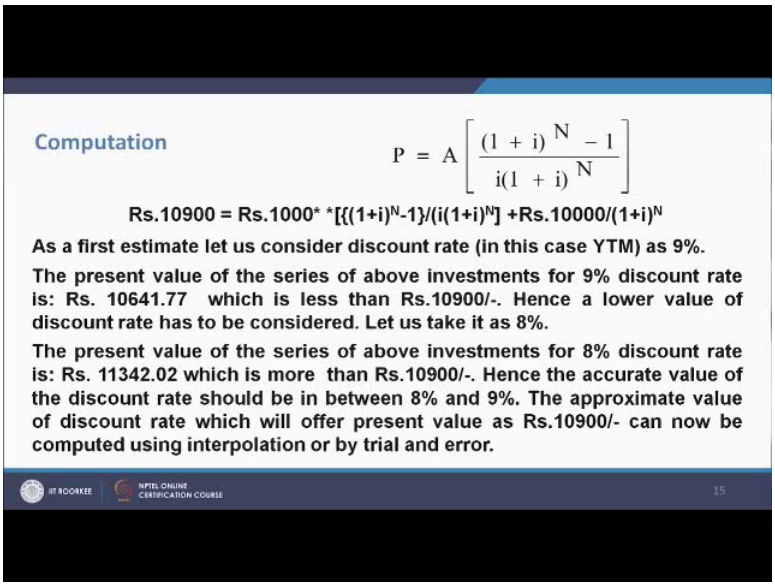

Hence a lower value of discount rate has to be considered and let us take it as 8 percent. The present value of the series of above investment for 8 percent discount rate is rupees 11,342.02, which is more than 10,900. Hence the actual value of the discount rate should be in between 8 percent and 9 percent.

The approximate value of discount rate which will offer present value of 10,900 can now be computed using interpolation or by trial and error. The difference between the bond values for 8 percent.

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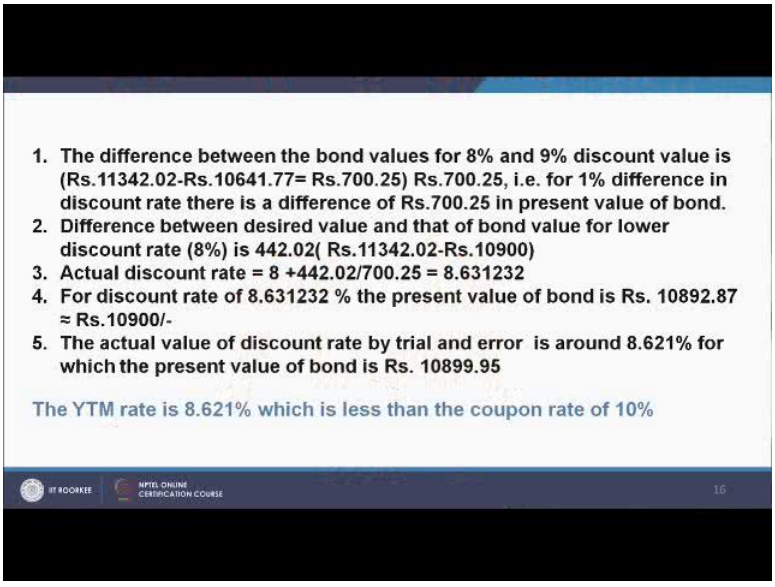

And 9 percent discount value is rupees 11,342.02 minus rupees 10,641.77, which comes out be 700.25. That is for 1 percent difference in discount rate, there is a difference of rupees 700.25 in the present value of the bond.

Difference between desired value and the bond value for lower discount rate 8 percent is 442 which comes out as rupees 11,342.02 minus rupees 10,900. So, actual discount rate will be 8 plus 442.02 divided by 700.25 is equal to 8.631232. Now for the discount rate of 8.631232 percent, the present value of the bond is 10892.87 which are almost equal to 10,900. Now the actual value of discounts rate by trial and error is about 8.621 for which the present value of bond is 10899.95. So, the YTM rate is 8.621 which is less than the coupon rate of 10 percent.

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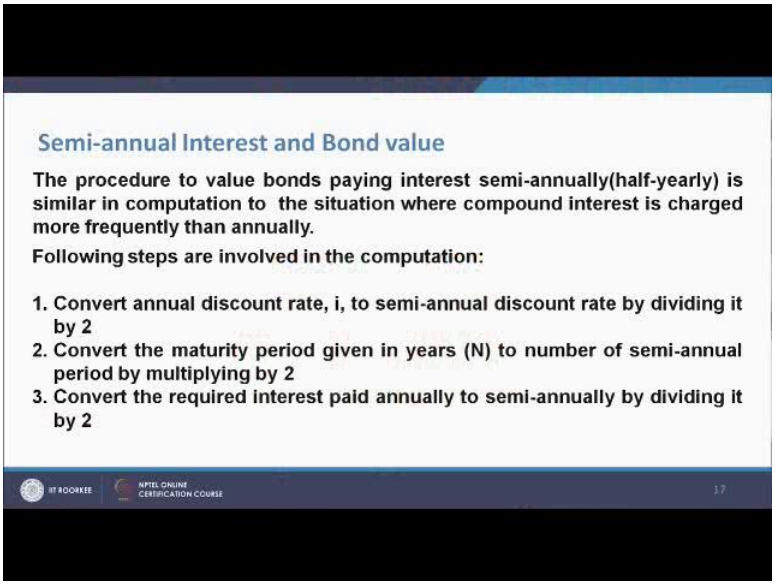

Now, let us see a third type of evaluation that is semi-annual interest and bond value. The procedure to value bonds paying interest semi-annually, that is half yearly is similar in computation to the situation, where compound interest rate is charged more frequently then annually. Following steps are involved in the computation. Convert annual discount rate to semi-annual discount rate by dividing it by 2. Convert the maturity period given in years N to number of semi-annual period by multiplying it with 2. Convert the required interest paid annually to semi-annually by dividing it by 2.

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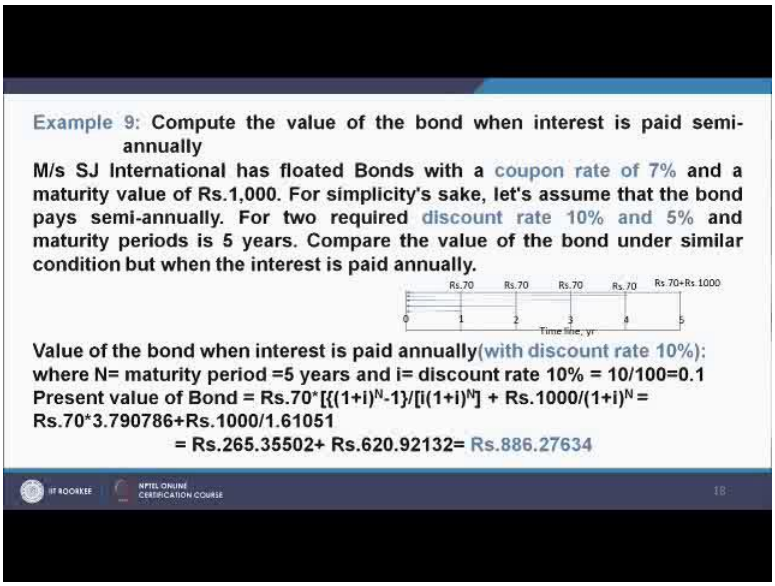

So, let us see an example to demonstrate this, example 9. Compute the value of the bond when interest is paid semi-annually. M/s SJ International has floated a bond with a coupon rate of 7 percent and the maturity value of 1,000. For simplicity, let us assume that the bond pays semi-annually for two required discount rates 10 percent and 5 percent and maturity period is 5 years. Compare the value of the bond under similar condition, when the interest rate is paid annually. So, there are two discount rates 10 percent and 5 percent.

So, let us see solution; the value of the bond when interest is paid annually. Now we will start the calculation with discount rate of 10 percent. So, my first calculation is with discount rate of 10 percent. Where N is the maturity equal to 5 years, i is the discount rate 10 percent, which is equal to 10 divided by 100 is equal to 0.1. Present value of the bond is 70. This is the interest per year and this interest per year is taken as annuity and is converted into the present value and this 1,000 which we are getting after 5 years is also converted into the present value. So, when we add the two present values it comes out to be 886.27634.

Now, the same problem we will evaluate with semi-annual interest rate and we will see; what is the value of the bond for semi-annual interest rate, the value of the bond when interest is paid semi-annually with discount rate of 10 percent.

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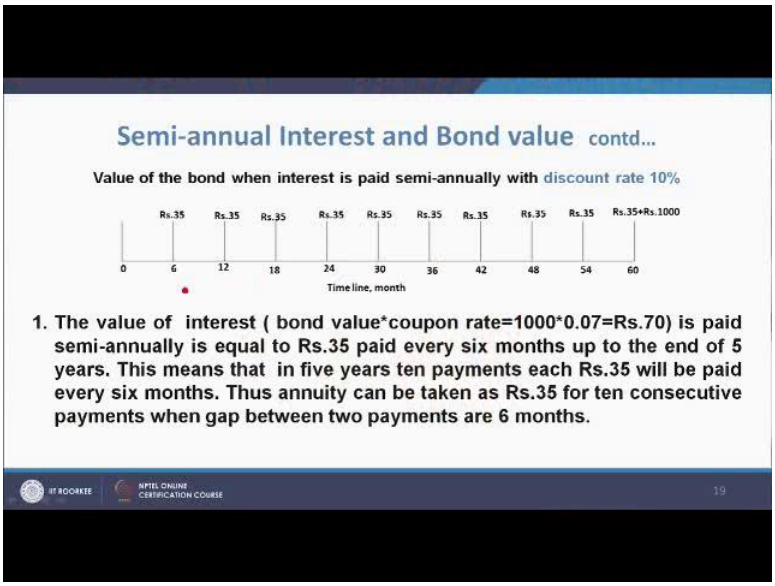

Now, here the value of the interest, the bond value into coupon rate is 1,000 into 0.07 is 70 rupees and this 70 rupees is paid semi-annually and thus it is equal to 35 because 70 divided by 2. So, it will be paid over to 6 months that is, why it is rupees 35 paid every 6 months up to the end of 5 years. This means that in 5 years 10 payments each 35 will be paid every 6 months. Thus annually it can be taken has 35 for 10 consecutive payments when gap between the two payments are 6 months.

Please note that the present value of annually formula has been changed to accommodate compounding semi-annually in this case. When compounding is semi-annually, compounding every 6 months and twice annually and hence N is equal to 2. For this problem N is equal to 5 years and i equal to r equal to 10 divided by 100 is equal to 01.

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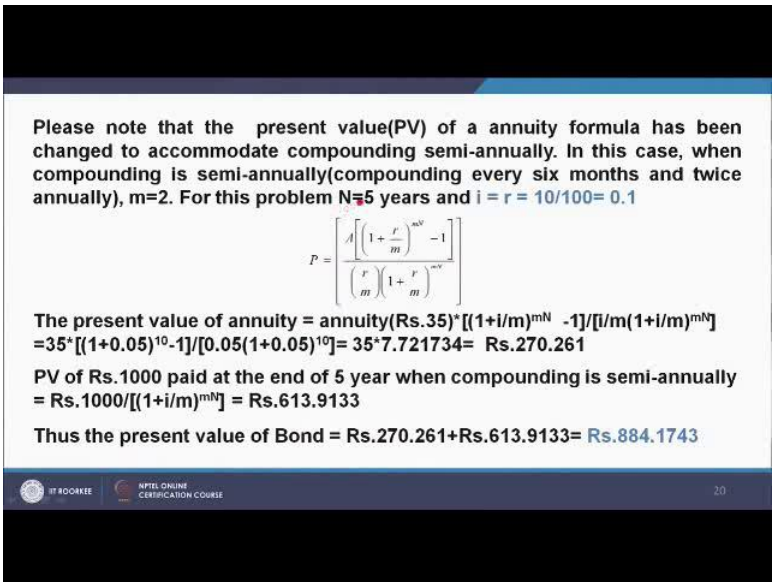

So, for compounding semi-annually this formula is used. Which is a discrete compounding formula and this is the P, that is present value and this is the A which is the annuity value. The present value of the annuity is equal to annuity that is 35, 100 into this group which comes out to be, when we plays the value it comes out to be rupees 270.261. Now again the PV; the present value of the 1,000 paid at the end of 5 years, when compounding semi-annually is this 1,000 divided by in brackets 1 plus i by m to the power mN which comes out to be rupees 613.9133. So, thus the present value of the bond is this 270.261 plus this one, which is 613.9133 which comes out to be 884.1743.

The value of bond when interest rate is paid annually with discount rate 5 percent, so we have calculated for 10 percent discount rate. Now we will calculate for 5 percent discount rate and here, the present value of the bond is 70 into this formula, which is annuity formula.

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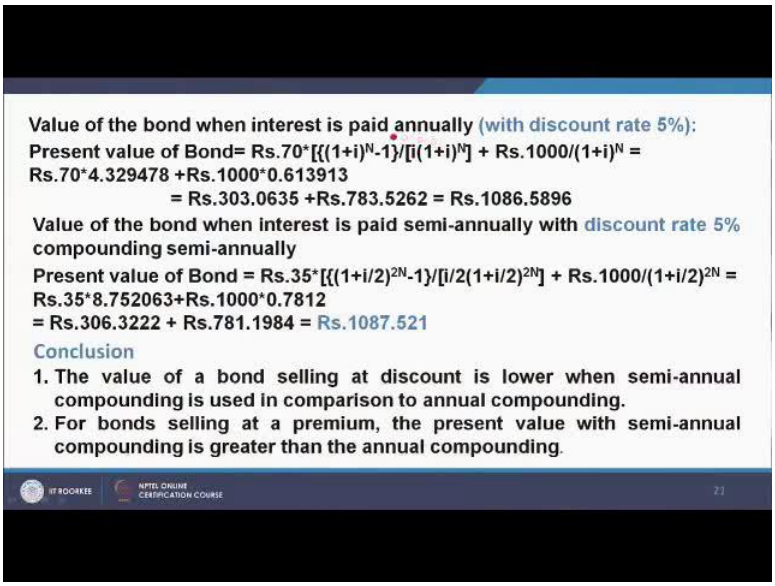

And compounding is annually and this is 1,000 is transferred to present value by dividing it with 1 plus i to the power N. So, this is annually. So, it comes out to be rupees 1,086.5896.

So, when discount rate is 5 percent and the compounding is annually, the value of the bond is 1,086.5896; the value of the bond, when interest is paid semi-annually with discount rate 5 percent and compounding semi-annually. So, the semi-annually compounding formula is used and this comes out to be 1,087.521. So, what are the conclusion we draw out of this the conclusion is, the value of a bond selling at discount is lower, when semi-annual compounding is used in comparison to annual compounding.

And number 2 conclusion is for bonds selling at premium, the present value with semi-annual compounding is greater than the annual compounding. And we already know that what is the discounting bond and what is a premium bond. In the discounting bond the discount rate is more than the coupon rate and in a premium bond the discount rate is less than the coupon rate.

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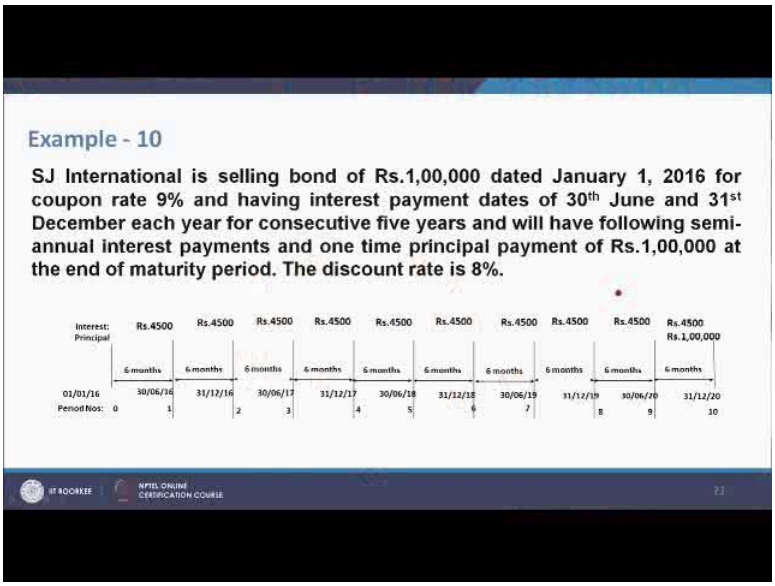

Now, let us take another example. Example number of 10; S J International is selling bond of 100000 dated January 1, 2016 for coupon rate of 9 percent and having interest payment dates of 30'th June and 31'st December each year for consecutively 5 years and will following semi-annual interest payments and one time principal payment of rupees 100000 at the end of the maturity period.

And discount rate is 8 percent. This time line very clearly shows this numerical. So, here in the first period which is 6 month away from the t equal to 0 time line, that is on 30 6 2016, 4,500 will be paid and then after 6 months again 4,500 will be paid and this will move till the 10'th period comes where 4,500 plus 100000 will be paid.

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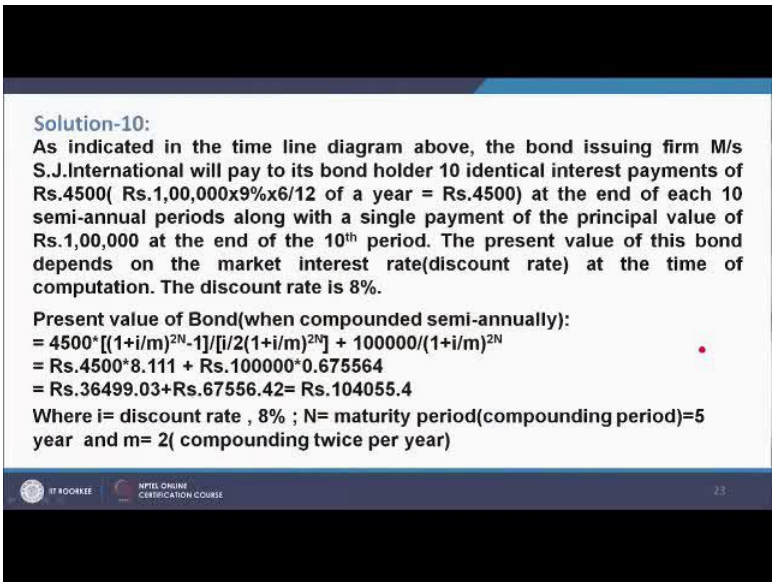

Let us see the solution. As indicated in the time line diagram ever the bond issuing from SJ International will pay to it is bond holder 10 identical interest payments of rupees 4,500. From where it has come, Rupees 100000 into 9 percent into 6 by 12 of a year that is equal to 4,500; at the end of each 10 semi-annual periods along with a single payment at the principal value of 100000 at the end of 10th period. The present value of this bond depends on market interest rate that is discount rate.

At the time of computation the discount rate was 8 percent. So, the present value of the bond compounded semi-annually and by given by this. This is the annuity of 4,500 for 10 consecutive payments and this is converted into the present value using a discounting method, which is compounding semi-annually and this is 100000 is converted into the present value using a discounting method, which is compounded semi-annually. This comes out to be rupees 104055.4

Where i is the discount rate, 8 percent, N is the maturity period, which is 5 years and m equal to 2, that is compounding twice per year; the present value of 36,499.03 of the annuity. Now from where this 36,499.03 has come, you see this is the 36,499.03 and this comes from the annuity payment of the annuity, the present value of the annuity, this group, so the present value rupees 36,499.03 of the annuity 4,500 for 10 periods. So, that if an investor invests rupees thirty six 36,499.03 on January 1, 2016 will in return will get 10 semi-annual payments of rupees 4,500 with first payment on 30'th June, 2016

when discount rate is 8 percent. The difference between the 10 semi-annual future payments of 4,500 is 450000.

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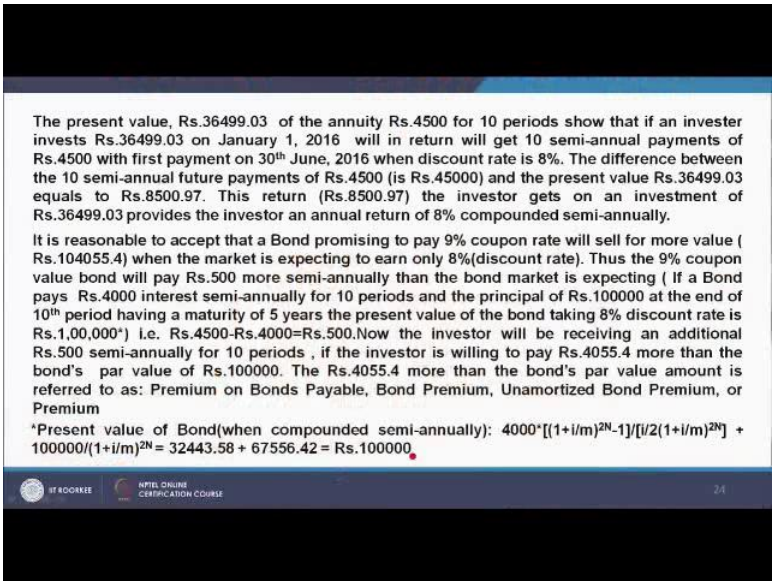

And the present value of rupees 36,499.03 equals 8,500.97. This return that is 8,500.97 the investor gets on an investment of rupees 36,499.03 provides the investor an annual return of 8 percent compounded semi-annually. It is reasonable to accept that a bond promising to pay 9 percent coupon rate, will sell for more value. That is rupees 104055.4 when the market is expecting to earn only 8 percent, that is discount rate.

Thus the 9 percent coupon bond value will pay 500 more semi-annually, then the bond market is expecting. If a bond pays 4,000 interest semi-annually for 10 periods and the principal is 100000 at the end of 10'th period having a maturity of 5 years, the present value of the bond taking 8 percent discount rate is 10000. That is rupees 4,500 minus rupees 4,000 is equal to 500.

Now, the investor will be receiving an additional 500 semi-annually for 10 periods; if the investor is willing to pay rupees 4,055.4 more than the bonds par value of rupees 100000. The rupees 4,055.4 more than the par value amount is referred to as premium on Bonds Payable, Bond Premium, Unamortized Bond Premium or Premium. The present value of bond, when compounded semi-annually is equal to 4,000, that is the annuity and then we use a formula for semi-annual compounding plus 100000 is converted into the present value comes out to be 100000. What is a perpetual bond? A perpetual bond is a

bond with no maturity date. Perpetual bonds are not redeemable.

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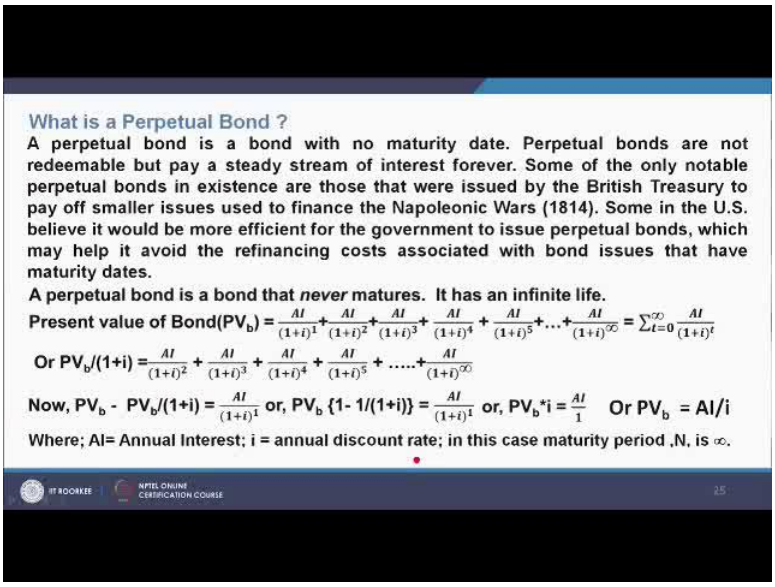

But pay a steady stream of interest forever. Some of the only notable perpetual bonds in existence are those that were issued by the British Treasury to pay off smaller issues used to finance the Napoleonic War in 1814. Some of the US believes it would be more efficient for the government to issue perpetual bonds which may help it avoid the refinancing costs associated with bond issues that have maturity dates. A perpetual bond is a bond that never matures. It has an infinite life. So, perpetual bonds present value PV is equal to AI divided by 1 plus i to the power 1 plus AI divided by 1 plus i to the power 2 like this up to AI divided by 1 plus i to the power infinite. So, it is summation of t equal to infinite AI 1 plus i to the power t. So, AI is per period interest, which the bond is paying and i is the discount rate.

Now, you can divide the PV b by 1 plus i. So, it becomes AI 1 plus i whole square plus AI by 1 plus i whole cube so on so forth plus AI 1 plus i to the power infinite. Now if I deduct Pb minus Pb divided by 1 plus i it is AI divided by 1 plus i to the power 1 or I can write down PV b is equal to AI divided by i. Where, AI is the annual interest, i is the annual discount rate and in case maturity period N is infinite. Example 11, valuation of perpetual bond; A bond P has a rupees 1,000 par value having a annual coupon rate of 8 percent, if the discount rate is 10 percent. Find the value of the perpetual bond solution.

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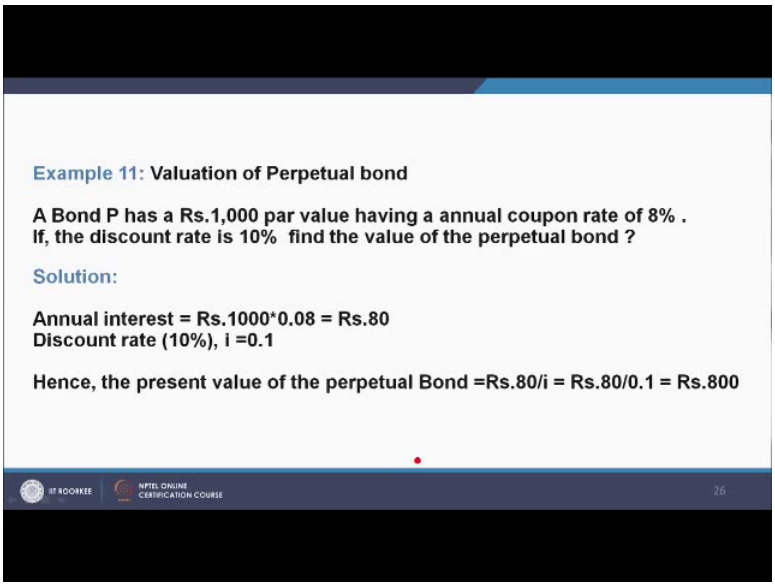

The annual interest is rupees 1,000 into 0.08 is equal to 80 rupees, discount rate is 10 percent, that is i is equal to 0.1. Hence the present value of the perpetual bond is equal to rupees 80 divided by i is equal to rupees 800.

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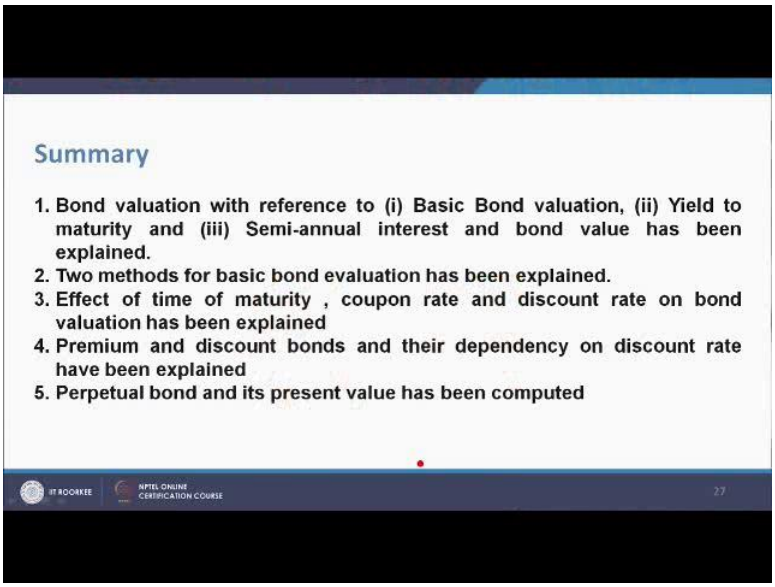

Summary, what we have had read in this lecture, number 1; bond valuation with a reference to basic bond valuation. Second, yield to maturity and third semiannual interest and bond value has been explained. Number 2, two methods for basic evaluation has been explained. Number 3, effect of time of maturity, coupon rate and discount rate on

bond valuation has been explained. Number 4, premium and discount bonds and their dependency on discount rate have been explained. Number 5, perpetual bond and it is present value has been explained.

Thank you.