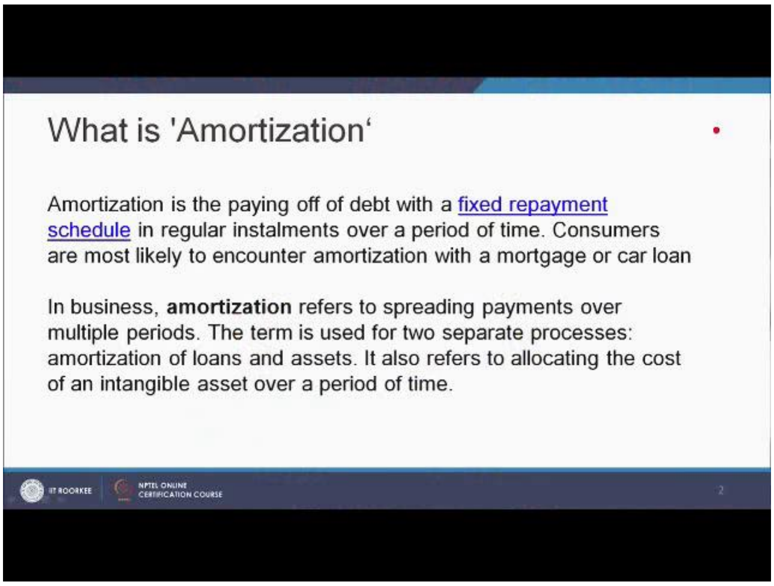
Time value of money-Concepts and Calculations
Prof. Bikash Mohanty
Department of Chemical Engineering
Indian Institute of Technology, Roorkee

Lecture - 12
Amortization

Welcome to the lecture series on Time value of money-Concepts and Calculations. Present lecture is on Amortization.

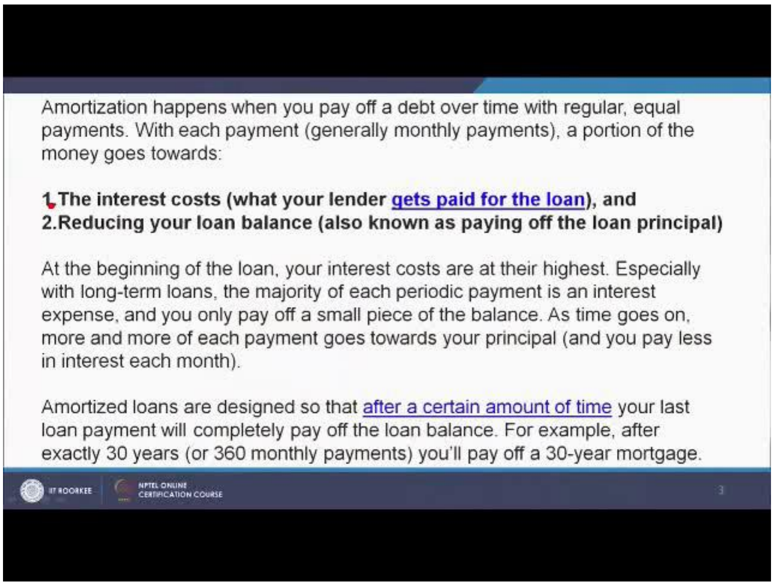
(Refer Slide Time: 01:00)



What is Amortization? Amortization is the paying of depth with a fixed repayment schedule in regular installments over a period of time. Consumers are most likely to encounter Amortization with a mortgage or car loan.

In business Amortization refers to spreading payments over multiple periods. The term is used for two separate processes; Amortization of loans and assets, it also refers to allocating the cost of an in intangible asset over a period of time. Amortization happens when you pay of a depth over time with regular, equal payments.

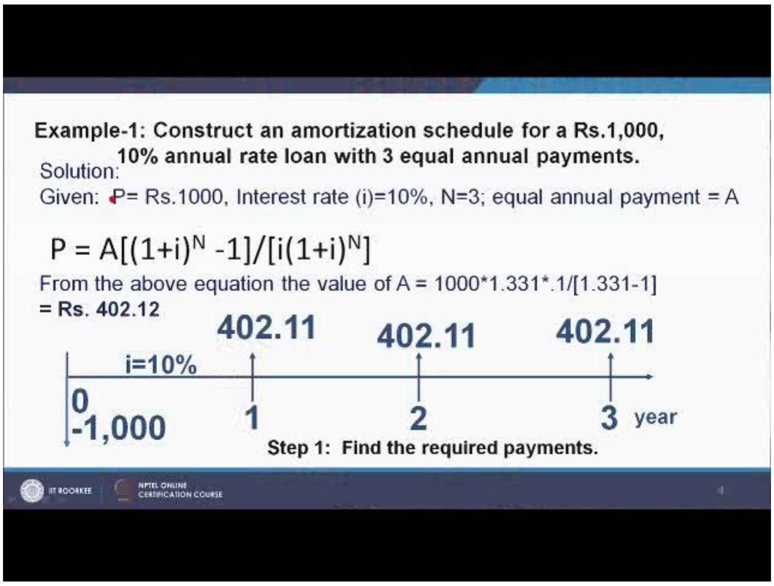
(Refer Slide Time: 01:50)



With each payment generally, monthly payments a portion of the money goes towards; one be interest costs what your lender gets paid for the loan, and second reducing your loan balance as known as paying off the loan principal. At the beginning of the loan your interest cost are at their highest especially with long term loans. The majority of periodic payment is an interest expense and you only pay of a small piece of balance. As time goes on more and more of each payment goes toward your principal and you pay less in interest each month.

Amortized loans are designed. So, that after a certain amount of time your last loan payment will completely pay off the loan balance. For example, after exactly 30 years or 360 months payment you will pay of a 30 year mortgage.

(Refer Slide Time: 02:59)

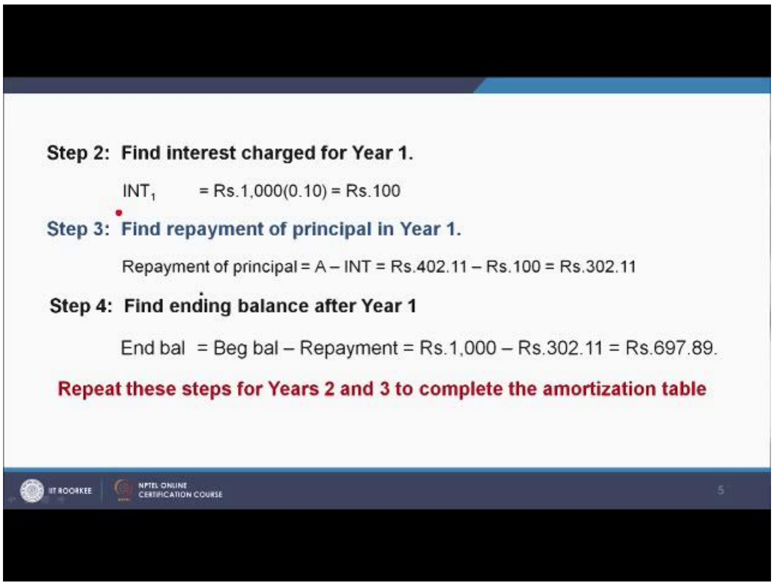


Let us take an example through which will explain this. Construct an Amortization schedule for a Rupees 1000, 10 percent annual rate loan with 3 equal annual payments. So, P is here 1000, interest i is called 10 percent, N is equal to 3 equal annual payments say A. That means, I am paying A amount each year. From the above equation the value of A is 1000 into 1.333 into 1 divided by in brackets 1.331 minus 1; this comes out to be Rupees 402.12. Basically this is a equation which was taken from Annuity and the principal is 1000 and we are finding out annuity for 3 years and 1.33 to the power basically this is 1 plus i to the power N is equal to 1.331 and 0.1 is the i.

So, here at P equal to 0 I am seeking 1000, and at the end of the first year I am getting 402. That means, if I take 402.11 at the end of the first year, at the end of the second year, and at the end of the third year then I can pay a 1000 Rupees loan. This is the equation which is been used P is equal to A in brackets 1 plus i to the power N minus 1 divided by i 1 plus i to the power N, and this equation we have modified because we want the value of A not P. So, A is equal to P into i 1 plus i to the power N divided by in brackets 1 plus i to the power N minus 1.

So, the first step is to find out the annuity, that we have already find out. Now in the step two find interest charge for year 1. So, interest charge for year 1, If I say INT 1 is equal to 1000 into interest rate that is 10 percent is 0.1 comes out to be 100. Now step 3 find replacement of principal in year 1. So, replacement principal is equal to A minus interest.

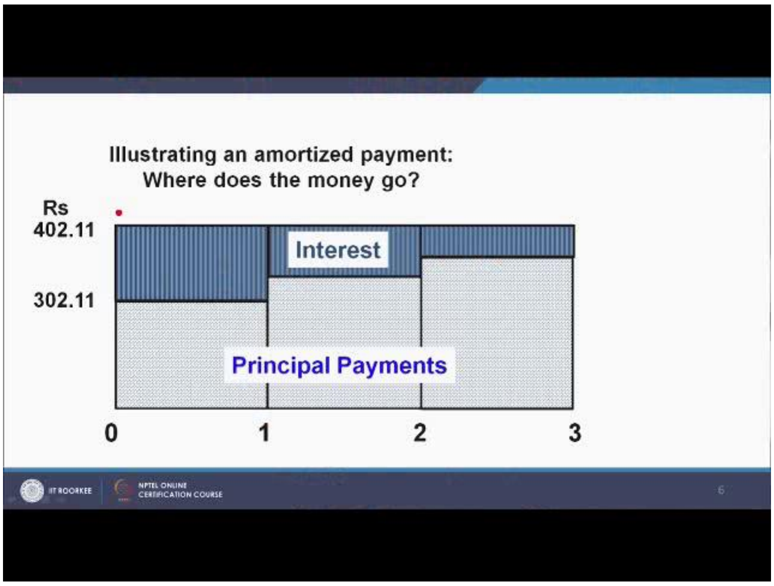
(Refer Slide Time: 05:05)



So, A is my 402.11 minus 100 comes out to be Rupees 302.11. Step 4 find ending balance after 1 year, ending balance is the beginning balance minus repayment beginning balance is 1000 out of this 1000 the repayment Rupees 302.11 has already be done. So, the end balance becomes 697.89.

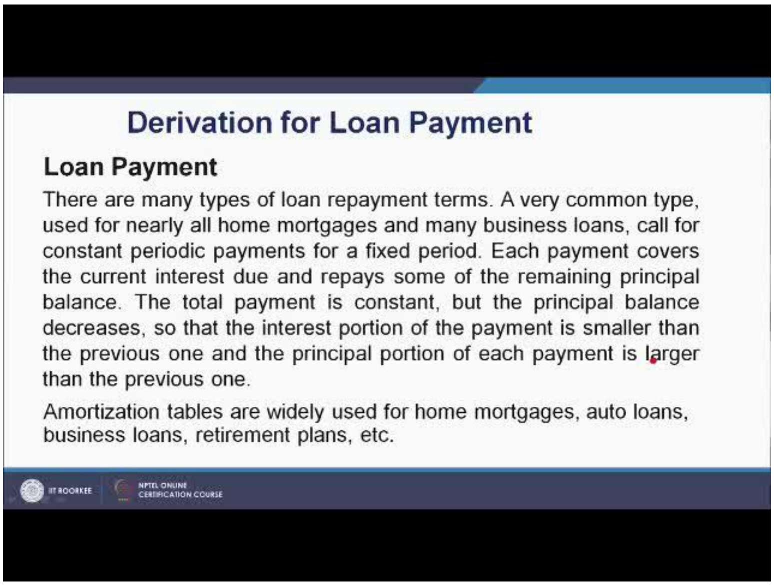
Now, we will have to repeat this steps for year 2 and 3 to complete the Amortization table.

(Refer Slide Time: 05:52)



This illustrates the Amortization payments, where the money goes and the first year out of 402.11 which we have paid this much of amount goes to the principal this much amount goes to the interest and the second payment this much goes to the for the payment of principal this much amount goes to the return a interest and in the third year this much amount goes for the payment of principal and this much of amount goes for the payment of interest. So, this is how constant payments are divided per year between the interest and the principal and the payment.

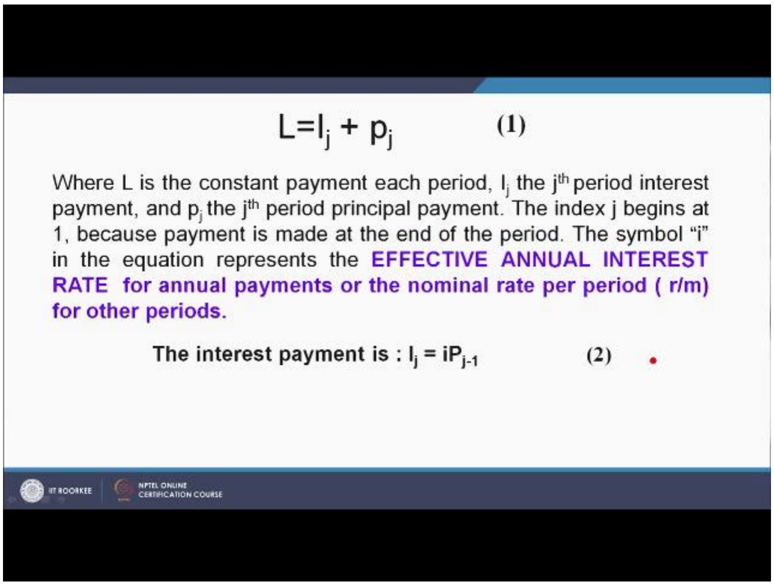
(Refer Slide Time: 06:33)



Let us see the derivation of loan payment. There is much type of loan repayments terms a very common type used for nearly all home mortgages and many business loans call for constant periodic payments for a fixed period. Each payment covers the current interest due and repays some of the remaining principal balance. The total payment is constant, but the principal balance decreases. So, that the interest portion of the payment is smaller than the previous one and the principal portion of each payment is larger than the previous one.

Amortization tables are widely used for home mortgages, auto loans, business loans and retirement plans.

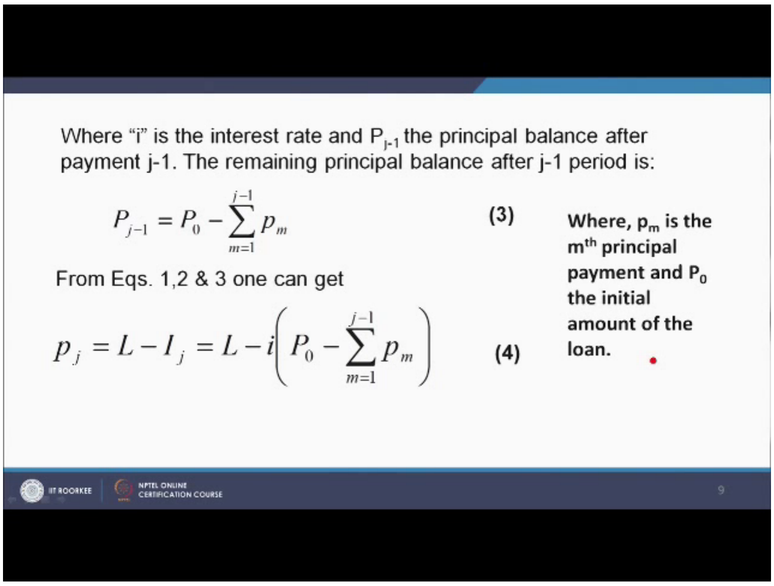
(Refer Slide Time: 07:26)



It is derived it let us L is equal to I j plus P j I can write down, where L is the constant payment each period I j the j th period interest payment and P j the j th period principal payment. The index j begins at 1 because payment is made at the end of the period. The symbol I in the equation represents the effective annual interest rate for annual payments or the nominal rate per period r by m for other periods.

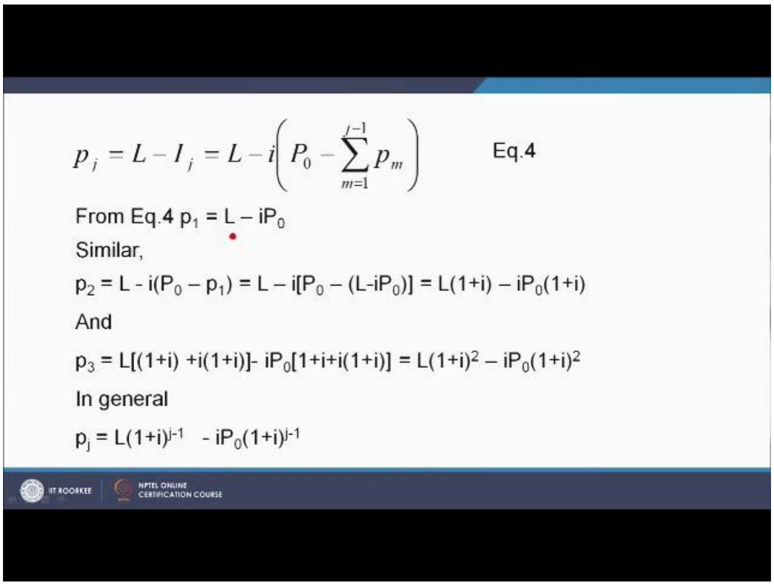
So, the interest payment I can write down I j is equal to I into P j minus 1, where I is the interest rate and P j minus 1 the principal balance after payment j minus 1. The remaining principal balance after j minus 1 period is P j minus 1 is equal to P0 Minus summation m equal to 1 P j minus 1 Pm. Where Pm is the m th principal payment and P0 is the initial amount of the loan. Then based on the equation 1,2 and 3 we can write down this P j is equal to l minus I j equal to Ll minus I j can replace by this quantity 1 into this quantity P0, this is the quantity because this I j into P j minus 1.

(Refer Slide Time: 08:28)



Now P j 1 minus can be replaced by this quantity. So, this is replaced here. So, this is equation four. Now P j equal to L minus I j equal to L minus I j into this factor this is equation number four I am rewriting it.

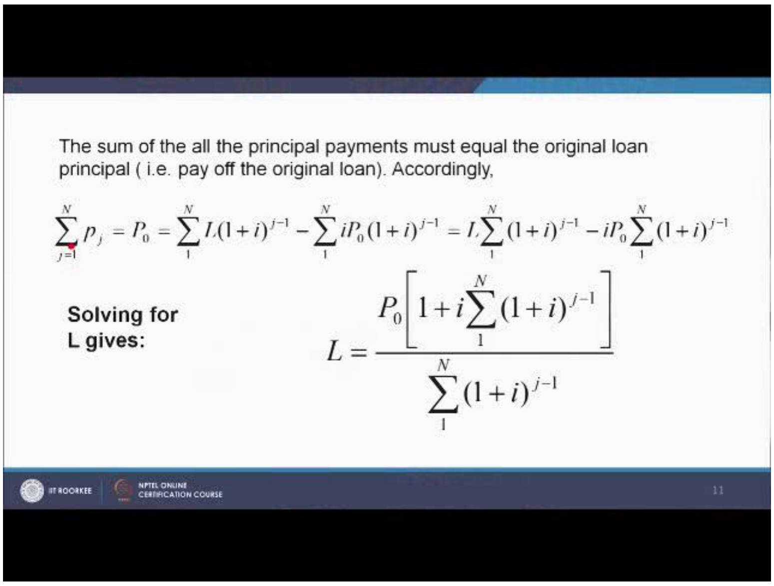
(Refer Slide Time: 09:23)



From equation number 4, P1is equal to L minus I P0. If I put this j is equal to 1, this is L minus I1 equal to L minus I and this m is equal to 1 to j minus 1. So, it become 0. So, it is only P0. So, I into P0 now for P2, this comes out to be L minus I in brackets P0 minus P1 is equal to L minus I then P0 minus P1 is replaced by this quantity, P1 is replaced by this quantity.

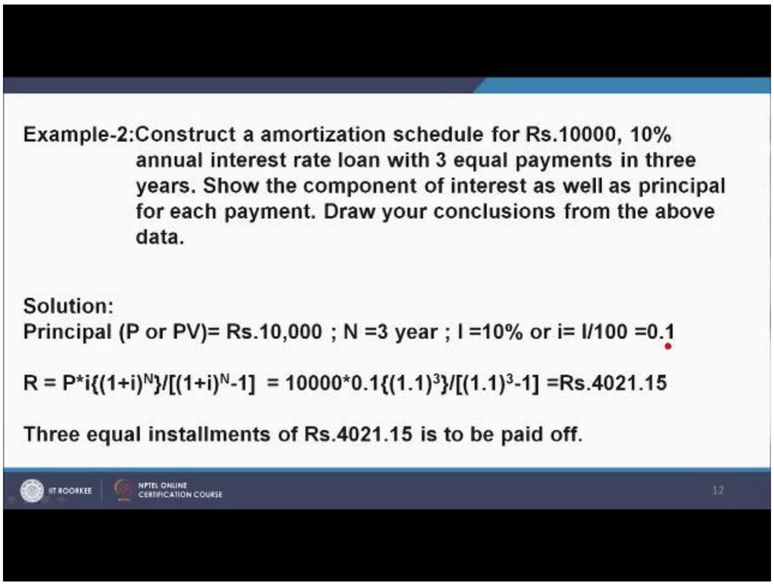
So, finally it comes out to be L1 plus I minus I into P0 in brackets 1 plus I , and similarly for P3, it becomes L1 plus I whole square minus I P0 1 plus I whole square and for general we can write down P j is equal to L into 1 plus I to the power j minus 1 minus I P0 1 plus I to the power j minus 1, because if I see P3 here, the power of 1 plus I here is 2 that means, 3 minus 1. So, when I take j here, it would be j minus 1. Similarly here also to 2 that is 3 minus 1. So, this would be j minus 1. The some of the all principal payments must equal to the original loan principal that is pay off the original loan.

(Refer Slide Time: 11:06)



Accordingly summation P j j 1 to N is equal to P0. So, I can write down for this, the summation of L 1 plus I to the power j minus 1 from 1 to n minus summation 1 to N, I P0 into 1 plus I to the power j minus 1 . So, this comes out be this, L, I can take out because it is a constant values similarly I and P0 I can take out. So, this becomes this. I solve this for L. So, L becomes this, P0 in brackets 1 plus I summation 1 to N 1 plus I to the power j minus 1 divided by summation 1 to N 1 plus I j minus 1 .So, this gives me the yearly or monthly payment what so ever be for the loan.

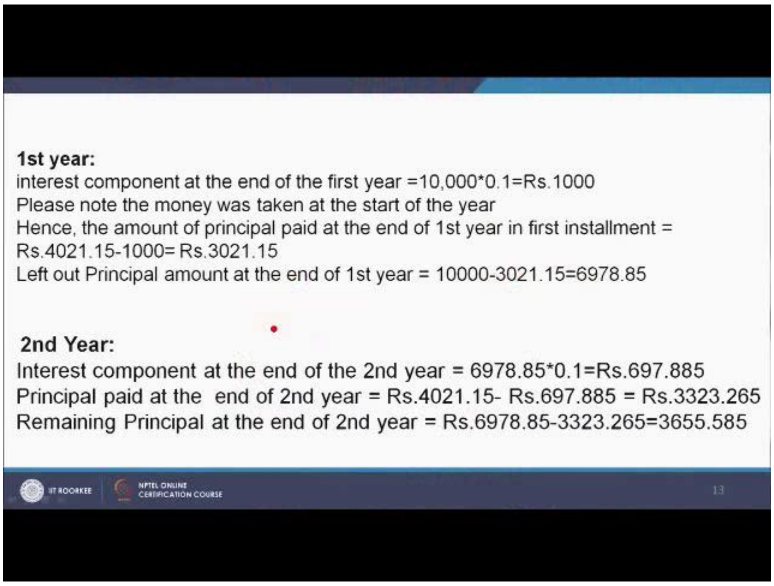
(Refer Slide Time: 12:10)



Now, let us take an example, construct a Amortization schedule for Rupees 10000, 10 percent, annual interest loan with 3 equal payments in 3 years. So, the component of interest as well as principal for each payment draw you conclusions from the above data. Solution principal P or PV is equal to 10000, N is equal to 3 years, i equal to 10 percent. So, i is equal to I by 100 is 0.1. So, R or the annuity this is the annuity R is equal to P into this formula, basically is a annuity formula and we are finding out the annuity value which is represented here by R and many exercise we are represent this by A also. So, equal to 10000 into 0.1 into this factor comes out to be 4021.15. So, 3 equal installment of 4021.15 is to be paid off to get 10000 loan.

In the first year interest component at the end of the first year is 10000 into 0.1 which comes out be 1000, please note the money was taken and start of the year.

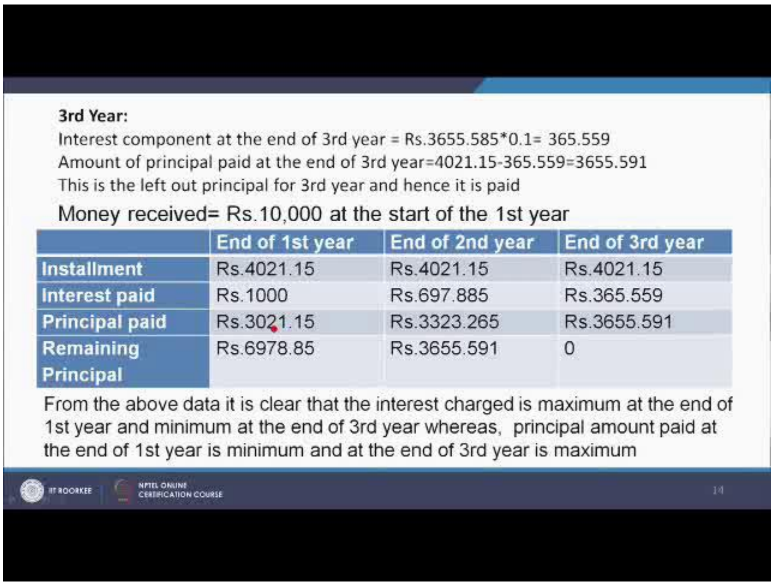
(Refer Slide Time: 13:25)



Hence the amount of principal paid at the end of the first year, in the first installment is Rupees 4021.15 minus 1000, which is the interest. So, comes out to be Rupees 3021.15. So, left out principal amount at the end of the first year is 10000 minus 3021.15 it comes out to be 6978.85; that means, at the start of the first year the principal was 10000, but at the end of the first year, when we pay the annuity it comes out to be 6978.85.

Now, for the second year interest component at the end of the second year is 6978.85 into 01 is comes out to be 6978.85, principal paid at the end of the second year is Rupees 4021.15 minus the interest value that is Rupees 6978.85, it comes out to be Rupees 3323.265. So, the remaining principal at the end of the second year is, the principal was left out at the end of the first year which is 6978.85 minus this value 3323.265. So, it comes out to be 3655.585; that means, this is the principal which is left out at the end of second year. For the third year interest component at the end of the third year is 3655.585 into 0.1 is comes out to be 365.559.

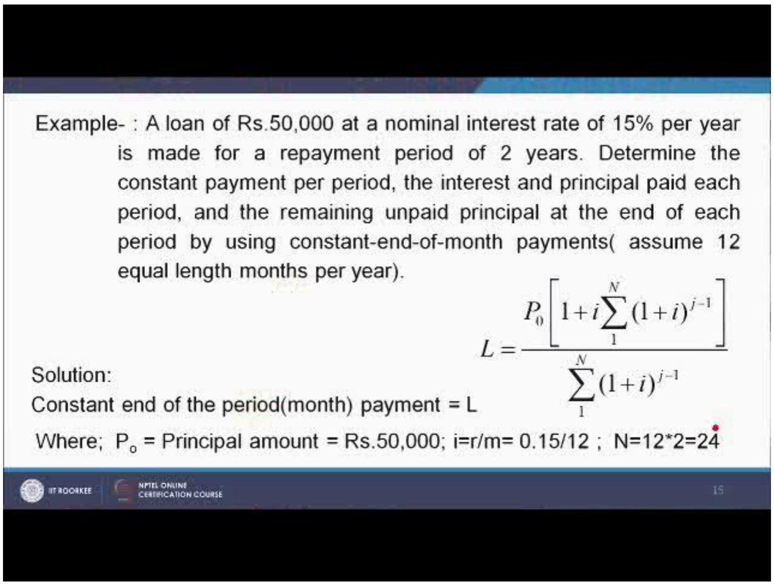
(Refer Slide Time: 15:08)



So, amount of principal paid at the end of the third year is 4021.15 minus 365.59. So, amount paid for again principal is 3655.591. So, this is the left out principal end of the third year and hence it is paid. Now if I draw the Amortization table then i divide into 3 columns, end of the first year, end of the second year, end of the third year. So, installment is the same for all the 3 years, that is 4021.15, 4021.15, 4021.15 and 4021.15 interest paid is Rupees 10000 at the end of the first year at the end of the second year the interest draws down to 6978.85 and the end of the third year drops down further to 365.559.

Principal paid on the first year is 3021.15 in the end of the second year rises to 3323.265 and it further rises to 3655.591 at the end of third year and remaining principal at the end of first year is 6978.85 at the end of second year is this becomes 3655.591 and end of the third year the remaining principal becomes 0; that means, loan is paid totally. From the above data is clear that, the interest charge is maximum at the end of the first year and minimum at the end of third year. Where as principal as amount paid at the end of the first year is minimum but at the end of the third year is maximum.

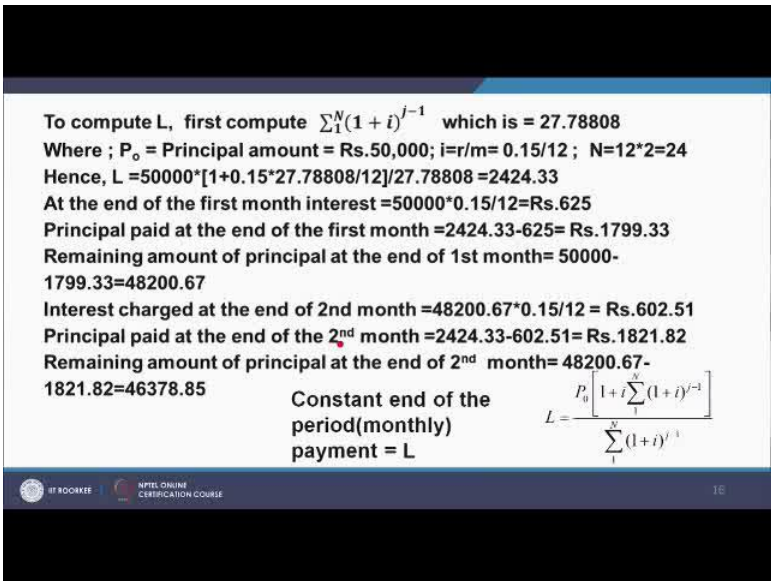
(Refer Slide Time: 16:58)



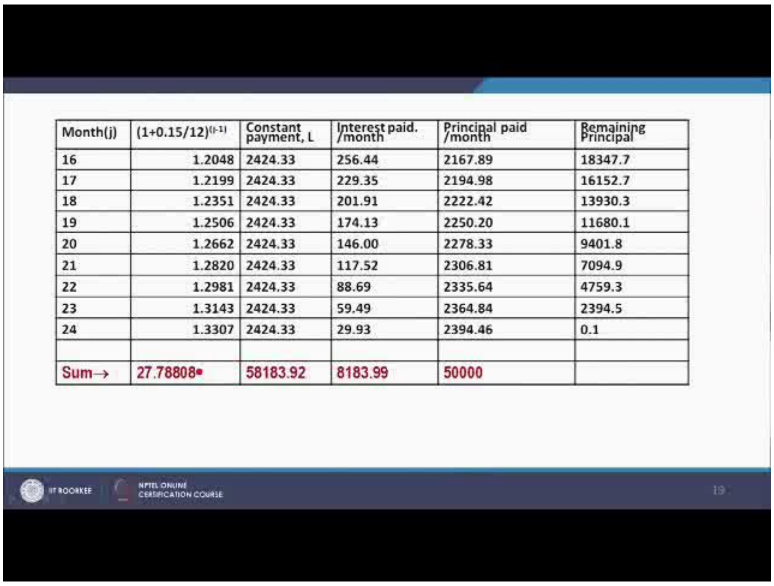
Now, take an example where we calculate the loan a loan of Rupees 50000 at a nominal interest of rate of 15 percent per year is made, for a repayment period of 2 years determine the constant payment per period, the interest and principal paid each period and the remaining unpaid principal at the end of each period by using constant end of the month payments assume 12 equal length months per year. Now you will be using the equation which we have derived for the payment.

Now constant end of the period that is month payment is equal to L the principal amount is 50000 I equal to R by m is 0.115 divided by 12N is 12 into 2 is 24. This is a discrete compounding problem now to compute L first compute summation 1 to m 1 plus I to the power j minus 1. This comes out to be 27.78808. So, we will see that this is here I have computed this factor.

(Refer Slide Time: 18:01)



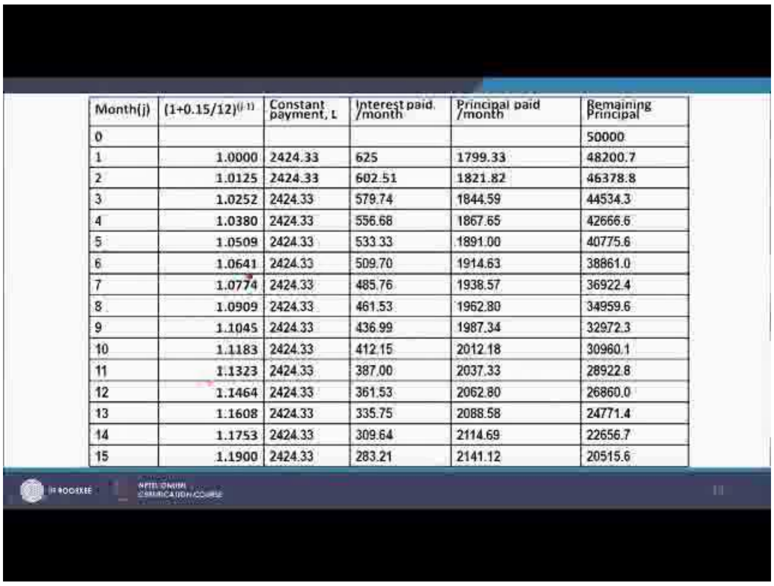
(Refer Slide Time: 18:11)



Month(j)	$(1+0.15/12)^{(j-1)}$	Constant payment, L	Interest paid. /month	Principal paid /month	Remaining Principal
16	1.2048	2424.33	256.44	2167.89	18347.7
17	1.2199	2424.33	229.35	2194.98	16152.7
18	1.2351	2424.33	201.91	2222.42	13930.3
19	1.2506	2424.33	174.13	2250.20	11680.1
20	1.2662	2424.33	146.00	2278.33	9401.8
21	1.2820	2424.33	117.52	2306.81	7094.9
22	1.2981	2424.33	88.69	2335.64	4759.3
23	1.3143	2424.33	59.49	2364.84	2394.5
24	1.3307	2424.33	29.93	2394.46	0.1
Sum→	27.78808	58183.92	8183.99	50000	

So, first compute this factor and do the summation and here in the summation it is 27.78808.

(Refer Slide Time: 18:21)



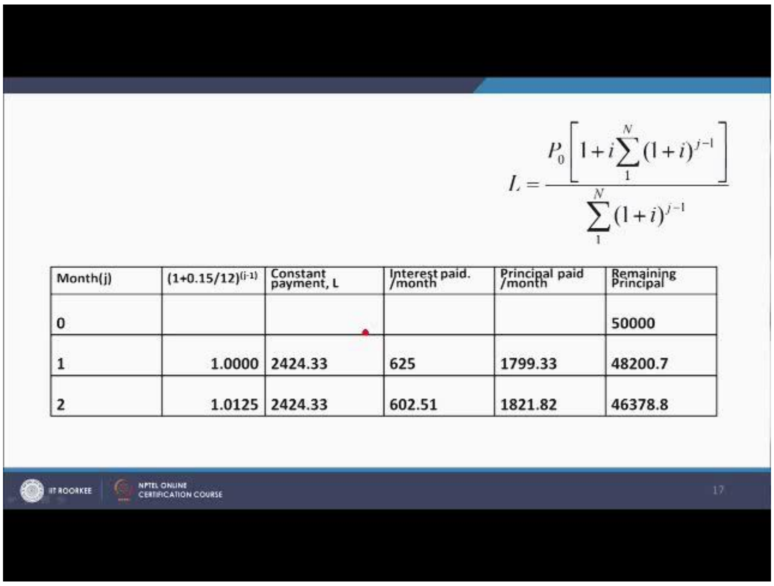
Month(j)	$(1+0.15/12)^{(j-1)}$	Constant payment, L	Interest paid /month	Principal paid /month	Remaining Principal
0					50000
1	1.0000	2424.33	625	1799.33	48200.7
2	1.0125	2424.33	602.51	1821.82	46378.8
3	1.0252	2424.33	579.74	1844.59	44534.3
4	1.0380	2424.33	556.68	1867.65	42666.6
5	1.0509	2424.33	533.33	1891.00	40775.6
6	1.0641	2424.33	509.70	1914.63	38861.0
7	1.0774	2424.33	485.76	1938.57	36922.4
8	1.0909	2424.33	461.53	1962.80	34959.6
9	1.1045	2424.33	436.99	1987.34	32972.3
10	1.1183	2424.33	412.15	2012.18	30960.1
11	1.1323	2424.33	387.00	2037.33	28922.8
12	1.1464	2424.33	361.53	2062.80	26860.0
13	1.1608	2424.33	335.75	2088.58	24771.4
14	1.1753	2424.33	309.64	2114.69	22656.7
15	1.1900	2424.33	283.21	2141.12	20515.6

So, this factor can be very easily computed without the information of others parameters only takes into a count of value of I and other fixed parameters. So, I can calculate this very easily 27.78808. Now where P0 is the principal amount 50000 plus I am using this formula this 50000 in brackets 1 plus i this is 0.15 into summation this. This already I have calculated. So, it is 27.78808 divided by 12 because this i will be divided by 12. So, the 12 is here I divided by 12 because here I equal to r by m this is 1.5, 0.15 divided by 12 and then whole divided by this values which is 27.78808 divided by 27.78808.

So, the value of L the monthly payment comes out to be Rupees 2424.33, at the end of the first month the interest would be 50000 into 0.15 divided by 12 which comes out to be 625. So, principal paid at the end of the first month this, this L value subtracted by this interest is comes out to 1799.33. So, this is the principal amount which would be paid at the end of first month.

So, remaining amount of principal at the end of first month is 50000 minus this value which comes out to be 48200.67. Similarly interest charge at the end of the second month will be interest will be charged on this amount because this is the left out principal. So, 48200.67 into this is the rate 0.15 divided by 12 because this is the month period of interest comes out to be 602.51, principal paid at the end of the second month is the value of L that is payment this is each month minus this interest comes out be 1821.82. So, remaining amount of principal at the end of second month is this is the remaining amount at the end of the first month minus this 1821.82 comes out to 46378.85.

(Refer Slide Time: 21:04)



Now, we see that the we fill up this table, this is the parameter we have computed here this is 1 and this is 1.0125 this is the constant payment which is being made 2424.33 the interest paid when the first month this we have already calculated principal paid for the month this we have already calculated and remaining principal we have already calculated. For the month 2 this values we have already calculated.

So, once this is calculated we can fill up this values in this table in this table and then we can sum up this is 27.788 and this is value is this value is 58183.92 that is through constant payment for a principal of 50000. I will be paying 58183.92 this is the interest paid total interest paid is 80183.99 and the total principal is 50000 Rupees and this was due to the remaining principal.

Thank you.