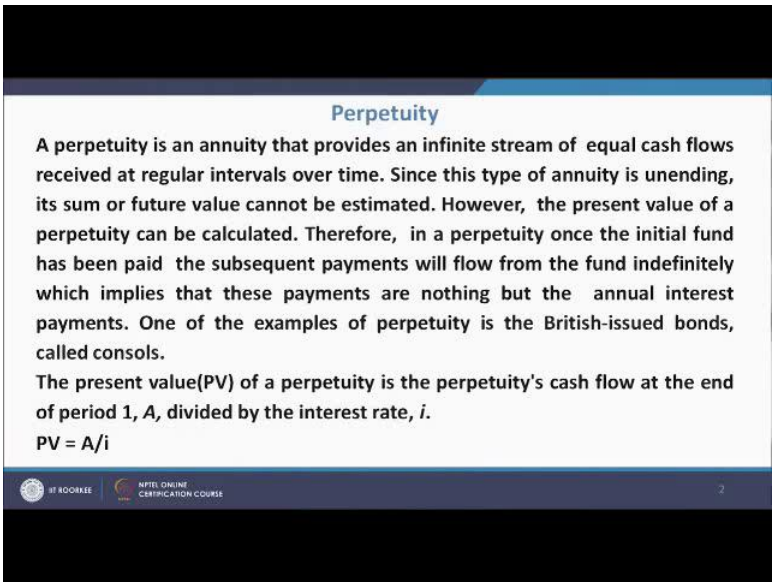
Time value of money-Concepts and Calculations
Prof. Bikash Mohanty
Department of Chemical Engineering
Indian Institute of Technology, Roorkee

Lecture - 11
Perpetuity

Welcome to the lecture series on Time value of money-Concepts and Calculations. The topic of the present lecture is Perpetuity.

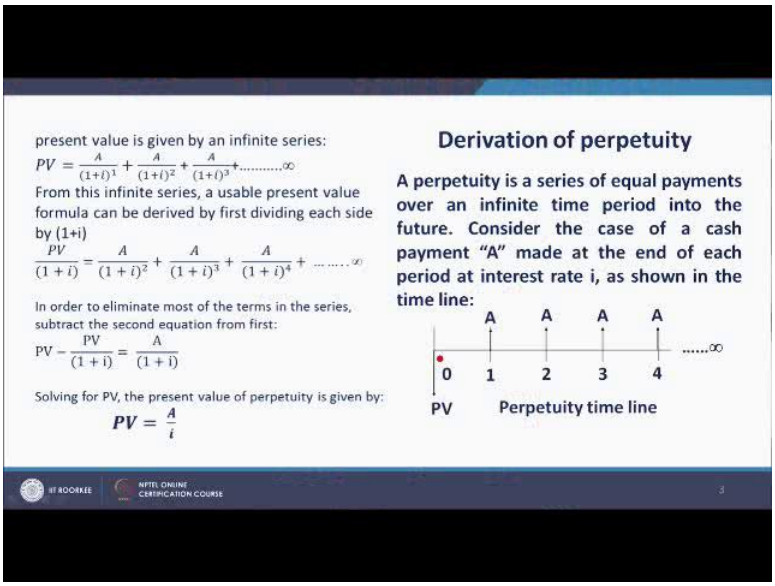
(Refer Slide Time: 00:40)



Perpetuity is an annuity that provides an infinite stream of equal cash flows received at regular intervals over time. Since this type of annuity is unending, it is sum or future value cannot be estimated. However, the present value of perpetuity can be calculated. Therefore, in perpetuity once the initial fund has been paid the subsequent payments will flow from the fund indefinitely which implies that these payments are nothing but the annual interest payments.

One of the examples of perpetuity is the British-issued bonds called Consoles. The present value of perpetuity is the perpetuities cash flow has the end of the period 1 A is divided by the interest at i. So, PV is equal to A by i.

(Refer Slide Time: 01:56)

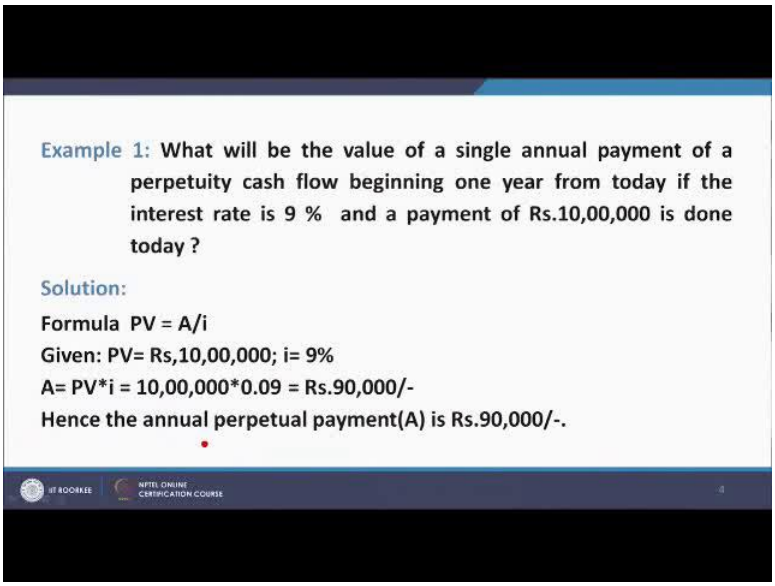


Derivation of perpetuity; a perpetuity is the series of equal payments over an infinite time period into the future. Consider the case of cash flow here have the end of first year there is a payment A, end of second year there is another payment A, end of third year there is another payment A and this goes on of to infinite. Now if you want to find out what is the PV that is present value of this perpetuity this derivation is made. Present value is given by an infinite series. So, PV is equal to a divided 1 plus i. So, when I transfer this amount 2 present value, it will be A divided by 1 plus i and when I transfer this amount to the present value, this will be a divided by 1 plus i whole square.

Similarly, it will be a plus i whole q up to infinite. From this infinite series a usable present value formula can be derived by first dividing each side by 1 plus i. So, when I divide PV by 1 plus i this is PV 1 plus i and this becomes a divided by 1 plus i whole square, this becomes A divided by 1 plus i whole square q and so on so forth. In order to eliminate most of the terms in the series, subtract the second from the first. So, it becomes PV minus PV 1 plus i is equal to A divided by 1 plus i and when we solve this equation we get PV is equal to a by i.

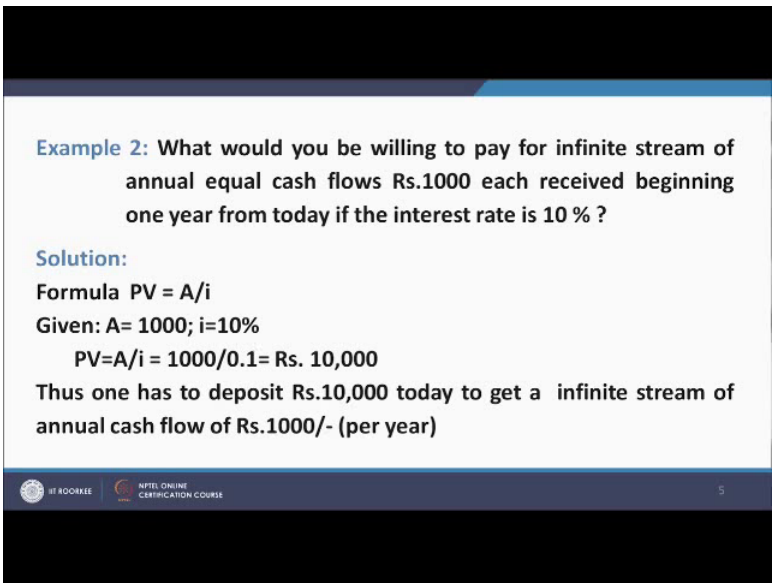
Now, let us takes few problems.

(Refer Slide Time: 03:54)



Example 1 what will be the value of a single annual payment of a perpetuity cash flow beginning one year from today, if the interest rate is 9 percent and the payment of 100000 is done today. So, PV is equal to a by i. So, PV is in this case is 1000000 i is nine percent. So, A is PV into i. So, it is 1000000 into 0.09 is equal to 90000. Hence the annual perpetual payment A is 90000.

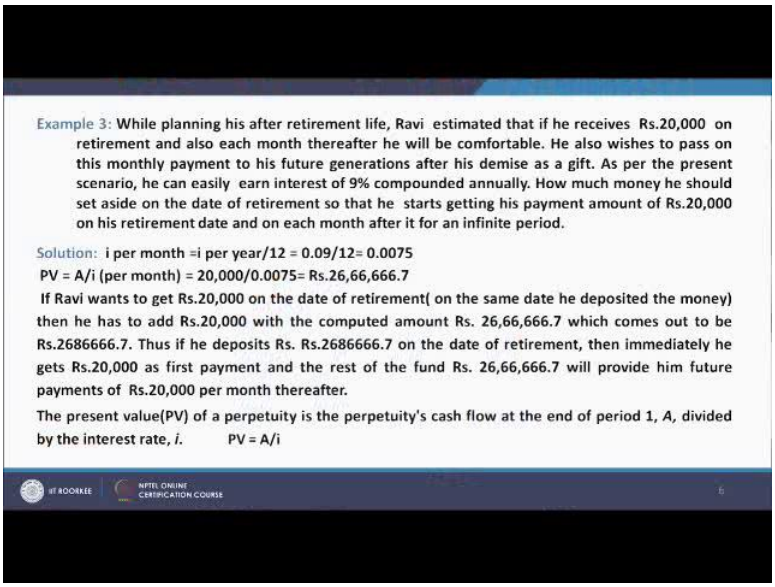
(Refer Slide Time: 04:33)



Now, question number 2, what would you be willing to pay for infinite stream of annual equal cash flows of Rupees 1000 each received beginning 1 year from today if the

interest rate is 10 percent. So, the formula is PV A by I, A is 1000, i is 10 percent. So, the formula is PV is equal to A by i this is 1000 divided by 0.1 is equal to 10000. Thus 1 has to deposit 10000. Today to get a infinite stream of cash flows of 1000 per year.

(Refer Slide Time: 05:27)



Let us take the example 3, while planning is after a retirement life Ravi estimated that if he receives Rupees 20000 on retirement date and also each month there after you will be comfortable. He also wishes to pass on this monthly payment to his future generations after his demise as a gift as per the present scenario.

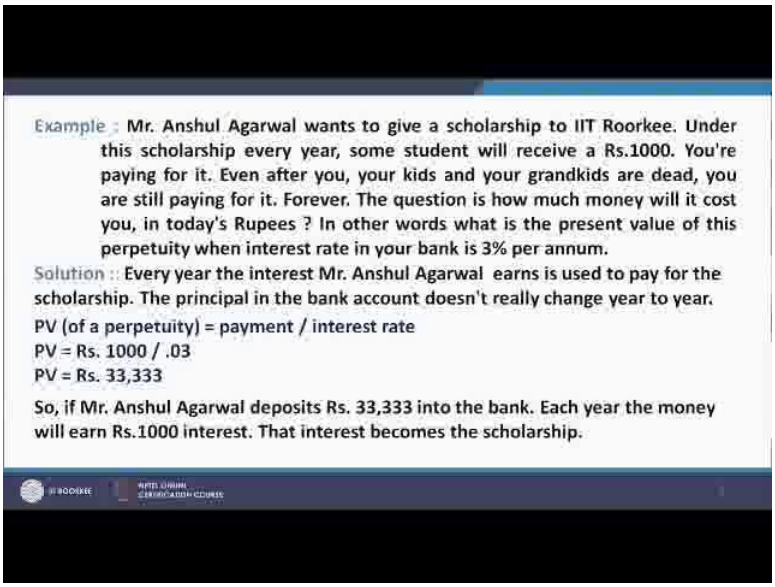
He can easily earn interest of 9 percent compounded annually. How much money he should set aside on the date of retirement? So, that he starts getting his payment of amount Rupees 20000 on his retirement date and on each month after it for an infinite period.

Now, if we see here the i per month is equal to i per year divided by 12. So, this is 0.09 divided by 12 is 0.0075 and PV is equal to A dived by i is 20000 divided by 0.0075 which comes out be 2666666.7. If Ravi wants to get Rupees 20000 on the date of retirement, that is on the same date he deposited the money then he has to add Rupees 20000 with the computer amount of Rupees 2666666.7, which comes out be 2686666.7.

Thus if he deposits this amount on the day of retirement, then immediately he gets Rupees 20000 as first payment and the rest of the fund that is Rupees 2666666.7 will

provide him future payments of Rupees 20000 per month thereafter. The present value of PV of the perpetuity is the perpetuities cash flow at the end of period 1 which is A divided by the interest rate.

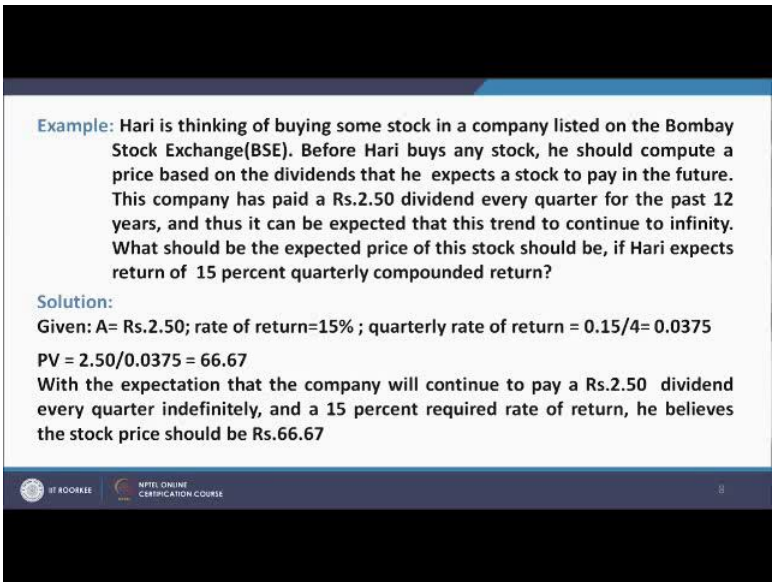
(Refer Slide Time: 07:19)



Let us taken example Mr. Anshul Agarwal wants to give a scholarship to IIT Roorkee under this scholarship every year, some student will receive Rupees 1000 and you are paying for it even after you, your kids and your grand kids are dead you are still paying for it forever, the question is how much money will it cost you in today's Rupees, in other words what is the present value of this perpetuity when interest rate in your bank is 3 percent per annum.

Now solutions every year the interest Mr. Anshul Agarwal earns he is used to pay for this scholarship. The principle in the bank account does not really change year to year. So, PV of perpetuity is called payment divided by interest rate. So, PV is equal to Rupees 1000 divided by 0.03 is comes are to be 33333. So, if Mr. Anshul Agarwal deposits Rupees 33333 into the bank. Each year the money will earn Rupees is 1000 interest and that interest becomes the scholarship.

(Refer Slide Time: 08:39)



Another example, Hari is thinking of buying some stock in a company listed on the Bombay stock exchange BSE. Before Hari buys any stock he should compute a price based on the dividends that he accepts a stock to pay in the future. This company has paid day Rupees 2.50 dividend every quarter for the past 12 years and thus it can be a excepted that this trend to continue for infinity.

What should be the excepted price of this stock? If Hari excepts return of 15 percent quarterly compounded, the solution is giving that A is equal to Rupees 2.50, rate of return is 15 percent, quarterly rate of return is 15 percent divided by 4, which is 0.0375. So, PV is equal to 2.50 divided by 0.0375 which comes off to be 66.67, with the exception that the company will continued to pay Rupees 2.50 dividend every quarter indefinitely and a 15 percent required rate of return he believes the stock price should be 66.67.

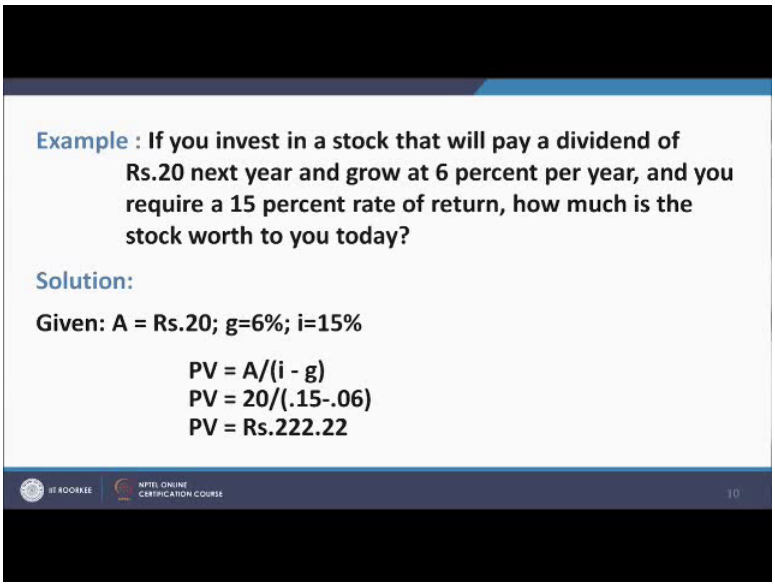
Now, we have to see the derivation of another type of perpetuity, which is called growing perpetuity. Sometimes the payment in perpetuity is not constant, but rather increases at a certain growth rate G, as depicted in the following time line. If you see this time line at the end of 1 year we are getting an amount A, but end of second year we are getting an amount A into 1 plus G and third year we are getting an amount A into 1 plus G whole square and so on and so forth up to infinite period this is a time line for a growing perpetuity.

Let us try to find out what is the PV that is present value of this type of perpetuity. So, because this cash flow continues for ever, the present value is given by an infinite series PV is equal to A divided by 1 plus i. So, if this A is converted to present value. So, the present value of this A will be A divided by 1 plus i the present value of this A into 1 plus g will be A into 1 plus G divided by 1 plus i whole square and so on and so forth up to infinite. From this infinite series a usable present value formula can be derived by first multiplying each side by 1 plus G divided by 1 plus i. So if I multiplied this P with one plus G divided by 1 plus i, so this is PV 1 plus G divided by 1 plus i. So, this becomes a factor becomes A into 1 plus G divided by 1 plus i whole square and so on and so forth.

In order to eliminate most of the terms in the series subtract the second equation from the first. So, it is PV minus PV 1 plus G divided by 1 plus i is equal to A divided by 1 plus i. So, this gives PV is equal to A divided by i minus G, where i is greater than G. So, this is the formula for growing perpetuity to find out present value.

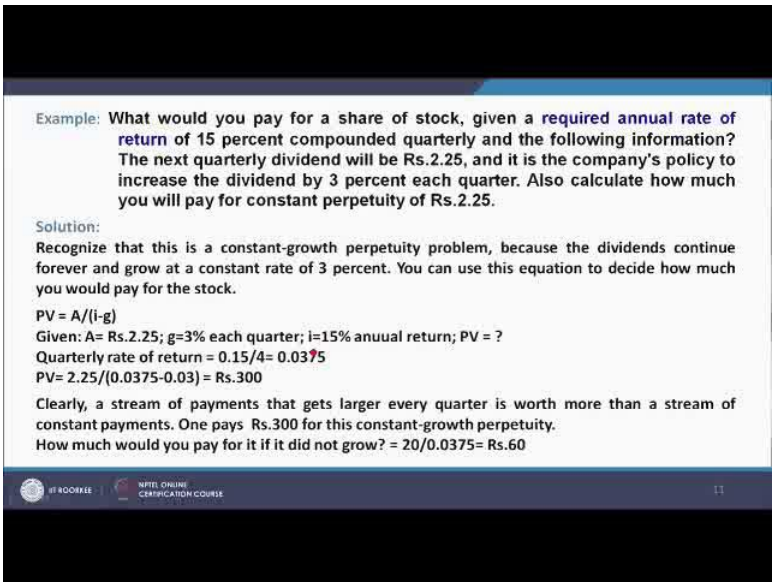
Now, let us take some numericals. If you investing stock that will pay a dividend of Rupees 20 next years and grows at 6 percent per year and you require a 15 percent rate of return how much is the stock worth to you today.

(Refer Slide Time: 12:33)



So, here A is twenty g is six percent i is 15 percent; so PV is equal to a divided by in brackets i minus G, which comes out to be Rupees 222.22.

(Refer Slide Time: 12:49)

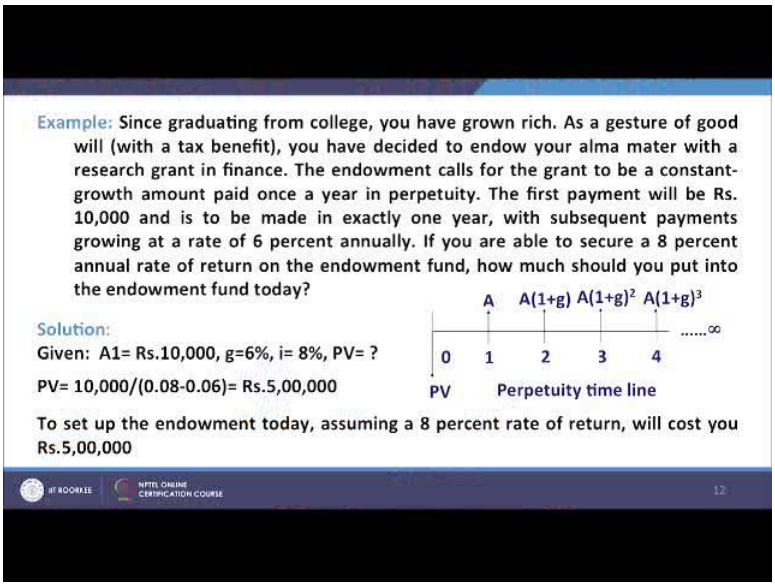


Let us take other example, what would you pay for a share of stock given a required annual rate of return of 15 percent, compounded quarterly and the following information the next quarterly dividend will be Rupees 2.25 and it is the companies policy is increase the dividend by 3 percent each quarter, also calculate how much you will pay for constant perpetuity of Rupees 2.25.

The solution is recognized than this is a constant growth perpetuity problem, because the dividends continue forever and grow at a constant rate of 3 percent. You can use this equation to decide how much you would pay for this stock. So, PV is equal to a divided by i minus G and what are given is A is 2.25, G is 3 percent each quarter and i is could 15 percent annually return and PV is what is. So, I want to find out what is the value of the PV.

Now, quarterly rate of return will be i is equal to 15 percent. So, 0.15 divided by 4 it should be 0.0375. So, PV is equal to 2.25 divided by in brackets 0.0375 minus 0.03 which comes out to be Rupees 300. Rarely a stream of payments that gets larger every quarter is worth more than a stream of constant payments, one pays Rupees 300 for this constant growth perpetuity and how much would you pay for it if he did not grow is 20 divided by 0.0375 which comes out to be 60.

(Refer Slide Time: 14:45)



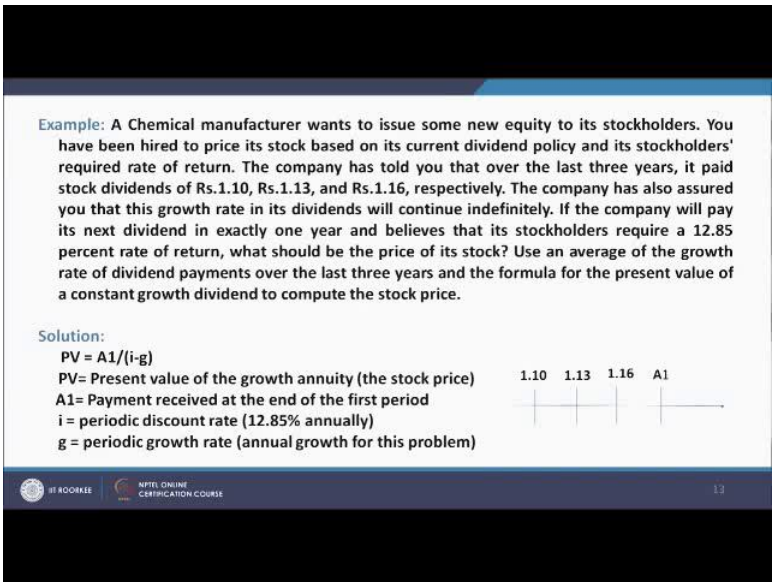
So, if we see that for a constant perpetuity you have to pay 60 Rupees and for a growing perpetuity we have to pay 300 Rupees.

Now another example since graduating for college, we have grown rich as a gesture of goodwill you have decided to endow your alma mater with a research grant in finance. The endowment calls for the grant to be a constant growth amount paid once in a year in perpetuity the first payment will be Rupees 10000 and is to be made in exactly 1 year, which subsequent payments growing at the rate of 6 percent annually.

If you are able to secure a 8 percent annual rate of return on the endowment fund, how much should you put into the endowment fund today. So, we have a growing perpetuity at the end of 1 year the perpetuity is a then at the end of second year this is a into 1 plus G and so on and so forth up to infinite period.

So, in our case a one is 10000, G is 6 percent, i is 8 percent and what is the PV that is present value which I should invest to get this growing perpetuity. So, PV is equal to 10000 divided by 0.08 minus 0.06 which comes out to be 500000. To set up the endowment today assuming 8 percent rate of return it will cost you Rupees 500000.

(Refer Slide Time: 16:15)



Now, let us take another complex problem a chemical manufacturer wants to issue some new equity to it is stock holders. You have been hired to price it is stocks based on it is current dividend policy and it is share holders requires rate of return. The company has to told you that over the last 3 years it paid stock dividends of Rupees 1.10 Rupees 1.13 and Rupees 1.16 respectively. The company has also assured you that this growth rate in it is dividends will continue indefinitely.

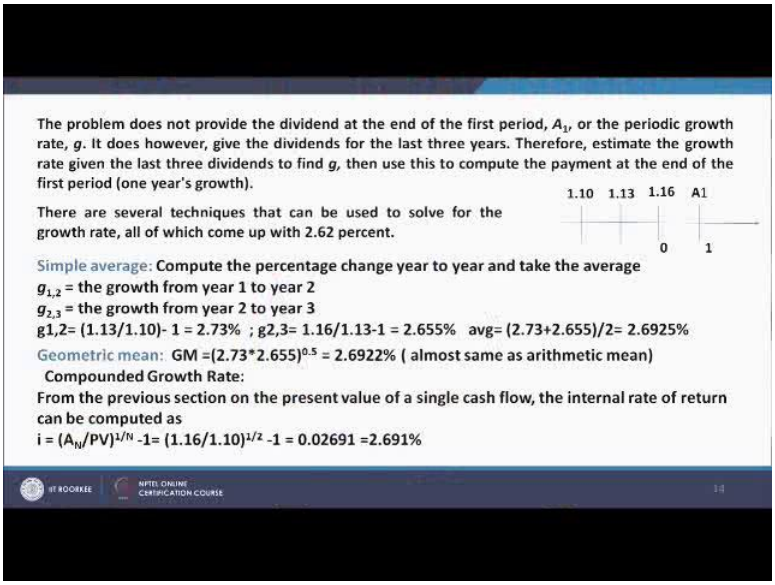
If the company will pay it is next dividend in exactly 1 year and believes that it is stockholders require a 12.85 percents rate of return what should be the price of it is stock, use an average of the growth rate of dividend payment over the last 3 years and the formula for the present value of a constant dividend to compute the stock price.

Now, in the last 3 years, if you see here in the last 3 years it has to paid 1.10,1.13,1.16 and 1 year hence from this point it will pay a one which we do not know. So, the solution is PV equal to a one divided by i minus G, PV is the present value of the growth annuity is the stock price and a one is the payment received at the end of the first period, i is the periodic discount rate which is 12.85 annually and G is periodic growth rate annual growth for this problem.

The problem does not provide the dividend at the end of the first period that is A1 or the periodic growth rate G it does; however, give the dividend for the last 3 years, Therefore,

estimate the growth rate given the last 3 years dividends to find the value of G. Then use this to compute the payment at the end of the first period that is 1 year's growth.

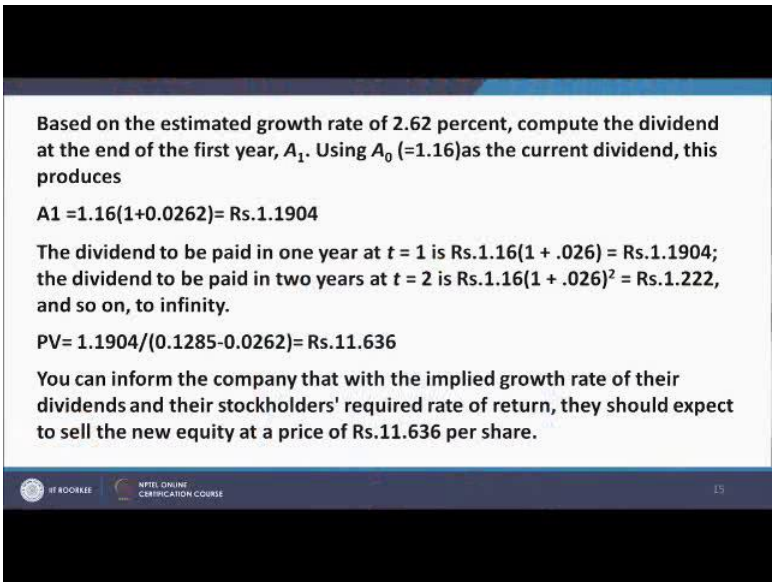
(Refer Slide Time: 18:25)



There are several techniques that can be used to solve for the growth rate of which one come up with 2.62 percent, by using simple average compute the percentage change year to year and take the average if G1,2 is the growth rate from year 1 to year G23 the growth from year 2 to year 3 and G1,2 is equal to 1.13 divided by 1.10 in brackets minus 1 which comes out to be 2.73 percent. Similarly G23 comes out to be 2.655 percent. So, average is 2.73 plus 2.655 divided by 2 comes out to it 2.6925 percent. Now if a take a geometric mean of this percent change, this is 0.23 into 2.65 and root of this 2 multiplications comes out to be 2.6922 percent almost same as a arithmetic mean.

Now, the compound growth rate from the previous section on the present value of a single cash flow, the internal rate of return can be computed as this, i is equal to AN divided by PV to the power 1 by N minus 1 on this, if i use the last one and the first one that is 1.16 and 1.10. So, it is 1.16 divided by 1.10 to the power half minus 1, it is comes out to 2.691, based on the estimated growth rate up to 2.62 percent compute the dividend at the end of the first year that is A1 using a 0 is equal to 1.16 as the current dividend.

(Refer Slide Time: 20:23)



This produces A1 is equal to 1.16 into 1 plus 0.0262, that is 1 plus G is equal to Rupees 1.1904. The dividend to be paid in 1 year at t equal to 1 is Rupees 1.16 and brackets 1 plus 0.26 is comes out to be Rupees 1.1904.

The dividend to be paid in the 2 years at t equal to 2 is 1.16 in brackets 1.1 plus 0.026 whole square comes out to be 1.222 and so on up to infinity. So, PV is equal to 1.1904 which is the value at t equal to 1 divided by i in brackets 0.1285 which is the rate of interest minus the growth is 0.0262 which comes out to be Rupees 11636.

So, you can inform the company that with the implied growth rate of their dividends and their stock holders requires rate of return, they should expect to sell the new equity at a price of Rupees 11.636 per sale.

Thank you.