Review of Basic - Mathematical Concepts

Hello everyone, welcome to module 4 on introduction to the concepts of biomechanics. In the previous section, we looked at the various applications and scope of quantification of movement. So, in this section, we will revisit the definition of biomechanics and then we will work towards the concept of motion. We will look at how to define motion and then we will move forward towards the linear and angular kinematics followed by few examples like general examples and some examples from sports applications to look at various kinematic parameters and how to derive them. So, to start with, we will start with what is biomechanics. So, as the name itself indicates, it is made up of two words which includes bio and mechanics.

So, from the word itself, we can see when the word comes bio, so it is something our mind works in a way like it is something related to living or biological system. Whereas, mechanics is the analysis of forces and their effects using the principles of physics and mathematics. So, Herbert Hadze in 1974 defined biomechanics as a study of movement of living things using the science of mechanics. So, what we look in mechanics particularly, we look at the distance.

For example, what is the distance between the right and left toe or we look at the location of joint or we look at the joint locations. Similarly, we can look at the angle between different segments. For example, over here between the forearm and the upper arm. So, how to define these parameters. Another thing which we look in biomechanics is location or the effect of body weight on the movement or what is the force being produced by the athlete when it is hitting a ground or what is the friction between the surface and the athlete or a person or what is the effect of air resistance while performing a movement.

So, now let us move to sports biomechanics. So, what is sports biomechanics? So, in order to understand how body moves and how to improve athletic movements, we use the concepts of sports biomechanics. So, it is nothing but mechanics of human movement in sports. So, by understanding how the body of athletes move, we can make adjustment to their technique which can lead to improved performance. Sports biomechanics also help coaches to develop training programs that are more efficient and effective.

And also, while understanding the various forces acting on the athlete's body during various movements, we can identify the areas where the athletes might be at the risk of injury and develop strategies to prevent those injuries. So, in general this mechanics, understanding the mechanics of movement in sports will help us with injury prevention, rehabilitation or performance enhancement. So, before moving to the analysis, first of all we need to understand the concept of reference system. Why? Because whenever we are talking about motion, we need to define it with respect to a reference system. So, in

biomechanics, there are many options in regards to a reference system, but most commonly used is the Cartesian coordinate system or rectangular coordinate system, which can be applied in two dimensions as well as three-dimensional motion analysis.

So, what we do in this, we define one horizontal and denote it as x axis and another axis in a vertical plane which is defined as y axis. So, this is a 2D coordinate system. So, in case of 3D, what will change is you will have a third axis which is coming out of your screen towards you. So, that will be defined as or going inside the screen itself. So, that is the third axis and it is being denoted by z.

So, for this course, we will limit ourselves to the two-dimensional case only. So, why we need to understand the reference system? For example, in a football match, there is an athlete with the ball at this corner. So, if you have to define the location of the ball at this point, so how you will define which can be repeatable in nature. So, we can choose either of the edges and then define its location from that edge in horizontal plane and in vertical plane. So, that in that sense, we can define the unique location of that particular point and it is helpful to remember the positions and to identify the strategies to get the maximum output.

So, now let us move towards the concept of mechanics. So, first we will start with the kinematics. So, in kinematics, we describe the motion without reference to the cause of motion. For example, in this example, here we are talking about distance, but we are not talking about how that distance, how we reach to that particular distance or the location of joint. The location of joint will change due to the action of muscle on the skeletal system.

Similarly, the joint angles, the angle between two segments of the body is dependent upon the muscular action. So, in general, we can define kinematics as a description of motion or in simple terms form or technique of a movement. The next type of analysis which we do in sports biomechanics is known as kinetics. In kinetics, we also reference to the causes of motion. For example, during the same activity, how much forces or torques are being generated at various joints, which will help the different segments to move in a particular pattern or fashion.

So, it is nothing but the explanation of motion or generally we can call the effort or power required for the movement. So, let us start with an example which we have seen in our day-to-day life that is known as a drop vertical jump. So, in this what happens is an athlete or participants jumps off a box which is of certain height most in most common scenarios it was it is approximately 31 centimeters and then the athlete or the person has to jump off it and then as soon as they land, they have to make a counter movement jump or a vertical jump up. So, to understand this we had to analyze this movement in two

planes. In frontal plane, we can see the different time points or different critical points which are required from a performance or injury prevention point of view.

Like when the athlete touches the ground and similarly, we can look at those in the sagittal plane like all three movements and these movements the first one is the touchdown, when the athlete first touches the ground, second is the maximum knee flexion and third would be the feet off or the when the foot of the athlete left the ground to perform a vertical jump. So, these three time points are based upon the requirements from the coach or a performance analyst. So, they are more interested into these things. So, let's discuss the various concepts of kinematics first and then see how these are being applied in a drop vertical jump. So, to apply these concepts first of all we need to understand the concept of motion.

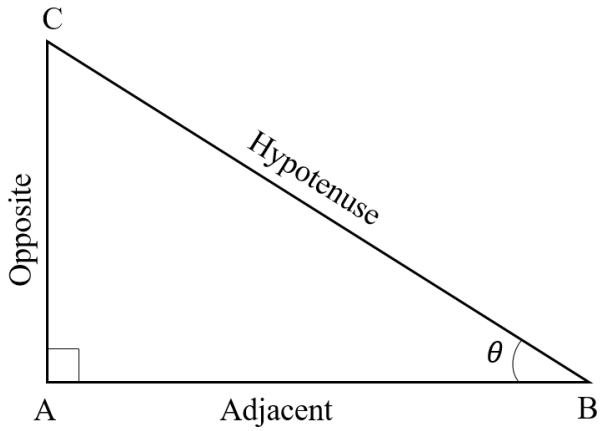
For example, when you are throwing a ball in a basketball, so how to define a motion? So, motion is nothing but change in position with respect to time. So, an athlete might have thrown a ball from this location and with time it followed this path towards the basket. So, the necessary conditions for motion to occur are space and time, for space to move and time required to complete the event itself. Now, let us look at the different classifications of motion. So, motion can be classified into different types and each one provides a unique insight into an athlete's movement patterns.

Linear motion, first of all the motion can be classified as linear, angular or general motion. So, why we need to classify motion? Because we can break down the complex movement and made our analysis easier. So, in linear motion what happens? Linear motion refers to the movement in a straight or a curved line or like movement in one direction or a combination of different directions also. For example, while lifting this weight, if we look at the weight itself, it moves in a curved path like athlete will lift it up and moves the whole thing in a like all the points on the mass moves the same amount of distance in the same amount of time. Whereas in angular motion, which involves the rotation around an axis or a fixed point, for example, the wheel of the bicycle are rotating about its axle.

So, in this what happens is the different points on the circumference of the wheel cover different distance or in a fixed amount of time. So, in this case, like the whole body is not moving, like all the points on the body are not moving the same distance in the same interval of time. However, the general motion is a combination of linear and angular motion. So, understanding the classification of motion in biomechanics provides a structured framework for analyzing and enhancing the athletic performance, whether it is a linear motion or angular motion, they provide a valuable insights and empower athletes and coaches to optimize technique and help athletes to reach their full potential. For example, in this example in cricket bowling, it involves both linear as well as angular motion.

So, in generally speaking, in every day-to-day activity, it is always a general motion, the isolated linear and angular motion are very difficult to mimic in the real life. So, to understand the motion, let us review some mathematical concepts. However, we are not going into too much detail, but just to give you an overview of these mathematical concepts, which can be very handy while performing biomechanical analysis. So, in this we will discuss about the trigonometry, Pythagoras theorem, scalars and vectors and central difference method. So, let us move towards these one by one.

So, trigonometry is made up of two words like trigonon, which means triangle and metron means measure. So, it is a branch of mathematics, which deals with the measurement of the sides and angles of triangle and their relationship with each other. In this particular course, we are more interested into a specific type of triangle, which are known as right angle triangle. So, right angle triangle means one of the angle between two sides of the triangle is 90 degrees. So, let us define an angle theta between two sides of the triangle and name these sides as the longest side as hypotenuse, the side between hypotenuse and the other side with which it is making an angle it is known as adjacent and the side which is opposite to the angle is known as opposite.



So, the main trigonometric identities, which we will use repeatedly in this course or in particular biomechanical analysis are

$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\tan\theta = \frac{Adjacent}{Hypotenuse}$$

So, these are very helpful trigonometric identities, which we will be using over and over again during this section of the course on human movement science. So, the next concept is Pythagoras theorem. So, it is again applicable in right angle triangles. So, we have already seen the nomenclature for right angle triangles.

So, in this what we will do is according to Pythagoras theorem

$$Hypotenuse^2 = Adjacent^2 + Opposite^2$$

Sometime adjacent side is also known as base and opposite side is known as perpendicular.

$$Hypotenuse^2 = Base^2 + Perpendicular^2$$

And perpendicular sometime is denoted by a horizontal and a vertical line on it.

$$Perpendicular = \perp$$

So, whenever you see this sign between like (⊥), if we say this is A, B and C.

$$AB \perp AC$$

This is the general mathematical notation to understand this concept. So, Pythagoras theorem the square of hypotenuse is equal to the sum of square of the other two sides.

This principle is also very useful while performing a biomechanical analysis. Now, let us move the discussion towards the scalars and vectors. So, these are also very important things. So, these mathematical quantities that are used to describe our motion can be divided into broad two categories.

The first one is known as scalar. In this, these quantities only have magnitude, but no direction. For example, distance, speed, mass, temperature and time. We will look further into these concepts during further, we will look into these concepts in next few slides. And similarly, the vectors are defined as the quantities which have both magnitude and direction. For example, displacement, velocity, acceleration and force.

These concepts particularly are very important to perform a quantitative biomechanical analysis. So, now let us see the various characteristics of a vector. So, why we need to discuss further about the vectors because vectors play a very crucial role in biomechanics in general and sports biomechanics also, because they allow us to perform a detailed analysis of forces, velocities and acceleration involved in various movement, whether athletic or day to day activities. So, let us look at how a vector looks like. So, it is nothing but it resembles like an arrow starting with the point which is known as point of application and then the point from where it is starting, the end of the vector is known as

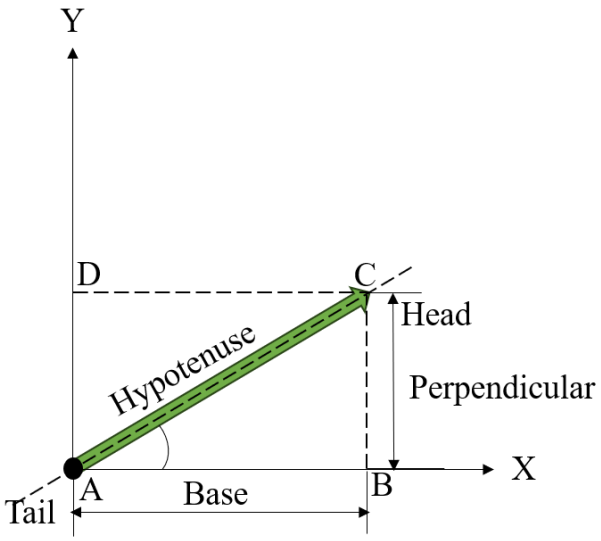
the tail of the vector and where the arrowhead is present it is known as the head of the vector.

The total length of the vector defines the magnitude of the vector when it is in vector representation. So, another thing which we will be using in advanced biomechanical analysis, which is which are like out of scope for this course as of now. So, the line which is extending from head to tail that is known as the line of action to look at the application of like for example, if this vector represents a force. So, what is the line of action of force and what are its effects on the motion. So, next thing which we will be more interested into to know how to resolve vectors.

So, over here we take the same vector and we place a reference system with x axis as a horizontal and y axis as a vertical axis. So, let us consider this vector makes an angle theta with the horizontal axis. What we will do is we will draw a vertical line from here like 90 degrees. So, this whole thing becomes a right-angle triangle. So, the vectors can be resolved into horizontal component.

So, let us define this vector AC. So, in vector notation, a vector is represented by an arrow on the name of the vector. So, AC is a vector. So, it can be resolved into two components and similarly, if we draw a line or a projection of point C on the y axis. So, let us name this point as D. So, the horizontal component of a vector is AB and it is a scalar quantity.

Similarly, the vertical component of the vector is AD. So, if I represent horizontal by H, it represents the magnitude of AB and vertical by V as the AD. So, using the right-angle triangle and trigonometry, we can easily find the so, since this is the base, this is the perpendicular and this is the hypotenuse of the right-angle triangle.



So, based upon that, base will be:

We know

$$\cos\theta = \frac{Base}{Hypotenuse}$$

$$Base = Hypotenuse * \cos\theta$$

$$Base = AC * \cos\theta$$

Similarly, for vertical component so, AD is also equal to BC. So,

$$\sin\theta = \frac{Perpendicular}{Hypotenuse}$$

$$Perpendicular = Hypotenuse * \sin\theta$$

$$Perpendicular = AC * \sin\theta$$

So, let me differentiate this from here. So, this process of converting a vector quantity into its horizontal and vertical components in case of two-dimensional analysis is known as resolution of vectors or the components of the vector also. So, now, let us see how to calculate the magnitude of a vector. So, in this like let us take the same example and same nomenclature. So, if this is A B C and D. So, what we will do is we will try to find out the magnitude AC.

So, here we can use the Pythagoras theorem which tells

$$Hypotenuse^2 = Base^2 + Perpendicular^2$$

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (Hypotenuse * \cos\theta)^2 + (Hypotenuse * \sin\theta)^2$$

$$AC^2 = (Hypotenuse)^2[\cos\theta^2 + \sin\theta^2]$$

In trigonometry this identity is equal to 1. So, from here like what also we can we can calculate the value of hypotenuse in terms of its components. So, we can convert from the if the components of the vector are given the values in terms of these two things, then we can calculate the hypotenuse or other way around. But another way of looking at magnitude of a vector from numerical point of view like if we have this vector let us represent the

$$AB = a$$

$$BC = b$$

So, this is the value of this whole quantity like

$$\cos\theta * hypotenuse = a$$

$$\sin\theta * hypotenuse = a$$

$$AC^2 = a^2 + b^2$$

$$AC = \sqrt{a^2 + b^2}$$

So, this is generally what you will see in case of numerical examples. So, that is how you will calculate the magnitude of a vector. Now, let us look at another mathematical concept which is known as central difference method. So, this is a type of numerical method which is used to calculate rate of change or change with respect to time.

So, since biomechanical analysis involves the movement which occur over a time. So, we might be interested into finding how those parameters change over the time. So, according to this

$$Rate\ of\ change\ for\ x_i = v_i = \frac{x_{i+1} - x_{i-1}}{2\Delta t}$$

So, there is one assumption over here that delta t is constant. So, let us assume an example where an event starts at t_0 and then t_1, t_2, t_3 and t_4 are different time values which are present during that particular movement and the value for a one-dimensional case.

Let us look at the position in x direction that would be x_0 at time 0, x_1 at time 1, x_2 at time 2, x_3 at time 3 and x_4 at time 4. So, according to central difference method if we are interested to find the value of velocity or rate of change of x_3 that is given by

$$Rate\ of\ change\ for\ x_3 = v_3 = \frac{x_4 - x_2}{t_4 - t_2}$$

So, this way we can calculate the velocity from a numerical data for position or displacement. The same concept can be utilized to find the acceleration. So, acceleration at i^{th} component would be nothing but

$$a_i = \frac{v_{i+1} - v_{i-1}}{t_{i+1} - t_{i-1}}$$

So, if we plug in these values, what we will get is if we plug in these values using this expression what we will get is in terms of values of x would be

$$a_i = \frac{x_{i+1} - 2x_i + x_{i-1}}{\Delta t^2}$$

So, the knowledge of these equation will help us to find the velocity and acceleration given the numerical data for position. So, this is an important concept where we learn how we can calculate the velocity and acceleration provided we have the time variable data for the position or displacement. Thank you.