Energy Economics and Policy
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Week - 05
Energy Supply – Part II
Lecture – 04
Economics of Renewable Energy Supply

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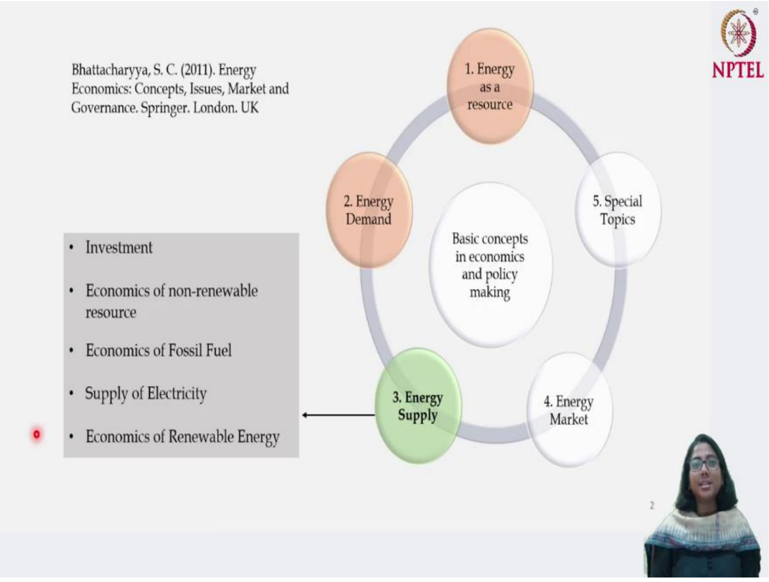

This is the last lecture of week 5 and in this lecture, we are going to discuss the Economics of Electricity Supply.

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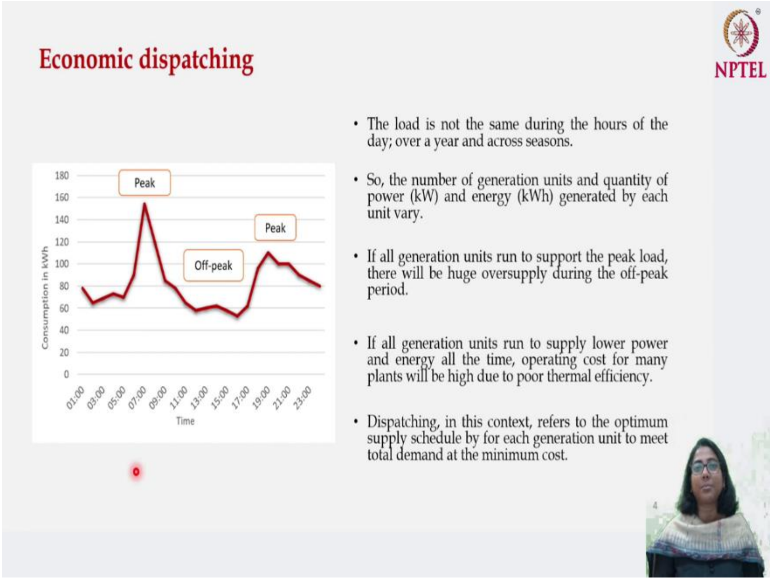

Starting with the concept of the load curve that has already been discussed in the context of electricity demand. The diagram shows how the load curve looks like at different time points of the day and the consumption at various time points of the day. This particular load curve is constructed for one household; however, we can also construct the load curve for a particular area or number of households and so on or can also construct it for a year.

The economics of electricity supply or production of electricity with the help of fossil fuel begins with the concept of load curve. This is because the load is not the same during all the hours of a day or over a time period and also varies across different seasons. There is a variation of electricity demand at different time points. Therefore, the number of generation units and the quantity of power which is measured as kilo Watt and the energy that is measured as kiloWatt hour generated by each generation unit varies. Each of the generation units in a power plant generate different amounts of electricity at different points of time.

What if they do not do so and keep on producing the same amount of electricity which is required to support the peak load. If all the generators are running in the capacity in order to supply energy which is required to support the peak load; then during the off-peak period there will be high supply of electricity with very low demand. There will be a lot of electricity generated which will be wasted and this also has implications for the running cost.

We are incurring a running cost which is not fetching any revenue and is a complete waste and this electricity which is coming from a non-renewable resource and therefore the depletion premium is going up and not only that it also leads to a lot of carbon dioxide and equivalent emission. We do not want to produce electricity when it is not required and so cannot keep all the generation units running at a scale, where it can support the peak load.

The other alternative could be to run all of them at a very low rate. But if all generation units run to supply lower power, the operating cost may become very high because there will be very low capacity utilization and poor thermal efficiency. Therefore, we have to find some sort of a solution where not all generating units are working at full-fledged neither all of them are working at very low capacity. There has to be some kind of an optimum solution.

In this context comes the notion of dispatching. Dispatching is a perfect case where you need to understand both the electricity sector as well as the economic context. This is where interdisciplinary work takes place in the context of economic dispatching of electricity. Dispatching refers to the optimum supply schedule by each of the generation units in a power plant in order to meet the total demand at a particular point of time at the minimum cost.

There are three things that are considered, the first decision is about the amount of electricity that each of the generators will produce and supply. The other two constraints are minimum cost and it should be able to supply as much power as required to satiate the demand at a particular point of time. This is economic dispatching.

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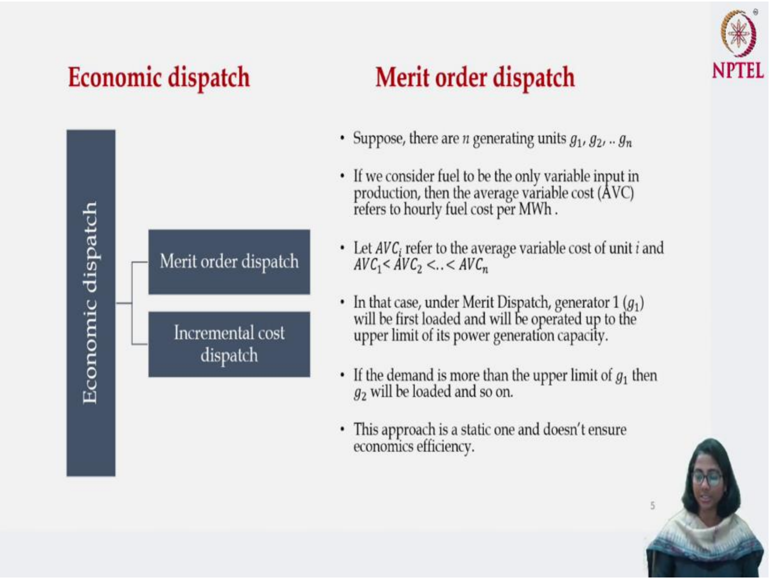

There are majorly two types of dispatching discussed in the context of energy economics or the economics of the electricity sector. One is called the merit order dispatch and the other is called the incremental cost dispatch or marginal cost dispatch. The incremental cost and marginal cost can be used interchangeably at a conceptual level although it is called incremental cost dispatch.

Let us first have a look at the functioning of the merit order dispatch. Suppose there are n generating units in a power plant which we are denoting by g_1, g_2 up to g_n. If we considered fuel to be the only variable input of production although there may be other variable inputs of production for example raw material, labour and so on but for the time being let us consider that fuel is only the raw material that is coal or gas or oil are the only variable cost that the electricity generation sector has to incur. In that case the average variable cost of this plant will be given by the hourly fuel cost per megaWatt hour. In order to generate 1 megaWatt hour what is the hourly fuel cost that the plant has to incur. This is given by average variable cost.

We rank the generating units based on their average variable cost and assume that AVC_1 is less than AVC_2 and in the ranking AVC_n comes least. The first generating unit has the least average variable cost of production. The second-generation unit has the second lowest average variable cost of production and the nth generation unit has the highest average cost of production.

If that is the case then under merit order dispatching generator 1 will be loaded and it will be operated up to a point which touches the upper limit of power generation capacity of generator 1 that is generator 1 which has the lowest average variable cost will start working and it will work up to the point where it touches the maximum capacity utilization. If generator 1 cannot meet the total demand generated at that point of time then generator 2 will be loaded.

This way you keep on adding generators one after the other in a sequence so that you use the generator with the least variable cost first and the generator with the highest variable cost last up to the point where the total supply meets the total demand. The problem with this approach is that this is sort of a static approach and this does not ensure that the cost is minimized. The other problem is that when we are doing this, we are sort of assuming that at every level of production the cost is going to stay unchanged. The marginal cost is going to stay unchanged which may not be the case. Although merit order dispatch is followed in some of the cases it is unable to deliver the optimum outcome both financially and efficiency wise.

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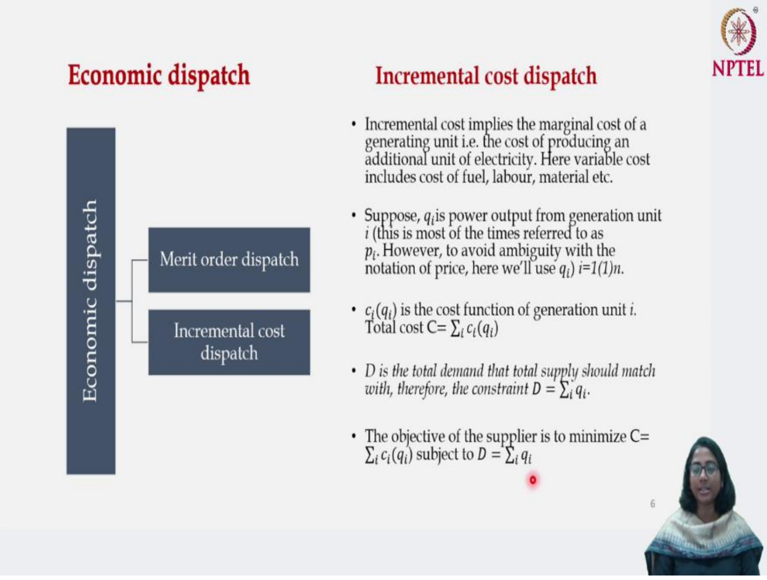

What is the alternative to merit order dispatch? The alternative to merit order dispatch comes in the form of incremental cost dispatch where the decision is taken based on the marginal cost of the generating units. Incremental cost is the marginal cost of a generating unit that is the cost of producing an additional unit of electricity. When we are talking about the marginal cost you may remember the discussion that the marginal cost is determined only in terms of the variable cost. When we are increasing the production of electricity it is only the variable cost of production that is going to go up, nothing is going to happen to the fixed cost of production. The fixed cost of production is not going to change while there will be an increase in the variable cost of production which will increase the marginal cost.

What are the different variable costs? In case of merit order dispatch, we only considered the cost of fuel but in this case along with the cost of fuel we will also consider the cost of other variable inputs for example labour, raw material, services and so on. Let us come up with the notations that we are going to use. Suppose q_i is the power output from generation unit i. When you look at the textbooks you will find that in most of the cases the power output of a generation unit is usually denoted by p_i. That is the case in the context of electrical engineering literature. However, we are not writing it as p_i because we have used p as a notation of price and therefore, we are using q_i. Generally, you can think about it as the output produced from the generation unit i. There are n generating units and i will run from 1 to n.

We next define the concept of cost $c_i(q_i)$ that is c_i as a function of quantity produced is the cost function of generating unit i. What is the total cost of the power plant where the electricity is being generated? The total cost of the power plant is given by C which is the sum of the cost of individual generating units. $C = \sum_i \quad c_i(q_i)$, the total demand is marked as D and this acts as the constraint in the optimization problem. The production should be such that you are able to meet total demand at every point of time.

What is the total supply of the firm? The i[th] generating unit is supplying q_i. If there are n such units the total generation that is the total power supply will be $\sum_i \quad q_i$. The constraint is that this $\sum_i \quad q_i$ has to be equal to D. If it is higher than D by a little bit it is fine but for optimization problem, we will keep $D = \sum_i \quad q_i$.

What is the objective of the supplier? We again go back to the production theory. The objective of the supplier or the producer is to minimize the total cost of the power plant which is given by $C = \sum_i \quad c_i(q_i)\ subject\ to\ D = \sum_i \quad q_i$. This is the objective function of the power plant. If this function is satisfied then we will see that the economic efficiency will also be ensured.

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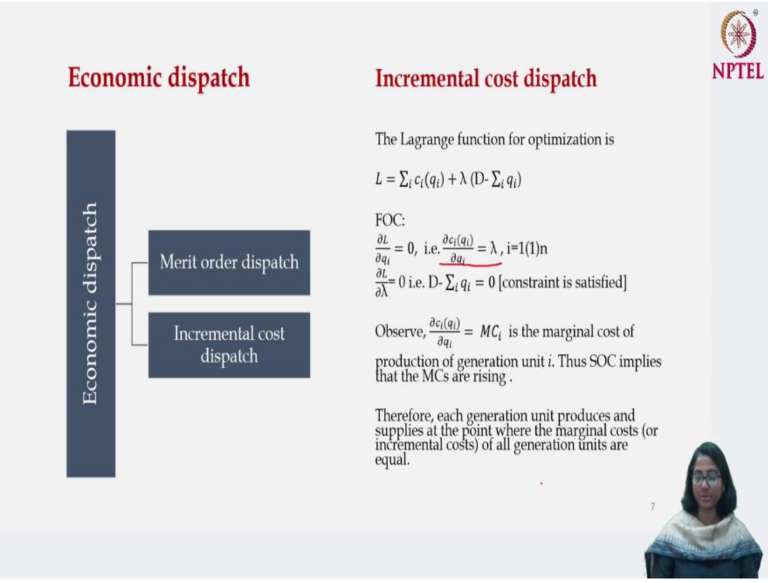

For the optimization problem let us set up the Lagrange function in the following manner. So, L is the Lagrange under consideration, we need to minimize this total cost therefore, the Lagrange is written as $L = \sum_i \quad c_i(q_i) + \lambda\,(D - \sum_i \quad q_i)$. If $D - \sum_i \quad q_i = 0$ that implies that the constraint is satisfied. There will be n plus 1 first order conditions and the second order conditions as well.

The first order conditions for this optimization problem is given by $\frac{\partial L}{\partial q_i} = 0$. Taking the partial differential of the Lagrange with respect to qi which is in the domain of the cost function and which is here as well in the constraint that should be set to be equal to 0. The first order derivative has to be set to be equal to 0. If $\frac{\partial L}{\partial q_i} = 0$ that means, $\frac{\partial c_i(q_i)}{\partial q_i} = \lambda$ because the first production unit is only considered. In that case I have $c_1(q_1)$ and I am taking the derivative only with respect to q_1. If I take the derivative with respect to q_1, then $c_2(q_2)$ $c_3(q_3)$ all of them are constant. The differentiation is happening only for $c_1(q_1)$ if I am taking the differentiation with respect to q_1.

The first order condition which is given by $\frac{\partial L}{\partial q_i} = 0$ which means $\frac{\partial c_i(q_i)}{\partial q_i} = \lambda$ because we are getting $\frac{\partial c_i(q_i)}{\partial q_i} - \lambda = 0$ and therefore the expression $\frac{\partial c_i(q_i)}{\partial q_i} = \lambda$. There are n first order conditions because the first order condition holds for all i running from 1 to n. The n plus 1th first order condition is given by $\frac{\partial L}{\partial \lambda} = 0$ which means $D - \sum_i \quad q_i = 0$ which means that constraint is satisfied.

What is $\frac{\partial c_i(q_i)}{\partial q_i}$? This is the increase in the cost of production for the ith generation unit when there is one unit increase in the production. This is the marginal cost of the ith generation unit or ith production unit. Thus, to satisfy the second order conditions we have $\frac{\delta^2 L}{\delta q_i^2}$ since we are minimizing the cost so that should be negative. If that is negative it tells that the slope of the marginal cost curve has to be positive, so the marginal cost curve has to be upward rising.

This is the same kind of condition that we discussed when we were discussing the producers supply behavior. The second order condition typically is associated with the fact that the marginal costs are rising. Therefore, if you look at the first order conditions you will see that the marginal cost of all the generating units are equal to λ. We are saying that the optimum allocation or the optimum decision is achieved where the marginal cost of all the units are equal, that is the point of generation given the constraint that the demand is met.

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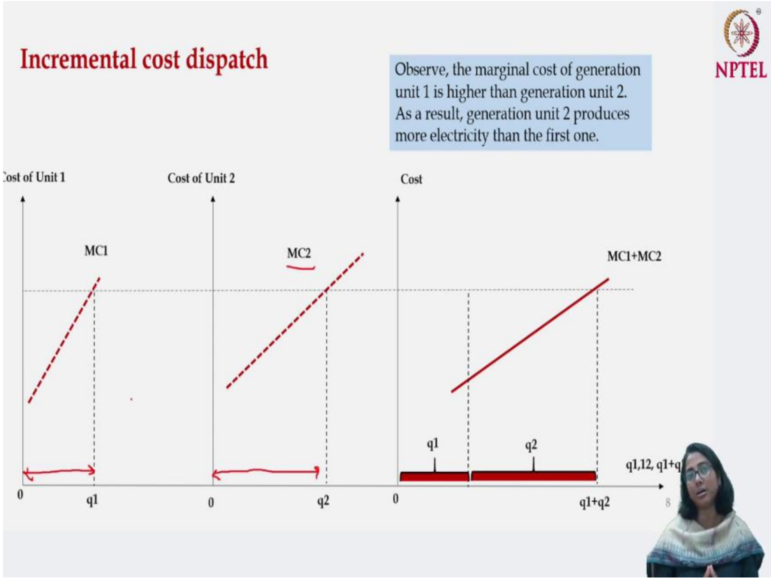

The diagram shows three panels and we suppose there are two generation units. The first panel shows the first plant and the second panel shows the second plant. The third panel is the add up of unit 1 and unit 2 that is the added-up result over quantity. The diagram depicts the marginal cost of the first-generation unit and the marginal cost of the second-generation unit. Clearly, the marginal cost curve for the second-generation unit is flatter. That implies in order to produce the same amount of electricity, the cost that the second unit is going to incur is less than that of the first unit. In terms of the variable cost the second unit is more efficient as compared to the first unit.

We will see how the dispatch decision will be taken. If I add up the marginal cost 1 and marginal cost 2, the supply curve of the market which is the added up MC_1 and MC_2 over quantity is the supply schedule in the market and this broken lines MC_1 and MC_2 are the supply schedule of individual generating units or is the supply schedule of the power plant which consists of 2 units.

Suppose the power plant has to supply $q_1 + q_2$. This is the total output or electricity the plant has to supply in order to meet the demand. If that is the case then the marginal cost the plant is going to incur is captured by the horizontal line. By the profit maximizing or the optimization condition we have seen that at the point of supply $MC_1 = MC_2$.

If the level of marginal cost is given, how much does the first unit produce at this marginal cost and how much does the second unit produce at this marginal cost. The first unit will produce

q_1 amount of output given this marginal cost and the second unit will produce q_2 amount of electricity given the marginal cost.

The total output $q_1 + q_2$ is divided among the first-generation unit and second-generation unit based on their marginal cost curve schedule. We have said that the second unit is probably more efficient as compared to the first unit and therefore whatever is being produced by the second unit is much bigger than what is being produced by the first unit. The generation unit with lower marginal cost will actually produce more.

This is the observation in case of incremental cost dispatch problems. We equate the marginal cost of all the generating units and therefore determine the supply accordingly.

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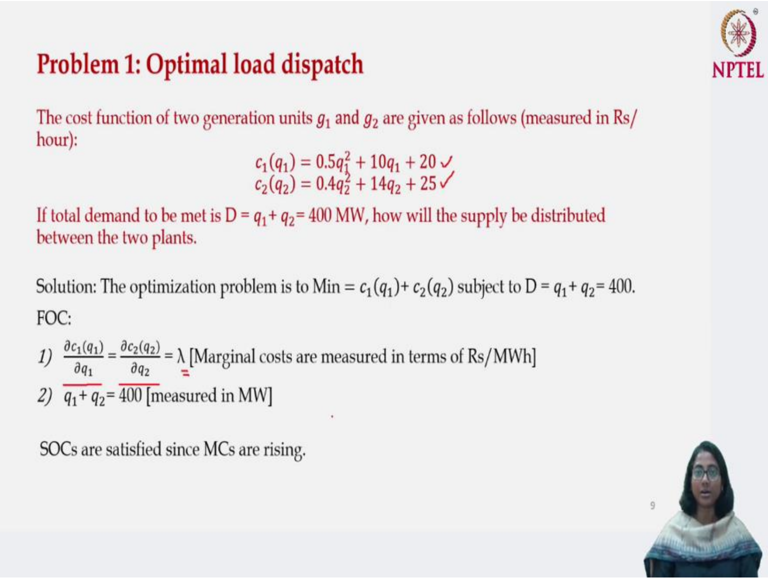

Let us have a quick look at one of the simple problems on optimal load dispatch. The cost function of two generating units g_1 and g_2 are given as follows. The costs are measured in terms of rupees per hour. The cost function is $c_1(q_1)$ for the first generating unit and $c_2(q_2)$ for the second-generation unit. For the first-generation unit, the cost function is given as $0.5\ q_1^2 + 10\ q_1 + 20$. This has the quadratic form.

The cost function of the second-generation unit is given by $0.4\ q_2^2 + 14\ q_2 + 25$. We can do some calculation in order to know the shape of marginal cost curve, shape of average cost curve, shape of average variable cost curve, the average fixed cost curve and so on.

The total demand that this plant has to meet through these two generation units is $q_1 + q_2 = 400$ mega Watts. This is the total amount that the plant has to produce. The question is how the plant is going to distribute this 400 megaWatt between the first generating unit and the second generating unit. Let us start with the problem of optimization that we have been discussing. The optimization problem is to minimize $c_1(q_1) + c_2(q_2)$ that is the total cost subject to the supply constraint $D = q_1 + q_2 = 400$. Let us see what the first order conditions look like. In case of the first order conditions we have seen that the marginal cost of the first generating unit will be equal to the marginal cost of the second-generation unit. This is the marginal cost of the first-generation unit equal to the marginal cost of the second-generation unit which is equal to the Lagrange multiplier lambda.

It is important to notice that when we were talking about total cost, we were measuring it in terms of rupees per hour. When we are talking about marginal cost, we are talking about rupees per megaWatt hour and the first order condition is given by $q_1 + q_2 = 400$. This is derived by setting $\frac{\partial L}{\partial \lambda} = 0$. The second order conditions are satisfied because the marginal costs are rising. We can derive the marginal cost functions and then take the derivative of the marginal cost functions with respect to q1 and q2 and see whether that is rising or not. We have to calculate the marginal cost for the first- and second-generation unit and have set them to be equal to lambda.

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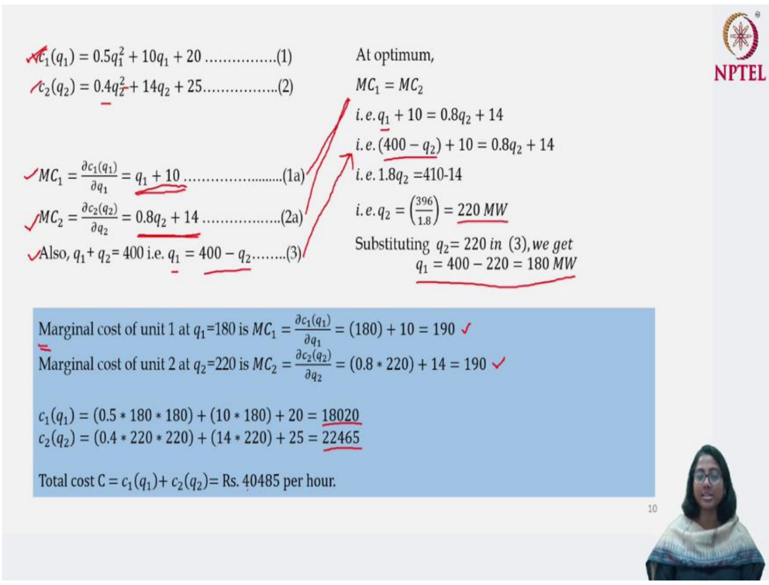

Let us first have a look at the marginal cost curves. Given is the cost curve of the first-generation unit and the cost curve of the second-generation unit. From $c_1(q_1)$ we derive the marginal cost which is $MC_1 = \frac{\partial c_1(q_1)}{\partial q_1}$ this is equal to $2 * 0.5\ q_1$. So, this is $q_1 + 10$. Taking a derivative with respect to q_1 this becomes 10 and since 20 is constant, it goes away from the marginal cost. In the cost function 20 is the cost which does not vary with the level of output, so 20 is the fixed cost of production. In the calculation of marginal cost, the fixed cost has no role to play. It is only the variable part that plays a role in determining the marginal cost. Similarly, if you calculate the marginal cost of the second generating unit, the $MC_2 = \frac{\partial c_2(q_2)}{\partial q_2}$ which is equal to 0.8. 0.8 is $0.4 * 2$ and therefore, we have $0.8\ q_2 + 14$ and 25 disappears.

These are the two marginal cost curves given for the first generating unit and the second generating unit. The third condition optimality say first order condition that I have by setting $\frac{\partial L}{\partial \lambda} = 0$, that $q_1 + q_2 = 400$ or can express q_1 in terms of q_2 by writing $q_1 = 400 - q_2$. At optimum I know that $MC_1 = MC_1 = \lambda$. Plugging the values, we get $q_1 + 10 = 0.8\ q_2 + 14$.

This is the equation in terms of q_1 and q_2 by just changing the relationship between q_1 and q_2. Plugging this relationship that is $q_1 = 400 - q_2$. Instead of q_1 I am writing $(400 - q_2) + 10 = 0.8\ q_2 + 14$. If q_2 is taken to the right-hand side, we get 1.8 $q_2 =$ (400 +10) -14. We get $q_2 = 396/1.8 = 200$ mega Watt.

The second generating unit is going to produce 220 megaWatt of electricity. Substituting this value in the relationship between q_1 and q_2, this tells that if q_2 is equal to 220 then q_1 should be equal to 180 mega Watt. 400 megaWatt is distributed between q_1 and q_2 that is between the first generating unit and the second generating unit. The second generating unit is going to produce more output because the marginal cost is lower and it's more efficient as compared to the first one. This is the optimal distribution of load in two generating units.

Let us have a quick look whether we are satisfying the condition of $MC_1 = MC_2$ or not. This is just a cross check to see whether your calculation has gone right or not. The marginal cost of the first unit at q_1 is equal to $180 + 10 = 190$. The marginal cost at the optimum production is 190 for the first plant, for the second one we $0.8 * 220 + 14 = 190$. The marginal cost of the first plant is equal to the marginal cost of the second plant at the point of optimum production and one check is done.

The second is let us also try to identify what is the total cost of producing 400 megaWatt. The cost that the first plant is incurring to produce 180 unit of output can be derived from $c_1(q_1)$ function. Plugging the values, we get the total cost of production of 180 megaWatt.

The total cost of production is also the fixed cost as well. Similarly, we can derive the total cost of production for the second plant at the optimum level of 220 megaWatt and that is 22465. The second plant is producing more however, the marginal cost is much lower. The total cost to produce 400 megaWatt of power is equal to $c_1(q_1) + c_2(q_2)$ that which adds up$18020\ and\ 22465$ and then we get 40485 per hour. This is the total cost.

There are certain interesting things that you can carry on along with these examples. This is the optimum allocation and we can check the total cost under merit dispatch. In case of merit dispatch, we set some capacity for both the plants and run plant 1 up to its full capacity if it is less than 400 then for the remaining amount run plant 2 and then you see the total cost. If you want to distribute the generation in the equitable fashion so both the generating units will produce 200 megaWatt, then what is going to be the cost implication? You will find that, if under merit dispatch as well as under this equal distribution the total cost of generation of the same amount of power will be higher than this 40485 per hour.

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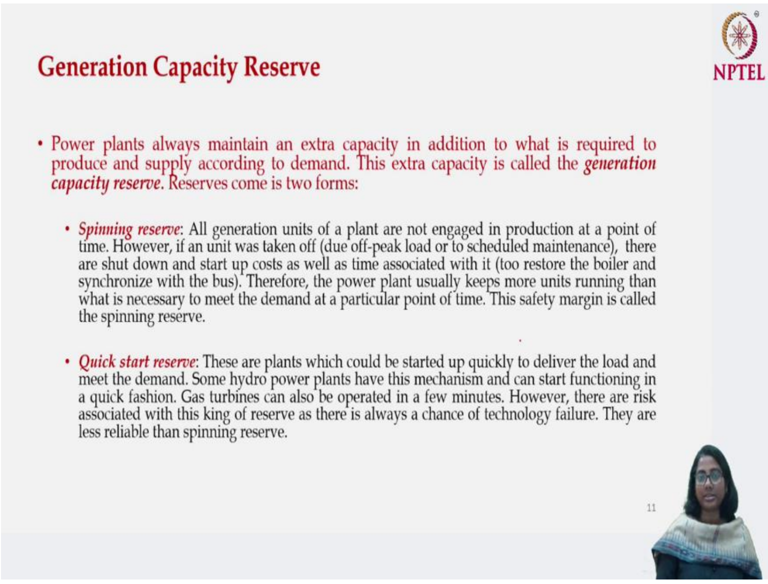

Another concept is called generation capacity reserve which follows the concept of the load dispatch. The power plants always maintain an extra capacity in addition to what is required to produce and supply according to demand. If my demand is 400 megaWatt, then I would try to be ready to produce a little more than that because of the possibility of malfunction. The extra capacity is called the generation capacity reserve. This is like a buffer, a cushion that you can use when it's needed and if not needed you don't use it.

The generation capacity reserve comes in two forms: one is called a spinning reserve and the other is called the quick start reserve. Spinning reserve is particularly important in case of fossil fuel-based electricity generation. All the generation units of a plant are not engaged in production at every point of time. Some generation units are loaded and some are not loaded. If one generation unit is taken off because there is not much demand or because of maintenance, then once the demand arises, immediately that generation unit can be run. It takes at least 8 hours to run the generation unit and also shut down and restart has some cost associated with it. Therefore, if we completely shut down one generation unit and then restart again it not only costs some money because of the restarting but it also costs some amount of time.

If the load is coming, we cannot immediately switch on the generator and start generating electricity, it needs some time. Therefore, the power plant actually runs a greater number of generators than it is required at a very low capacity. Some generators will work to a higher capacity and some generators as a buffer will work at a low capacity. This kind buffer, safety margin that the power plants keep is called the spinning reserve.

The second type of generation capacity reserve is called the quick start reserve and this is often noticed in the context of hydropower. These plants could start up quickly to deliver the load and meet the demand. Some hydro plants do have this kind of mechanism and can start functioning in a very quick fashion and you don't need to wait for 8 hours. Gas turbines can also operate in a similar manner.

However, in case of quick start reserve there is always a risk associated, if the generator unit doesn't start or if there is malfunctioning. Oftentimes it is said that the spinning capacity reserve is a much reliable option as compared to the quick start reserve. Although, they are often utilized in a very different context or with respect to different technologies.

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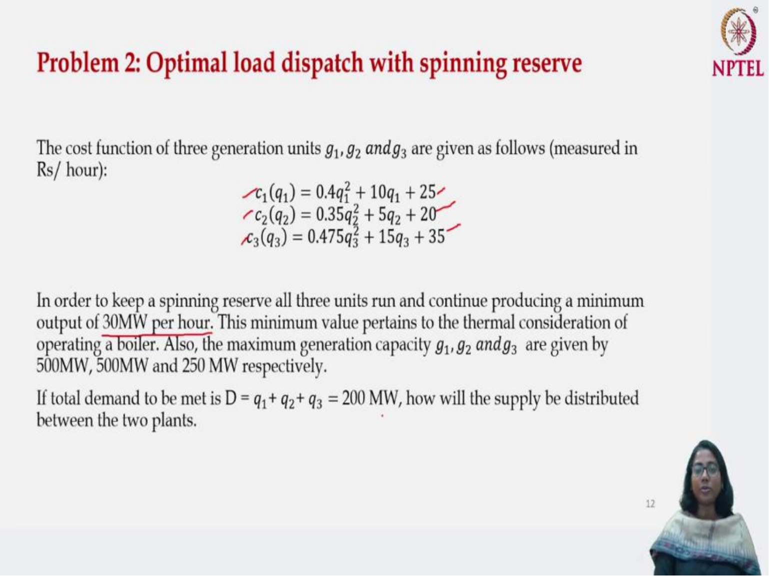

The optimal load dispatch problem becomes a little bit complicated if you think about the spinning reserve. It is not only the equalization of marginal cost but also having to ensure that all the plants under consideration that are supposed to run, have to run on a minimum load.

Suppose, one more generating unit has been added and the cost function of these generating units are given as follows. The cost function for first generating unit c_1 is given as $c_1(q_1) = 0.4q_1^2 + 10q_1 + 25$. The second generating unit has cost functions $c_2(q_2) = 0.35q_2^2 + 5q_2 + 20$ and the third generating unit has $c_3(q_3) = 0.475q_3^2 + 10q_3 + 35$. Again, notice 25, 20 and 35 are fixed cost of production and will not be reflected in the marginal cost.

In order to keep the spinning reserve, the condition is as stated. All three units have to run and continue producing a minimum output of 30 megaWatt per hour which is the minimum requirement. In order to keep the spinning reserve all these three generating units have to keep on producing 30 megaWatt per hour. This is generally determined based on the thermal consideration of operation of the boiler. These are some technical specifications. It cannot be run at capacity less than 30 megaWatt per hour, probably then the operation cost will go up and physically that is not quite possible. The maximum generation capacity of these three units are 500 megaWatt, 500 mega Watt and 250 megaWatt respectively. The first two units are relatively bigger whereas the third unit is relatively smaller.

The question is that if the total demand that you need to meet is 200 megaWatt how will you distribute the load? What will be the load dispatching when you have this added condition of that all three have to run minimum at 30 megaWatt per hour capacity?

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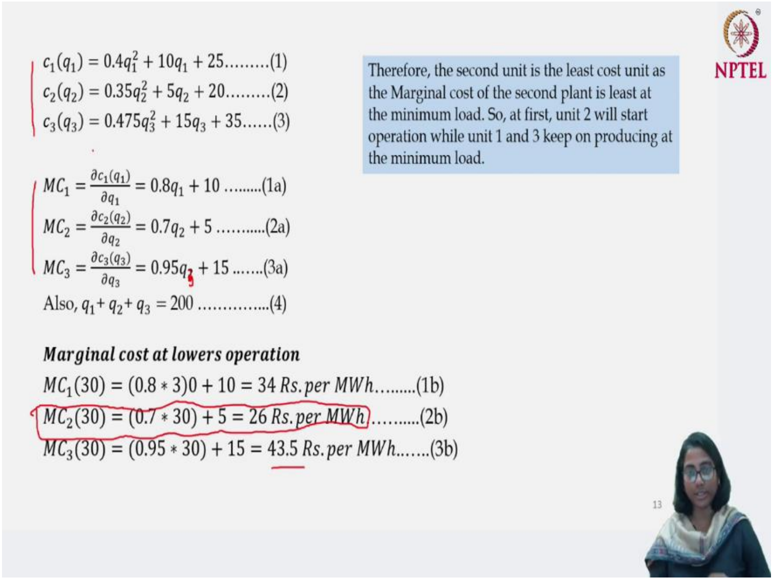

Let us have a look at the marginal costs first. Taking the derivative of these three-cost functions, we get the marginal cost functions. By taking the derivative of c_1 with respect to q_1, the taking the derivative of c_2 with respect to q_2 and the derivative of c_3 with respect to q_3 give us the marginal cost functions MC_1, MC_2 and MC_3.

We are denoting these conditions as 1a, 2a and 3a. We also have the condition which is $q_1 + q_2 + q_3 = 200$. We have 4 equations here but we will see that we actually have 3 equations to solve in order to determine the 3 variables q_1, q_2 and q_3 which is perfectly possible. Before we try to equate the marginal cost curves, we are going to see the marginal cost of each of these generating plants at 30 megaWatt that is the marginal cost of each of the plants at the lowest operation.

We calculate the value of the marginal cost at q_1 equal to 30 for the first one, q_2 equal to 30 for the second one and q_3 equal to 30 for the third generating unit. The marginal cost of the first plant at output equal to 30 is 34 rupees per megaWatt hour. The marginal cost of the second plant at 30 is 26 per megaWatt hour and for the third plant the same is 43.4.

If we compare 34, 26 and 43.5 the marginal cost of second plant is lowest at q_2 equal to 30. The second unit is the most efficient generating unit. The next efficient generating unit is the first one and the third one is the least efficient generating unit, as you can see that the marginal cost is highest at q_3 equal to 30 for the third generating unit.

The second unit is the least cost unit as the marginal cost of the second plant is least at the minimum load. The plant unit 2 will start operating while unit 1 and 3 keep producing the minimum load and it is the second unit that will be allowed to go beyond the minimum load. It does not mean that the first and the third generating units stop producing, they keep on producing 30 megaWatt. However, given the low MC, the second generating unit is allowed to produce more than that.

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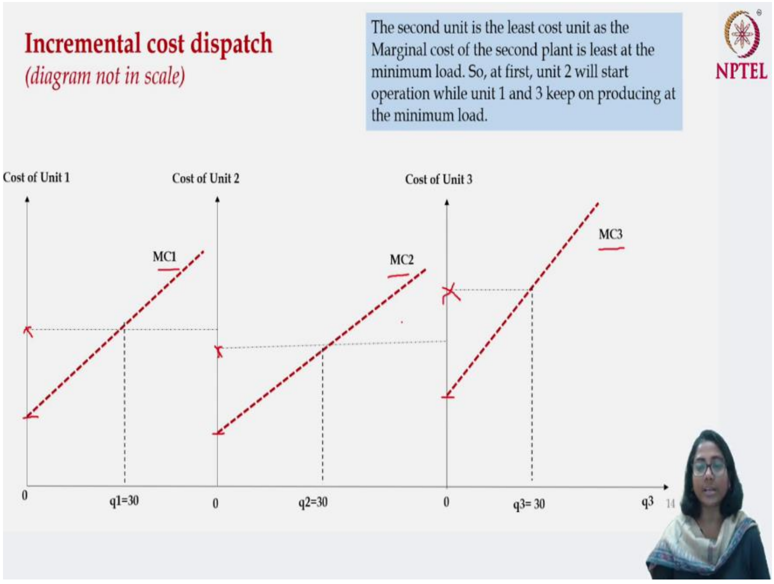

The diagram gives the general understanding of what is going on. It shows the marginal cost of the first generating unit, the marginal cost curve for the second generating unit and the marginal cost curve for the third generating unit. The intercept is lowest for the second generating unit followed by first and third generating unit.

The second thing is to look at the slope. The flattest one is MC_2 for the second-generation unit because it is most efficient, then the first one which is next and the third one is the least efficient. We can see that if all of them produce, 30 then the minimum marginal cost is obtained. Given is the minimum marginal cost that you have and this is happening for the second plant.

The next highest marginal cost is for plant 1 and the highest marginal cost which is actually quite high as compared to 1 and 2 is for plant 3. Second generating unit is the most efficient unit followed by the first and then followed by the third.

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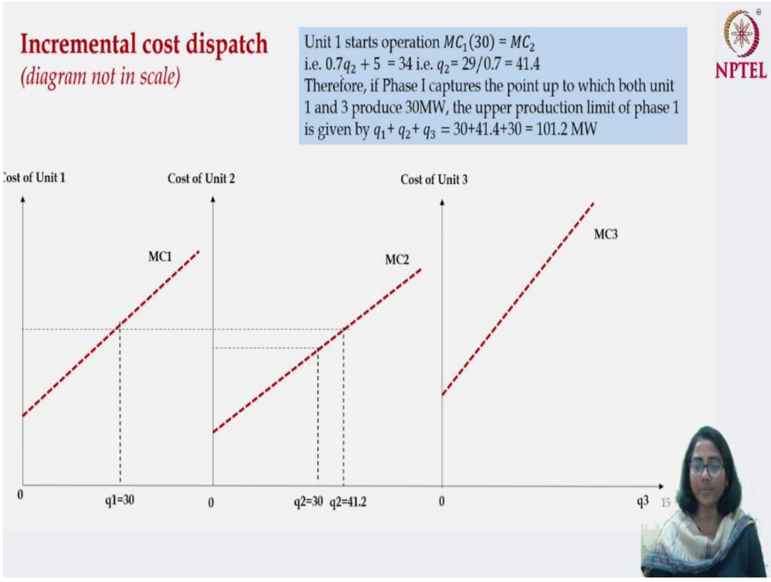

All the units will start producing 30 megaWatt that gives 90 megaWatt but the demand is more than 90 megaWatt. At least one plant has to produce more than 30 megaWatt. Now, the question is do you think that once this 90 megaWatt is produced, the rest 110 megaWatt all will be produced by the second plant or is it going to be distributed between the other plants as well?

The logic goes on like this; unit 1 starts operating where the marginal cost of producing 30 unit is equal to the marginal cost of the second plant whatever it produces that is $q_2 = 41.2$ is produced only by the second unit because if you produce anything here then the marginal cost of the first unit will be more than the second unit.

The moment the second unit produces $q_2 = 41.2$ beyond that the marginal costs are equalized. When marginal costs are equalized, the marginal cost that you need to produce 30 units in plant 1 is the same as the marginal cost that you need to produce 41.2 units in the second plant. Beyond that it doesn't matter whether you produce in the first plant or you produce it in the second plant. The condition is given by MC_1 at 30 should be equal to MC_2.

MC_1 at 30 is 34. MC_2 is 0.7 $q_2 + 5$. Equating $MC_1 and\ \ MC_2$ we get 0.7 $q_2 + 5$ =34. This gives $q_2 = 41.2$. If you think about the first phase which gives the point up to which both unit 1 and 3 produce only 30 megaWatt. The upper production limit of phase 1 is given by $q_1 + q_2 + q_3 = 30 + 30 + 41.2$ or you can say 30 from the first unit, 41.4 from the second unit and 30 from the third unit. The total production is 101.2 mega Watt. Even if you are producing at this point it gives only 101.2 mega Watt, we are almost left with some 99 mega Watt. The question is who will produce 99 mega Watt?

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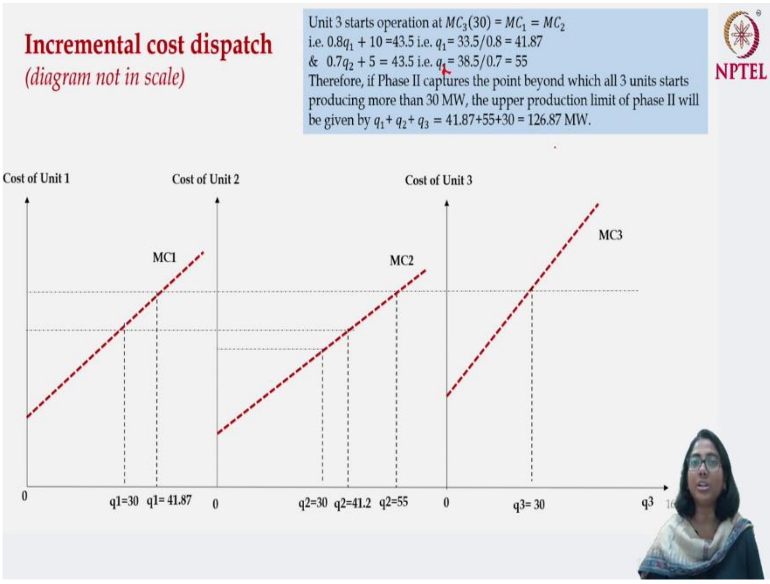

Plant 1 and 2 keeps on producing unless the marginal cost of production is equal to the marginal cost of production of the third generating unit at $q = 30$. Unit 3 starts operating at MC equal to 30 which is equal to $MC_1 and\ \ MC_2$. This MC_1 0.8 $q_1 + 10$ is being equated with 43.5 which is the marginal cost of the third generating unit at quantity 30. Also, we are equating $0.7\ q_2 + 5$ which is the marginal cost of the second unit equal to 43.5 which is the marginal cost at the level of production 30.

This gives us two equations $q_1 = 33.5/0.8 = 41.87$ and $q_2 = 55$. The third generating unit is brought into the process of production to the point where the second plant is producing 55 units, the first plant is producing 41.87 units and the third plant is producing 30 units beyond that any of the plants can produce depending on the next marginal cost.

Up to 101.2 it was only the second unit that was producing more than 30 units. After 126.87 mega Watt both the first- and second-unit start producing more than 30 units. Between 101 and

126.87, the first and the second units that are producing more than 30 units. Beyond 126.87 all 3 units are producing. The total amount of electricity generation always incorporates the 30 units that is minimum to be generated by each of the units. If your supply has to exceed 126.87 mega Watt then all 3 generating units have to run in an optimum manner.

It boils down to the simple incremental cost dispatch problem. Had total demand been 100 megaWatt then you would have seen that you don't need to use the third one beyond 30 megaWatt. The first and second one can go beyond 30 megaWatt and produce. However, since the total demand is greater than 126.87 all the 3 plants have to run optimally.

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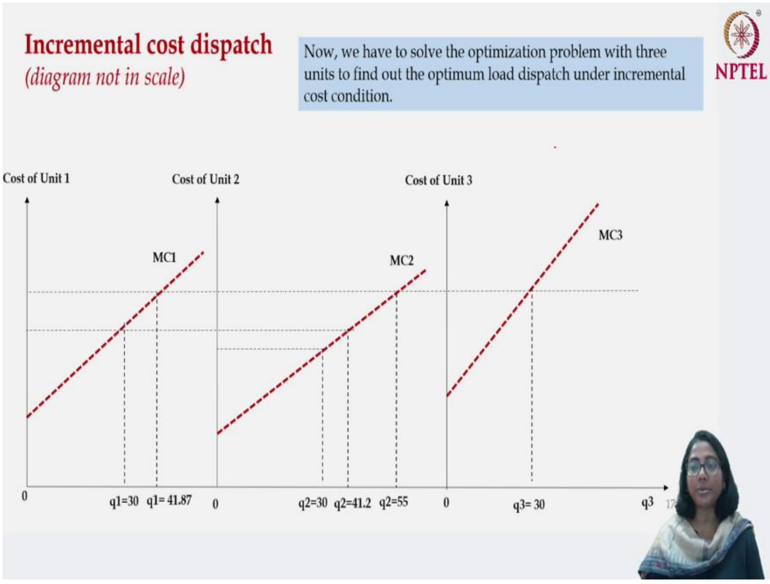

Let us look at what is the optimum load dispatch condition under the incremental costs.

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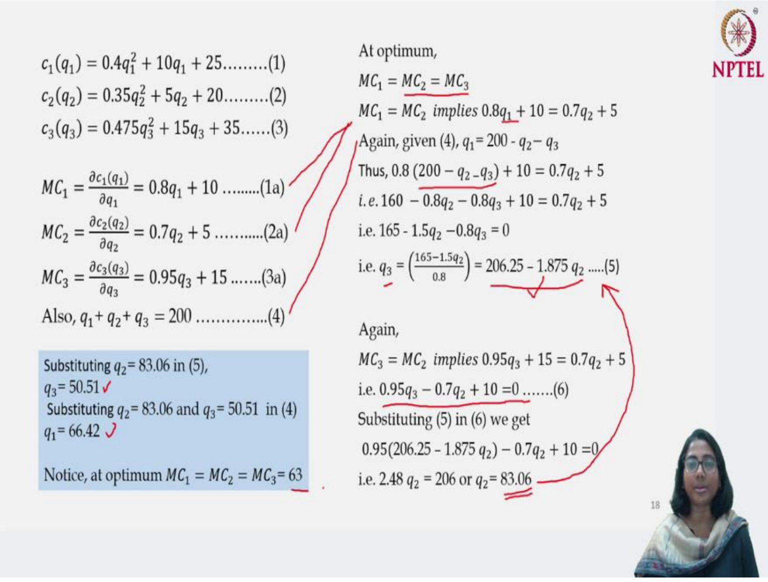

These are the three cost functions derived from three marginal cost functions and we also have the constraint where demand is equal to supply which is equal to 200. At optimum we have $MC_1 = MC_2 = MC_3$. We will first take $MC_1 = MC_2$ and then we will take $MC_2 = MC_3$ and try to solve the problem. $MC_1 = MC_2$ imply $0.8\, q_1 + 10 = 0.7q_2 + 5$. Plugging equation (4) that is $q_1 = 200 - q_2 - q_3$ we get $0.8\,(200 - q_2 - q_3) + 10 = 0.7q_2 + 5$.

This gives $160 - 0.8q_2 - 0.8q_3 + 10 = 0.7q_2 + 5$. It reduces to the equation $165 - 1.5q_2 - 0.8q_3$. If I want to express q_3 in terms of q_2 then I get $q_3 = (165 - 1.5\, q_2)/0.8$ which is approximately equal to $206.25 - 1.875\, q_2$. This is the relationship between q_2 and q_3 that we have derived.

Let us equate MC_2 and MC_3 the second condition, this gives me the equation in terms of q_2 and q_3 that is the equalization the marginal cost gives the relationship between q_2 and q_3. $0.95\, q_3 - 0.7q_2 + 10 = 0$. Plugging the value of q_3 we get, $0.95\,(206.25 - 1.875q_2)\; - 0.7q_2 + 10 = 0$.

Solving this we get, the total production by the second plant q_2 is equal to 83.06. Substituting the value of q_2 in (5). We have $206.25 - 1.875 * 83.06$. This gives $q_3 = 50.51$.

We have $q_2 = 83.06$, $q_3 = 50.51$, deducting both values from a total of 200 we get $q_1 = 66.42$. At optimum, we derive marginal cost and are approximately the same and all of them are equal to 63.

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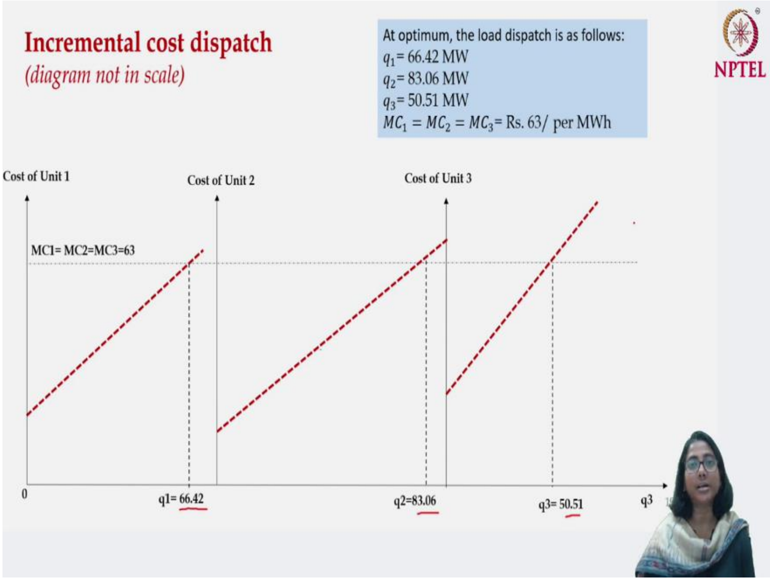

The diagram shows the production by the first unit which is 66.42, the production by the second unit and the production by the third unit and marginal costs is equalized at 63. The most efficient plant is unit 2 which is producing the most, followed by the next efficient unit which is the first unit and the third unit produces the least amount of output.

This is in a nutshell the problem of dispatching when you have the spinning reserve on and you can check what happens to the total cost. Here the total cost hasn't been calculated but the costs of different generating units can be calculated and can compare this situation with the situation where you distribute the total output equally among the three units and you can also see what happens to the cost under the problem of merit dispatch.