History of Economic Theory Prof. Dr. ShivaKumar Department of Humanities and Social Sciences Indian Institute of Technology, Madras

Lecture No. #22 More on Equilibrium: Cournot, Dupuit, Gossen, von Thunen

The study of the equilibrium picked up a lot faster among the French, and Germans in the middle of 19 th century, then it did among the British; until of course, Stanley Gevens started writing.

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But very interesting precedence to many modern usages in micro economics, new classical economics happened through these writings in France, and Germany. So what we shall do now, is to look at that, but before I go into that let me attempt to Saranya's question, can you say that again please, it is a very interesting remark.

Student: (()) answer was found in economics or outsiders.

Right. Is she is saying the answer to the question, why were there entrepreneur when there was the zero rewards for entrepreneurship in there in the Walrasian system, and she says, could the answer have been found outside of the economics. You are absolutely right.

In recent times attempts have been made to explain this institutionally; in terms of certain types of behavior patterns; in terms of certain social psychological rules which probably, the entrepreneurs were following. Whatever, you are right. I mean there is no logical economic answer to this question, and the answer is probably either not available within the economics of Walrasian systems or the answer is available through institutions outside economics is it right. One of the most interesting work pieces of work done at this point in time simultaneously, when people like Leon Walras was doing their thing. You had people like Cournot in France, a mathematician doing very interesting piece of work Cournot like J B Say, he was a follower of J B Say rejected any notion of absolute value in economics. In the sense that, we did not need any absolutes what J B Say argued and what others said even at his time and later was that.

You do not need the notion of an absolute value to talk about what happened in the market place, because what happened in market place happened between buyers and sellers, between demand and supplies. So, why do you need any of these abstractions in the market? Cournot was one of them, he said you do not need to believe in labor theory of value, embodied labor and all that sort of; all that stiff all that sort of stuff, you do not need that in the process of understanding what happened in market between players in the market.

What is enough is to see that, there were prices in the market which measured as an index of the way the economic actors behaved in the market. Secondly, Cournot went a step further and said, you do not even need the idea of utility in the market, not because he thought it was irrelevant, he certainly thought absolute values were irrelevant in economics to understand the market. So, he did not think that utility was irrelevant really, but he was one of the earliest ones to discover a very central problem with utility; very central problem which did not get sorted out till Hicks arrived in the picture. This is a problem which started with the definition of utility by the early utilitarian like Bentham.

Bentham was staunch believer in smith. So he believes that, the driving force is the preference for pleasure among people that is Bentham. He also believed that you could

measure the contribution of objects, situations and people to each others happiness or pleasure by a thing called utility.

You might do something for me and that might have utility for me. I might I might eat an apple and that has an utility for me. Utility then became a very central criterion among the utilitarians, not just in measuring the benefit through physical consumption of goods, but through whole lot of social situations. For instance, utilitarians were strongly supportive of Ricardo when he fought the landowners who work on law, because they believed that rent earnings have no social utility. So, all landlords must be discouraged gradually in the British economy, because they were dysfunctional. So, utilitarians talked about utility in a wide variety of situations most of them social. The central thesis which not all of them subscribed too, but it was a central thesis was that utility is measurable. You could measure the utility; there was calculus of utility as Bentham use to say.

Now, Cournot ran into this problem; how do I measure utility? He was a mathematician that, how do you measure utility? He did not know how, so he rejected utility. So he rejected utility, not because it is irrelevant. He rejected utility, because it was simply very problematic measuring it. The problem of measuring utility were very significant till the 1930s when Hicks found a way, and then later Samuelsson found another way of talking about utility without having to measure it. So when Hicks brought in his analysis of comparability of commodities and indifference among them, the problem of measuring utility was sides stepped. The cardinal measurement became ordinal measurement; we all know this in micro economics, but that to the long time incoming and when Cournot was writing, hicks was not around and offering the solutions. So when Cournot wrote he said, well I am not interested in utility, because I cannot measure it. So he rejected absolute values and he rejected the idea of utility underlying consumption.

So he looked at price formation in a strategic sense. So he was not looking for a general equilibrium solution as was Walras, he was looking for situations were pricing became a strategic decision making process. So he argued first of all that, if there is a monopolist supplying a product, how would he behave with respect to his customers, and he invented for the first time in economics, the marginality rule that goes for all firms today, that marginal cost equals marginal revenue; it came out in Cournot's writing first. Cournot says, a monopolist maximize his profit if he behaved in such a manner that, marginal cost equals marginal revenue and today you know that not just monopolist, but

everyone including those under perfect competition have to follow this marginality condition to maximize profits. So this comes from Cournot. But the thing which was very interesting and which today is coated as a standard paradigm is the study of duopoly. Which is suppose, there are two producers and each of them trying to control the market and neither of them being fully able to control the market. There behavior in the market would be a response to how they expect the rival to behave.

So they would quote a particular supply for a particular price hoping that their rival would be quoting something. So if the rival quoted so much I am going to quote this. So if this one quotes this the other one says, if he is going to quote that so you know, a succession of expected behavior of the rival prompted each participant in the duoploys market to make his own quotations in the market. So this was strategic equilibrium; this is not an equilibrium in the conventional sense, either in a partial equilibrium analysis or general equilibrium analysis. So, how does this happen?

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Let us look at Cornots basic model. Now, we have here two players in the market in the duopoly. The small a small a... There is a little mistake here, you must notice that the two curves are intersecting; the the the symbols are misplaced. So assume that, the flatter one is small a small a; and the steeper one is small b small b. So these are a's and b's response curves in the market. The two axis represents prices and supply. Now start with some price, initial price P 1 which is prevailing in the market and let us say a's response

is to b at his point D, which involves some supply, but if a is producing D then, at that price b is going to produce E, and if b is producing E then, a is going to produce F, and so on; it goes on till eventually, it will resolve itself at point G or Z right. At that point, there is a coincidence of expectations; each is expecting the other to be at G and therefore, that is the equilibrium. Is that clear? So Cournot's equilibrium is a strategic equilibrium even today this equilibrium, because it is stable.

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It ensures stability, because both producers, both suppliers have reached simultaneously the same expectation is called the Nash Cournot equilibrium even today. What was proved through the simple duopoly model of Cournot can be established game theoretically and therefore, it became Nash Cournot equilibrium. What is important in this analysis of Cournot is not simply that he was looking at duopoly. He was bringing in a very major factor into the reckoning. He was pointing out that, market responses by players in the market or supply responses by players in the market were often condition by each others behavior in the market. Which then means that, all market could be looked upon as strategic behavior, and that takes one step further the argument then, such strategic behavior must have other reasons as motivation then simply profit maximization. You see I am, suppose I am operating a taxi service between IIT and the airport and back, and I have rival whose operating a taxi service and I am constantly timing my schedules, my rates, everything in response to what I think my rival is scheduling them as. So, what would be my central most preoccupation? Do you get my question?

Student: (())

Again. I am talking about, when pricing becomes a strategic issue, the whole lot of motivations which are assume in a market economy such as profit motives may become irrelevant as a part of strategic decision making. And I am saying, as an example let us consider a situation where am running a taxi service from IIT to (()) and back. Yes.

Sir, how does profit decisions become relevant to the strategic decision making?

That is what am asking? That is what am asking? Will people be concerned with profits at all or would they be concerned with something else is what am asking? As an illustration, I am going into this particular taxi thing. Suppose, I am running a taxi business between airport to IIT and back, and I have another guy whose operating a similar taxi business between airport to IIT and back, then I am all the time scheduling my time schedules, scheduling my arrivals in airports, departure timings, my rates, tariff, everything in response to what I am expecting my rival to do. If he has 5 rupees per kilometers a rate; I say, sir I can offer you 4 rupees 75 paise. So he sees or if he is getting in time to airport to pick up passengers arriving in a particular international flight, then I look for more crowded international flight and make sure I get there in time first. So, all that is happening all the time is I am thinking as my opponent, my rival would think and trying to evolve my strategies right.

Sir, I do not think we can see the profit becomes irrelevant, because there will be a point beyond which, they will be under cutting of price I mean, you know there will be no price under cutting. So it does mean that somewhere you know, they have if the if the m c and the m r i I guess, they become equal beyond that they would go lower than that.

Student: (())

Yeah, so when am saying you still cannot say the profit is irrelevant; I mean, even the whole under cutting thing is happening with why. I mean that is mode of a the whole strategic game is happening keeping in mind (()).

Accept. I accept this which is why, I started with the question, is profit relevant? I did not say profit is irrelevant, but what is important at this point is to show that, there can be a vertex developing here; vertex of strategic behavior in which each is trying to get an advantage at some loss over the other.

Student: But, the vertex will converge it a point of equilibrium.

It may or may not. It would not go I mean, at least what do you studied does not go lower than that (()) below the.

Student: (()) they might calculate the point of.

So we are now talking of more strategic behavior. So you have a meeting with each other and say, listen my pal we do not want get destroyed completely, so let us assume that you know till somebody else comes will follow the rule that we will not charge below this per kilometer or will not operate less than; will not operate more than this size of a fleet each and so forth. In other words, you get together to create a collusive equilibrium right. One way to end the vertex is to have a collusive equilibrium, and that collusive equilibrium is a is fundamentally fragile equilibrium; fragile because, does it answer your question about your profits? We still have to answer. She says, in all the strategic behavior, why do you want to talk about profits at all? Is that about you ask?

Student: I was (()) but that is what she asked.

What was she asking?

Student: How can profit become irrelevant?

Relevant?

Student: Irrelevant.

Irrelevant. While that that is what we are discussing right now. Profit might be relevant or irrelevant, but survival is relevant here in this process, because they understand that fundamentally the best way to have profits is to have 100 percent control over the market, and that is not possible when the other guy is around. So you have strategic decision making, and the strategic decision making lees to a vertex of negativity and losses, and this is overcome when you reach your collusive equilibrium through an collusive arrangement, which sort of preserves you. So survival is possible in this market through collusion; growth is possible or profit is possible when one of them goes to some other market and starts making profits there.

Anyway, the point I am trying to make at this stage is the work of Cournot draws attention to the fact that when market is not competitive. There are whole lot of criteria which may come in as important considerations in the mind of the participants in market, which may not be profits alone. It is true that competitive markets ensure profits, but the moments markets are not competitive, then there is all kinds of predatory behavior which is possible in the market know. This whole approach to markets, to the market economies where individuals may not be concerned with profits, because the markets are not competitive and markets are following other rules, and there are whole lot of other considerations that are coming to the minds of the economic actors.

This idea lies at the heart of the theory of new theory of industrial organization. I had an occasion to mention this in one of the earlier classes. Did I not? Some of you were around in that class. I was talking about Bane; I was talking about market shares. Certainly all of you were not absent in that class. Anyways so, I will quickly recap. The theory of new industrial organization commences around 1960 when Hayward economist called Bane, produces a classic called entry barriers in American industry.

Entry barriers you remember right. And he started a whole lot of debate on what is industry in doing in the US? What is business doing across the world? And you found that, the prime consideration for people in the market, in that kind of a situation was market share, and we discussed a whole lot of reuses or strategies or tactics which are used in order to maximize market shares, or to hold down to market shares right. What is crucial here is to understand that everybody seems to be in the modern corporate world as perceived by Bane. They seem to be thinking very much like Cournot's duo policies, not so much in terms of supply response, but in terms of each is trying to response to the anticipated or expected behavior of the rival. So Cournot model is one of the earliest works in economics, which talks of this kind of behavior, which is been substantially picked up by this new theory of industrial organization, and there is lots and lots of work done in that. One of the important aspects of this is, there is an extensive work done, the theory of contracts, implicit and explicit contracts. You might leave, you might sign an explicit contract with a particular business partner, but your market situation might be such that, you might need a lot of other guarantees, then simply what is explicitly mentioned in these contracts. So you had a lot of lines in the middle which will imply more guarantees for you. Now the theory of contracts tells you that, the more and more and more elaborate a contract gets the more and more expensive is to enforce the contract. So it is a optimal trade of between the size and detail of, and the complexity of a contract and the cost of enforcement of that contract. Now all this literature in a sense follows the line developed by Cournot, so that is how important Cournot's work is.

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Now let us look at another economist from France Dupuit. Dupuit is a pioneer in yet another sense; Dupuit says that, all public utility, he focused on public utilities in the sense that, what are public utilities can you tell me?

Student: Natural (())

Natural.

Student: Monopolies.

They would not be entirely natural monopolies. For instance the road is a public utility. (()) I mean. A bridge is a public utility.

Student: (()) non rivalries.

Yes, non rivalries and public. Even that is a public good thing; even that is a public good argument. Public utility is more a case where, there is a very elastic demand curve facing the producer, and sunk cost concepts come very heavily into public utilities. You have to incur the expenditure, and recover a revenue over a very long time, over a large clientele base. You dig, you inference you make a road; you recover the cost of having made the road through tolls; as more and more number of people are using the roads, you collect more and more tolls and recover more and more of fairs of costs right.

Now, public utility have this kind of a problem bridges, roads, dams, many of these things. Some of them have specific ways through which you can recover cost, but some of most of the public utilities, the problem is always been, how do I recover cost when the number of customers in the long run will be open ended. So, if I know there is a finite number of users of this utility, then I shall be able to a plan some tariff through which I can recover; if I do not know then, what tariff shall I plan? What is the best way in which the government which uses public funds to start a public utility, how can it maximize public welfare in this process.

Dupuit found it a very simple rule. He says, use marginal utility pricing. Whenever marginal utility is declining, you can keep pricing the product lower and lower and lower and lower and lower and maximize public welfare. We say something like a surplus here, a rent here. In this diagram for instance, you find there is a marginal utility of the utility, which is shared by all the customers and as the price stops from P 1 to P 2.



You find that the total collection of revenue is P 2, C 2, Q 2, O. but something like a rent remains with the public, because the total utility gain is R, C 2, Q 2, O right. So maximizing public welfare by pricing public utilities, Dupuit made this illustration and showed, the region R, C 2, P 2 which is a surplus utility gained by the public goes on in increasing as a prices go on dropping. So he says, when marginal utility of a public utility is is is declining, then they lower the price, the higher the public welfare. This is Dupuit's argument.

Now, Dupuit is truly the pioneer truly the pioneer of an extremely complex issue in modern economics pricing a public utilities. For instance, question is I am running a bus system for the city; I have 20,000 buses traveling all kinds of routes, and all kind of distances, with all kinds of stoppages; how do I plan the tariff? How should the bus fares be decided? It is a big issue or alternatively, I built a huge road, there will be any number of users who come and use this road. How do I decide what tariff the tolls will charge? So you have or I built an airport, a number of planes will come and land there and take off. How do how do I charge the tariff for the aircraft companies which use the airport?

Most important, I generate electricity and vast number of customers are using this electricity. How many rupees should each customer pay per unit of electricity sold to him? The problem of pricing public utility is a huge problem which is continuously theoretically challenging economist worldwide over the last 50 years. Various answers

have been found, various solutions have been found, various models have been tried from very simple calculus based mathematical models to entire computer simulation models involving 15, 16 variables. All kinds of things have come into existence, but for all this the pioneer is Dupuit's work. That is how important Dupuit's work is.

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Let us look at the couple of Germans, Gossen. Gossen's work was very profound, very simple, very elementary. He is the one who actually formulated that, utility at the margin is declining through consumption. And second, he is the one who also formulated the idea that, you can maximize your satisfaction by trading of consumption of commodities according to the levels of their marginal utility. You go on consuming the two commodities till such time as the utilities, marginal utilities of the two commodities become equal. This is something very basic we know now in micro economics called the equimarginal utility principle, but Gossen is the first profounder of this rule.

So you can see that, some of the things which are very common to us today, they have their origin quite some time back. So if schools are teaching equimarginal (()) principle today for instance, when I was taught equimarginal (()) principle in my school, I just assume it just came from the heaven; you know like everything in economics yes.

Student: David told that it was (()) by Albert, Albert Marshall.

No, he was long before Marshall. See Marshall put a lot of things together, but the first idea of equimarginal utility writing is in the writing of Gossen some decades before Marshall. It is true that Marshall synthesized everything in very nicely, but if you want to look at the provenance of the idea, the provenance goes to Gossen

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Another concept in economic which is very very common today without which, you cannot have micro economics at all, the concept of opportunity cost. Von Thunen was a one who first pronounced the idea of opportunity cost. What is opportunity cost today as you know it? Krishna?

Student: The cost of giving up one good for another. The cost of the alternative that (()) the other.

Yes.

Or the loss of the alternative used know. Now, the idea that resources could have competing uses and competing ways of utilization, and if you choose one particular way of utilizing resources, the opportunity cost of that which is the alternative which you gave up, is an index of the value of that utilization. This idea of a opportunity cost came from the works of Von Thunen. Von Thunen also formulated a fairly, clear cut theory of distribution; he argued that all factors of production were paid in accordance with their marginal products. They earned their rewards in accordance with their marginal products. So he generalize this to say that, different segments of societies such as capitalist, workers and so forth, they earned their rewards in accordance with the performance of the resources which they supplied into the process of production, and the way the marginal product behave. So, Von Thunen introduced this theory of distribution as a fair and a just theory of distribution.

In contrast with many socialists and at the (()) time, what devotes student sorry student of Marx were also arguing that capitalism is unjust, unfair and so forth. So, Von Thunen was the one of the earliest one's to say, well if you are getting the equivalent of your marginal product, you cannot expect anything better; it is a fair and just system. So the the the articulation of the first version of the marginal productivity theory of distribution must also be attributed to Von Thunen.

Finally, Von Thunen was also trying to explain when firms have different level of efficiency, when they have different level of competence dealing with the markets, then different firms are doing differently in the markets. Some firms are making more than other firms right. So they might include their cost; they might include their normal profit in their cost function. but they might make something much more over and above the normal profit purely, because relatively more efficient. So, how do you explain this phenomenon? Von Thunen thought this was very much like the problem of rent. You had land of different qualities, varying qualities, when they were cultivated, we know that land which was more efficient, more fertile leased hire rent and that we know Malthus Ricardo both of them argued that it was a differential rent. That is a rent that comes about due to difference in level of fertility right. Either extensively, you can talk about more fertile land being cultivated first then, there less fertile land and the difference in earnings between net earnings, between the most fertile and less fertile is the rent we know that.

Von Thunen was saying pretty much in the same way, your firms was varying levels of efficiency in the market. So he did not assume perfect competition and all those things; he said, firms could have varying levels of efficiency. So the firms too make a differential rent right. So he introduced a way looking at the industrial economy, which use very creatively the idea of sorry excuse me which very creatively use the idea of the rent.

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So Von Thunen's rent is something like this; suppose you have firms A, B, C, D in the market and the price is let us say some C 2, equivalent to some cost C 2 of firm B; so firm B is selling in the market such that, its costs and price are actually equal. In a Marshallian sense, this is a firm which is just making its normal profit right. but the other firms look at A, C and D, their costs are all below the price. Is not it? And each one of them is making a little rent; A for instance is making a less rent than C; and D is making even less rent, but all three of them are making a rent while B is not making any rent. So, Von Thunen was talking of differential rent among industries, among firms as occurring due to differences in levels of efficiency and competence. This was a much earlier idea then the idea of abnormal profits which you find in Marshallian writing.

In fact, it might appear that the Marshall, the discussion of Marshall on the optimum and representative firm; are you aware of that discussion? Marshall talked of two types of firms. He talked of an optimum firm where, marginal cost equals marginal revenue in the short run, and an average cost equals average revenue in the long run. There are no abnormal profits. So this is an optimum firm, but Marshall also said, at any point of time a firm which is representative of firms in general, might not be an optimum firm, something like a median firm. You know what is a median? On a model firm is better, something like a model firm which represents a very large number of firms in the industry. May be making an abnormal profit; may be making a little bit of loss we do not know, because the economic conditions are constantly changing, and the response of

firms are changing economic conditions, the process of continuously adapting themselves that is going on.

So a firm might hit an optimum point at one point, might slip of it and go into a lost next time or go into an extra normal or abnormal profit. So Marshall distinguish between representative firm and optimum firm. According to him, a representative firm is a firm which may or may not be making normal profits, but it might be making abnormal profit, it may be making losses, it all depends upon the context and the situation in which the firm find themselves, how they are adopting themselves to changing conditions. So Marshall's optimum firm was a kind of a theoretical best, and he was very clear to emphasize that in actuality, a representative firm which was a firm, which was some kind of a model average need not necessarily be behaving like an optimum firm at all.

Now, this interesting distinction is slightly preceded by Von Thunen's argument about differential rent. He is also talking about efficiencies and performance levels of firms in the market. He is also talking about some firms making a little extra which is Marshallian abnormal profit, and some firms which do not make anything. So he says, buy and large, this is important. According to Von Thunen, most firms are firms which are varied in their performance usually the price in the market is a price which equals the least efficient firm costs.

At any point in time, firms have different levels of efficiency, the least efficient will have the highest cost. According to Von Thunen, most of the time in the market, the prevailing market price equals the cost of the least efficient firm which is highest cost. In this particular case, it is firm B which has the highest cost. So he says, all other firms earned differential rent. So once again you find a very interesting situation here. An argument which certainly predates the discussion on representative firm, which Von Thunen says, you may not have a representative firm in the picture at all, and we do not know Von Thunen says, we do not know what criteria of efficiency will measure levels of efficiency in firms. It was left to Marshall to bring up this concept to bring in the notion of u shaped cost curve in the short run, long run and so on and so forth. but Von Thunen was a head of Marx, so he could not say all those things. In his time what he could say was, the most the prevailing market price is equivalent to the highest cost prevailing in the market which is the least efficient firm. All other firms which are more efficient end up making something like a rent, differential rent because, it is a rent going to differences in their skills, just as differences in fertility of land existed so little comparison.

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See this is how Ricardian Malthus differential rent will work. You have different levels of fertility in a declining order there, and you have wages determine by O, W and profits determine by P, W and the whole region above P is rent, and you can see that lands with different levels of fertility are making lesser and lesser rent.

Finally, you get to a land, piece of land the 1, 2, 3, 4th piece of land which is just enough, making enough to cover profits and wages. Any of the 5th piece of land which does not even make profits, it just covers the wages. So the idea of a differential rent is basically Malthus and Ricardo acknowledges although, it is called Ricardian rent, Ricardo acknowledges that, it is Malthus who first wrote about it. So which is why I said is Malthus-Ricardo rent, so Von Thunen's differential rent as you can see is substantially similar to and comparable with Ricardian Malthus-Ricardo rent. Only thing is, Von Thunen's application is much wider than the case of land, which is what Ricardo-Malthus application was.

So with this we kind of come to an end of discussion of different kinds of equilibrium theories were thrown up. At this point in time, we came from, we started with Say and went on to Walras to show, how the orthodox is developed, but outside of that orthodox we saw a few illustrations of how people were thinking in terms of equilibrium and efficiency in the market economies, and how they were achievable. Now, I am open to questions, remarks. Anybody? Then, I will go one step further and explain this idea of differential rent, and how it is applicable in again in modern economy.

You see most firms in modern economy corporates, their pricing strategies are influenced by two sets of factors. One cost plus margin, which is a pretty Von Thunen kind of a situation. So, every firm plans its pricing strategy in terms of what is my total cost of production including provision for my margin and then, I mail a make a little extra margin on top of that, which is my cost plus basis. So all commodities are fundamentally priced on a cost plus basis, but how much more plus? Here the criterion that comes into operation is what is the pricing strategy of my rival? So I want to make it a cost plus strategy, but I want to make sure that I do not lose out my market to my rival.

So once again, you have everybody trying to fix the highest possible cost plus as a price within the limits, which is very reminiscent of Von Thunen, what Von Thunen saying right that,

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I try to for instance, maximize my differential rent here. So if I am a firm whose able to cut cost, minimize my costs then, I can have this differential rent, which is not possible for others who are not able to cut costs. So the economy of costing and pricing in modern business is not so much like the marginal cost pricing or average cost pricing which goes on in the theory.

They were composite notion of costs which includes a certain margin for management, entrepreneurship and so on, and so forth. Certain provision for contingencies; this composite cost is what is available to the firm, and the firm is now looking for a pricing strategy over and above this, and while it is doing that, it is looking around at the market see what the rivals are doing right. Pretty much, what should be happening in Von Thunen kind of a market, they look at firm B, and say we do not want to be like firm B, we want we want to be a lot better, so let us cut cost and become more efficient. So they may become like firm A or C or D.

So once again, the application of this idea that firms are making rent is very significant, because in actual pricing strategy most modern firms are actually trying to make something like this Von Thunen rent. Will break it up today, and get back on Saturday, and starts with socialist and Carlmarx.