**Logistics & Supply Chain Management**

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**Lecture 51 : Assignment Problem Programming**

Hello dear friends, welcome back to NPTEL online course on logistics and supply chain management. So, today we will start discussion on very another interesting topic which is taken from obviously the operation research which is assignment problem programming right. So, in this chapter we will see how we can assign different task to different people or other resources or different task on different machines or we can say this can be extended to how we can assign different markets to different factories from where we can you know transport the material to those from those factories to those markets. So, that the overall scenario will be optimized, optimized in the sense means the total cost can be minimized or if we are talking about the profit function. So, profit function can be maximized. So, this is the content, we will just introduce what is the concept of assignment model and then we will talk about some mathematical representation, how we are representing in matrix form.

We will use Hungarian method to solve this assignment problem. A couple of examples, numericals I will be showing you and then we will discuss about advantages and what are the basic assumptions which are posing a kind of limitations on this assignment algorithm right. So, as I told you the assignment problem is a kind of you know case of transportation problem only and you if you remember in transportation problem what we discussed from where that the material should be transported from that particular factory to particular market location. So, the overall transportation cost can be minimized same way here we are assigning resources to different activities like I told you manpower is also resources if we have 5 different activities we will allocate those 5 different activities to 5 different manpower like 5 different jobs on 5 different machines right so this is the basic concept under assignment problem.

and because why this is required because in terms we are limited with the resources if you are talking about manpower if you are talking about machines if you are talking about factories we are limited with the capacity right so how to assign workers to the machine how to assign markets to different factories from where the material can be you know transported and then salesman to different sales area now it's not only the requirement of that particular activity or job it is also the conditions are lying with the

resources side as well if we are giving you a particular task to different manpower you should know about the skills the talent of all the manpower available to you right only, then you can match whatever the job requirements are, and then what that person can do right so in that way you can see if we have n jobs to be performed and n persons are available this is another very important point that number of task should be equal to the number of allocation means where we are allocating to any if we are allocating to people if we are allocating to machines right. and then once we have done the allocation before that we should know that what is the cost if I will assign job A to person 1 if I will assign job A to person 2 3 4 5 all the available manpower similar is the case with the machines also if I am assigning this job on this machine what will be the total cost If I am assigning this particular factory to this particular market what is going to be the overall cost when we are serving those locations markets from those factories right. So, the very basic assumption we are assuming here that the per unit cost is known to us right. So, this is the assignment model again because this is I told you the operation research part which is taken from linear programming problem and will try to convert this overall problem into LPP problem right. So, here only one assignment is done that means one person will be assigned only one job.

We cannot assign two jobs to one person. Similar is the case with other resources as well right and here what we are doing because we are knowing with the what is going to be the cost of you know performing that particular function by individual machine or by individual person and then the objective is how we can minimize the overall cost. So, before going further you can just go through these couple of videos I have shared on this assignment problem how we are you know shifting the material from one point to other point and then we can go further with the next slide. So, this is how we are representing this assignment programming problem into matrix form. See all the resources are written on the row side so if you are let's say you are scheduling manpower so manpower will be written man m1 m2 mn right and all the tasks job 1 job 2 job 3 job n are written on the column side right this is c11 any c is representing the cost of assigning job 1 to man 1 or assigning job 1 to machine 1 see 1 2 is representing assigning task 2 to machine 1 similarly we are having all the possible combination pair wise comparison what is the cost if i will assign job 1 2 3 4 5 and jobs on ah machine one right and then all these cost on machine two three four five all the available machines right so if we are knowing what is the cost right of performing that particular task on that particular workstation then we can you know optimize our function So, as I told you Cij is representing the cost of resources i to task j.

So, if we are assigning, let us say if we are assigning job 3 on machine 1, we will write 1 here and rest of the cell will be 0 because only one assignment will be done in one row

and one column. So, one machine, one job can be assigned only on one machine. right so same is the case with the other resources so in that way if I am saying that this Cij is the cost and if I will say Xij is assignment if I am assigning to particular resource then I will write 1 if I am not assigning I will write 0 so in that way the total overall function will be Cij into Xij which we need to optimize either minimize or maximize. So, this is the objective function CIJ is the cost and then per unit and then whether you are assigning that task to that person or that machine accordingly you will put 1 or 0. So, this is the function we need to minimize.

see it can be maximization problem as well if this is not cost it is representing p and n profit of assigning job n on machine 1 so then that means we need to maximize it right so we have two different cases one is minimization another one is maximization so maximization problem we can convert into minimization we will see how we can do that right so you can follow rest of the algorithm same way right so we have discussed that only one machine can have one task or one task can be allocated to one person or one machine only. So, that means if I will assign all the allocations are done So, if I will sum up the column values or row values it will be always 1. So, because only one allocation is done. So, same is the case with the rows, so same is the case with the columns. So, another important concept here is whether we are getting the balanced problem or not.

If we are having 3 machines, 3 jobs that means it is square matrix. So, whenever we have square matrix, this is a balanced problem. When we have this m is not equal to n, that means, i is not equal to z, means number of rows are not equal to number of columns, this is known as unbalanced problem. In that case, we need to convert the problem first into balanced. How we can do that? We can add rows or columns and then we will make the order of matrix equal.

square matrix we need to confirm first right. So, we will see that special case how we will make the unbalanced problem into balanced problem. So, the Hungarian method is used to solve this assignment problem. So, this is the first step. First, we need to develop the cost table.

All the assignments cost table means all the values we need to find out. So, what if we will assign task 1 on machine 1, task 2 on machine 1, task 3 on machine 1. Similarly, all the tasks on machine 1, all the tasks on machine 2, these cost values we will record in the matrix. We will fill these cost cells. So, then if the problem is not balanced, we need to add dummy row or dummy column.

Let us say we have this problem. So, here because we have job 1, we have job 2, we have machine 1, we have machine 2, machine 3 because it is not square, this is not balanced, I need to add the dummy job. So, here the cost value will be assigned 0 0 0. right this is the concept how we can convert the unbalanced problem into balanced problem so we will add either you need to add column if number of jobs are less than number of resources or if we need to add rows if number of resources are less than number of jobs vice versa right so this is the first step second step is find the opportunity cost table now what we will do along that matrix we will go row wise first row will check and we will find out which one is the minimum value in that row and we will subtract that minimum value from rest of the values and that value itself So, when we will do that, we will get at least one 0 in each row. Let us say this value is 1, 2, 3.

So, what is the minimum value? Minimum value is 1. We will subtract this 1 from all these three values. So, here it will be then 0, here it will be 1, here it will be 2. So, we will repeat these steps for all the rows. Once it is done for the row, then we will do the same thing for the column as well.

So, from column 1, 2, 3, n column will go 1 by 1 will find out the minimum value and then will subtract that value from rest of the values in that particular column. so when we will be completing step two in each row and column there will be at least one zero because that we are ensuring that this is opportunity cost table right then the second thing is make assignment in the opportunity cost matrix so how we will do that first we will examine the rows successively first we will find out the first row right and we will see whether there is one only one zero if more than one zero you leave that row and go to the next one right if only one zero is there you will circle that zero and assign that particular cell let's say here only one zero is there and here value is one and two So, you will assign this value that means job M is assigned to machine 1 and rest of the zeros in that particular column you will strike. rest of the 0s in that column or row you will strike off that means those 0s are now not available for the allocation. So, similarly you will go for row 1, 2, 3, 4 n number of rows it is done and then we will go for columns successively. So, the concept is very simple how we will make the assignment.

We will go for first row, we will check whether only one 0 is there, if it is there then we will you know encircle that 0 and we will make that assignment that this cell value xij will be 1 and all other 0s in that particular row or column you will strike off. Same is the case for all the rest of the rows and for all the columns. Now, this you will continue this task until all the value, all the rows or columns are assigned right, all the zeros are circled. Now, we will check. so once you will encircle what is the meaning of encircle

that that means if I am putting this square over that 0 that means this particular assignment is done similarly if I am doing writing this 0 is here and if I am putting square here that means job 1 is assigned on machine 3 Similarly, we will make all the assignments.

Once the assignment is done, obviously what was our objective function? Our objective function was summation of Xij Cij. So, wherever we are assigning, we will write 1. If it is not assigned, we will write 0 and particular cell value will be multiplied. So, we will find out the total cost. This will be total cost and once we will achieve the optimal solution, this cost will be minimum cost.

So, now how we will check whether the solution is optimal or not? If the assigned cell values are equal to the order of the matrix, if you have done 4 allocations, order of matrix is 4 means number of rows or columns, then this solution is optimal solution. if it is less than 4 the order of matrix then that solution is not optimal obviously you cannot have more than 4 order of matrix right that is not feasible right that is not possible so once you will be having the allocation number of allocation equal to order of matrix that means each row and each column must be having single allotment only. That means only one job is assigned to one person that you will ensure. Let us say this is optimality condition your solution is ended here. So, you can give this is the optimal minimum cost.

of that assignment whatever jobs activities we are assigning to different resources now the second thing is what if these are less than 3 if it is less than 4 the order of matrix then again thing is we need to revise the opportunity cost table and how we can do that there is procedure first for each row in which no assignment was made you need to tick that row right second examine the marked rows wherever you put the tick if any zero occurs in those column tick the respective rows that contain those assigned zeros right you need to follow this algorithm then repeat this process until no more zeros or columns can be marked then draw a straight line through each marked column and each unmarked row. The purpose here is we need to draw the minimum number of lines covering all the 0s. Let us say this was the matrix right, I can draw another matrix otherwise, let us say this is the matrix. So, let us say 0 this is how we have done the 0s. So, these are that means these are the final allocation.

We have other 0s also, but these are the final allocation. So, now let us say here also 0 is there, here also 0 is there like this 0s are there. Now, we need to draw the minimum number of lines. covering all the zeros that means you either horizontally you can draw or vertically you can draw right but minimum number of lines should be there so if it is small matrix thus just by visualizing that matrix may be you can draw the minimum

number of lines but the order of matrix is bigger then you need to follow these steps right and these steps already I have listed right we will do one numerical how we will follow these steps then how will revise the opportunity cost cell among the cells not covered by any line choose the smallest element right where there is no line we need to pick those cells where there is no line and where there is no line like here i think i have crossed all the cells but this will not be the case because we have allocations less than the order of the matrix we must be having some cells where these lines are not passing through all those cells you will find out we will find out the smallest element in that and what we will do we will subtract this smallest element from those cell values which are not covered by the line that means from those cell values from where this line is not passing we need to subtract and we need to add this value where two lines are intersecting let us say on this cell these two lines are intersecting we need to add the value here whatever value is here we need to add into that minimum value so minimum value we will find out we will subtract that value from those values from where the lines are not passing and we will add that value at the intersection of two lines right this is done now the elements which are not covered or cell values which are not covered by any of the line will remain same So, this is another opportunity cost cell table and again we will go for step 3 and we will make the allocation. How we made the allocation? We went through the first row, we will find out whether only 1 0, if 1 0 we will uncircle that and we will strike off all the other 0s in the row and column.

This is how we will keep on going, we will keep on iterating the solution until we will reach the final stage. so this is just the flow chart how we are doing already I have discussed is the problem maximization you need to convert the maximization problem into minimization problem that how you can do that if it is maximization problem you find out the largest value and subtract all the values of that matrix from that largest value and that is minimization now you that problem is converted into minimization rest of the algorithm will be same this is how you can do that other way how you can do that you multiply all the cell values by minus 1 that is another way out how you can convert the maximization problem into minimization and rest of the algorithm from here will be same how we did that balanced problem if it is not balanced we need to add dummy row dummy columns and rest of the procedure we will keep on assigning row wise column wise and we will see if it is allocations are equal to order of matrix so that means optimal solution if it is less than the order of matrix it is not optimal we will again go for revised solution and how we will find out the revised solution we will draw the minimum number of lines and then we will find out the minimum value that value will subtract from all those cell values from where the lines are passing and add those values where lines are intersecting and those cell values will remain unchanged from where those lines are not passing through this is simple algorithm of assignment problem see this is the problem so just will not see this whatever is written this is the problem we have programs we have

programmers here it is mistake programmers right now i want to decide a b c which is the program i should assign to these three different programmers right because i know the capabilities of three different programmers i know how much time they will take in that way i can calculate how much i am paying to that person how many days he will spend on that and then I can find out if I will assign program A to person 1, he will, that cost will be 120 rupees. If I will assign B to program A, 100 rupees. If I will assign C to programmer 2, the cost will be 110 rupees. This is known to us, this is cost matrix.

Now, the first step is whether the problem is balanced or not. Yes, it is balanced because number of rows are equal to number of columns. Now, the next step is we need to find out the minimum value in the row and we need to subtract that value from all other values. So, in here the minimum value is 80 if I will subtract 80 from 120, 100, 80 it will be 40, 20, 0. Next minimum value is 80 will subtract from the rest of the values 0, 10, 20.

Next minimum value is I think 110 will subtract from 110, 140, 120 value will be 0, 30, 10. Similarly, we will go for now column. So, column here is 0. So, value will remain unchanged.

Here 10. So, here I will subtract 10, 10, 10. Here 0. So, if I will subtract 0, value will remain same. So, this is the next table after calculating the opportunity cost table. up to this fine now what we will do we will find out we will do the allocation and how we decided we will go first row whether we can make the allocation yes we can make the allocation only one zero is there we will encircle this zero all other zeros will strike off because there is no zero will not strike anything second now second you can see So, we have two 0s.

So, for the time being you do not touch the second row because we have two 0s. Third, we have exactly one 0. Again we will assign this 0 and we will strike off all the 0s in that particular column and that particular row where we have done the assignment. Done? Now, you come to the second row again because this is already we have strike off.

So, we have only one 0. This assignment is done. Now, this if I will follow this algorithm, program C should be assigned to person 1, program B should be assigned to person 2, program A should be assigned to person 3. Now, we will find out what is the cost, cost will be just whatever the cell value here is you need to sum up those values. So, you can from the initial solution you can see here 80, here 90, here 110, you sum up this value this will be the total cost. Now, we will find out whether this total cost is minimum cost means optimal solution or not.

So, that means number of allocations 3, order of matrix 3 equal to that means this is the optimal solution. done. So, in the end, you can find out this total, I think this cost was not given, time was given. So, we need to minimize the time.

If it is cost, we need to minimize the cost. If it is distance travelling from one market to the factory, then we need to minimize the distance. So, that is why the initial slide we discussed that this is a special case of transportation problem, right. Another problem is we have 5 machines we have 4 jobs now this is not balanced problem we need to add one more job and the cost of that will be 0 0 0 so again now it is balanced what we will do we will find out the minimum value here and we will subtract from first row here we will find out the minimum value is 10 we will subtract from the row here the minimum value is 1 we will subtract from all these values here minimum value is I think 6 will subtract from all these values and here 0 it will be 0 because we have added one dummy row. So, there is no need to find out the opportunity cost table for the column because the minimum value is already 0.

So, if you will subtract 0 it will be same. so this is how it is done now column and opportunity that is done now we will go for the assignment if we will make the assignment here only we can do that first row we can go through we will make this assignment all other zeros will strike off Second, we have exactly one 0, all other 0s will strike off. Third, we have any 0, we do not have any 0. Fourth, we have 0, yes we have two 0s, we will skip this.

Fifth, we have 0. Yeah, we have three 0s. We cannot go for that. Again, now we will go for column wise. So column B, we have exactly one 0. We will make the assignment and we will strike off all the 0s in that particular row and column. again we will go for next column c and we have exactly one zero we will strike off all other zeros done so how many allocations are done one two three four four allocations are done and what is the order of the matrix five it is less than the order of the matrix that means this is not optimal solution Now, what we need to do? We need to find out the minimum number of lines.

So, I told you the concept of drawing the minimum number of lines. covering all the zero. I will just show you how we can do that. So, first is mark the rows that do not have any assignment. So, I think mark the row that do not have any assignment.

So, I think row 3 is not having any assignment. Rest rows all are having assignment. Second is we need to mark the column that have zeros in the marked row. Mark the

column that have 0 here also 0 is there I think here is 0 yes it is 0 we need to mark that column where there is 0 only one 0 is there right ok then next is we need to mark the row that have assignment in the marked column so that means this first row so this also we have done so instead of writing here i am marking here so we need to this we need to keep on repeating until we will draw the means we will tick all these points right row or columns now we need to draw the minimum number of lines through all unmarked rows unmarked rows what are unmarked rows in these lines we need to draw unmarked rows this is also unmarked this is also unmarked right and marked column so these are the minimum number of lines right how many lines four lines we have drawn then after doing that now what we need to find out we need to find out all those uncovered elements those are not covered by any line these are uncovered this is also uncovered this is also uncovered we'll find out the minimum value and what is the algorithm saying we need to subtract this minimum value from all those elements from where line is passing that means here also here here here here here from where the line is passing right and add these this minimum value at the intersection how many intersection this is one two three so at these three points we will add that minimum value once we will do that we will be having this matrix right again now we'll go for allocation how we'll do that we'll find out that row and column where we having we are having only one zero first row we cannot do that because we have more than one row zero second also we cannot do that third also we cannot do that fourth also we cannot do that fifth also we cannot do that let us go by column so only one zero is here we can assign that and all other zeros in that row and column we need to strike off second we can make the assignment no we cannot do that third we can know we have three zeros now fourth we can do that no we cannot do that fifth again we have two zeros so we cannot do that so because now we have striked off one row again may be we can check for that row or column and then when we can see that exactly where it is happening So, if you see that we have exactly this one, we can make the assignment and we can strike off the zeros in that row and column. then again we can pick the arbitrarily here and we can depending upon whatever is the minimum cost out of that if you are having two different alternatives right that means cost in two different cell is same so that means you can assign arbitrarily because if cost is 15 rupees here 15 rupees here any one assignment if you will make the cost will be same. So, you will continue this procedure until all the assignments are done.

So, now these are assignments and how many? 5 assignments are there. We have 5 order f matrix that means this is optimal solution. So, this way we can assign all the machines for all these 4 jobs. So, if you will add the cost, the total cost will be the minimum cost.

So, this is how we can deal with the unbalanced problem. Now what are the advantages

obviously you can maybe solve some more numericals will have more understanding of that how you can do that right. so advantages assignment problem first is efficiency minimization of cost obviously we are considering the transportation cost when we are saying this particular market should be allocated to that particular factory or these ten markets should be supplied from this particular factory one right so that means we will find out the cost we will find out the time how much time it is taking to ship that product and then we can Find out the final sell value of making that allocation right and then maximization of benefits obviously when we are considering all the costs transportation cost labour cost time consuming for you know you you are consuming time for doing that activity all we are doing that so in the process of optimization obviously you are minimizing the cost overall profit will be improved dynamic optimization this works in the dynamic environment you keep on changing as the real world scenario is changing you keep on updating your constraints you keep on updating your resources capacities because it's not fixed so today whatever person is capable maybe tomorrow he is more capable he is more trained that machine may be more enhanced capacity if you are updating the machine updating the technology so that dynamic optimization what today is the optimal solution tomorrow may change so you need to work in the dynamic optimize reduction of waste obviously when we are optimizing the function we are targeting to reduce the waste only and waste here extra time if you are consuming extra labor extra resources if you are consuming that is waste enhanced productivity because we are ensuring the proper utilization of the resources so when you are ensuring the minimum time minimum consumption of manpower minimum consumption of material you will be efficient clarity having said that mathematical formulation objective function already we have defined is xij cij right we need to minimize this function So, we are including all the associated cost. So, this is clear objective we are having that this very precise criterion in terms of the constraints, in terms of the evaluation we are making, the decision making we are doing based on this particular equation. so optimization criteria obviously we know that we are optimizing the time minimizing the time minimizing the cost or maximizing the profit right quantifiable matrix subjectivity is not there because whatever the best solution is coming out we will see that is the objective solution we are getting out of this right that this particular job should be assigned to this person and cost will be 150 rupees very clear right integration with constraints because we are considering all the constraints what are our resources capable of so that way we can find out the constraints and we can amend those constraints on the objective function then easiness because we know the decision variables right and how we can control only binary decision if you are assigning you will assign 1 if you are not assigning you will assign 0 no other intermediary value is there right tabular representation which is very easy to understand because matrix we are preparing the cost matrix we are preparing the allocation matrix and in the end we can find out just multiplying and summing up all the values we can find out the minimum cost scalability if your business is operating at low small scale you

can you know optimize at that level if it is large scale you can optimize the large scale resources right accessibility so easily you can access it and because we have a decision making software as well you can use this algorithm and you can quickly find out the you know information based decisions like which person should be assigned with which job so that the overall cost can be minimized. Flexibility having said that when i told you we have two different zeros in the same row and the minimum cost is same means 15 15 like i told you so if i will assign job a on machine 1 cost is 15 rupees if i will assign machine c on job c on machine 1 again 15 rupees that means i am having that flexibility i can assign any job on that particular machine because cost is same so alternative You know alternatives also you can find out so constant also you can keep on changing the constant today whatever is the constant tomorrow the constants will be different right if your technology is changing so today you are limited by capacity maybe tomorrow you are limited by the demand you are producing enough, but you are not able to consume that so then constant is changing today you are limited by manpower tomorrow you are totally automating the production again so constant is changed tailored solution it is providing you because when you are changing the real world constants it will change the solution also so iterative improvement because step by step we are finding the opportunity cost table and then we are improving the solution so this is iterative approach and scenario analysis what if this will not happen right that is important to find out if this job c is not assigned on machine 1 what is the cost if i will assign the job c on machine 2 i have to spend extra 5 rupees to do that job right so sensitivity of solution that also you can find out in that way may be sometimes that machine is not available may be broken down it will take time so you anyhow you have to process that on some other machine so extra cost will be there right real time applications because immediate response i told you you can configure your all the constant functions and then real time data you can fit iot and sensor based data you can keep on tracking all the capacity related or resources availability requirements uh job requirements your resources capabilities right all those you can record in the real time and then you can find out the best solution so it's not that today if you are assigning that particular job C on machine 2 tomorrow also the same solution will come right if those things are changing your capabilities are changing in terms of resources if your job requirements are changing so in terms of customer demand so then you have to do in that dynamic workforce management right so obviously if that because you are considering the staff skills also manpower skills also today you are lacking with that maybe tomorrow you will train your manpower and then you will come up with the improved skills optimal routing and scheduling because we can know if I will serve market 10th using factory 1.

So, that will be the optimal route. So, I can schedule in that way. So, that is optimal decision. so these are some of the assumptions deterministic demand and supply this we should know what are the column side values in terms of your demands in terms of your

number of jobs and what are the row side values in terms of your resources right this is first assumption but in real world it is very difficult to accurately predict how many jobs are required right homogeneous task and resources this we are considering every task is homogeneous which is not every resource is homogeneous which is not if I'm doing the same job using machine and if I'm doing the same job using manpower difference is there obviously fixed assignment cost what we are saying if today i am doing may be 15 rupees is the cost tomorrow also same cost will be there which is not same if you are upgrading your technology may be cost will come down if raw material cost is increasing may be cost will increase right no intermediate stops or transshipments we are considering direct assignments from factory 1 to market 10 right intermediary we are not considering right linear relationship this is the basic assumption under linear programming that the relationship between the variable should be linear relationship non-linear relationship non-linear programming methods will be there capacity constraints obviously we should know what are the capacity constraints and then this is single period planning because right now if i am running the model iterations whatever solution is coming is right now is best right i cannot say that this is for long term clearly we are defining the objective function because we need to minimize the transportation cost we need to minimize the time spent on doing that activity, minimize the cost, maximize the profit, maximize the revenue, maximize the share. So, objective function is very clear. Another very important limitation of any linear programming problem is we cannot enter the non-negative values.

Obviously, minus 10 man power cannot exist. right so non negative variables cannot be there fixed origins and destination this is another assumption that we are saying this is only the market this is only the demand this is only the customer this is only the resource but that is not the situation because locations or customer locations are also changing so in that way you need to change you need to consider the dynamism in the environment so these are all assumptions which are obviously coming as the limitation of this problem right so these are the references you can go for further reading so that's all for for this assignment programming problem so thank you very much