Logistics & Supply Chain Management

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Lecture 49 : Transportation Problem Programming

hello dear friends welcome back to NPTEL online course on logistics and supply chain management so in the last session we started discussion on transportation management and we discussed that how by picking different mode of transportation how we can address all the parameters like your cost like your efficiency like your delivery time like what are different services level of services are expected from the customers how we can hit those parameters by using different mode of transportation and then we ended that discussion that it's the intermodal transportation where we can you know implement all the different types of mode of transportation depending upon the availability infrastructure and then the requirement based on the requirements right how we can maximize this customer experience right in terms of efficient and effective delivery of the quality products right. So, in this session we will continue this discussion and we will go little mathematically we will try to sort out which one is the best route right how we can minimize the total transportation So, this chapter is transportation problem programming. So, we will discuss about how we can formulate the problem in terms of linear programming problem, where we will design the objective function and then how we can go for optimizing that objective function. We have some set of assumptions for solving that linear programming problem. We will talk about the mathematical derivation of that, different types of transportation problems. then phases of transportation problem, how we can find out the initial basic feasible solution and then we will use the, find out the optimal solution towards the end, right.

And we will discuss some special case of transportation problem and obviously I will show you some numerical demonstration also. the transportation problem is classical optimization problem right where we are talking about what we need to minimize the transportation cost so that means there will be some transportation cost function objective will be there and we need to minimize that cost right if it is profit we need to maximize that profit function right so obviously when we are talking about transportation decide it is important to decide the most effective way to transport the goods from multiple sources we have multiple sources in terms of let's say factory one is manufacturing factory two manufacturing factory three factory four all these different factories are there and in terms of markets there may be n number of markets may be more than thousand markets in india and then abroad as well right so which market should be served from factory one Maybe you are targeting 200 markets from factory 1 depending upon the distance, depending upon the load factory 1 can take, depending upon the demand coming from the market, those 100 or 200 markets. So many different factors are there. when talking about transportation cost it is the overall cost of ownership right we talked about total cost of ownership TCO right so we will include the transportation cost wear and tear cost operational cost the depreciation cost of the asset everything will include and will say if we are saying that 10 rupees per unit that means 10 rupees if we are shifting one product from that market from that factory to one market.

It is not only the factory, if we will include vendors also the same algorithm can be evaluated for the vendors that vendor 1 is serving factory 1 and factory 2, vendor 2 is serving factory 2 and factory 3 then also we can apply the transportation problem. So, this is a linear programming problem and we when talking about linear mathematical programming problem obviously we have decision variables. And decision variables are here how many units when we are talking about factory 1. will fulfill the demand coming from 100 markets, how many units will be shipped from factory 1 to factory 1 to those 100 markets, that is decision variable. So, factory 2 to other 100 markets or may be some markets may be common.

depending upon because factory 1 is not able to produce that much so same market is also served by factory 1 and then factory 2 so I told you the capacity of the factory is also very important this is one point decision variable other point is objective function what is the objective function here is cost total cost of ownership so this objective function of cost we need to optimize means we need to minimize the programming this mathematical programming will ensure the minimum transportation cost keeping all the constraints in mind now keeping all the constraints in mind means constraint in terms of supply in terms of demand see supply from factory 1 is limited if we are running that 24 x 7 without any failure the maximum capacity we can expect from factory 1 same is the situation with factory 2 factory 3 factory 4 we have designed different capacities for all these different locations Then talking about market 1, 2, 3 to market 1000. all markets are not coming with the same demand requirement right so different market different demand different factories different supplies so we need to keep these constraints in mind keeping these constraints in mind we need to maximize the objective function and in this case not maximize means maximize in terms optimize so here we need to minimize the cost function this is the complete picture of the mathematical modeling right so assumption i will quickly talk about the assumption fixed supply and demand so this model is based on that assumption the demand is fixed and supply is fixed which is not practically the case may be from market one today you are getting demand for ten thousand products tomorrow you may get for twelve thousand may be from factory 1 today you are producing 10000 products tomorrow you are running in may be extra shift or double shift may be you are producing 80000. may be you are going for expansion and then earlier the capacity was 10000 only now you have expanded and now the capacity is 25000 so this demand and supply is not fixed having said that our planning should not be the fixed one we need to come up with the routine planning process what we are deciding today will not be valid for tomorrow already we have discussed this is the dynamic environment Single community, another very strong assumption is that we are transporting only single product. If we are talking about transporting mobile phone, we are talking only about the mobile phone. transporting car we are talking about that product only single homogeneous so that means we are keeping that cost fixed 10 rupees transporting per unit so obviously transporting one mobile phone the cost will be different transporting car will be different transporting some other product will be different so that means if we are talking about single commodity, so this transportation model may not be applied as it is in the e-commerce industry.

So, when we are talking about dealing with the uniform products, may be in industries, when we are talking about where one may be Maruti Suzuki plant is here in Calcutta, so transporting cars from Calcutta to different markets, then we can apply this model. where our the agencies who are you know doing the retailing things are fixed right so cost linearity is another assumption which is again practically not true It says that if I am transporting 1 unit, the cost is 10 rupees. Transporting 2 unit, cost is 20 rupees. Transporting 10 unit, cost is 100 rupees. Transporting 100 unit, cost is 1000 rupees.

Uniform cost, but this is not the case. When we are transporting more unit, per unit cost will be reduced. May be if I am transporting 1 unit, 10 rupees. Simple example, if 1 courier delivery.

.. or pizza delivery boy is transporting one pizza from the McD to IIT Kharagpur campus. So, cost may be 100 rupees, but if he is transporting 2 pizza, so that will be divided into 2. So, I do not think any extra cost will be there, fuel consumption will all will be same, but yes, because delivery location may vary, may be he needs to travel 2 kilometers more, but then also the cost will be anyhow will be less than this 100 rupees per unit, may be 80 rupees, may be 70 rupees somewhere per unit. Economies of scale concept does not hold good here in this case. Then capacity constant, already I talked about the maximum capacity from factory is also fixed, the maximum demand from the market is also fixed.

Known negative decision variable, obviously we cannot say that factory 1 is transporting minus 100 products to market 1. So, decision variables cannot be negative. What is decision variable? How many units we are transporting from one end to other end. that is x that cannot be negative full shipment goods are fully transported from source to destination so again limitation for e-commerce industry if i am putting order for fifty grocery items may be within same day they are delivering me forty items so that means partial fulfillment is there so we are in transportation problem we are discussing about the fully transported goods from one end to other end these are some of the assumptions Obviously, we are working on minimizing the cost function and the cost function includes the distance between the two location, the path followed, mode of transportation, number of units are to be transported and speed of the transportation. These many things you can configure in your cost function and we will try to minimize the total transportation cost without compromising the constraints which are on supply and demand.

right so the linear programming problem is there because linear constraint we have put that the cost per unit is linear right so that's why linear behavior of the decision variables as well as your all the independent variables so linear programming we are saying it so what we are doing here transportation deals with transportation of a commodity single product homogeneous product from m number of sources to n number of destinations right so It is assumed that level of supply of each source and the amount of demand at each destination are well known. the demand from the market is fixed and known and supply is also in the same way and unit transportation cost is also known so the objective is to determine the minimum amount of transportation cost that is required to ship the product from one end to other end let us understand and configure this into mathematical form this is a kind of matrix these all are sources let's say these are factories where we are manufacturing the product factory 1, factory 2, 3, 4, 5, factory M, all rows are representing factory. So, that means this is supply from factory 1, supply is A1, from S2 factory supply is A2, from third manufacturing unit supply is A3 likewise. These are destination you can say these are the markets from where the demand is coming order fulfillment center, retail locations, whatever you can call. So, demand from market 1 or maybe destination 1 is V1, demand from this is B2.

So, these are known. This is what we were saying. The supplies are known and demands are known. So, in between C11 is representing the cost. So, here if I am saying C23, it is representing cost of shipping the product from S2 source to destination 3 market 3, cost is C23.

C3n is representing the cost of shipping 1 unit from S3 to n market. So, this is the cost. So what I am saying the decision variable is let us say xij for any xij means if I am writing here let us say if I am writing here x32 that means x32 is representing source

How many units I am shipping from factory 3 to market 2? This will represent the number of units. So, obviously by solving this linear programming problem we will decide how many units we will ship from factory 3 to market 2. So, that means if I want to design this objective function of cost, how many units I am shipping from i to j and the cost of i to j will be the total cost.

These many units I am shipping and this is the cost. If I am talking about I am shipping x to 3 units from factory 2 to market 3 and cost of shipping per unit to from factory 2 to market 3 is C23, then total cost will be X23 multiplied by C23. So, same way if I will do the allocation in different cells right here, here, here, here, I can just multiply the cost with the number of units and keep on adding. and then I will get the total transportation cost this is the objective function and we need to minimize this objective function now constraints like we talked about the supply is known demand is known and this should be equal total supply should be equal to total demand right if it is so then we call it balanced problem if it is not so the problem is not balanced right then we need to balance the problem we will talk about the special case in that case when it is not balanced so here we are assuming supply is also fixed demand is also fixed and this is known so another concept is if I will say total if I will sum any row that should not be greater than a1 if I am summing up all the allocation done in row 1 that should not be greater than a1 because the maximum supply in row 1 is a1 only. If I am talking about AM, so sum of all the location should not be more than the supply.

That is obviously. SM cannot supply more than what SM is supplying, maximum capacity. Same is the case of the demand. If I am talking about any particular column, total sum should be equal to, like if I am talking about third column, total sum should not be greater than B3. It can be less than, but it cannot be greater than. right so this is the total structure of your mathematical transportation problem this is how we have designed the transportation this cost function and we can apply this transportation algorithm to minimize this cost function and this is the restriction put on the supply side because total supply in any row cannot be more than the allocation should be equal to the supply here the allocation in the column should be equal to simple concept should be equal to the market demand if I am talking about B3 market obviously you will supply only those many units whatever is the requirement from the B3 so this is the mathematically how we can represent the same There are some terms we will just go through and when we will solve the numerical we will understand more clearly.

Feasible solution is any solution that satisfy the total condition. Let us say I am having 10 meters of cloth very simple example. So, if I will make the shirt I need 2 meter per shirt. If I need to make the trouser may be I need 3 meters. per trouser now the decision I need to make may be if I am making shirt per shirt 5 rupees is the profit if I am making trouser may be 7 rupees is the profit so then I need to make the decision how much I need to make the shirts how much how many I need to make the trouser so that the total your cost can be minimized and profit can be maximized right so obviously if you are saying that per trouser giving I am getting 7 rupees profit let us make as many as possible but then the restriction is we are having only 10 meter of clothes so resources is limited right so that means the feasible solution is that solution within that 10 meter how many I should make shirts how many I should make trouser so that the total profit can be maximized and we can ensure the full utilization of the resources basic feasible solution in this case transportation problem when we having m plus n minus 1 number of allocations then it is non-negative independent location we will discuss in the numerical here it is difficult to understand but only thing the number of xij's should be equal to your order of matrix minus 1 order of matrix means this matrix whatever is the order of this matrix minus 1.

Non-degenerate, we will discuss in detail when we will solve the numerical. Non-degenerate basic feasible solution. Degenerate solution is when any decision variable in the basic feasible solution, the basic variable is getting 0 value, then it is degenerate solution. right so we need to remove the degeneracy so if this transportation cost if we have exactly n plus n minus 1 non negative allocation at independent position then we will say non degenerate basic solution and optimal solution is the feasible solution which is giving you the minimum cost right transportation cost meeting all the requirements right. So, right now may be these terms may be little bit confusing for you, but once we will go through the numericals it will be very clear to all of you what are the independent position, what is the meaning of this m plus n minus 1 allocations we will see.

So, types of transportation problem, I discussed balanced transportation problem, where your supply is equal to demand, this is balanced problem. So, whatever supply is coming from the factories and whatever demand is coming from different destinations, if it is equal, it is balanced problem. when it is not equal then it is unbalanced transportation problem so we need to make it balanced only the first step is just check whether this transportation problem is balanced or not if it is not we need to make it balanced so phases of transportation problem first phase is to obtain the initial basic feasible solution right here we will go for the we have different methods right we'll opt for those method any of the method we have three methods northwest best corner method least cost method or vogel's approximation method you can use any of this method to get the initial basic feasible solution it is that solution which will give you the initial solution that you can implement it but it is not the optimal solution not going to give you the highest profit So, once you get the initial basic feasible solution, then we will go for the optimal solution.

We will do the iterative process, we will opt some algorithm and we will see how much we can minimize the cost. And in your operation research, these two are the basic steps.

First, if you talk about transportation problem, assignment problem, simplex problem, all these algorithms, linear programming problems. we are initially finding the initial basic feasible solution and then we try to maximize maximize if we are talking about profit minimize if we are talking about the cost right this objective function either we will maximize or we will minimize only two things are there The optimal basic solution provides the valuable insight for decision making in supply chain management, helping to minimize the transportation cost, maximize resource utilization and meet the customer demand effectively. so optimality condition in this case transportation see in all the algorithms linear programming these optimality conditions are different steps are same right so here we if we are getting m plus n minus 1 number of allocations decision variables right so then and those are at independent positions known negative then we are getting the optimal solution so the optimality condition in context of optimization problem such a linear programming indicates that a feasible solution has been reached that either maximize or minimize the objective function satisfying all the constraints so it's not that always cost function is defined in transportation sometimes we can define the profit as well if I'm shipping from factory 1 to market 3 per unit product I am shipping may be 5 rupees is the profit, same way same matrix I can highlight the instead of cost matrix now I will write from factory 1 to factory 3, 5 per unit. So, that means now it is profit maximization problem, now we need to maximize this, but if it is cost problem we need to minimize the cost function simple concept only you need to reverse the algorithm. So, simple thing is you can convert maximization problem into minimization and minimization problem into maximization just multiply the matrix by minus 1 or you can do one other thing you subtract all the values from the maximum highest value in that matrix even then you can convert maximization into minimization or minimization into maximization right.

So, let us explore how we can find out the initial basic feasible solution. So, this is the solution that satisfies the constraints, supply and demand constraint of the problem while ensuring that both the supply and demand requirements are met. So, this solution will serve as the initial point from where you will launch, you will go for the optimal solution. So, before going for that your problem should be balanced, again we need to check whether your demand is equal to supply and as I told you we have three methods to get this initial basic feasible solution in transportation one is north west corner method another is Vogel's approximation method and third one is least cost method so one by one we will go through these methods north west corner method this starts with allocating the number of units from the northwest corner so in this matrix northwest corner will be this one right so once we will allocate in this cell then if either we are going to strike row or we are going to strike column or both also we can strike. So, if we will strike this the next in the remaining matrix the next north west corner will be this one.

So, we will keep on doing this and we will keep on allocating the number of units. So, this is just algorithm you can go through, but I will just start solving the problem and will give you the clear idea on that. Let us say these three destinations are there factories Jaipur, Udaipur and Mumbai. We have three location where we are manufacturing the product and we need to serve to Kanpur market, Pune and Delhi and per unit cost is given from Jaipur to Kanpur it is 4 rupees, from Jaipur to Pune it is 5 rupees.

for transporting per unit. Now, the supply from Jaipur is 40, from Udaipur 60 and Mumbai 70. Total supply we will add is 170. So, demand from the Kanpur market, Pune and Delhi is also given. So, you can just sum up and it is also 170. So, first step is to check the problem is balanced or not.

So, here you can see our problem is balanced. So, now we can go for finding the initial basic feasible solution. So, this first step is checked. It is initial basic feasible solution.

We can go for northwest corner. This is the northwest corner. So, let us go for this. Now, you can see in this matrix, just forget about these lines, in this matrix which one is the northwest corner? This one is the northwest corner. Now, for this cell, I will find out what is the supply? Supply is 40.

What is the demand? Demand is 70. So, how much I can maximum allocate in this cell? I can maximum allocate 40 because I cannot go for more than 40 because in that case supply is only 40. How I can allocate in that cell 50 because supply from Jaipur is only 40 and the constraint we said that the total allocation in the first row sum should be equal to the number of allocation. equal to the number of supply and that should be same throughout the all the columns all the rows now we have allocated 40 in the first cell that means this supply is totally gone now Jaipur is not available to supply to some other location either Kanpur, Pune or Delhi now because we have exhausted demand of Kanpur by 40 only 30 is remaining so next in northwest corner is this one now how many maximum we can allocate because demand is 30 and supply is 60 but we cannot allocate more than 30 because if i will keep supply in mind and will allocate 60, so Kanpur requires only 70, but if I will supply 60, supply to Kanpur will be 100, because 40 already we have assigned in the first cell, that also we need to keep in mind, this sum of the column should also be equal to the demand. So, now after allocating 30, this Kanpur market is whatever demand is completely met. So, and we have exhausted this Udaipur supply by 30, so this Udaipur is now left with 30 more supply.

Next north west corner is this one, how much maximum we can allocate? 30, because maximum supply from Udaipur is 30, so that is this Udaipur factory is also gone and we are left with 10 more units of demand from the Pune market. Now, we cannot do much because we can assign 10 here simply no need to check further because these only two options are there. So, you cannot shift 10 to 20 or 60 to 50 because then it will you know. we need to meet the condition total sum of the column should be equal. Now, you can sum up all the allocations we have done: 40 here, 30 here, 10 here, 30 here, 60 here; you sum up any column, any row that will be equal to the number of supplies coming from that particular unit or the demand coming from that particular market.

So, this step by you can see just I have shown you in the first slide only you can see step by here how the allocation is being done and in the last two only options were left like we did this is the total allocation. So, this is the complete allocation we did in the first you just you can verify here what we have done is this total solution. Now we have allocated that means we have not done allocation Jaipur factory will not supply to Pune and Delhi because cost is somewhere increasing if we will supply from Jaipur to Pune to Delhi right. And then Udaipur is also not supplying to Delhi, Udaipur is supplying 30 units to Kanpur and 30 units to Pune. Then Mumbai is not supplying to Kanpur, Mumbai is supplying to Pune and Delhi.

So, now these units we have already decided and per unit cost we know and if I will ask you to calculate the total cost, obviously from Jaipur to Kanpur per unit cost is 40, you are transporting 40 units. So, 4 into 40 will be cost of transporting Jaipur to Kanpur. Cost of transporting Udaipur to Kanpur per unit is 3, total units we are transporting 30. Udaipur to Pune per unit is 4 we are transporting 30 Mumbai to Pune we are transporting 10 units per unit cost is 2 Mumbai to Delhi per unit cost is 8 and we are transporting 60 right if you sum up this this will be total 870 rupees cost so this is northwest corner method how we can apply this method to minimize the total cost so we have seen how we can apply the northwest corner method to find out the initial basic feasible solution so this is one method to make the initial allotment and Then obviously we will go for the optimization in the next session we will discuss about other two methods that is your Vogel's approximation method and least cost method how those methods are also used to find out the initial basic feasible solution and then we will see one algorithm of modified distribution method how we can use that algorithm to get the optimal cost right. So then again allocation will be changed and then we can find out the minimum transportation cost.

Again these are some of the books you can refer and thank you very much.