Petroleum Economics and Management
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Module - 11
Management of Petroleum Wealth
Lecture - 54
Model of Economic growth with uncertainty

Hi everyone, I am Dr. Anwesha Aditya, your instructor for the course Petroleum Economics and Management that we are offering in the NPTEL program. So, if you remember we are in module 11 of our course where we are discussing the Management of Petroleum Wealth and this is the last lecture in module 11. So, overall, this is the lecture number 54 in the course where we will be discussing the Model of Economic growth with uncertainty.

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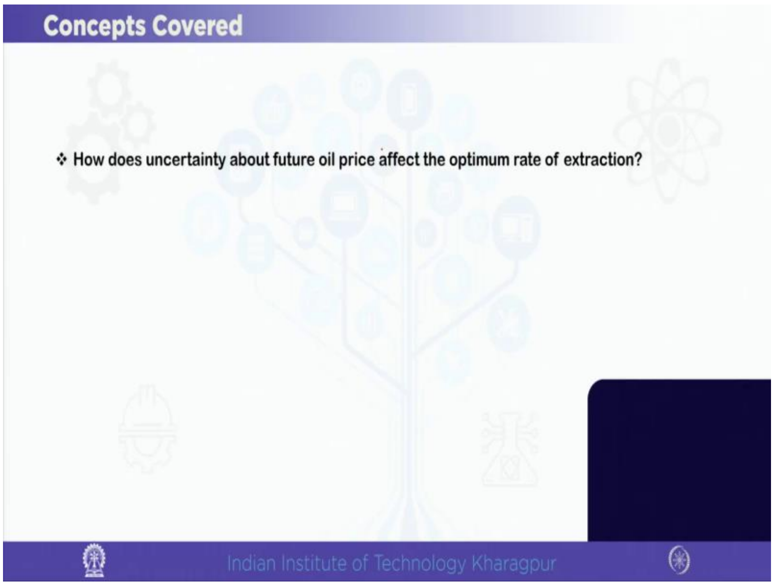

If you remember in this module 11, we are discussing theoretical model of how economic growth is affected when natural resource like petroleum is suddenly discovered. So, for our purpose we are sticking to the example of petroleum, but this same model can be applied to any other type of non-renewable or exhaustible natural resource which is in fixed supply.

So, if you remember this module is spread over 4 lectures in the first 2 lectures, we have structured the theoretical model and without means before the discovery of the resource. So, we solve the model using dynamic optimization and the Hamiltonian function, but due to time constraint we could not do a step by step solution, but I have already mentioned the references.

So, those who are interested you can go through those references and the mathematical economics related books like the book of Dynamic Optimization by Chiang and we have also referred to the books of Macroeconomics like the book of Abel Bernanke and books on Economic Growth like Barro Sala-i-Martin.

So, these are the references, but how about the model that we are discussing is largely from the Petroleum Economics book of Hansen so, which we are following more or less consistently throughout our course. So, in this module in the first lecture we discussed some empirical evidences. Earlier also we have discussed the hypothesis of resource curse and the Dutch disease.

So, we have investigated the country experiences. We have also studied a theoretical model in which what happens with allocation of resources after some natural resources discovered and we have seen how that may lead to a recession and stagnation in the economy. So, we have discussed many country experiences, but we have found that natural resource endowment may be good or may be bad also it is not that always on a country which is endowed with natural resource it is a good performer.

So, we in this context we discuss the resource curse hypothesis and we have seen the examples of the Middle-East countries. However, it is also true that Dutch disease can be cured as we also saw the experiences of Netherlands and even you see Norway.

So, Norway also suffered from balance of payment crisis after oil was discovered in 1987, but then Norway bounced back and it is a very good performer. It belongs to the category of high income and developed countries the HDI Human Development Index rank is very good.

Even Netherlands also later performed very well the name Dutch disease was due to the what happened in Netherlands after the discovery of natural gas, but again Netherlands performed very well. So, these countries were already much more developed than the

Middle-East countries, but there are countries like Venezuela who actually went into a crisis because those economies got completely dependent on oil and they could not grow the other sector.

So, when after the shale oil revolution, the oil price started declining. So, in 2018 Venezuelan currency also depreciated a lot. So, it is not about just getting endowed or discovering the resource how to manage the resource wealth is very important because if the resource wealth is only enjoyed by very few in the country, then there will be increased inequality, dualism and in the extreme case it may lead to a mono-economic which is completely dependent on the natural resource sector.

So, these things we have already discussed in module 11 we are discussing how the optimum capital stock and consumption rates will be affected. So, in the first lecture we discuss some evidences from the literature and then we outlined our theoretical model. Then in the third lecture we introduced a natural resource discovery.

So, how a natural resource discovery might change the optimum part of consumption and capital accumulation and the steady state level of capital accumulation and savings rates. So, we have seen that the paths of consumption and capital accumulation may change after the discovery of natural resource.

So, natural resource discovery may ease the burden of capital accumulation, countries can increase the consumption a bit, but it is not that the country should consume everything in the present because we have to think about the future also because this type of resources are in limited stock and we should not be running out of the resource before the future generation can benefit from the resource.

And also, we should not leave the resource entirely for future consumption in that case the resource may become useless also. So, we need to maintain a balance. So, we have already studied in module 10 what should be the optimum rate of extraction and pricing we have studied the Hotelling rule. Now, see in module up to lecture 3 in module 11 we have considered the cases under certainty.

So, what happens after the resource discovery to the economic growth of a resource endowed country, but we know we have already seen many circumstances where oil market is subject to lot of volatility and uncertainty because oil there are many instances where the oil price trend suddenly reversed like the shale oil revolution with a great price collapse even means those things which are beyond the control of the oil market also let us say the Asian currency crisis in 1997 and the latest example is the COVID-19 pandemic.

So, things beyond the control of the suppliers of oil can also influence the oil price. So, how does this uncertainty about the future oil price can affect the optimum rate of extraction. So, we need to consider this because till now we have not incorporated uncertainty in the oil market in our theoretical models.

But uncertainty is a very important component. So, it is because of this uncertainty and volatility in the oil prices that the countries have to take into account. So, what should be the optimum rate of extraction given the uncertainty in the oil price?

So, that is what we are going to discuss in today's class. So, we will be sticking to the same model. So, I am not going to structure the model again. We already know the structure of the model pre and post discovery of resource. So, what we will d o in today's class we will bring only the we will introduce the uncertainty component. And as I already mentioned that this is an advanced theoretical model.

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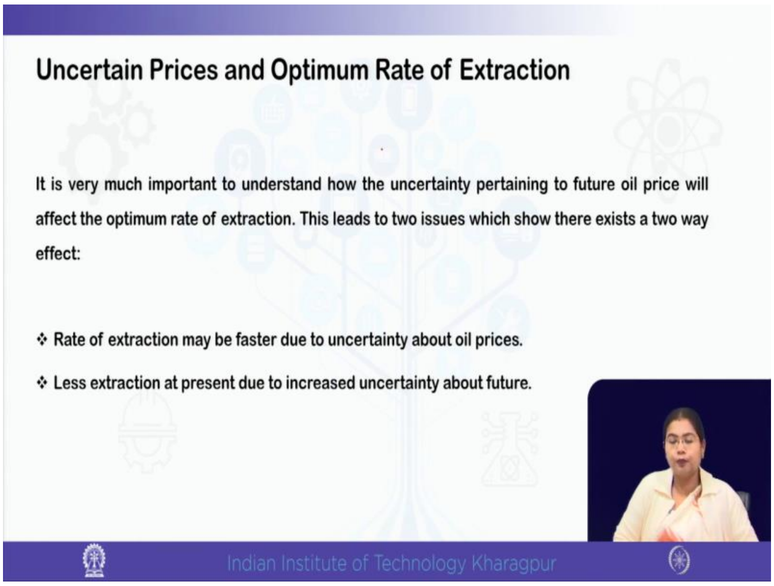

So, we are keeping it very simple we are just mainly focusing on the results. So, those who are interested please go through the references at least you can go through the relevant chapter in the book of Hansen. And there are also many numerical examples worked out in Hansen. So, due to time constraint we cannot devote more time in module 11 because we have one more module left in our course.

Now, see how uncertain prices can affect the optimum rate of extraction. So, there can be two possibilities either the rate of extraction may be very fast due to uncertainty about oil price because if you from the point of view of a resource endowed country if the country thinks that price of oil may fall in the future. So, the country may speed up the rate of extraction right.

But the other possibilities also there the country may also think that the future price can increase because we have already studied the Hotelling rule with or the R percent rule, which says that the price of a natural resource increases at the rate of market interest. So, at the market rate of interest so, in that context if a country thinks that the future price may increase because the world demand for oil is growing.

We do not have very close substitute of oil at least in the transportation sector. So, if a country believes that the oil price will increase in future. So, it may reduce the rate of extraction at present and it will keep a greater amount to be extracted in future. So, both the options are there. So, which one will be true? So, let us seek to find an answer to this question. So, how uncertainty about oil price can affect the rate of extraction?

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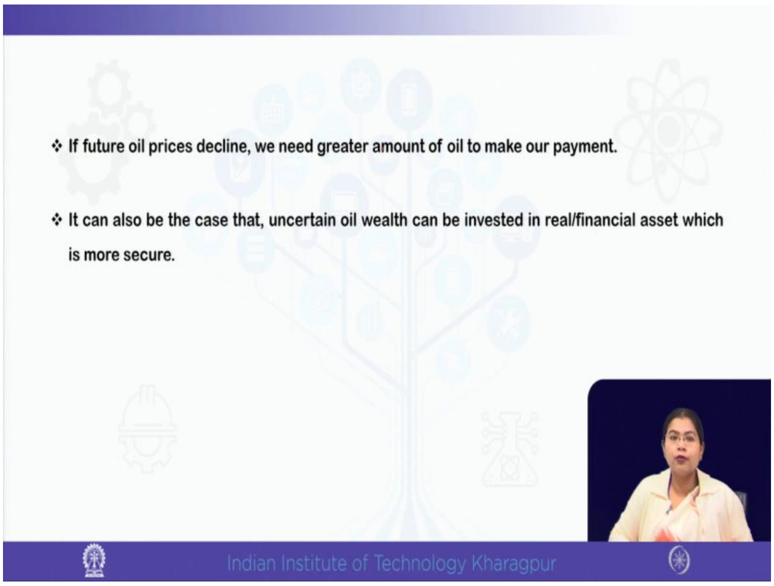

So, if future oil prices fall therefore, we have to make a greater amount of oil to make our payment. You see we have already seen the examples of the OPEC countries mainly the Middle East countries and they finance their import spending from the export revenue earning from oil because we know that these countries are very much dependent on oil and when oil was discovered like in Saudi Arabia, it was a poor country.

So, these countries often have to import lot of high technology good pharmaceutical products. So, how do they finance because when you are buying something from abroad or you are importing you have to pay in terms of the foreign currency. So, one major source of their import spending is of course, the export earning they earn by selling oil abroad right. So, that means, in future if the oil price falls, they need to spend more amount of oil to finance their import bill.

So, a more wiser option can be then to invest the oil wealth because you remember in the previous lectures we have discussed about the investment fund the owner of the resource remember we are discussing a planned economy ok. So, in the owner of the resource can think of investing the oil wealth into real or financial asset and that is more secured because the oil market is subject to much more volatility.

So, what the country can do the country can extract the resource can sell the resource and earn the revenue and whatever revenue is not required for current consumption the country can invest that in investment fund like government stock, bond, market even financial asset and real estate also are creating physical infrastructure which may yield return in future. So, investing a part of the oil wealth in investment fund can be a better option and that reduces the volatility and uncertainty.

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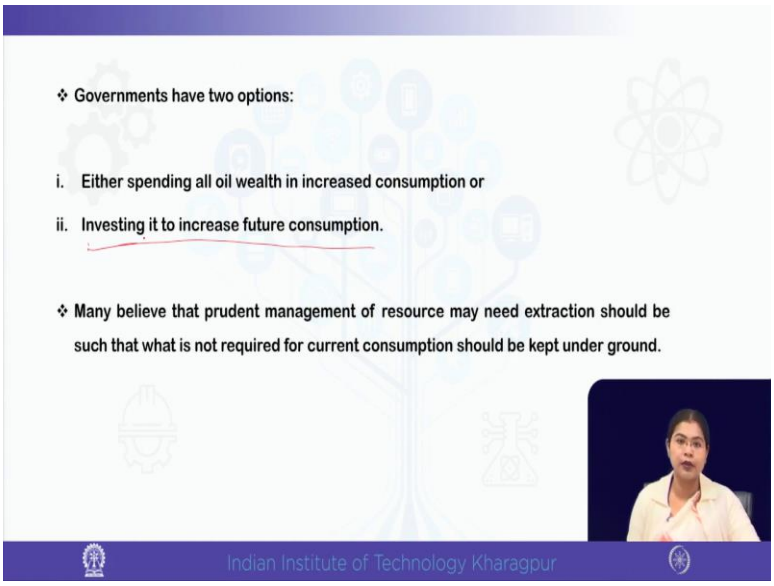

Because oil market is subject to more volatility. So, governments or the policy maker is really having two options. So, see this model is from the point of view of a country which is endowed natural resource. So, not a country like India which is a net importer of oil. So, you should keep in mind. So, there are two options for such a government which is dependent on a natural resource.

So, first is either to spend all the oil wealth in increased consumption, increased consumption and even see that means, the see the government has to spend its means government has to finance its expenditure like paying wages and salaries financing the social welfare schemes, creating infrastructure also spending for public investment in education and health like human capital formation and creation of physical infrastructure like electricity irrigation road fly over railway.

So, the government can find can be tempted to use a greater part of oil wealth in increased consumption at the present ok. Or the other option can be to invest the oil wealth to increase the future consumption because then you are sacrificing the current consumption.

And you are investing a part, which can be used means when you invest you get a return in future. So, when you get the return in future so, what will happen your future generation can be benefited they can use that return from the investment in their consumption ok.

Now, many believe that many economists think that wise management or prudent management of resource may need extraction should to be at a such a level that whatever is not needed for present consumption should not be extracted because often many economists believe that it will be hard difficult for the policymakers to resist the temptation of not using the wealth.

That means, the oil wealth in increasing in financing its current consumption expenditure because often what happens the policymakers are interested, they have to think about the majority for voting purpose. So, they want to spend more for current consumption.

So, it may be difficult for the governments and the political parties and the politician's bureaucrats to resist the temptation of not investing a greater part of wealth in present consumption so, not spending a greater part of wealth for the purpose of present consumption.

So, that means, if a greater part of wealth is extracted now most of the oil wealth will be used for financing present consumption unless will be available for future even in extreme case like Venezuela did a typical example is Venezuela almost all the part was exactments used for increasing the current consumption Venezuela imported a lot and the import was financed by the export earnings from sale of oil abroad.

So, this should not be the case. So, that in that case a better solution can be just to extract the amount which will be required for present consumption the government should not extract greater amount. So, if the government extracts the greater amount then a larger part will be used for current consumption.

So, according to this economist a better option a prudent management of the resource may need the government to extract only that part, which is required for current consumption. And whatever is not required for current consumption should be kept underground because once a greater part of resource is extracted. So, the second option that we have written over here that investing into future consumption.

So, according to this economist it will be a difficult option because the governments will be tempted to use more of the wealth in increasing the present consumption. So, investment in future consumption may not be achieved. So, in that case the policy makers should try to extract only a part which is required for present consumption and the rest should be kept underground ok. And then in future the rest can be extracted, but of course, in that case also you see there is some uncertainty because in future oil price can fall.

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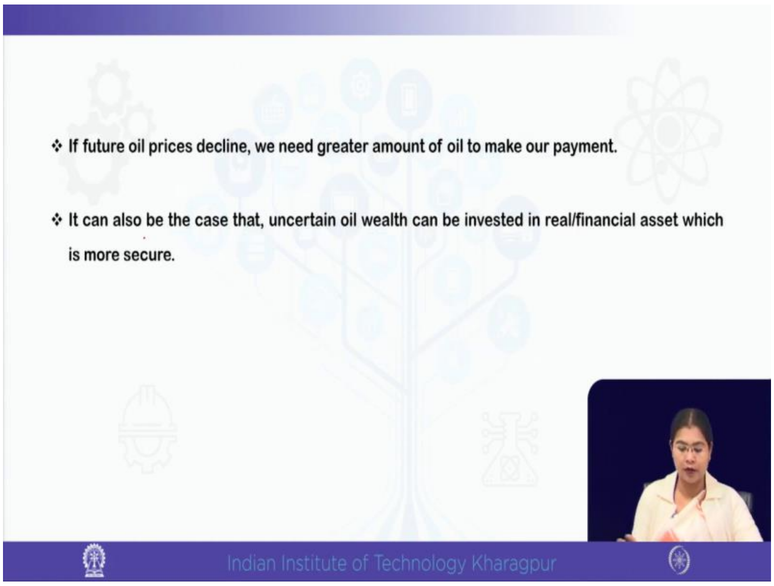

And as we mentioned that if oil price falls in future therefore, the country will need a greater amount of oil to make the payment right. So, you should keep these things in mind.

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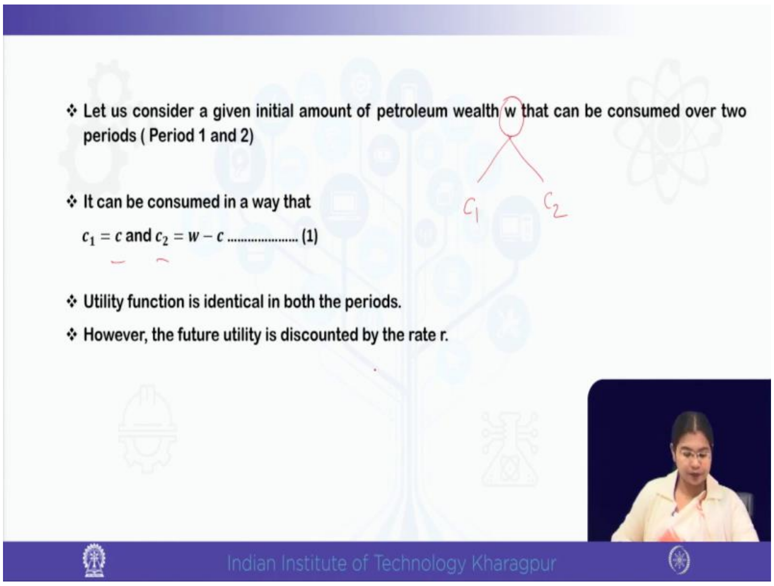

Now, in this structure let us now consider uncertainty. So, first we start with a given initial amount of a resource oil. So, for our purpose we are sticking to the example of petroleum. So, let us consider a given initial amount of petroleum wealth denoted by w. So, this w can be consumed over two periods c1 and c2 ok.

So, in period 1 and period 2. So, suppose we are denoting the consumption in period 1 is c1, c1 is equal to say the amount c and c2 is the rest of the amount. So, c2 is basically w minus c because c is the consumption in period 1. So, consumption in period 2 will be the total amount of endowment of petroleum wealth and less of the amount consumed in period 1.

Now, we also assume that we consider the same utility function. If you remember we have already specified our utility function in the previous lecture. So, utility is a function of the amount of consumption. Because we also assume that the oil wealth or the resource can be either consumed or can be used for capital accumulation and these two possibilities can also be done without incurring any additional extra cost ok.

So, these ways we divide our consumptions and your utility function depends on the amount of consumption and we assume identical utility function. However, we already know that in the present we are considering the utility. So, we have to consider the discounted value of the utility. So, or the present value of utility because we do not put equal weightage for present and future consumption right. So, we are considering the model with a discount rate. So, the future utility is to be discounted by the rate r.

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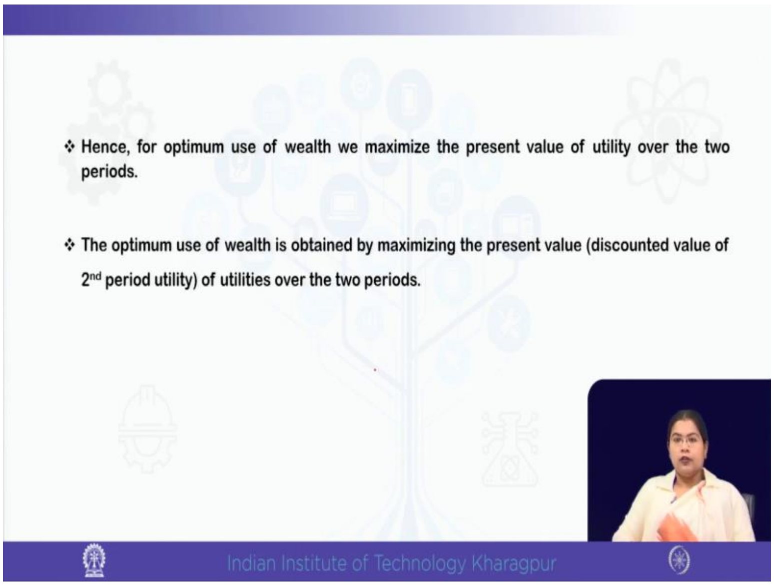

So, what should be the option our means the choice of the policy maker so, to use the wealth optimally. So, what the policy maker should do the policy maker should maximize the present value of utility over the two periods. And the optimum use of wealth that means, what should be the rate of extraction that is to be obtained by maximizing the present value of utility over the two periods.

So, present value of utility is the discounted value of 2nd period utility and the first period utility is of course, the current utility from the current consumption. So, because you are calculating your utility at the present for the future periods also. So, that is why we consider we discount the future values of utility.

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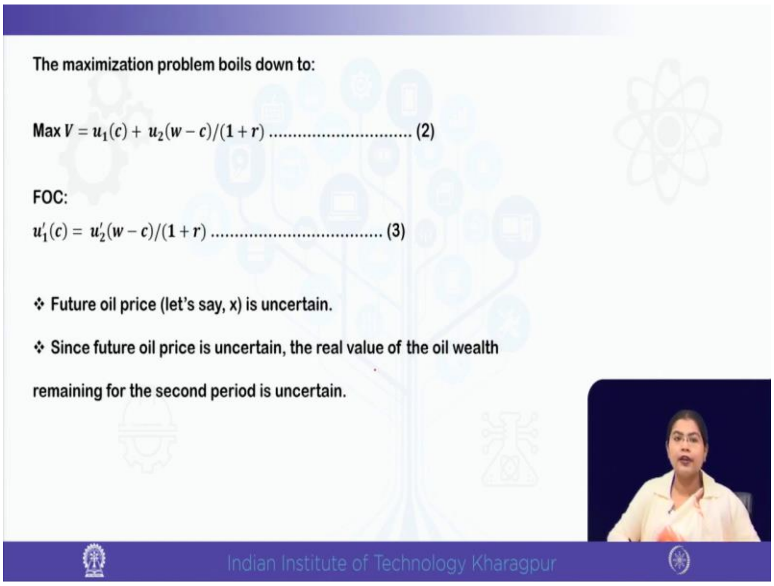

So, finally, our maximization problem can be written in this way. So, you remember we have already written the maximization problem, but now the second period utility is a function of the amount of consumption in second period that is c2 and c2 is equal to w minus c where, c is the amount of consumption of the oil wealth in the first period, ok.

So, accordingly we are writing the utility functions. So, the problem of the planner is to maximize the value function V which is equal to u1 c plus u2 as a function of w minus c and this is discounted ok by 1 plus r. So, what is the first order condition? So, this is our first order condition if we maximize the utility subject to how much we are going to consume. So, u 1 dash c is equal to u 2 dash w minus c divided by 1 plus r.

So, now let us bring an uncertainty, let us bring the uncertainty explicitly in the model. So, far we have considered the model with certainty even in our entire course we have not introduced uncertainty explicitly. So, suppose now we introduce the variable x which denotes the future oil price which is uncertain.

So, since future oil price is uncertain the real value of the oil wealth that is remaining for the future consumption is also uncertain because you know what will happen you do not know what will happen to the future oil price x. So, its x can increase or decrease. So, that is why the real value of the oil wealth which is remaining in the ground for the second period that is also uncertain.

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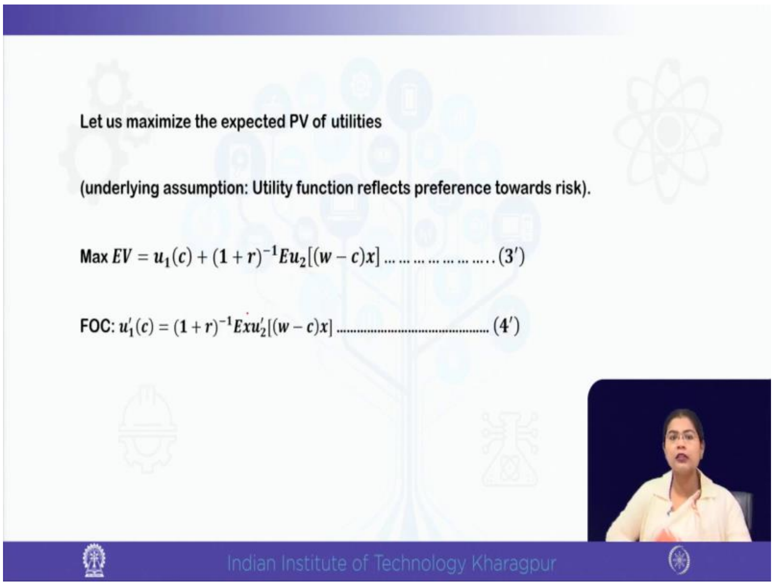

So, under this context of uncertainty let us maximize the expected present value of utility. So, we have you remember we have already discussed about the expected utility in game theory when we discussed rational intelligent agent. So, we discussed about expected utility. So, here we are using the expected present value because now at period in the current period you are discounting when you are calculating your utility you have to use the discount factor.

And also, we under the we assume that our utility function reflects the preference towards risk means the economic agent that is the planner can be a risk lover, can be a risk neutral or even it can be a risk averse. So, what is the preference towards risk? It may prefer to take more risk or it may be averse risk or it may be just indifferent to risk. So, these are the three possibilities. So, under this context we maximize the expected present value of utility by introducing the uncertainty variable that is the future oil price.

So, the expected present value of utility is equal to u1c plus 1 plus r whole to the power minus 1. So, this is the discount factor into E of u2. So, this is the expected value of second period utility E of u2 w minus c into x because x is now introduced x is the future oil price which is uncertain, ok.

So, now you see how the first order condition is changing. So, we are denoting these equations with 3 dash or 4 dash because we have already got this as 3 means. These are the corresponding equations of our previous lecture if you see the first order condition.

So, 3 and 4 so, these are now changed slightly with the uncertainty. So, that is why we have not named them separately means we have used this dash 3 dash and 4 dash. So, what is the first order condition now? First order condition becomes u1 dash c is equal to 1 plus r whole to the per minus 1 expected value of Exu 2 dash w minus cx, ok.

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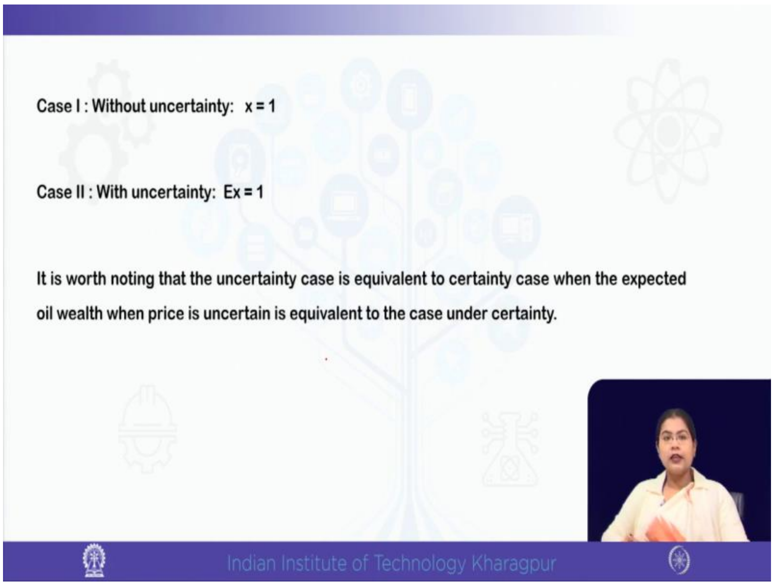

Now, let us consider two cases. One is without uncertainty where x is equal to 1. So, it is entirely certain and the 2nd case is with uncertainty where the expected value of x is equal to 1. So, there is entirely the market is uncertain the future oil price is completely uncertain, ok.

So, it is worth noting here that the uncertainty case is same as the certainty case when the expected oil wealth under price uncertainty is equivalent to the case under certainty. So, obviously so, if the expected oil wealth under uncertainty is equal to the amount of oil wealth under certainty so, obviously, the two will give us the same result it is obvious it is trivial right.

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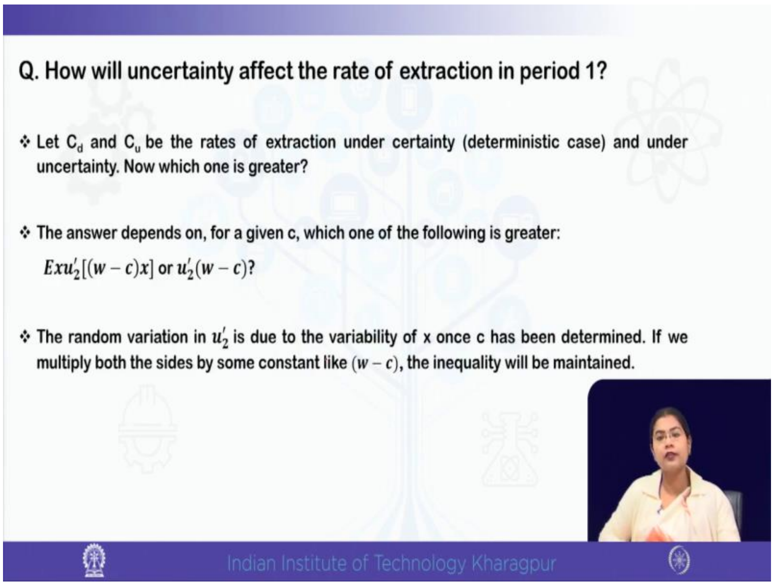

Now, let us focus on our question that we are addressing in this lecture. So, how will uncertainty about the future oil price affect the rate of extraction in period 1? So, you are deciding the rate of extraction in period 1 so, that we should remember. Now, suppose we introduce two variables one is Cd which is the rate of extraction under certainty that means, it is a deterministic variable ok and Cu be the rate of extraction under certainty sorry under uncertainty.

So, Cu is under uncertainty and say there is a random variation and Cd is under certainty; that means, deterministic. So, that is why you see we have used the subscript d and u. So, d stands for the deterministic case and u stands for the case under uncertainty, ok. Now, the answer depends means how uncertainty will affect the rate of extraction in period 1. The answer to the question depends on for a given c out of the following two expression which one is greater, ok.

So, you can see whether Exu 2 dash w minus cx or u2 dash w minus c which one is greater? Now, you see the random variation in u 2 dash; that means, marginal utility of period 2 its arising because of the variability of x once we determine c. Now, if we multiply both the sides by some constant like w minus c the inequality will also be maintained.

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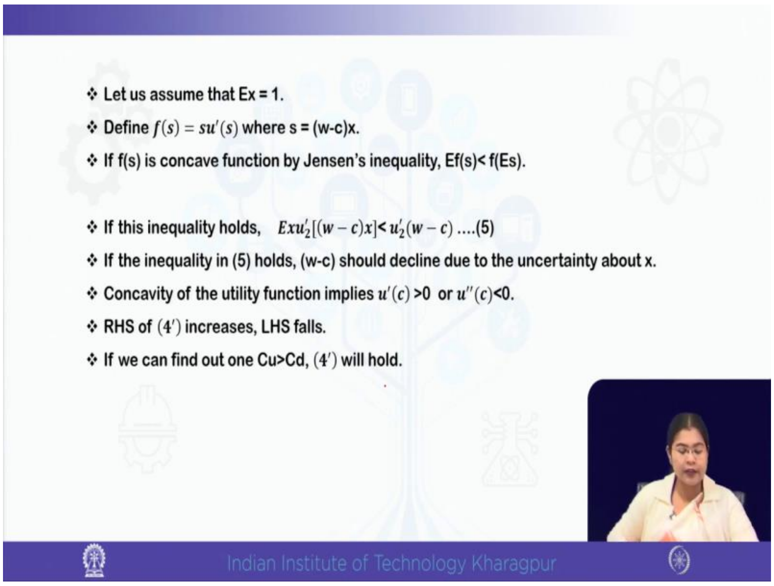

Now, let us assume that Ex is equal to 1. So, expected value of x is equal to 1. So, that means, we assume means we consider the case under uncertainty that is case 2. So, this result is under case 2. So, if Ex is equal to 1 now suppose we define a variable fs which is s. s is what if you remember in the previous lecture, we defined s is the amount of resource or the mineral wealth discovered, ok.

So, fs is s into the marginal utility of su dash su dash means it is the marginal utility del u del s. So, we are defining a function fs which is s into marginal utility of s that means, fs is equal to s into u dash s where s is what? s is w minus c into x w minus c is what w minus c is your 2nd period. You see if you go back w minus c your is your second period consumption.

So, second period consumption into the uncertain price of oil, ok. So, S is basically the value of the second period consumption of oil, ok. So, fs is equal to the value of the second period consumption of oil into the marginal utility. So, now the result means the answer to this question that how will uncertainty affect the rate of extraction in period 1. It entirely depends on the functional form of fs or; that means, to be more specific depends on the underlying preference that the utility function describe.

So, the utility function can take various forms. So, as I mentioned the planner can be a risk lover risk neutral or risk averse. So, the whether the optimum rate of extraction will increase or decrease that entirely depends on the underlying preference so, now if fs is contained function by Jensen's inequality the expected value of the expectation of the functional value of s.

So, E of fs is less than the functional value of expectation x, ok. This is by Jensen's inequality and this is the definition of a concave function. So, now if this inequality holds now if in place of fs we are putting these values we have already denoted s. So, we can write.

So, if the inequality holds, we can write equation 5 means condition 5. So, E of expected value of x into the marginal utility of second period consumption u2 dash into w minus c into x is strictly less than the marginal utility of consumption means the amount of consumption in period 2.

So, u2 dash w minus c. So, this is we are denoting as condition 5. So, if the inequality in condition 5 holds w minus c should fall due to the uncertainty about x ok. Because x is uncertain so, w minus c; that means, the amount of second period consumption should fall.

So, that means, we already know that concavity of the utility function it will imply that marginal utility is positive, but it is diminishing. So, u prime c is positive, but u double prime c is negative. So, that means, if we go back to our previous condition 4 dash if you refer to condition 4 dash by this condition the right-hand side should increase and left-hand side should fall. So, the right-hand side this one should increase and the left-hand side should fall.

Now, we have to mean this condition 4 dash this will hold with equality. So, that means, we have to find out one Cu means the consumption under uncertainty. You see the rate of extraction under uncertainty Cu such that the rate of extraction under certainty is greater. So, Cu greater than Cd so, that 4 dash will hold with equality sign because you see by this definition of concavity as we just saw that inequalities means the equality is not maintained in condition 4 dash.

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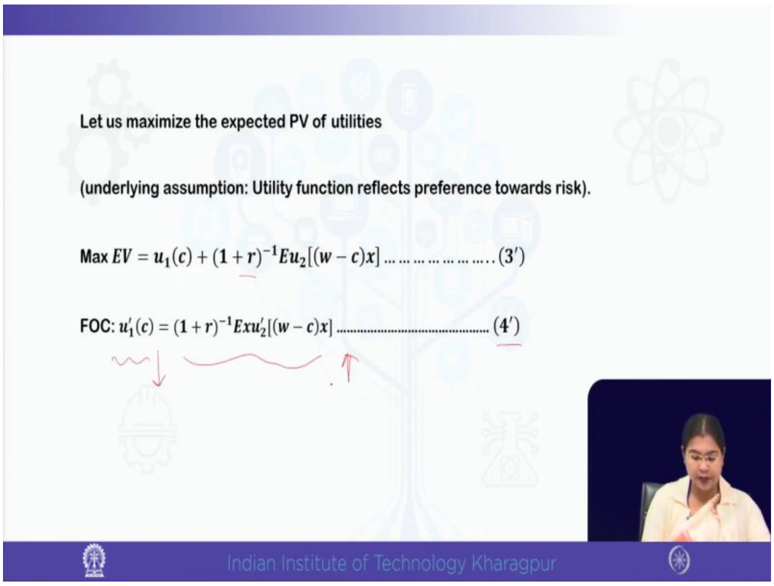

So, the RHS is increasing LHS is falling. So, that means, we have to find out one Cu which is greater than Cd. So, that 4 dash will again hold with equality sign.

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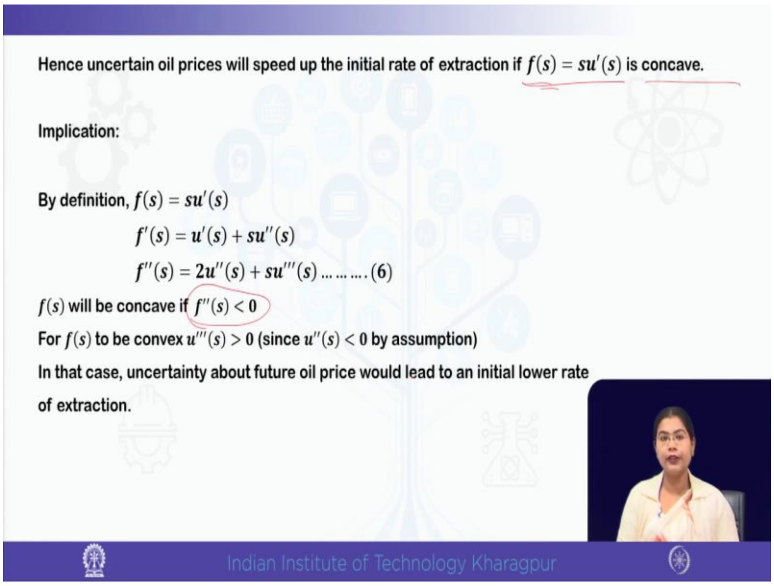

In that case what will happen uncertain oil prices will increase the speed of initial rate of extraction. So, this is happening if the function fs is concave. So, in that case you see if fs is concave. So, in that case your w minus c is falling. So, that means, you are actually increasing your future consumption is falling because you are increasing the initial rate of extraction. So, this is one possibility and you we can find out that this possibility is arising from the fact that the function value this fs is basically concave.

Now, what is the implication? Now, if we just rewrite the functional form f s is now, we have defined it fs is equal to s u dash s. So, what is f double f prime s? F prime s is if you now differentiate with respect to s the RHS it is u dash s plus s u double dash s then what will be f double dash s that will be u double dash s plus again u double dash s which we are writing as twice u double dash s plus s u triple dash s.

Now, you see fs will be concave if and only if this f double dash s is negative, ok. So, this is the condition for fs to be concave which is the case we have already discussed in under that circumstances the initial rate of extraction will go up. But however, if fs function is convex so, u triple dash s is positive because by assumption the marginal utility is diminishing u double dash is negative by assumption. In that case you see now you are getting opposite result uncertainty about future oil price will lead to initial lower rate of extraction.

So, you increase your future consumption. So, you see entirely what is happening in our model when you bring the uncertainty variable regarding the future oil price, we can see that whether the initial rate of extraction will increase or decrease that depends on the underlying preference which is reflected in terms of the functional form of f s.

So, f s if the f s function f s is what? The amount of the resource wealth into the marginal utility of the resource in for future consumption right into the means the value of marginal utility of the resource in future consumption because s is equal to w minus c into x. So, it depends on the answer entirely depends on the functional form of s which in turn again depends on the underlying preference structure.

So, if fs is concave then uncertain oil prices will increase the initial rate of extraction. So, future consumption will fall because if you extract more now, you will be left with less amount for future consumption. But if the function fs is convex in that case you will like to consume more in future and you will like to reduce your initial rate of extraction ok.

So, these are the possibilities that we can see over here. So, we cannot definitely say because you see that is an uncertain case. So, there are different possibilities. So, it entirely depends on the underlying preference structure.

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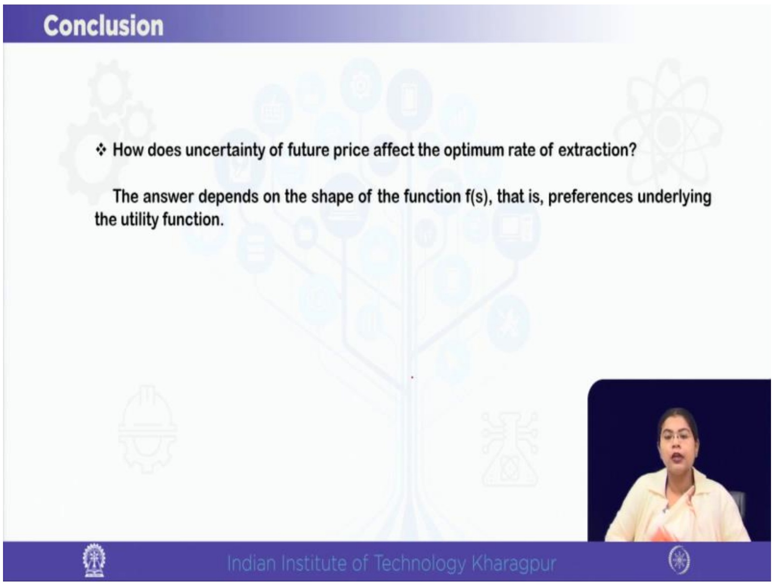

So, what we summarize for today's class? We continue with our theoretical model that we have structured in module 11. So, we started in module 11 we started with a theoretical model on dynamic optimization. So, first we started without resource discovery.

So, we found out the golden rule and modified golden rule we compared. And then we introduced resource discovery and we saw how the steady state consumption and the optimal paths of consumption and capital accumulation can be affected by resource discovery.

And finally, in this lecture we address the question that how uncertainty about future oil price will affect the rate of extraction. But we see that the answer depends on the underlying preferences which is again reflected in terms of the shape of the function fs ok.

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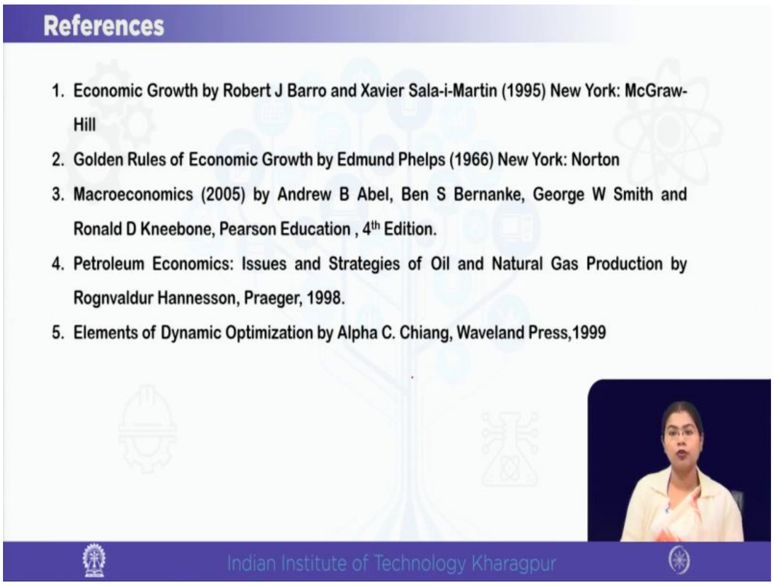

So, as I already mentioned the main important references for this lecture. So, for the Dynamic Optimization part 1 should definitely study the book of Alpha Chiang on Dynamic Optimization you would have rather preferred to do a in detail solution because of time constraint we are unable to do so, we have mostly followed the book of Petroleum Economics by Hansen for this theoretical model.

But those who want a more in-depth analysis they should of course, start with the Macro Economics or Economic Growth-related books like the book of Barro and Sala-i-Martin and the book of Macroeconomic by Abel Bernanke of course, the book of Phelps on Golden Rule of Economic Growth. So, these are the background reading you can say, but the reference for the Theoretical Model is Hansen's Petroleum Economics book.

And due to time constraint, I could not cover the numerical examples and there are different cases discussed in the book of Hansen in the relevant chapter where the model is worked out with different forms of the utility function and the paths of consumption and capital accumulation are also plotted.

And how they will change with resource discovery and uncertainty are also discussed in detail. So, those who are interested should go into deeper and study the relevant chapters of these books that are mentioned over here.

So, thank you very much. I see you in the last module.