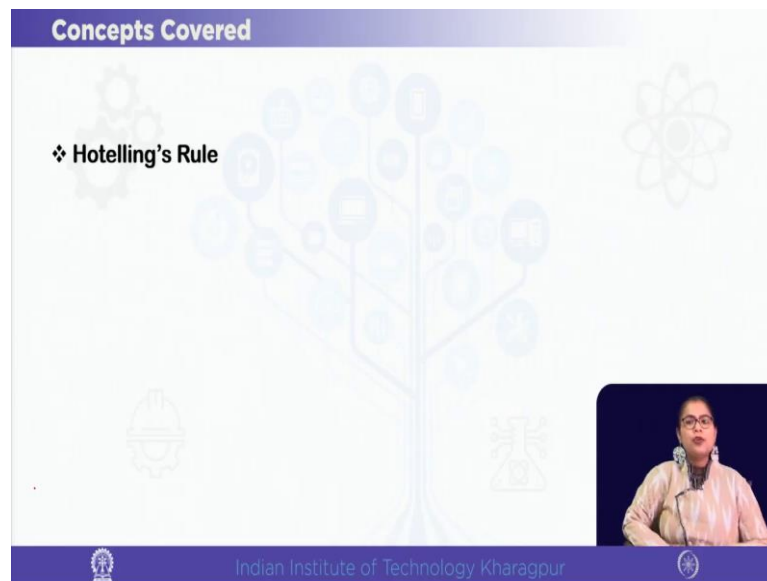


**Petroleum Economics and Management**  
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**Module - 10**  
**Theories of Price Formation of Petroleum**  
**Lecture - 49**  
**Hotelling's rule**

Hi everyone. I am Dr. Anwasha Aditya, your instructor for the course Petroleum Economics and Management. So, welcome back to our NPTEL course. We are in module 10 of our course, where we are discussing the Theories of Price Formation of Petroleum Products. This is lecture number 49 in our course, where we are discussing a very important rule called the Hotelling's Rule of Price Formation.

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So, if you remember, we are in module 10 discussing about the optimal allocation of a given stock of a natural resource. And this model can be applied in for any type of non-renewable or exhaustible natural resource like coal, natural gas or oil. So, for our purpose we are giving the example of oil, but it can be extended to any type of natural resource which is in fixed supply at a given point of time.

So, how that resource is to be consumed over time because at a given point of time, we have almost a fixed supply of the resource, so we have to distribute its consumption over

time. Because we know that these resources come from the savings of the past millennium.

And the current rate of formation is very low, so we should not be using the resource at a very fast rate, so that we run out of the resource because the current formation is very low. So, if you are using, if you are depleting the resource very fast, so we will be running out of the resource.

And at the same time, we have to also make a balance because if we keep the resource for future purpose, it may happen that the resource can become useless because of technological breakthroughs, innovation of some very cheap substitutes. So, the resource can become useless, and that will mean the owners of the resource will be incurring huge loss.

So, you have to make a balance between the allocation of the resource between now and future present and future. So, with this motivation, we are studying module 10 how the allocation of resource is done over time. And in market mechanism we know that it's the forces of demand and supply and price which actually signals how much to use at present and future.

So, we see we have already derived the pricing rule and the optimal allocation of a natural resource in a very simple framework. If you remember, we have been studying a two period model, period 1 and period 2. So, period 1 is the present and period 2 is the future. So, in this simple two period model, we are discussing how the given stock of the natural resource is to be distributed between now and then.

So, we have got the rule already, and we have justified the rule from both the consumers perspective and the owners of the resource perspective. So, we saw that from the consumers perspective, we equated the marginal utility of the present and the present value of marginal utility of future. And we got the pricing rule that price increases at a rate of interest over time or which is also equal to the rate of discount, ok.

So, price of the natural resource increases over time and we also validated that pricing strategy from the owners of the resource perspective, because from the owners perspective the owner will be indifferent between supplying the resource at present and in future.

So, if the owner supplies the resource at present the owner will earn a return of, we have already studied that  $rP_1$ , and if the owner waits for the future price increase. So, the future price will be increased to  $P_2$ . So, the profit will be  $P_2 - P_1$ , whereas  $P_1$  is the current price.

So, the owner will be indifferent between supplying now and future, when this to,  $rP_1$  which is the return that the owner will make by investing the profit in the current period. So,  $rP_1$  is equal to  $P_2$  minus  $P_1$ . So, then also we get the same pricing strategy that price of a natural resource increases over time and this rate of increase is given by the discount rate or the rate of interest.

And we also saw, we discussed the implication that even if price is equal to, means marginal cost is equal to 0 the price is positive and it is increasing over time it is because of the scarcity of the resource. Now, this the price of this finite natural resource is increasing over time this is called the Hotelling's rule.

So, in today's class, what we will be discussing? Will be discussing about the Hotelling's rule, its application, its implications and also the limitations. We will be studying the price path of a natural resource over time.

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**Hotelling's R Percent Rule**

- ❖ The rule that the price of a finite resource must increase at a rate equal to the interest rate is called the r% rule or Hotelling rule.

$$\frac{dp}{dt} = rp$$

- ❖ In a perfectly competitive market, market price (and marginal costs) should rise at the interest rate.

Handwritten derivations:

$$u_1' = \frac{u_2'}{1+r} \Rightarrow \frac{u_2'}{u_1'} = 1+r$$

$$\Rightarrow \frac{P_2}{P_1} = 1+r \Rightarrow \frac{\Delta P}{P} = r$$

$$rP_1 = (P_2 - P_1) \Rightarrow \frac{\Delta P}{P} = r$$

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So, what is the Hotelling's rule or which is also commonly known as the R-percent rule?  
 So, this is the rule that states that the price of a finite resource must increase at a rate

which is equal to the interest rate. So,  $\frac{dp}{dt} = rp$ , where  $t$  is the time and  $p$  is the price of the resource, and  $r$  is the market interest rate or which is also the discount rate. So, price of a finite resource increases at a rate equal to the interest rate.

Now, if you remember we have got it from both consumers and producers, perspective from the consumer side we equated  $u'_1$  marginal utility of period 1 with marginal utility present value of marginal utility of period 2. So, we got it as  $\frac{u'_2}{u'_1} = (1 + r)$ .

Now, if you remember from consumers optimum condition,  $\frac{u'_2}{u'_1}$  is basically  $\frac{P_2}{P_1}$ , right.

Because we in one of the lectures I think in lecture number 47 we have derived the condition of  $MRS = \frac{P_1}{P_2}$ . So, this  $\frac{u'_2}{u'_1}$  that is ratio of marginal utility is equal to the price ratio is equal to the 1 plus the rate of interest.

So, here we see that price of the resource increases at a rate equal to  $r$ . So,  $\frac{\Delta P}{P} = r$ , and the other we also validated the same rule from the owner's perspective ah. For the owner there are two choices either to invest means they extract the resource now; if the owner extracts the resource now, the owner will get the price for one unit of extraction, it will be  $P_1$ , the owner can invest the money in the capital market and the owner will make a return of  $rP_1$ .

And what is the other choice for the owner? Other choice of the owner will be just to wait; that means, delay the extraction to period 2 in that case the owner will earn a return of  $P_2 - P_1$ . So, we can see that here also the rule is  $\frac{\Delta P}{P} = r$ , right. Because  $\frac{P_2 - P_1}{P_1} = r$ .

So, price of the resource increase is positive and it is increasing over time, and the rate of increase is given by the market interest rate. So, in a perfectly competitive market, the market price and marginal cost it should rise at the interest rate, because in perfect competition price is equal to marginal cost.

So, if you remember even, we assume that the cost of extraction is 0, then then also we see that price is positive, and it is increasing over time, it is because of the scarcity of the resource. So, this is the famous Hotelling's rule.

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**The Economics of Exhaustible Resources by Hotelling, 1931**

- ❖ Finite resources and their supply is limited and the exploitation of these resources including minerals, fossil fuels and other exhaustible assets are produced and consumed too rapidly at a cheap rate.
- ❖ The resources are irreplaceable and can only be replaced with long delays and difficulties or costly.
- ❖ Even after 90 years the theory still has some relevance and states that the shadow price of non-renewable resources, should grow at the rate of interest.

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So, Hotelling, in 1931 proposed the economics of exhaustible resources. So, according to Hotelling the finite resources and their supply is limited because we know that these are from the past and millenniums of centuries, and we have a fixed given supply as it because at present also say petroleum or coal is being formed, but that rate is very less.

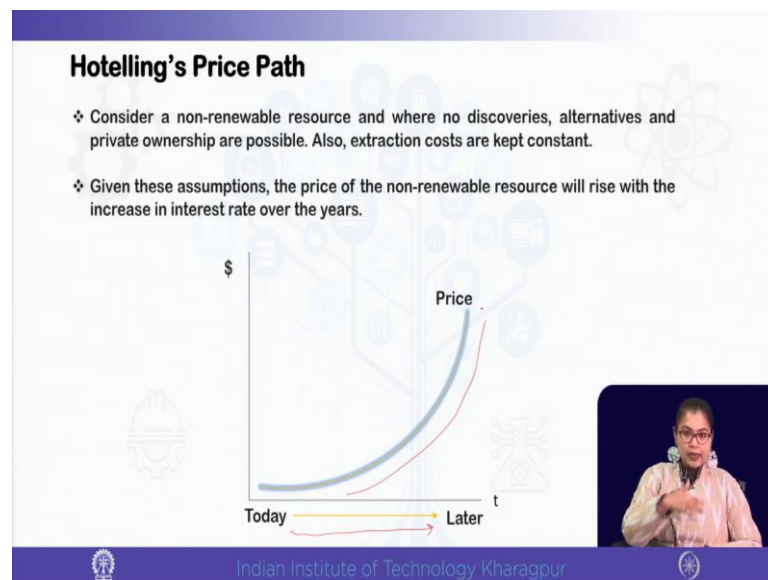
So, we can consider that at given point of time, the endowment is almost fixed it is limited. And the exploitation of these resources including minerals, fossil fuel and other exhaustible assets, these are produced and consumed very rapidly because the pricing is also less, sometimes because when supply increases price is less. So, they are used up at a very fast rate. And because we have also already studied how our present human civilization depends on this type of mineral resources and especially on petroleum.

And you see these resources are almost irreplaceable because we cannot replace them in a very short run. They can be replaced with long delays and difficulties or they are even costly and cheap easy substitutes are till now not available. We have already many times discussed that in transportation entirely depends on oil. We do not have any close substitute of oil in the transportation sector.

So, even after almost 90, years you see the rule was rule the Hotelling rule was proposed by Hotelling in 1931. So, almost after 90 years, the theory has still some relevance and it states that the shadow price of non-renewable resources should grow and the rate of growth is given by the market rate of interest.

And it is this high price of resource that actually discourages the future consumption. At the same time, it is because of the high price of the resource, the owners of the resource will not supply everything in the market at present. The owner will keep some amount for future because future price is increasing.

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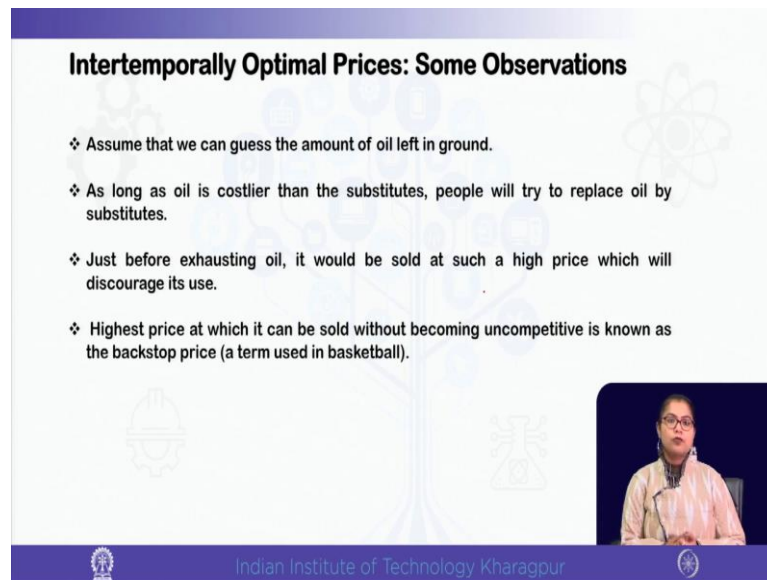


So, what is the Hotelling's price path? So, if you plot the price of the resource on the vertical axis because we know always price is on the vertical axis and if we plot time on the horizontal axis. So, over time how price of the resource changes? So, if we consider a non-renewable resource, and we let us for the time being we assume that there is no discovery alternatives and private ownership is possible.

So, we are considering only government ownership of the resource, and we also assume that extraction cost is constant. So, under these assumptions we can show that the price of the non-renewable resource, it increases with the increase in interest rate over time. So, and we have already got this rule that price of the resource increases at a rate given by the rate of interest  $r$ . So, this one we are actually plotting in this figure. So, we are plotting the price of the resource over time.

So, you see that as you time passes by, over time you see the price of the resource increases. So, you have a upward rising curve. So, this is basically the pricing path of the non-renewable resource like oil or other type of mineral resources, so over time price increases, ok.

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**Intertemporally Optimal Prices: Some Observations**

- ❖ Assume that we can guess the amount of oil left in ground.
- ❖ As long as oil is costlier than the substitutes, people will try to replace oil by substitutes.
- ❖ Just before exhausting oil, it would be sold at such a high price which will discourage its use.
- ❖ Highest price at which it can be sold without becoming uncompetitive is known as the backstop price (a term used in basketball).

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So, this one I just have drawn for your understanding. So, over time price increases as an present price is less, but as time passes by price increases you see and the rate of increase is given by the market interest rate. Now, let us discuss some observation regarding the pricing strategy or the inter temporal pricing strategy. Inter temporal means over time. So, over time how the pricing is done? We have seen, how the pricing is done. So, let us discuss some more observations and implications.

So, suppose for the time being if we can make a right guess about the amount of oil or any natural resource left in the ground, so suppose we know more or less how much of the proved reserve is there. So, as long as the natural resource or for our purpose we are just taking the example of oil. So, as long as oil is costlier, it is more expensive than other substitutes like coal or natural gas, we will try to replace oil by the substitutes, right.

And we have already seen that when a price of oil started rising mostly after mid-1970s, and there were the oil price shocks of 1979, you remember already. So, Saudi Arabia and other OPEC members also restricted the oil supply because the price increased very much and they were making profits. But we saw that over time this high price policy had to be abandoned by Saudi Arabia and other OPEC members because the elasticities of both in the demand side and the supply side increased.

So, by 1985 over Saudi Arabia and other OPEC members, they increased the supply and we entered the phase of great price collapse in 1986 because over time there were more substitutes of oil. Because from mid-1980 onward whatever new power stations were built, they were built using mostly cheaper substitutes like coal and natural gas.

And also, we started depending more on the hydroelectric power plants; that means, the renewable sources of energy, tidal power plant, solar energy. And more recently in 2014, we had the shale oil revolution. Even the nuclear power is also used in in very developed countries.

So, that means, if oil becomes costlier than its substitutes, we will try to replace oil by the substitute and that replacement has already been started from mid-1980 onward, at least in electricity. But for transportation we do not have any substitutes for oil. So, now assume that just before we exhaust oil, the oil price will be very high, right. Because the oil, if the oil price becomes very high, so it will discourage means this high price of oil will discourage the use of oil.

So, suppose we consider this highest price at which oil can be sold without becoming uncompetitive it is called the backstop price. And this is a term this backstop price, is a term which is borrowed from a basketball, the game basketball. So, we consider a highest price a very high price at which oil will become, means people will stop using oil. So, just before exhausting oil, oil price will become very high. So, oil becomes uncompetitive.

So, till oil becomes uncompetitive, so the highest possible price at which oil will till be sold is called the backstop price. You remember we already have studied the peak oil hypothesis which had a very pessimistic view over the use of oil according to the proponents of peak oil hypothesis like Campbell. They consider that oil price will become so high that the oil production will culminate, means people will stop using oil.

So, they were quite pessimistic. But they were proved wrong because of the shale oil revolution. But if you now consider a similar situation that oil price is increasing very much. And just before oil will be replaced by other alternatives, let us consider this highest possible price to be the backstop price.



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**Use of Intertemporal Pricing Rule**

- ❖ Now with the help of the  $r$  percent rule, we can back calculate price from the end to the present.
- ❖ In this framework the only unknown is the length of time required to run out of oil.
- ❖ But if we calculate price, we can calculate quantity demanded.
- ❖ Finally by summing up the demand in all the periods and equating it with the amount of oil remaining in the ground, we can find out the time duration required for exhausting oil.

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So, now how do you use the intertemporal pricing rule under these circumstances of backstop price? Now, with the help of the  $r$  percent rule what we can do is that we can back calculate the price from the end to the present, right, because you know that over time price increases.

Now, you know that at some point of time say at period end for the just an example. So, price reaches the backstop price let us say  $P^*$ , ok. So, with the help of the  $r$  percent rule what we can do? We can now back calculate price from the end. So, we can start from the last period and we can calculate the price till the present.

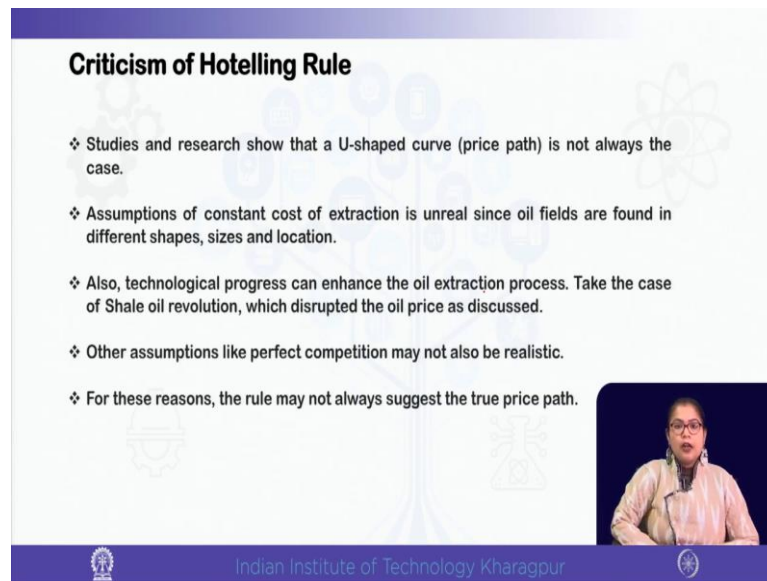
So, the in this framework, you see the only unknown is what, only unknown is the length of the time which is required to run out of oil. So, how much time is required to exhaust the given stock of the oil. But if we calculate price, we can also calculate quantity demanded because we already know that we, can we have the demand function, quantity demanded is a function of price in the inverse form, the price as if becomes a function of quantity demanded, right.

So, if we can calculate price, we can also get the quantity demanded. So, finally, what we can do? We can add up the quantity demanded. So, that means, finally, we can sum up the demand in all the periods. And we can equate it with the amount of oil that till remains in the ground. So, by equating these two, the total demand in all the periods

equal to the amount of oil or the natural resource left in ground, we can find out the amount of time which is required for exhausting oil, ok.

So, you see we can use the inter-temporal pricing rule to find out what will be the time span required to run out of oil. So, this is the use of inter-temporal pricing rule.

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**Criticism of Hotelling Rule**

- ❖ Studies and research show that a U-shaped curve (price path) is not always the case.
- ❖ Assumptions of constant cost of extraction is unreal since oil fields are found in different shapes, sizes and location.
- ❖ Also, technological progress can enhance the oil extraction process. Take the case of Shale oil revolution, which disrupted the oil price as discussed.
- ❖ Other assumptions like perfect competition may not also be realistic.
- ❖ For these reasons, the rule may not always suggest the true price path.

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Now, it is not that the Hotelling's rule is without any criticism or limitations or what are the limitations or criticism of the Hotelling rule. Till after 90 years the Hotelling's rule which have discussed that it is very much relevant, but till there are some criticism. So, first and foremost the existing literature shows that this upward rising price path is not always the case because you see the way we have plotted the price path means price of the natural resource.

Over time we see that it is upward rising. But this upward rising price path is not always felt. We you remember we have plotted the price of oil, the WTI Brent Crude price, this price, prices over time when we have seen there are so many temporary dips also. Like, in the phase of great price collapse, during the Asian currency crisis of 1997, even during the Shale oil revolution, and more recently during the COVID-19 pandemic where the WTI price future price became negative.

So, this U shaped curve is not always the case. Second is, second limitation of the Hotelling rule is that the assumption of constant cost of extraction is not very realistic

because the oil fields are found in different shape, size and location. So, it is not that always the cost of extraction is 0, right.

Means, if you have to go deeper, if the quality of the crude oil is not very good, so you have to use refineries. Or even we saw that for the WTI benchmark price the oil is extracted in the landlocked areas means then the cost of transportation is also very high. So, you see that due because the oil fields are unevenly distributed and they are in different places, they are in different size and shape. So, the constant cost of extraction is not very realistic assumption, cost of extraction can also change.

Third is that, technological progress can enhance the oil extraction process, right. Because if you are assuming that price of oil increases over time, now if you just we I just mentioned about the peak oil hypothesis also, but then we know that the peak oil hypothesis proponents, they were just means they had to change their opinion about the future of oil because of the shale oil revolution.

Because the proponents of the peak oil hypothesis, they did not know about the research and development going on for inventing the shale oil. So, the shale oil revolution just it reverse the price trend. So, it disrupted the oil price as we already discussed. Therefore, technological breakthrough like the shale oil revolution can enhance the oil extraction process. And we know by if supply increases price will fall.

So, this upward rising price path is not always the case for a mineral resource and even for a very important and scarce resource like oil. So, we may have dips also. It may become downward sloping also sometimes. And then other assumptions like perfect competition also, it not it will not be very realistic. So, for these reasons the Hotelling's rule may not always suggest the true price path. So, these are some of the limitations or criticism of the Hotelling rule.

But even with the even after taking into account the limitations and criticism, we can say that the Hotelling rule its provides a very good intuition about how the finite stock of the resource has to can be divided between present and future. And how; without any intervention by the third party how the price only acts as a signal of how to use, how much of the oil to be used in present and in future, ok.

Now, you see what we will do in the next class, we will be relaxing some of the assumptions. Like, we will be relaxing the assumption of constant cost of extraction. Means we will now bring a positive cost of extraction. Because if you remember, so far we considered in the previous lecture when we derived the Hotelling's rule, we considered no cost of extraction. But till price was positive due to the scarcity of the resource, but this is quite unrealistic.

So, in the next lecture, we will be considering a nonzero or positive cost of extraction, and we will be seeing how the Hotelling rule will change. And also, we will be considering a deviation from perfect competition. So, we will go to the other extreme. So, if we consider a monopoly supplier of oil. So, how the Hotelling rule will change?

So, in the next two next lecture, we will be just relaxing these two assumptions. So, one is the zero cost of extraction, so we will be considering positive cost of extraction and we will be also considering imperfectly competitive market structure, more specifically we will be considering a monopoly.

And we will see, means we will be taking into account or we will be addressing these limitations of the Hotelling rule. But till now we cannot deny the fact that the Hotelling rule is very much relevant and it is used. Because it shows that the price increases, even if the cost of extraction is 0 price in is positive and it is increasing over time and the rate of increase is given by the market rate of interest.

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**Conclusion**

❖ Hotelling rule: derivation, intuition and limitation.

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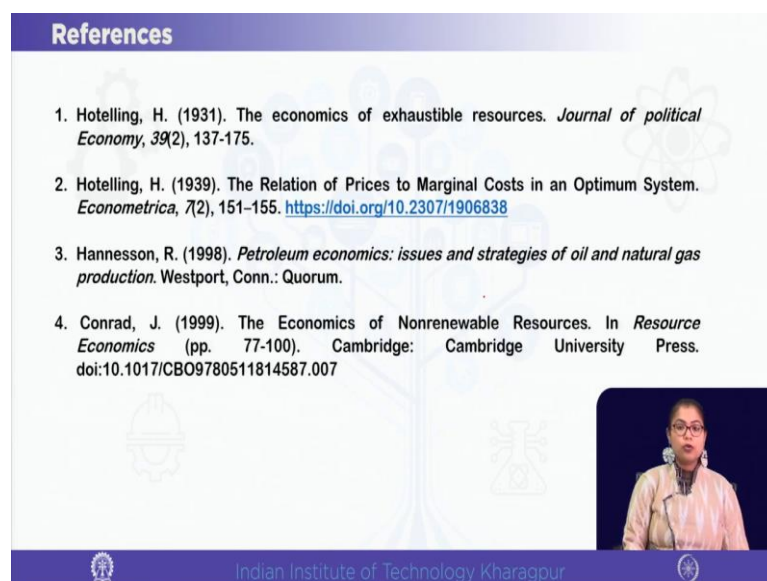
So, in today's class, what we did, we provided the intuition and we derived the Hotelling rule. We have discussed the implications and the use of the Hotelling rule. So, we saw that using the Hotelling rule, we can back calculate the price and we can also find out the time which is required to run out of oil. So, this is the case where you have a finite given stock of oil supply.

So, we can calculate what will be the time frame required to run out of the oil. So, that may be approximation because we can we know already by this time because technological innovations or breakthroughs can actually change the result. But till it will give you a rough idea of the time frame required to run out of the oil or the given stock of the mineral resource.

And then we discuss the limitations, but as I already mentioned that we will be relaxing some of the assumptions of the Hotelling's original model and we will be addressing the limitations. So, two are, one is the positive cost of extraction and the second is a monopoly market structure. So, how the Hotelling rule will change? So, that I will discuss in the next class.

So, for our time constraint we are just giving a very brief overview of the Hotelling optimal allocation and the pricing rule.

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But those who are interested will be providing you with the materials, the original paper of Hotelling you can see. And two papers of Hotelling are there, one is 1931 and another is 1939. And another paper by Conrad which discusses the economics of non-renewable resources.

And of course, the book of Hannesson is very important in this case. So, we have mainly followed the book of Hannesson. And there are many examples with numerical values with particular forms of the utility function, how to find out the optimal time required to run out of the resource.

So, all these are discussed with examples and numerical values and particular forms of the utility function. So, those who are interested they should go for some advance in-depth study which we cannot do because of constraint in time.

So, thank you very much. See you in the next class.