Petroleum Economics and Management
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Module - 10
Theories of Price Formation of Petroleum
Lecture - 48
Optimal Division of Natural Resources

Hi everyone, welcome to the NPTEL course Petroleum Economics and Management. And I am your instructor Dr. Anwesha Aditya. So, we are in module 10 of our course where we are discussing the Theories of Price Formation of Petroleum products. Now, this is our lecture number 48 in the course where we will be discussing in today's class the Optimal Division of Natural Resources over time.

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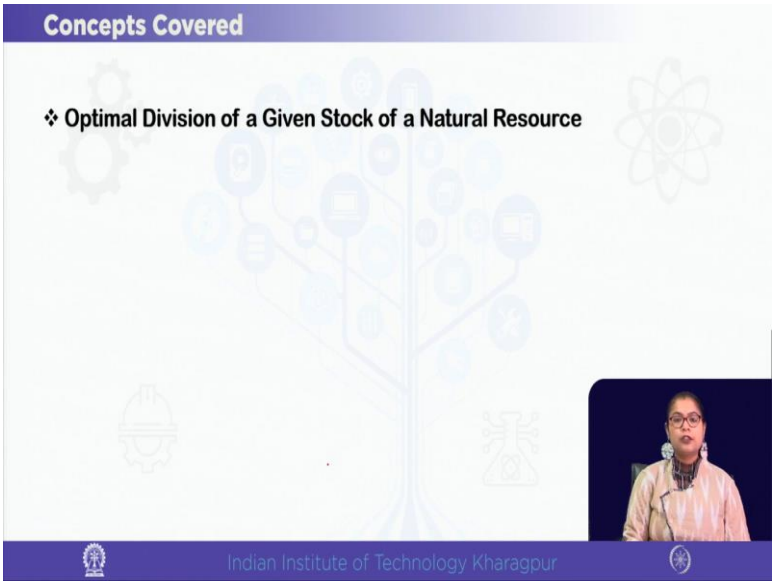

So, if you remember we have devoted a module 4 to study how oil or any other mineral resource which is in given supply. So, how that will be used over time and what should be the pricing rule? Because we know that this type of resources, they are used generation after generation. And these are from the past savings of the past millennium like we know that petroleum is fossil fuel.

So, coal also is from the fossil. So, these natural resources these are in given supply and means at the current period also they are being formed, but the data formation is much

less than the rate at which they are used at present. So, we have to use these resources prudently. Now, how to use the resource means, its not that we should use of all the resource then we should not be having the resource for the future generation.

And if we are too miser, if we do not use the resource currently it may also happen that due to some technological breakthrough some cheap easy substitute is available and the resource may become useless in the future. So, if we do not use it at the present and if we keep it for future that will not be a wise decision. So, how to decide how much of the resource to be used at present and how much will be used in future?

And what should be the pricing between now and future present and future, ok. So, with this motivation we are studying module 10 of our course where we are discussing the inter temporal pricing and the optimal division of a given stock of a natural resource. So, this theoretical model that we are studying currently it is not only applicable to oil, but any type of mineral resource which is in given supply means these are the non-renewable resource or exhaustible resource.

So, over time we are using and we are depleting the resource because current formation is very less. So, this can be applied to any type of such natural resources. Now, how the pricing is done? Because in if we leave everything to the market forces of demand and supply, we know that price will signal means price will act as a signal of how much to use in present and in future.

So, already we have developed the very simple theoretical model so, for our convenience we are studying a simple model of 2 periods. So, period 1 and period 2. So, that means, period 1 refers to the present and period 2 refers to the future. So, in this 2 period framework we will be studying the distribution of the resource between now and future and what should be the pricing at the present and in the future.

So, we are studying the optimal division of a given stock of a natural resource and first we are studying the model under certainty means we know. Suppose we know how much amount of the resource is there and in our next module when we will be studying the model of economic growth and its impact on resource discovery.

If suddenly a resource is discovered then how economic growth is disrupted or if it is positively affected and then we will bring uncertainty. So, for the time being we are

considering a fixed given amount of supply of the resource which is suppose of a known amount.

So, if you remember we have already outlined our theoretical structure of a 2 period economy and we consider the use of the natural resource in the 2 periods and we also one in one of the lecture we studied some basic of consumer theory, we derived the consumers optimum choice how the consumer decides how much to consume and what should be the rule. So, we will be using that condition for consumer's optimum in today's class.

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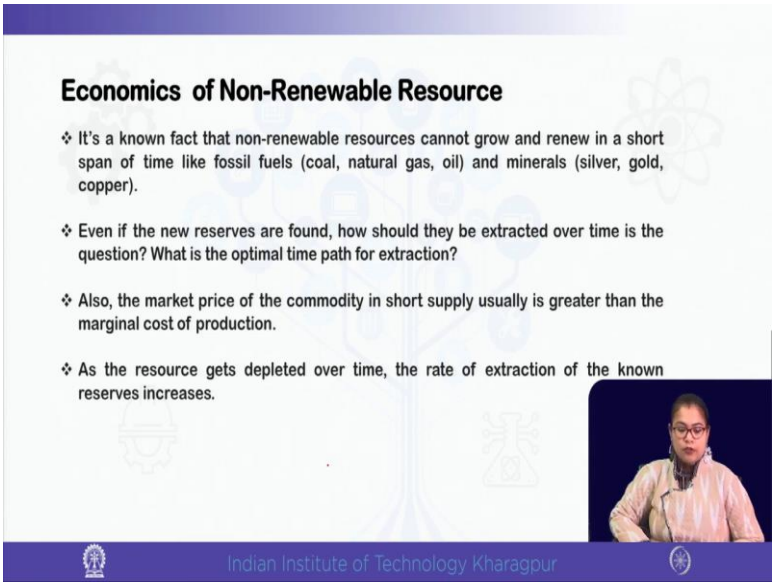

So, we know that this non-renewable resources like coal, natural gas and oil they cannot grow and they cannot renew in a short span of time ok and even if the new resources are found. Suppose you are depleting the resource from a particular region where you know that there is proof reserved. So, you are depleting the resource you are using of the resource and at the same time new reserves can also be found.

So, the total reserve for the entire world may be more or less unchanged or it is falling at a slower rate, but how should they be extracted over time that is the very important question because we should not be running out of the resource and at the same time, we should not be leaving the resource entirely for future use.

Because then we know that due to some technological breakthrough the resource may become useless. We have already seen let us say the oil market we have seen how shale oil revolution reverse the price trend.

Now, suppose if nuclear power also becomes cheap and more technologically available in the developing countries or you can consider of other technological breakthroughs then also oil price can fall. So, if we do not use the resource presently if we keep it for future, it may become useless so; that means, we have to have a balance, ok. So, that we should not be running out of the resource for our future generation and we should not be keeping in the entire resource for future use.

So, what should be the optimal time path for extraction? Now, as I was mentioning that in market economy the pricing will decide what should be the optimal rate of extraction. So, we will be deciding the market price of the commodity ok and generally we see that the market price of the commodity which is in short supply is greater than the marginal cost. So, these are some of the interesting observations that as we proceed will be getting the results in this theoretical model.

And as the resource gets depleted over time the rate of the extraction of the reserve it increases. So, let us now move to a theoretical model.

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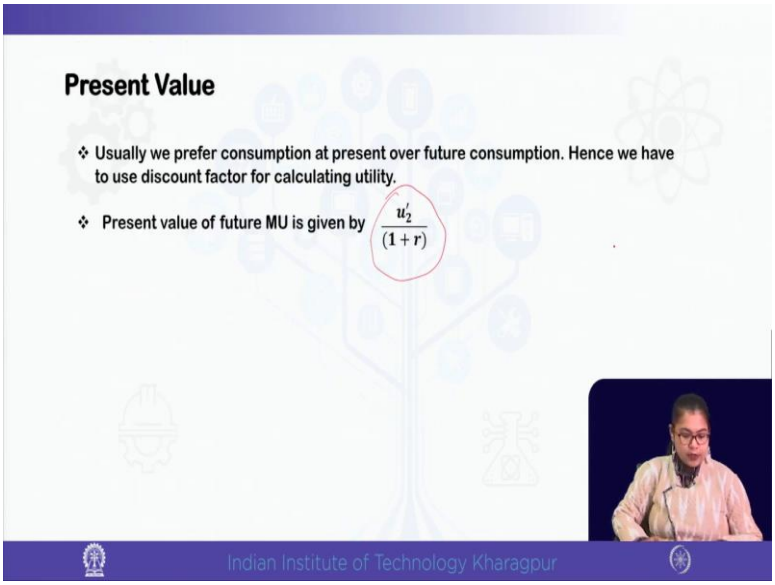

So, simple two period economy. So, if you remember in the first lecture of this module; that means, lecture number 46 we have discussed how the resources will be divided between now and then. So, we have drawn the marginal utilities right we used a box type of diagram where we used the left origin to denote period 1 and the right origin to denote period 2 or the future.

And then we plotted the marginal utilities without discounting the future. That means, in that lecture we used we put equal weightage on present and past and we plotted the diminishing marginal utilities means the marginal utilities curve and by the law of diminishing marginal utility we know that marginal utilities are positive, but diminishing as we use consume more and more of a good.

And without discounting the future we saw that the price was same in the two periods P_1 was equal to P_2 and the total amount of the given stock of the resource was almost equally divided between the two periods. But that is not true right because we usually prefer current consumption over future consumption.

So, we put greater weightage on our present than on future right because we see we are human beings are mortal. So, we think more about our present. We obviously, we do care for our future generations, but we prefer current consumption more than the future consumption.

So, we will be putting greater weightage on the present. So, that is for that reason we will be using the discount factor for calculating the present value of future marginal utility. So, that means, marginal utility in period 1 is u_1' and marginal utility is period 2 is u_2'.

But in period 1 when we are calculating the marginal utility of period 2 we should be using a discount factor. So, therefore, the present value of future marginal utility it will be given by $\frac{u_2'}{(1+r)}$. So, this is the present value of future marginal utility.

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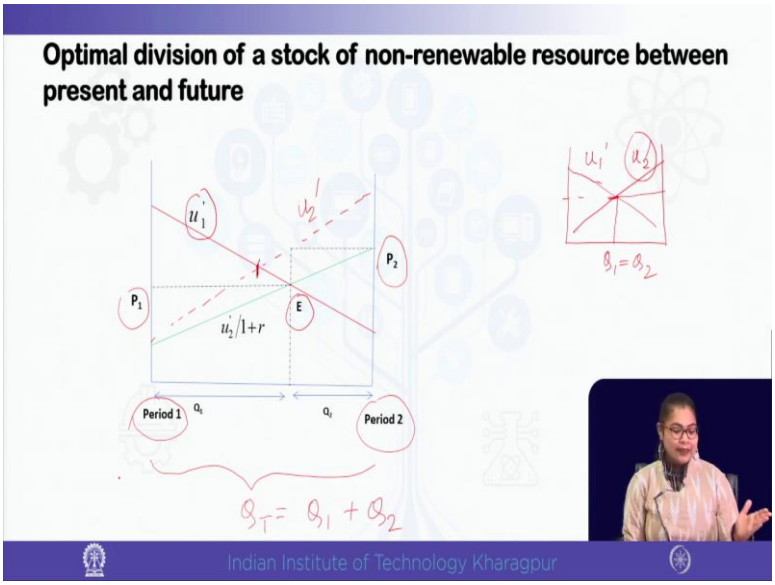

Now, what we will be doing? We will be studying we will be finding out the optimal division of the stock of a non renewable resource it can be oil, it can be coal or natural gas between present and future. So, as I mentioned just like in the lecture number 46 we are a plotting period 1 on the left origin and period 2 in the right origin ok.

So, and as usual we always know that price is on the vertical axis. So, and here is the price of period 1 and in this vertical axis we are measuring price of period 2 and the horizontal axis denotes the total supply. So, basically this length of this horizontal axis this is the total supply of the resource which at a particular point of time it is given. And for the time being we are assuming that it is also known.

So, this Q_T is to be divided between present consumption that is Q_1 and future consumption that is Q_2. So, Q_T is equal to Q_1 plus Q_2 and we have plotted the marginal utility. So, in this previous class if you remember we put equal weightage on u_1 and u_2. So, that means, the marginal utilities were plotted with equal weightage. So, we have drawn the marginal utilities like this and the price was same and if you remember.

So, P_1 was equal to P_2 and the allocation was also same more or less same. The same amount was allocated between the 2 periods. However, this is not true as we just now mentioned that we put greater weightage on present consumption. So, that is why at the

current period when we are calculating the present value of marginal utility, we are using the discount factor. So, the present value of marginal utilities you see it is $\frac{u_2'}{(1+r)}$.

So, we have plotted the present value of marginal utility of period 2 and u_1' is as usual the marginal utility of period 1 and in the figure that we used earlier the marginal utility of period 2 was not discounted and hence the prices were same and the same amount was. Means the out of, the total endowment almost equal amounts were used up in the current and future consumption.

However, this is not true. So, in this figure you can see we have discounted the marginal utility of period 2. So, it becomes $\frac{u_2'}{(1+r)}$. So, this is a shifted down version of the earlier marginal utility without the discount factor. So, Now, we can see that the optimal allocation occurs at the point of intersection of the two marginal utilities the marginal utility of period 1 and the present value of marginal utility of period 2 which is $\frac{u_2'}{(1+r)}$ and by the law of diminishing marginal utility the MU curves are downward sloping.

So, now the optimal allocation is given by point E and correspondingly we can also find out the two prices. So, we can see that at point E the price in the present period or period 1 corresponds to P_1 and the price of period 2 is P_2 ok. You can actually bring the marginal utility of period 2 over here yeah. So, this is basically u_2'. So, this is your price in period 2 and see how much is allocated between present and future.

So, you can see that Q_1 amount is consumed in the present and a lesser amount Q_2; that means, Q_T minus Q_1 is equal to Q_2 a lesser amount is kept for future consumption. So, you see what happens we can find out that P_2 is greater than P_1. So, a higher price in the future that discourages increased consumption in future whereas, in the present period price is less and that encourages more consumption in the present because you see we already know by law of demand if price falls quantity demanded increases.

So, if you compare the pricing in the two periods future price is greater than the current price which encourages current consumption and you are left with less amount of future consumption. So, we can now see how the optimal division is changed if we are using the discount factor that means, if we do not put greater weightage means, if we do not put equal weightage.

In this figure you see that one we are drawing and we have used in lecture number 46, we use the same amount for current and future consumption because we did not discount the utilities.

The present value we did not consider the present value of the marginal utility of period 2 we just plotted the marginal utility of period 2 and then this was the pricing and this was the optimal consumption which was divided equally between the two periods, but that is not actually the rule because we have to discount our future, we put greater weightage.

So, you can see over here that price in period 1 is less which encourages consumption in period 1. So, this is a very interesting result and more or less we see this type of pricing trend in the market for non renewable natural resources ok.

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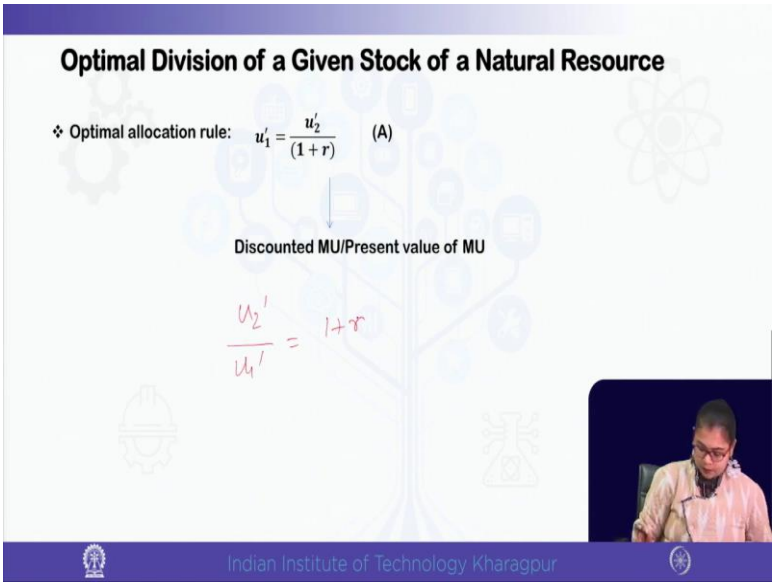

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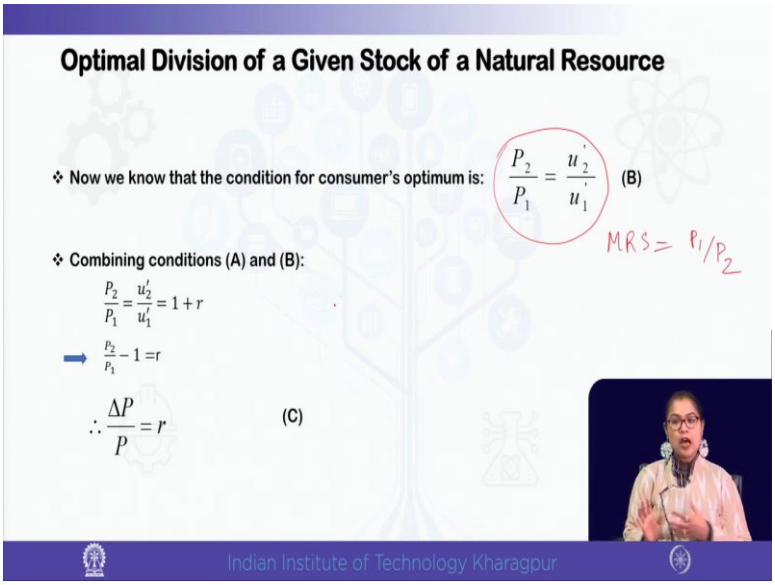

And so, we will be also getting the condition for the increase in price which is also known as very commonly known as the Hotellings Rule or the R Percent Rule how the price of a given stock of a natural resource changes over time. So, that means, based on this figure and our discussion so, far how we can write down the rule of allocation of the natural resource over time.

So, that means, the present marginal utility u_1' should be equal to the discounted marginal utility of period 2 right and that is $\frac{u_2'}{(1+r)}$. So, the optimal allocation rule will be the one where the marginal utility of present consumption is equal to the present value of future marginal utility. So, $\frac{u_2'}{(1+r)}$. So, you see that corresponds to basically point E in this figure u_1' is equal to $\frac{u_2'}{(1+r)}$. So, this is our optimal allocation rule of the resource between now and then or future ok. Now, we will be finding out the pricing rule based on this allocation rule. So, if we just recapitulate our previous lecture in which we derived the condition for consumers optimum if you remember we discussed it both intuitively and we also derived it mathematically using the Lagrange.

So, maxima the problem of representative consumer is to maximize utility where utility is a function of quantity of consumption of the goods and subject to the budget constraints. So, consumer is not a unconstrained. So, consumer maximizes utility, but the consumer is subject to purchasing power ok.

So, maximize utility subject to the budget constraint was the problem of the consumer and the consumer can maximize utility by choosing the amount of consumption of the goods ok. Because the prices are given in the market and we assume that the consumers are perfectly competitive.

So, and if you remember we wrote the Lagrange and finally, solving the Lagrange using the first order conditions we got the condition for consumer optimum as the relative price was equal to the ratio of marginal utility or if you remember we wrote it as marginal rate of substitution is equal to the price ratio.

So, again I am not elaborating we have discussed it in detail the intuition, the mathematical derivation everything and graphically it is the point of tangency between indifference curve and budget line under certain axioms and assumptions of preference like strict convexities. So, I am not going into that detail. So, basically based on our previous lecture this is the condition for consumers optimum ok that relative price is equal to the ratio of marginal utilities ok.

So, this we already know if you remember it was like MRS is equal to $\frac{P_1}{P_2}$ and MRS is the ratio of marginal utility. So, here we are writing $\frac{P_2}{P_1}$ is equal to $\frac{u_2'}{u_1'}$. Now, you see from the optimal allocation rule what we can see from condition A? $\frac{u_2'}{u_1'}$ is basically (1+ r) ok. So, in place of $\frac{u_2'}{u_1'}$

in this consumers optimum allocation rule we are putting 1 plus r from condition A.

So, that means, P_2 by P_1 is equal to now (1+ r). So, r is the rate of discount which is nothing, but the ongoing interest rate the market interest rate. So, if we do little bit of manipulation, we can write down that $\frac{P_2}{P_1}$ - 1 = r. So, that means, $\frac{P_2-P_1}{P_1}$ = r. So, that means, the rate of change of price that is $\frac{\Delta P}{P}$ = r. So, that means, price increases by the rate of interest or the discount rate ok.

So, this is condition C is the famous Hotelling rule. So, rate of change of price is given by the discount rate or the rate of time preference or the which is nothing, but the interest rate prevailing in the market. So, price of a natural resource is increasing over time and the rate of increase is given by the discount rate. So, this is a very famous law. So, we can see that how we derived condition C.

So, if you go back to the figure, you can easily find out that price in future that means, P_2 is greater than the current price P_1 ok. And this it is this pricing rule which also dictates the allocation of the resource over time since future price increases. So, we are discouraged to consume more in future whereas, current price is less so, consumers are consuming more in the present and keeping less amount for future consumption.

So, far we have not brought any intervention by any third party. So, without any intervention by any third party we know the allocation will be done by the pricing rule. So, you see price increases over time and that is why current consumption is more than future consumption.

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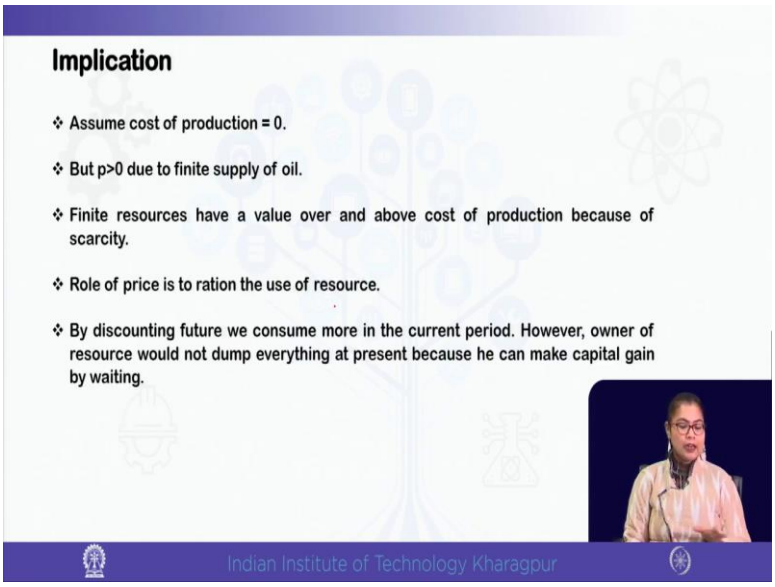

Now, there are certain very important implications that can come out from this pricing rule that price of a given stock of a natural resource it is increasing at the market rate of interest. So, what are the implications? See in this framework so, far we have not considered any explicit cost of production so, we have we are assuming that the cost of production is 0.

But even then, you see we can find out that price is increasing. So, even if the cost of production is 0 or cost of extraction is 0, but price is increasing. See if you remember in many classes, we have discussed about the pricing strategies under different types of markets. So, we know by this far that a price is equal to marginal cost in perfect competition ok. So, price is exactly tied to marginal cost in perfect competition.

In imperfect competition price exceeds marginal cost by an amount which is a mark-up we have mark-up pricing in imperfect competition. So, mark up depends on inversely on the elasticity. So, price exceeds marginal cost by an amount which varies inversely with the elasticity of demand.

But so, that means, imperfect competition price is equal to marginal cost, but then by that rule here if your marginal cost of production or extraction is equal to 0 price should be 0. But that is not happening price is positive even if you are assuming that the cost of production is 0 which is very interesting.

And that will not happen in other products which are based on a flow like the agriculture products you can grow crop season after season or even some manufacturing where you are producing the goods in the factories or even the service sector where you are using human capital it may be skilled or unskilled human capital to deliver some service.

So, if the cost of production is 0. So, price will be 0 at least in perfect competition. But here you see we can find out that even if cost of production is 0 price is positive and not only that the future price is increasing. So, this is because of finite supply of oil why? Because it is because of the scarcity of the natural resource that price is positive and it is increasing over time even if we are not bringing any cost of extraction.

So, finite resources have a value over and above the cost of production because of scarcity. So, its because of their scarcity it is not available everywhere we have many times so, far we have seen how petroleum is distributed unevenly the middle east countries are highly endowed, the OPEC countries are highly endowed whereas, countries like India we have very miniscule domestic production and supply of oil So, finite resources we can see over here that price is positive and increasing its because of its scarcity

And here comes the role of price in a market framework where the market forces of demand and supply decide the optimal allocation. So, role of the price is to use ration the use of the resource. So, price should increase over time so, that future consumption is discouraged because what will happen you see if the cost of production is 0 if price is also 0. So, then you will be using up all the resource.

But then you will not be left out with the resource for future consumption. So, that is why even if your cost of production is 0 price is positive and you are discouraging more and more consumption and price also increasing over time. So, the price here acts as a rationing instrument which limits the use of the resource otherwise you will be running out of the resource very soon.

So, it is the positive price and increasing price which limits the use of the natural resource which is in given supply and that is very important natural resource and by discounting future we consume more in the current period. However, the owner of the resource will not dump everything at the present because we can see that by waiting for the future the owner can make a capital gain. See the consumer wants to consume more in the current period because price is less in the current period, but what about the owners?

Now, you have to also see from the owner's point of view. So, if you are the owner what will you do? You will also put something for future because you can see that price increases in the future period right. So, as the supplier of the natural resource you will have enough incentive not to supply entire stock in the current period because you can make a profit by waiting till the next period. So, from the owner's point of view now you see what are the two choices for the owner?

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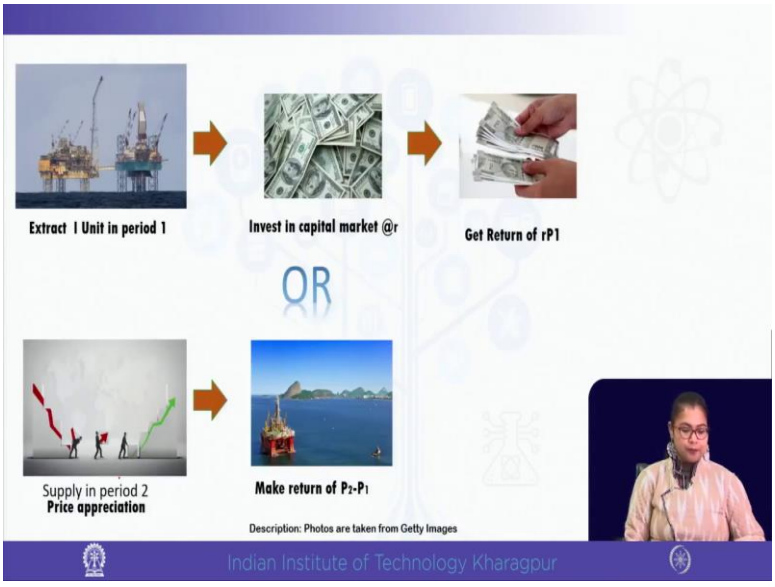

Either the owner can think of extracting one unit in period 1; that means, at present. So, if the owner extracts one period in period 1 and suppose the per unit price is P_1. So, what is the option for the owner? The owner can invest that amount in the capital market and suppose the ongoing interest rate is r which is the discount rate we are using over here.

So, what will be the return of the owner in period 1? So, if the owner extracts one unit in period 1 and sells the resource in the market at the ongoing price P_1. So, the owner's earns out of one unit I am saying the owner earns the return of given by the rate r. So, what will be the total return of the owner? It is rP_1 ok and what is the other option of the owner? Other option of the owner is to supply the one unit in the future

And in future we know that price increases price appreciates to P_2. So, if the owner waits and supply in period 2. So, what will happen? The owner will make a return of it will may he or she will make a profit of $P_2 - P_1$ because in future price is P_2 which is greater than P1 we have already seen right P_2 is greater than P_1. So, that will be the profit that the owner will make if the owner does not supply the amount in the present period and it supplies the amount in the future.

So, you see you now compare the return of the return and the profit of the owner from supplying now and then. So, by supplying at present the owner earns a return of rP_1. By supplying in the future, the owner earns a profit of $P_2 - P_1$ because future price is P_2 which is greater than the current price P_1.

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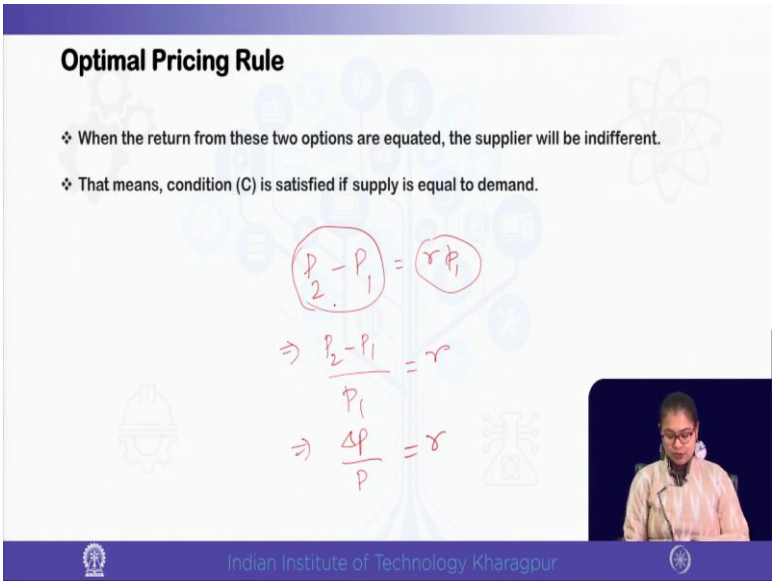

So, now the owner will be indifferent when means the owner ah. So, for the from the owner's point of view the optimal pricing rule will be the situation where the return from these two options are just equated. So, the supplier will be indifferent. That means what? So, that means, you see it is nothing but then if you just write down these two conditions P_2 minus P_1 is its P_2 minus P_1 because price in future increases.

So, $P_2 - P_1$ is equal to rP_1. So, you see the owner is just indifferent between supplying now and then. So, this is basically the tie breaking rule that we are using. So, what will be the pricing then? You see $\frac{P_2 - P_1}{P_1} = r$ which is nothing but $\frac{\Delta P}{P} = r$. So, that means, price increases at the rate r and this is what?

This is nothing but condition C that we got from the marginal utility. So, you see now what we are doing over here first we got the optimal allocation rule from the consumer's point of view. So, we are equating the present marginal utility and the present value of marginal utility of future period and from there we see that price is increasing over time which discourages future consumption.

But now we have come to the other side of the market because we know already that in market there are two sides the demand side and the supply side. So, we have got the pricing rule and the allocation rule that is condition C from the consumer point of view. Now, we are also getting the same pricing rule from the owner's point of view right because for the from the owner's point of view we can see that the owner has two option either to supply at present or to supply in future.

So, by supplying at present the owner is getting a return of rP_1 and by supplying in future the owner is making a profit of $P_2 - P_1$. So, by equating these two; that means, when the return from these two options are just equal the owner will be indifferent between supplying now and then.

So, if we just equate these two we get nothing, but condition C again. So, you see from both the suppliers point of view and the consumer's point of view price of the finite stock of a natural resource it is increasing over time ok.

And its because of the high price in future that discourages the consumption in future. So, you are consuming more at present, but you see its not that we are consuming everything at present because the owner will not dump everything at the present because the owner can make more profit by waiting till the future.

Because if the owner supplies more supplies some amount in future the price increases in future. So, it is the pricing strategy the market forces of demand and supply that dictates the price again the price will also dictate how much to be consumed now and how much will be consumed in future.

And even without assuming any cost of production we are getting a positive price and also over time price increases and that is due to the scarcity of the natural resource which is very important because otherwise in other markets we see that when we are producing some goods or services in agriculture manufacturing or in the service sector under perfect competition price is equal to marginal cost. But here price is greater than means price is positive even if the cost of production and extraction is 0.

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So, what we have done in this particular lecture is that we have studied the inter temporal pricing and the allocation of given stock of a mineral resource it can be applicable to oil, natural gas, coal any type of non-renewable or exhaustible resource which is in given supply its in fixed quantity at a given point of time. So, how that will be used at present and how much will be left for future consumption.

So, we see that without any intervention by any third party its the market price which distributes the allocation of the resource over time. And we see that even without assuming any cost of production price is positive because of the scarcity of the resource

and not only that price increases over time which encourages a present consumption and discourages future consumption.

But even if this price is low in period 1, the consumer would like to consume more in period 1, but the producers would not be supplying everything in period 1 because the producer can gain by waiting for the future because in future price appreciates or price increases.

So, the owners of the resource will not sell everything in the market in the present it will keep something for the future consumption. So, the owner will be indifferent between supplying now and in future the owner will compare the gains he or she will make by supplying at present and supplying in the future. And see that price increases at a rate which is given by the market rate of interest.

So, this is the famous Hotelling rule which we will be discussing in the next lecture and we will be discussing more about the optimal pricing path and the Hotelling rule its implications and the limitations of the Hotelling's rule.

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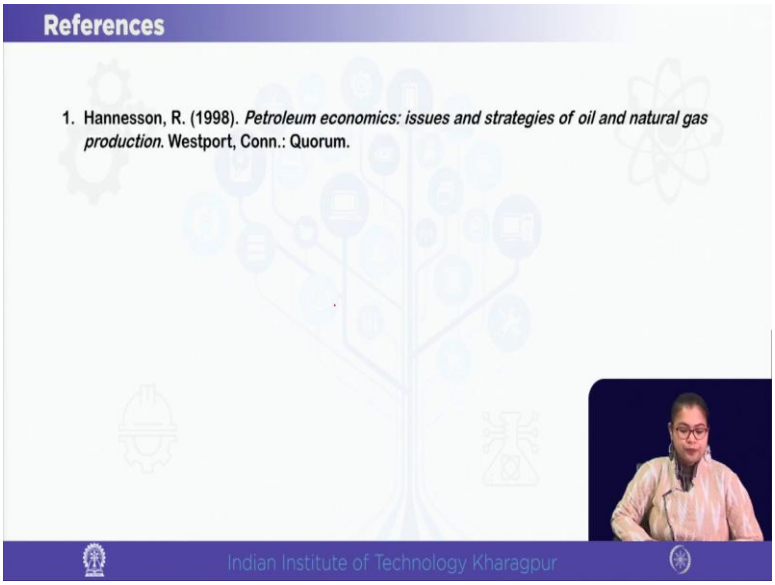

So, we have mainly followed the book of Hannesson on petroleum economics for this part of our course.

Thank you very much, see you in the next class.