Petroleum Economics and Management
Prof. Anwesha Aditya
Department of Humanities and Social Sciences
Indian Institute of Technology, Kharagpur

Module - 08
Oligopoly and Game Theory
Lecture - 40
Nash Equilibrium

Hi everyone. Welcome to the NPTEL course, Petroleum Economics and Management. So, we were in module 8 of our course where we are discussing Oligopoly and Game Theory. This is our lecture number 40 of the course, where, in today's class we are going to discuss a very important and interesting concept of Nash Equilibrium.

(Refer Slide Time: 00:47)

Now, if you just remember that, we are going to devote some time to understand the market structure of global petroleum industry. And for that we need to know the oligopoly models, the price leadership cartel model. Now, if you remember we have already discussed that finding out the equilibrium in oligopoly is tough, quite complex because we do not have any unique equilibrium in oligopoly. Because in oligopoly model the firms, the their strategies are interdependent.

So, when a firm is maximizing its profit, the profit of the firm, it not only depends on its own output or cost structure, it also depends on the rivals output cost structure. And

many other factors like whether the firms are taking decision at the same time or not, if they are producing identical good or not, whether the particular firm has information regarding the other firm or not.

So, there are means depending on these various aspect of the rival, various aspect regarding the timing of decision, we can have whole lot of possibilities. Even, we have seen we have discussed with example that just a change in the choice variable or the strategy variable; that means, a change from output to price can yield polar opposite outcome. So, that is why we do not have a single equilibrium or unique equilibrium or unique model in oligopoly.

Now, oligopoly models were developed much before, I mean the first oligopoly model was developed by French Economist Augustine Cournot, 1838. So, it deals with the simultaneous move output competition. But after the development of game theory, especially the concept of Nash equilibrium in 1950 and 51, nowadays we study the oligopoly models as application of game theory.

Because in previous classes, we have seen why we need to study oligopoly in game theory because oligopoly is just like when the players are taking decision or the firms are taking decision, the firm has to take into account its rival's action. So, it essentially like a game of chase where your particular move depends on your rival's actions, right. So, that is why it is helpful for us to study oligopoly models in a game theory framework.

So, that is why we are devoting some time to understand basic of game theory. But I have already mentioned that game theory itself it is a vast area and it is not only applicable in economics, but many other branches like mathematics and different branches of social science, international relation. Actually, many of the famous games are actually developed by mathematicians or other social scientist.

So, it has application beyond economics also. And that is why I hope that these part of the course, the game theory part of the course will actually be very interesting to you and you gain more interest and you study on your own also because for our time constraint we cannot devote much time on game theory. I would like to, but I cannot do that because we are really constrained by time. We have to discuss more about some petroleum related topics also.

So, here we are just discussing those models which will be required to solve the price leadership, dominant firm model and the cartel model. And we need to also know the cheating incentive of the OPEC members because many times we have seen with data empirical observation that the OPEC members they end up producing more or selling more oil than what they agree to.

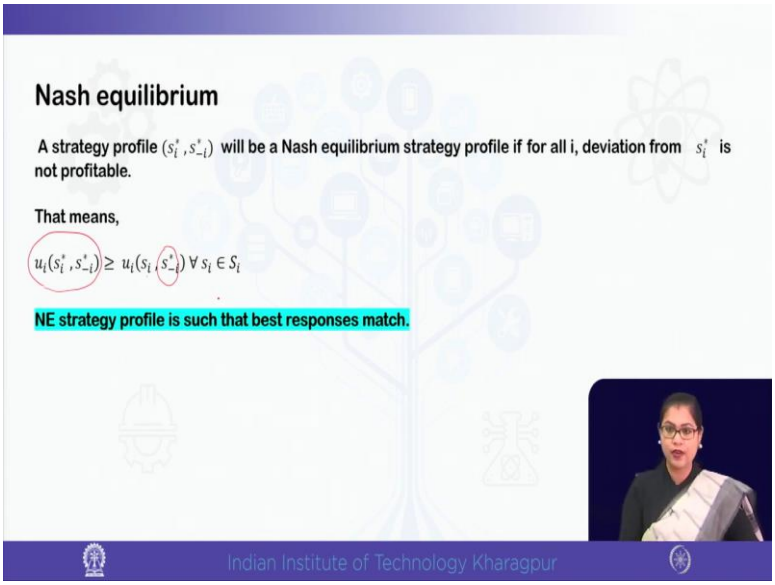
So, this is due to the prisoner's dilemma motive. So, we will be seeing that. I have earlier also uttered this term prisoners dilemma game, so we need to know about the game. So, for that we will be studying the concept of Nash equilibrium which is path breaking in the area of game theory.

So, in today's class we will be discussing about the definition and the properties of Nash equilibrium. Then, we will be seeing how to find out a Nash equilibrium using the very famous game of the battle of the sexes. There are many other very famous interesting games also, but I cannot do that. I cannot cover all these interesting games for time constraint, ok. So, we will be just finding out the Nash equilibrium in a static firm game.

You remember in the previous lecture we have shown how to represent game. So, there I have explained to you how to represent the dynamic firm game in terms of a game tree. But we are yet to show how to represent the static form game in terms of the matrix or the strategic form or the normal form.

So, in today's class we will be first showing how to represent a static firm game, then how to find out the Nash equilibrium, and once we obtain the Nash equilibrium how do we test that this strategy profile is particularly are Nash equilibrium. So, these are the main concepts to be covered in today's class, which is which I guess it will be very interesting and in helpful for you to understand the implications in the global oil market.

(Refer Slide Time: 06:07)



So, the concept of Nash equilibrium was of course, proposed by John Nash in 1950 and 51. There are two very famous papers. We have also referred to the original papers. Those who are interested can look at that. And also if you are interested you can also watch a movie based on the life of John Nash, the name of the movie, the academy award winning movie is “A Beautiful Mind”. So, you will be able to see the life story of John Nash.

So, what is a Nash equilibrium? Nash equilibrium is a strategy profile, let us say (s_i^*, s_{-i}^*); that means, s_i^* is the strategy profile of, means strategy of s_i^* is the strategy of one particular economic agent or players, say ith player and s_{-i}^* means the strategy of any other player, ok. So, a strategy profile (s_i^*, s_{-i}^*), minus i means the other player, ok.

So, suppose there are two players, so you can interpret it as s_1^*, s_2^*, ok. So, a strategy profile (s_i^*, s_{-i}^*) will be a Nash equilibrium strategy profile, if for all i deviation from this particular strategy s_i^* is not profitable, ok. I will explain with example.

So, Nash equilibrium is the one where from which deviation is not profitable. So, only Nash equilibrium has this great property that if it is a Nash equilibrium, no economic agent will have an incentive to deviate from that, ok. So, Nash equilibrium strategy profile can also be defined as a strategy profile where the optimal responses or best responses match. I will explain what do we mean by best response or optimal response.

So, just coming back to the definition of Nash equilibrium, the way we are writing here you see now in terms of the payoff we can write. So, it is very important that we understand the notation. So, u_i is the payoff of ith player, ok. So, u_i is the payoff of ith player, from playing this strategy profile is (s_i^*, s_{-i}^*). So, given that the other player is playing this strategy s_{-i}^* . So, you see the payoff of ith player from playing this strategies s_i^*, ok is u_i (s_i^*).

So, if it is a Nash equilibrium then you see that this payoff of ith player from playing this strategy s_i^* given that the strategy of the other player remains unchanged is greater than equal to the payoff of ith player from playing the other strategy s_i given that the other player is playing the same strategy you see. So, (s_i, s_{-i}^*) is same.

So, given the strategy of the other player, the strategy of the other player is unchanged. So, then you are comparing between two strategies of this ith player. So, you are comparing between strategy s_i^* and s_i. So, see here we are in this inequality, in this definition of Nash equilibrium, we are just changing the strategy of the ith player.

So, you are comparing between the payoff of ith player from playing two strategies s_i^* and s_i given that the strategy of the other player is unchanged. So, you see that the payoff associated with the strategy profile (s_i^*, s_{-i}^*) is greater than equal to the payoff associated with the other strategy profile (s_i, s_{-i}^*).

So, given that the other player is playing the same strategy, so if the player ith player is getting a hard payoff see s_i^*, the payoff will associated with s_i^* is greater than equal to the payoff associated with s_i. So, we call that this strategy profile (s_i^*, s_{-i}^*) to be a Nash equilibrium.

For all s_i^* small s_i^* means the particular i strategy belonging to the capital Si stands for the grand strategy set. So, you have a grand strategy space, right. So, a strategy space out of which you have many different Si means

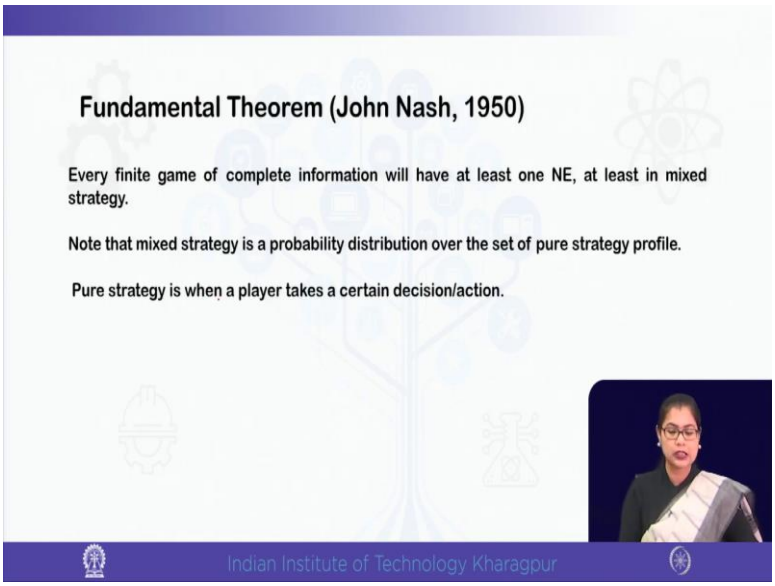
s_1 to s_n number of strategies. So, I will be discussing with example, then it will be clear. So, grand strategy set means it is the set of all possible actions or decisions or what we call in game theory jargon the strategies, ok.

So, a Nash equilibrium strategy profile, if it is a Nash equilibrium no economic agent will have any incentive to deviate because we are obtaining the Nash equilibrium by matching the best responses. So, what is the best response? So, best response means best response of ith player will be the strategy which gives higher payoff given the particular strategy of the rival player.

So, given that the rival player is not changing the strategy. So, you are comparing between say strategy 1 and 2 of a particular player, then which strategy is giving the best means higher payoff because see we are assuming that the economic agent is rational that means, one who is self-interested economic agent its maximizing either profit or maximizing utility, right. So, which strategy profile is giving you a better payoff, that is your best response given the particular strategy of the rival.

Why I am saying, at each stage I am saying given because you see if the strategy of the rival changes, so the best response can also change. Because a particular player's payoff depends on not only his or her action, but it also depends on the action of the rival. So, till now this may be a bit vague, so that is why we will be going to discuss the definition of Nash equilibrium with some example where it will be clear to you.

(Refer Slide Time: 12:12)



So, before going to the example part, so let us first see the fundamental theorem of John Nash. So, the theorem tells that every finite game of complete information will have at

least one Nash equilibrium. If the Nash equilibrium does not exist in pure strategy it will exist in mixed strategy.

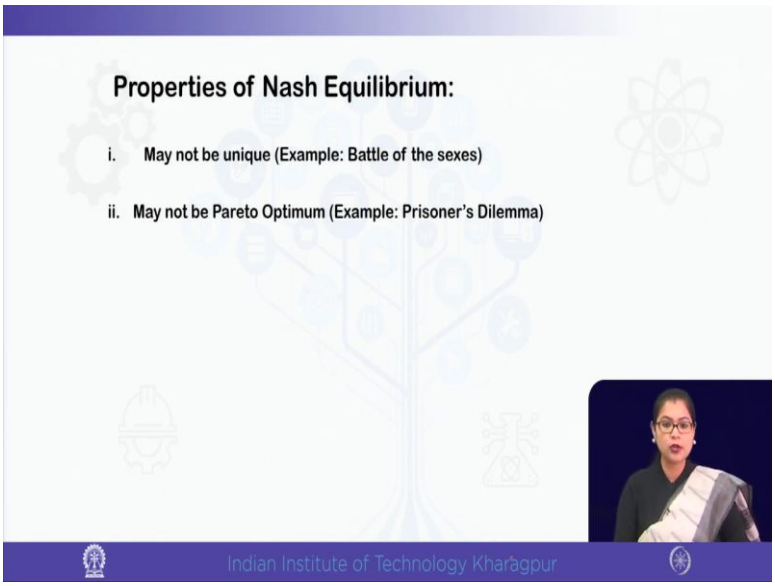
So, what is mixed strategy? Mixed strategy is a probability distribution over the set of pure strategy profile. And what is pure strategy? Pure strategy is when a player takes a certain decision or action, ok. And pure strategy is the probability distribution over the set of pure strategies, ok.

So, we are just confining our self to pure strategy. And as I already mentioned that we will be just discussing the games of complete information and simultaneous move static game, ok. So, because of time constraint we cannot go further. So, we are just exploring the solution concept of Nash equilibrium.

And there are also other refinements, other better solution concepts of refinements of Nash equilibrium which are applicable for dynamic game, sequential move game or multi-stage game or even games of incomplete information. But we are not going to that detail, we are just focusing on the concept of Nash equilibrium its applicability, its properties, ok.

So, we are just concentrating on static game of complete information where the solution concept is Nash equilibrium which has a very strong property that every finite game will have at least one Nash equilibrium. If not in pure strategy, then at least it will be mixed strategy.

(Refer Slide Time: 14:02)



(Refer Slide Time: 14:03)

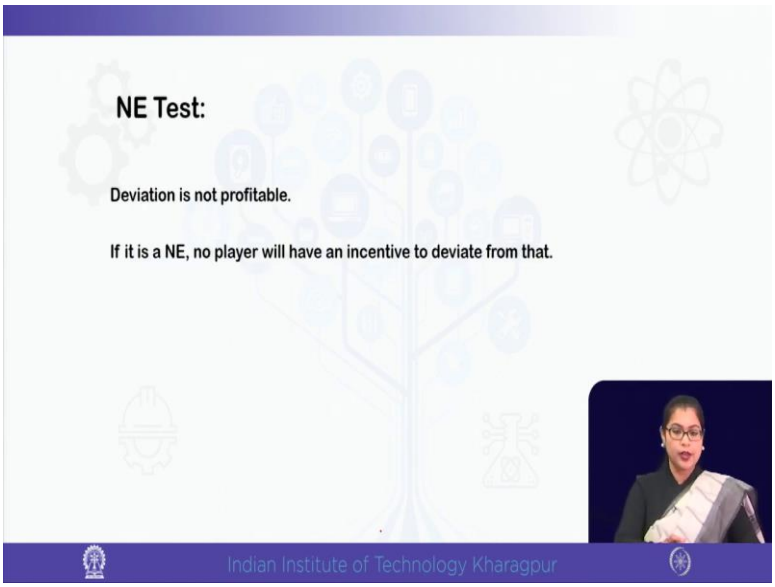
Now, I will come to the properties of Nash equilibrium because to show the properties actually I have chosen these two interesting games. But before that let us first find out how to represent the static form game and then how to find out the Nash equilibrium. Now, see what is a game?

As I mentioned game is when you what is game theory? Game theory is when you represent simple real life scenario a situation in terms of a mathematical model. Like, I took the example of this class. If the instructor takes a class he or she will give get some

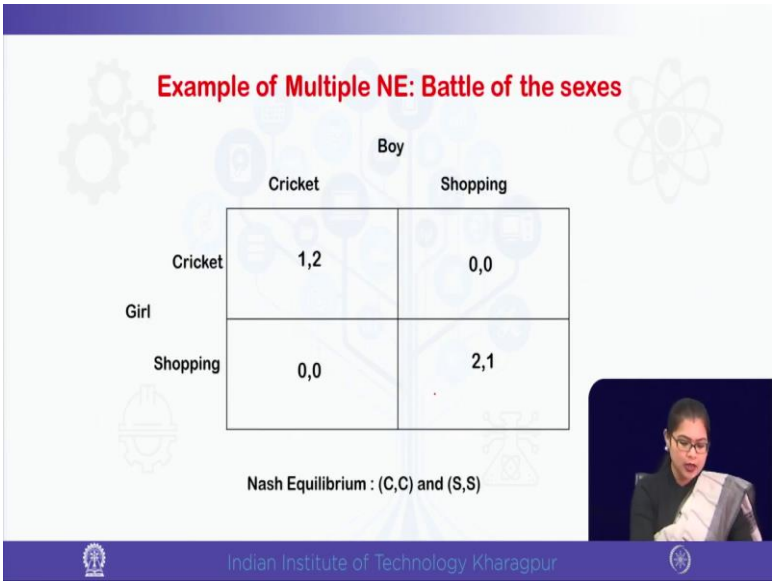
monetary as well as non-monetary benefit, and the audiences the students when they attend the class, they will also get to learn something. If they do not attend a class they will not learn.

So, so, you see there are some payoffs associated with this action. So, for the teacher the option is either to take the class or to not take the class, and for the student the option is to either join the class or to not join the class, ok. And you have the associated payoffs with each of them. So, this type of simple situation we can represent in mathematical terms. So, that is called a game theory model, ok.

(Refer Slide Time: 15:14)



(Refer Slide Time: 15:17)



So, we will be discussing a very famous game which is called the Battle of the Sexes it was developed by Luce and Raiffa in 1957 and the game the story goes like this. So, a couple is deciding where to go for a date in the weekend right. So, it is a very interesting simple example, you see often we decide where to meet your friend in the evening.

So, means the story was developed a long back. You can understand, the original story was regarding going to some ballet or going to some sports match, so we have changed the story a bit. So, suppose the option is that the couple can go to either a sports event like some cricket match or football match or they have the option of going for shopping, ok.

So, here there are two players, a boy and a girl, ok. So, you see in the environment, how do we what are the components number of players. So, first we see there are two players. So, we are defining the environment of the game in the Battle of the Sexes model. So, there are two players, girl and boy. Then, what are the other component in environment? So, number of players list of strategies.

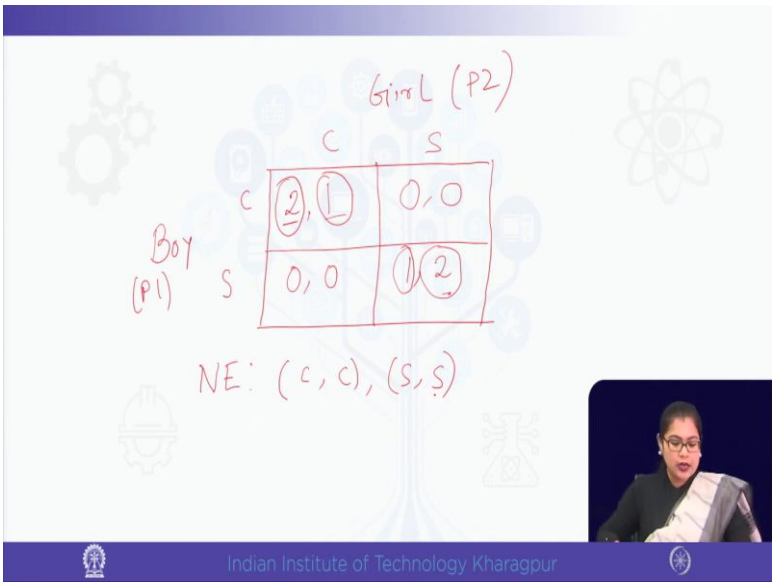
So, what are the list of strategies? Just I mentioned that two strategies, each of the player has two options, either to go for shopping or either or to attend a sports event like cricket match or football match. So, here we have written cricket and the other option is shopping, ok. You can also have some musical event or say anything else.

Then so, the number of players, list of strategies are already done in the environment and third is regarding the payoff. So, the story goes like this. If the player goes to his or her preferred event, he or she gets a higher payoff. If he or she goes to the other one's preferred event, then he or she will get a less payoff. And if they go separately, they do not receive anything, ok.

So, because the idea is to enjoy each other's company to spend the time together. So, if they go separately they do not get anything. So, they get 0, 0. Now, we are assuming that if the boy prefers to go for the sports event and the girl prefers to go for shopping. So, this is just an example there is no gender insensitivity associated with this. So, we can also change the story the girl can also prefer to go for a sports event and the boy can prefer to go for a shopping, ok. So, just this is a hypothetical example.

So, how do we represent such game? So, we are saying that if the girl and boy both of them attend the sports event say the cricket match. So, that is a preferred choice for the boy. So, the boy will get a payoff of say 2, and the girl will get a payoff of 1. So, if we now represent the game, ok, so it is already there, but I wanted to show because then it will be interesting for you, means you can also get to know how do we write down the environment of the game and then how do we find out the Nash equilibrium, ok.

(Refer Slide Time: 18:40)



So, this is a see this is a static one short game simultaneous move the players are taking decision at the same time, ok. So, there are 2 players and there are 2 strategies, ok.

Suppose, here we represent this is the boy and this is the girl. And there are 2 options, either to go for a football or cricket match or to. So, this is we can write in full for your understanding.

So, since there are 2 players and 2 strategies, so you see we are writing a as if a 2 X 2 matrix. So, this is called the normal form game or strategic form game or this is also known as the matrix form because there are 2 players and 2 strategies, ok. So, see how we are designing the game? We are saying that, we are assuming that if the boy will get a higher payoff by going for the cricket match, so the boy gets a payoff of 2. And the girl gets a payoff of 1.

So, this cell this upper left cell, this correspond to what? This correspond to when both of them are going for the cricket match. And you see the first one the first value or the number correspond to the payoff of the first player. Here we are writing the first player which is boy here and this is the second player which is girl here.

So, the boy by assumption the player gets a higher payoff, if he or she goes to his or her preferred position, but he or she is not going alone. The boy is being accompanied by the girl. So, the girl is also getting a positive payoff, but less. But the boy is getting a higher payoff because we are assuming that the cricket match or the sports event is a more preferred event for the boy.

Now, the other now let us see the other option where both of them are going for shopping, ok. So, if both of them are going for shopping so, you are assuming that the girl is getting a higher payoff of 2 and the boy is getting a lower payoff of 1. So, you can see in these cells, what is happening.

The first one correspond to the payoff of player 1 and the second value correspond to the payoff of player 2, ok. Here also the same thing. So, if you represent then? That means, the boy gets 2 by going to the cricket match, if the girl also goes to the cricket match. Here how do you represent?

Say the; if you interpret the payoff of the girl. So, the girl gets payoff of 2, if he if the girl is going for shopping, provided that the boy is also accompanying the girl in shopping. So, you see the payoff of a player what the boy will get that also depends on the strategy

of the other player; that means, girl, ok. Because the boy can also get 1 you can see over here, right.

Now, we are also assuming that they want to go together they want to enjoy each other's company. So, if they go separately they receive nothing, so 0, 0. So, this is the situation where the boy is going for shopping and the girl is going for cricket match. And the other situation is where the boy is going for cricket match and the girl is going for shopping.

So, you see, can we say definitely what the boy will get or receive if the boy goes for cricket match? No. Because if the boy goes for cricket match the how much the boy will receive that depends on the strategy of the girl. If the girl is also going for cricket match, then the boy will get a payoff of 2.

If the girl is going for shopping and the boy is sticking to the cricket match, both of them get 0, 0. So, that means, what a particular player will get that is dependent on the action of the other player. You see, now you can understand the element of strategic interaction. So, this is also it covers our earlier topic where I did not do that because I knew that we are going to discuss it in this class.

So, this is how we represent a static form a normal form game. So, there are 2 players with 2 strategies. You can have other options also more players, more strategies. So, this is how we represent. So, this is just the representation of game. Now, what is the second part? Second part is, next part is how do we find out the Nash equilibrium.

See, this is how we have defined the Nash equilibrium. So, Nash equilibrium is the one where the best responses match. So, how to find out the Nash equilibrium? So, what we will do? We are going to find out the optimal strategy or the best response of each player given the strategy or of action of the other player.

And then, we have to see what the best responses are matching. So, that is the Nash equilibrium. So, let us now find out the Nash equilibrium of the game. So, what is the Nash equilibrium? So, to find out the Nash equilibrium, we have to find out the best response of each of the player given the strategy of the other player or action of the other player.

So, first let us concentrate on the first player boy. So, what will be the best response of the boy given that the girl is going for the cricket match? Ok. So, that means, we are now comparing between the payoff of boy from playing C and playing S; that means, the what the payoff boy is getting from going to the cricket match and from going for shopping, given that the girl is going for shopping.

So, you can see that given that the girl is going for sorry, that given that the girl is going for cricket match, the boy gets 2 by going for cricket match, and the boy gets 0 by going for shopping. So, if we just compare between 2 and 0, given that the girl is going for cricket match. So, obviously, we know that 2 is greater than 0.

So, what will be the best response of the boy, given that the girl is going for cricket match? So, it is 2, ok. So, because best response means the strategy which gives higher payoff given a particular strategy of the other player. Similarly, we can also find out the best response of the boy from playing C and S given that the girl is going for shopping. So, now you compare between these two values 0 and 1.

So, given that the girl is going for shopping, you now compare what the boy is getting from going for cricket match and going for shopping. So, it is again 0 and 1. Now, which one is greater? Of course, 1 is greater. So, given that the girl is going for shopping. So, which one is getting giving higher payoff to the boy? You can see that shopping is the best response of the boy given that the girl is going for shopping. So, we have identified the best response of the boy given the actions or the strategies of the girl, ok.

Now, similarly, we have to find out the best response of the girl given the strategies or the actions of the boy. So, now we will be comparing with the second value, second numbers because that corresponds to the payoff of the other player that is girl over here. So, now let us see what is the best response of the girl given that the boy is going for cricket match.

So, now you see now we compare between 1 and 0, because given that the boy is going for cricket match, we can see that the girl is getting 1 by going for cricket match, and is getting nothing by going for shopping. So, 1 is of course, greater than 0. So, the best response of the girl is to go for cricket match if the boy is also going for cricket match. So, this is the best response of the girl.

Similarly, you see what is the best response of the boy? Sorry, the best response of the girl? So, we find out the best response of the girl, when the boy goes for shopping. So, then we compare the second row and the second values in the second row. So, given that the boy is going for shopping, if we now find out if we compare the payoffs of the player girl, so girl gets 0 by going for cricket match and gets 2 by going for shopping and 2 is greater than 0.

So, you see that we have identified the best responses of each of the player. We have circled the best responses, ok. So, this is the first step in identifying the Nash equilibrium. What is the second step? Second step is to we have to find out where the best responses match.

So, we can now see easily that there are 2 strategy profiles where the best responses match. So, these 2 will be the Nash equilibrium. So, you see in the battle of the sexes game, we have the multiple Nash equilibrium, but the uniqueness of Nash equilibrium is lost. But this is how we get the Nash equilibrium.

Now, how do you write the Nash equilibrium? So, Nash equilibrium strategy profile correspond to both going for cricket match or both going for shopping. Now, one thing to note here is that, we do not write the Nash equilibrium in terms of the payoff, but the Nash equilibrium is represented in terms of the strategies.

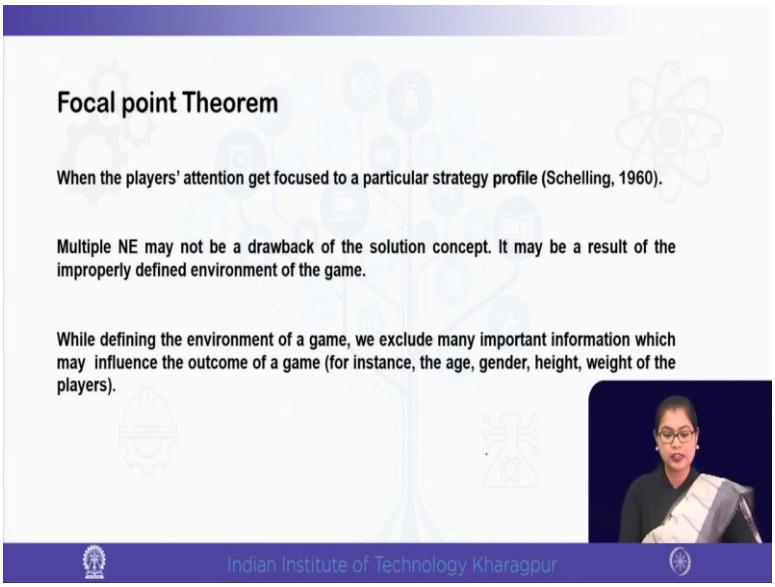
So, both going for cricket match or both going for shopping are the 2 possible Nash equilibrium in this game. So, this is how we find out the Nash equilibrium over here. Now, the game I have shown here I think payoff the boy and girl has changed. So, ok, we can continue with this example the one I have drawn over here because that will be easy for you, because that we have done in step by step. So, this is how we find out the Nash equilibrium.

So, here we see that the players will be gaining from each other, means they enjoy each other's company. So, if they go separately they do not enjoy anything. So, we have 2 possibilities, either both are going for the cricket match or both are going for the shopping.

Now, you see I will be doing the Nash equilibrium test with our next example, but one thing we would like to mention over here is that we can find out in the battle of the sexes

game that we have multiple Nash equilibrium. So, we have more than one Nash equilibrium. Now, what is the importance of Nash equilibrium? Because, we want to say which one will be the outcome.

(Refer Slide Time: 29:25)



Now, if there is a multiple Nash equilibrium, so some criticize that the essence of Nash equilibrium may be lost because we cannot definitely say what will be the outcome where they will meet, whether they will go for cricket or shopping we cannot say definitely from this, right. Because we have 2 possible equilibrium solution.

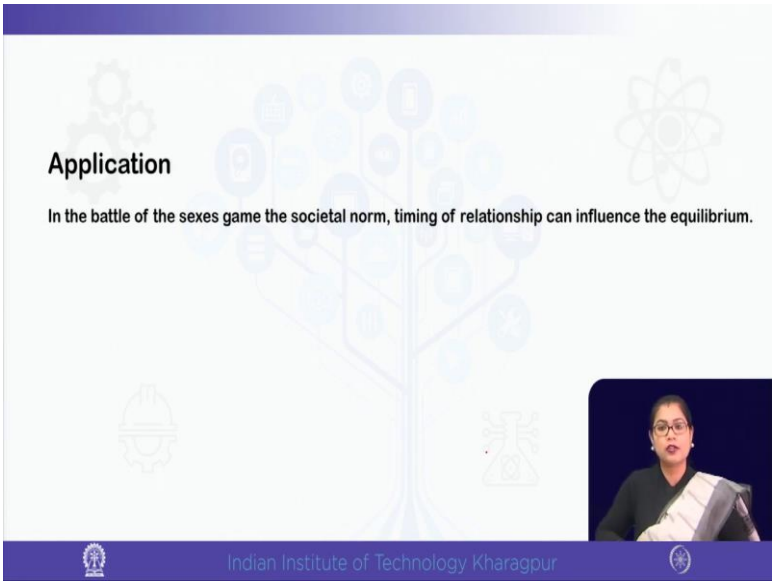
Now, there came a focal point theorem developed by or proposed by Schelling, 1960, who mentioned that it is not the limitation of Nash equilibrium that we have we are getting multiple Nash equilibrium. But it may happen that we are not properly defining the environment of the game. That means, we are only capturing only some information, but there are some other information which may not be captured in this type of information or the environment of the game.

You see environment consist of 3 things, number of player, list of strategies and payoffs. But we are actually representing a real life scenario, right. Then, real life scenario can be more complicated. There can be other factors that can influence the real life scenario which may not be represented or captured in the environment of the game.

So, according to many economists because we are defining the environment of the game and we are not including many important information which may influence the outcome of the game, ok. So, according to them, multiple Nash equilibrium may not be a drawback or limitation of the solution concept of Nash equilibrium or it may be a result of the improperly defined environment of the game because not all relevant information are captured.

So, Schelling proposed the focal point theorem which says that when a focal point means, when the player's attention gets focused to a particular strategy profile, ok. So, you see there are other means other factors which are not captured in the environment, let us say gender, age, the societal norm, that can also influence the outcome of a game.

(Refer Slide Time: 31:31)



Let us say for example, in the battle of the sexes, it can be the societal norm, whether the society is a patriarchal society or not, or if it is the timing of the relation whether the boy and girl, if they are married or not. So many things can influence the outcome, especially the culture and tradition can influence the outcome.

In a traditional society the girl may be following the boy, it may be other way round in other societies or it may also depend on the timing. Suppose, it is before the festive season, so both of them may be willing to go for the shopping or if it is some very important sports event is going on like the world cup football or cricket. So, both of them may be interested to go for the sports event.

So, these things the timing of the game, the timing of the relation of the boy and girl, the culture, the societal norms, these are not reflected or not captured in the environment of the game. So, according to Schelling, if we bring in this focal point analysis, so where the players attention gets focused to a particular strategy profile.

So, if suppose it is a very traditional society, again I am giving a very hypothetical example. If a traditional society and they are suppose the husband and wife, and the wife is following the husband. It may not be true also, this is just an example. So, in that society we can the attention can be focused to say cricket match.

Or what I say that if it is say before the festive season the attention can be focused to shopping, right. Or it can also vary depending on time depending on culture as the particular location, we are talking about the socio economic factors, the timing of the relation, ok. So, all these can influence, if the players attention get focused to a particular strategy profile.

Sometimes the players height, weight, these things can also matter. Like there is a famous game called the chicken game or the hawk dove game, in which, it is a very interesting game where two players are crossing they want to cross a one lane bridge. One lane bridge means if both of them try to cross the bridge to at the same time, so the bridge will crash on the in the river.

So, one player has to play a tough strategy. Tough mean one who moves first, and the other player will play a weak strategy, weak playing weak means you have to wait you have to be the second mover. So, you have to wait for the other one to cross first. So, if both tries to play tough, so they simultaneously try to cross the bridge and the bridge collapses and they get minus 1.

If both of them play weak, then they will just wait for each other. So, they will not cross the bridge, so they get nothing because the purpose is to go to the other side of the river. So, they are not getting anything. So, the Nash equilibrium, they are also we see in the chicken game or tough weak game we see that we have two Nash equilibrium, where one player will be playing a tough strategy and the other player will act like a weak one. That means, one player will cross the bridge first and then the second player will cross, ok.

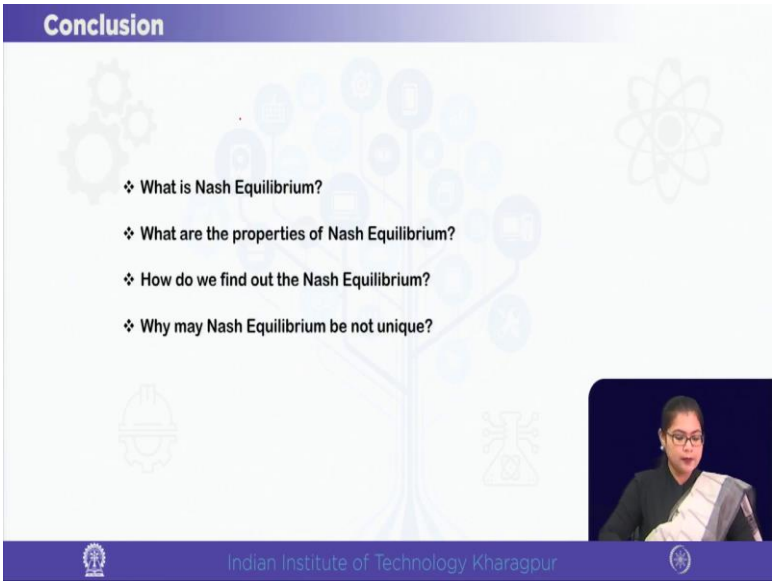
Now, you see, here also we have two strategy profile as Nash equilibrium, tough weak, weak tough. But now you see it here the height weight of your rival is also important. If you know that you have bodybuilder waiting out opposite side. So, obviously, you would like to not challenge the your rival, so you may decide to play weak, right.

So, you see this information is not captured. This information like the gender, age, height, weight of the players, the timing of the game, the cultural factors are not considered in the environment of the game. So, it is according to this economist, it is not the drawback of Nash equilibrium that we are getting multiple Nash equilibrium.

But we have to consider more factors while defining the environment of the game, and we can bring in the focal point analysis where the players attention gets focused to a particular strategy profile. So, this is one property of Nash equilibrium that Nash equilibrium may not be unique. The other property of Nash equilibrium is that it may not be Pareto optimal also. So, that one I will be discussing with the very famous Prisoner's dilemma game and which is very useful for our purpose.

So, with the Prisoner's dilemma game, I will be elaborating the second property of the Nash equilibrium may not be efficient. And also I will also be showing you how to carry out the Nash equilibrium test with the help of the Prisoner's dilemma game.

(Refer Slide Time: 36:21)



So, let me stop today for this class. So, in today's class, we have defined the Nash equilibrium. We have represented we have shown how to represent the strategic form game or a static game in terms of a game, I means a matrix form the environment, the strategies, the payoffs and then we have shown how to get the Nash equilibrium. So, first we find out the best responses, and then we find out where the best responses match that correspond to the Nash equilibrium.

So, we also saw in the battle of the sexes game, that we have multiple Nash equilibrium, but that may not be a deficiency or drawback of Nash equilibrium. With the focal point analysis we see that it is because of improperly defined environment of the game that we may get a multiple Nash equilibrium.

So, this was the main crux of the discussion in today's class which is very interesting. And we are going to continue the discussion and the other property of Nash equilibrium with the Prisoner's dilemma game.

(Refer Slide Time: 37:25)

So, thank you very much. So, we are following the important books that I have already mentioned in the previous lecture. So, the game theory books of Fudenberg, Tirole, and Gibbons, and the paper of Gibbons. And we have also referred to the original papers of John Nash, who are interested they can go through the original papers.

So, thank you very much. See you in the next class.