Petroleum Economics and Management
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Module - 08
Oligopoly and Game Theory
Lecture - 39
Introduction to Game Theory - II

Hello, I am Dr. Anwesha Aditya, your instructor for the NPTEL course Petroleum Economics and Management. So, we are in module 8 of our course where we are discussing Oligopoly and Game Theory and this is our lecture number 39 in module 8 means overall in our course. So, in today's class we will be discussing about some very brief overview of Game Theory.

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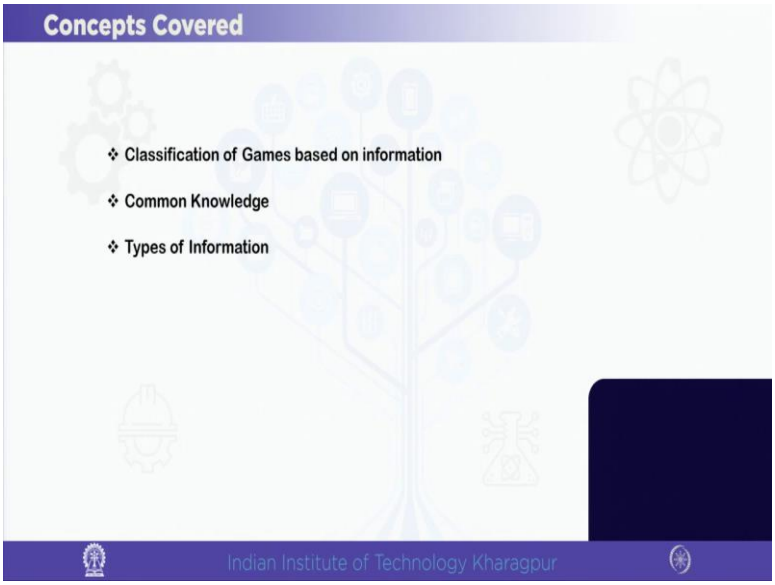

So, if we just motivate today's lecture, if you remember, we are going to study in detail the global oil market, the market structure of the world petroleum industry. So, we have already discussed whether OPEC is a cartel or not, we have discussed with empirical data and we have also analyzed the views from the existing literature. Now, time has come where we will be studying the cartel model, the price leadership dominant firm model.

So, these are the models which best resemble the global petroleum industry. Now, to study that we have already in other lectures of this particular module we have extensively discussed about different types of market structure, we have compared market structure ranging from perfect competition to monopoly and we have seen that depending on degree of market power we can classify different types of imperfectly competitive market.

Now, we our focus of market structure is on oligopoly because cartel is part of a collusive oligopoly. So, when we have discussed about the main characteristics of oligopoly, we saw that it is the strategic interaction which is only there in oligopoly market and it is not there in other type of market. And it is because of this interplay of strategies the firms' strategies the payoffs are interdependent and that is why the oligopoly models are just like game of chess.

So, where in which case at each move, we have to take into account the move of the rival. And that is why we study oligopoly models as an application of game theory framework. So, for that purpose we need to have some very basic understanding of game theory. Now, again I am saying that it is a very basic understanding because game theory itself is a very vast area, but we do not have any time and that is also not that purpose of our course.

So, it is just an overview of what is game theory, what are the means how we represent game and we will be focusing on the very important solution concept of Nash equilibrium. So, with this idea in mind we have designed we have devoted some time to understand the basics of game theory.

So, in one of the classes in the previous class actually we have already discussed the first part what is a game, how do we define the components in the environment of a game and we also discussed about different ways of classifying game. So, we are continuing with the that lecture because in the previous lecture we have classified game based on value of the game and information of the game if you remember.

So, information of the game is to be discussed in today's class. In the last class we have discussed about how to classify game based on its value and whether it is a static game or dynamic game, but information is very important in the context of game theory. So,

today we will be discussing about the importance of information and how do we classify games based on information.

So, we will be coming across a very important concept known as the concept of common knowledge. And finally, we will be also discussing about how to represent game. So, these are the concepts to be covered in today's lecture.

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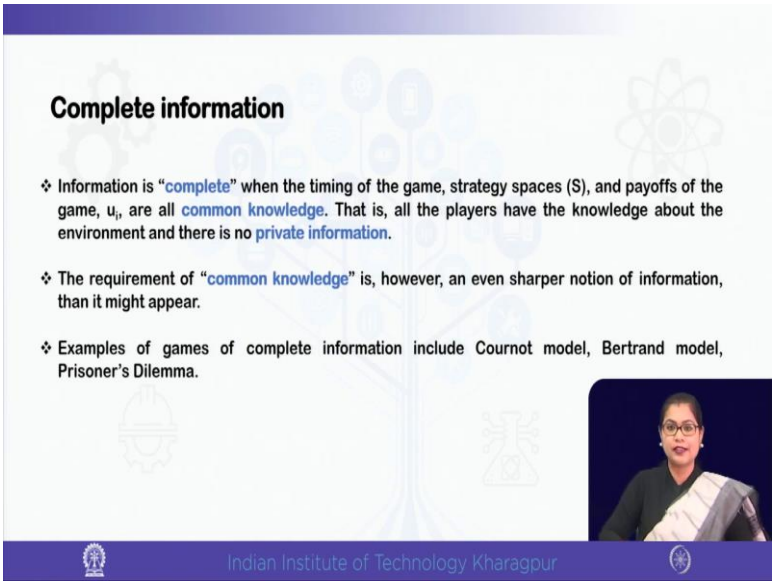

Now, when I am saying information. So, information regarding what? So, information regarding basically the environment of a game. So, if you remember if we just briefly recapitulate how do we define the environment of a game? Environment of a game has main three components. The number of players, the list of strategies or actions and the payoffs associated with the strategies or the actions of the players.

And also another component can be the timing of move or whether the players are taking decision at the same time or one player is following the other player is a sequential move or not. So, these are the main components of the environment of a game. So, when we are talking about information, how do we classify games based on information?

So, it is about the information regarding the environment, information regarding the observability of actions. So, in this way that is why information plays a very important role. It is a very critical part of any game whether it is a static game or dynamic game; we can divide the game based on information.

So, just for your understanding, if we are classifying games based on information, one part is information regarding the environment of a game and the other part is regarding information about observability of action, ok. So, in this way we are classifying games either as games of complete information or incomplete information or games of perfect information or imperfect information. So, we will be starting with complete information game, ok.

So, when do we say a game is a game of complete information? So, information is complete when the environment of the game is a common knowledge. So, environment means the timing of the game, the strategy profile, the number of payers, players, the payoff associated with the strategy. So, these are known to everyone.

However, when I say that these are known to everyone, the environment of the game is known to everyone. That is not equivalent to saying that the environment of the game is a common knowledge. So, what do we mean by common knowledge? The requirement of common knowledge is even a very stricter notion of information than what it may just appear if I say it is a common knowledge.

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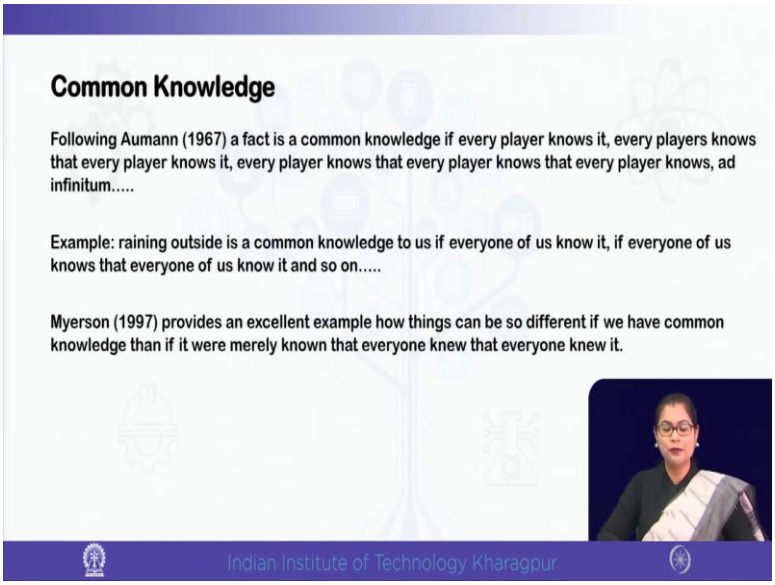

So, it is a very interesting one. So, following Aumann 1967, we can define a fact to be a common knowledge if every player knows it, every player knows that every player knows it, every player knows that every player knows that every player knows it and

infinite times. So, is not it interesting? So, let us say if we say that it is a sunny outside is a common knowledge.

So, I know that its sunny outside, you know that I know that its sunny outside, I know that you know that I know that its sunny outside and infinite time you see. So, just everyone knows that it is sunny outside that is one thing, but a fact being a common knowledge is different.

At each stage when the player is taking the decision, the player should know that the other player also knows that this player knows the fact and it applies in even in the next stage. So, you see the implication the concept of common knowledge, it is a bigger concept than if a fact is known to everyone, ok. So, we have given the example of its raining outside, it is a common knowledge if all of us know that it is raining outside.

If all of us know that all of us know that it is raining outside in this way. So, that means, if we say that environment of the game is a common knowledge. So, every player knows the environment of the game, every player knows that every player knows that the environment of the game is a common knowledge. So, at each stage we need this. So, it is a much bigger criteria than a fact just being known to everyone.

So, you should keep in mind and Myerson 1997 actually provided a very good example of things can be different. If we just assume that things are common knowledge or with just everyone knows. So, the two things are different. So, as I pointed out that its common knowledge means it is a much bigger criteria. So, it may be clear when we will be discussing with particular specific examples of game.

So, due to time constraint I cannot go into very detail, but we will be discussing about how to get the Nash equilibrium, how to represent a game static form game. So, we will be discussing with some examples, some very famous games we will be discussing, ok. So, examples of complete information game include the Cournot model, Bertrand model, the Prisoner's dilemma, the battle of the sexes.

So, where information at the environment of the game is a common knowledge so, there is no private information, it is not that some player has some private information which is not known to others, private information regarding the environment of the game. So, then if a player has any private information then what will happen?

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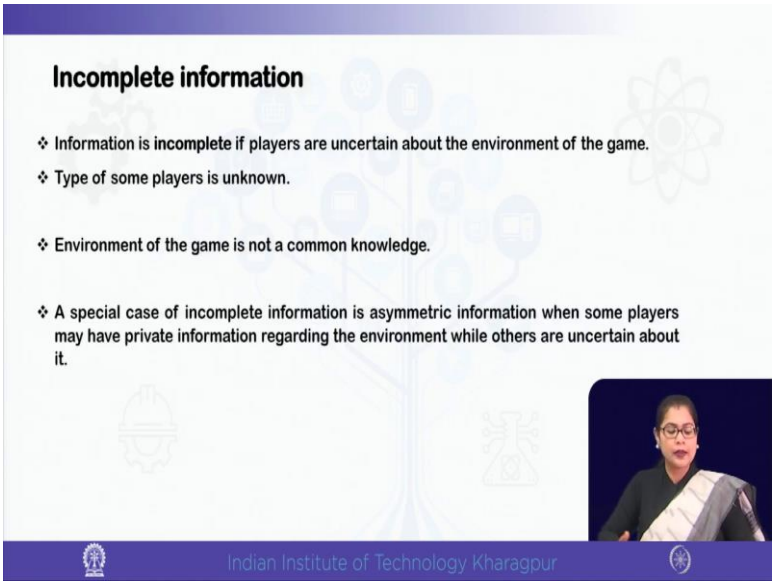

Then the game will no longer be a game of complete information it will become an incomplete information game. So, information is incomplete if players are uncertain about the environment of the game, ok. Let us say the player does not know about some of the strategies. So, environment of the game means the number of players, list of strategies the payoff.

So, some players may not know about the payoffs associated with some strategy or some players may not know about the list of strategies also. So, we can also sometimes say that some players do not know the type of the other player. So, these are all examples of games of incomplete information.

So, in this case you see the environment of the game is no longer a common knowledge. One particular case of incomplete information occurs when there is information asymmetry. That means, some players or some parties have private information regarding the environment of the game whereas, others do not know. Some players do not have any idea. So, some players have more information or private information. Private information means that information is not shared with everyone, ok.

So, information asymmetry some players have more information compared to some other players. So, this is also an example of incomplete information. So, there is wide application of asymmetric information. There are very interesting models, theoretical models and empirical applications of asymmetric information.

Let us say if you take a simple example of a principal agent problem or a firm when a manager is hiring workers. You see the firm does not know the type of the worker right. The worker can be lazy type or hard-working type. That means, when the firm is suppose software company is going for a placement drive in some institute, ok.

So, it does not know about the type of the student that the company may be hiring that can be hard-working or lazy. So, you see the type is uncertain. The particular candidate knows his or her type whether he or she is hard working or lazy, but the firm does not know. So, the firm has to use some criteria. So, the certificates, the recommendation sometimes, the certify means the assure whether the student or the candidate is hard working or not.

So, we have wide application regarding incomplete information in reality or asymmetric information or having private information. You can take example of say companies or the firm. So, suppose a foreign firm is venturing into a domestic market, ok. And the foreign firm it is a MNC it has a huge investment fund.

So, it will do a market survey before entering the means some other countries market, ok. So, it will know the cost structure of a domestic firm, but the domestic firm may not be in a position to know about the cost structure of the foreign firm it is whether it is a low cost firm or high cost firm.

So, you see there is asymmetric information. The foreign firm carries out market survey and knows about the domestic demand condition and also gathers information regarding the cost structure of the domestic firm, but the domestic firm may be constrained by fund and will not be in a position to know the cost structure of the foreign firm. So, you see there is information asymmetry. The foreign firm has more information than the domestic firm, ok.

So, it gives rise to the principal agent problem, the problem of moral hazard and adverse selection. So, you are not going to the detail we do not have time we are just seeing that how do we classify games based on information. So, if the environment of the game is not a common knowledge, if some players have private information. So, those games are called the games of incomplete information. So, for our purpose we will only be considering games of complete information.

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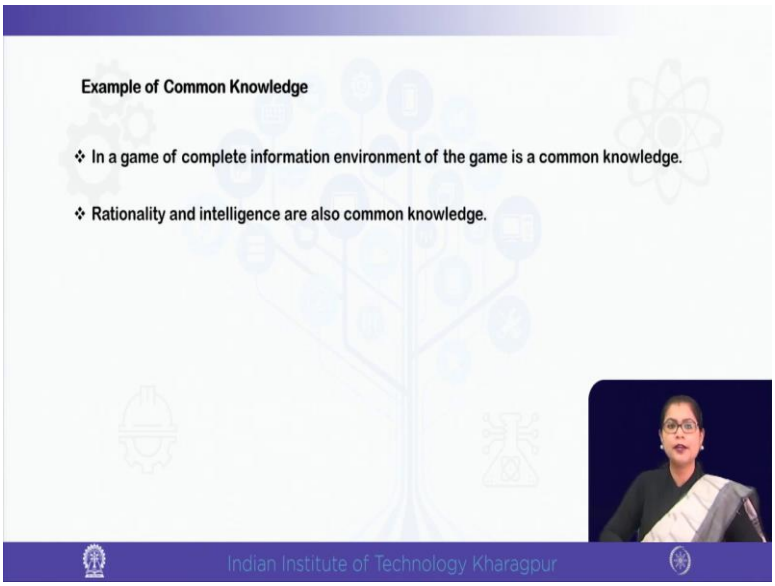

So, in games of complete information we assume that the environment of the game is common knowledge, even you see the assumptions of rationality and intelligence those are also assumed to be common knowledge. So, you have already discussed an individual is rational if he or she is a pursuing his or herself interest; that means, maximizing his or her expected utility.

And in when economic agent is said to be intelligent if he or she knows the rule of the game just like the game theorist know and each if we assume that all the players are intelligent. So, that means, they can replicate each other's thought process. So, that means, put in the same situation they will take the same decision, because everyone is driven by his or her self-interest.

So, individual economic agents are self-interested individuals ok. So, we assume that the assumption of rationality and intelligence are also common knowledge. So, at each place that means, at each point of time when a player is taking a decision, he or she has to credit his or her rival with rationality and intelligence.

So, that means, put in each other's situation they will be taking the same decision. So, they are able to replicate each other's thought process. So, we will be continuing with the assumption of complete information game. So, where the environment of the game is a common knowledge and rationality and intelligence are also common knowledge. So, the

first classification of game based on information is regarding whether the game is of complete or incomplete information.

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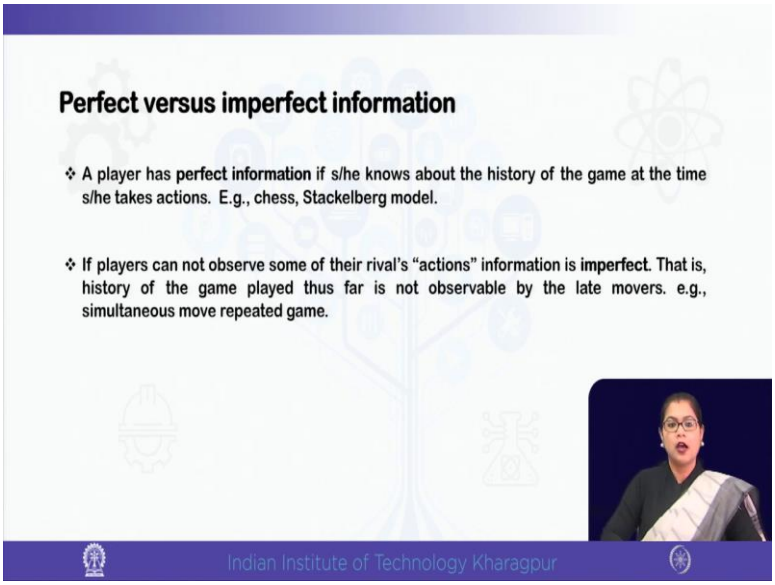

Now, there is another type of classification of game based on information that is regarding whether the actions are observable or not. So, this is called the distinction between perfect information versus imperfect information. So, a player or decision maker is said to have perfect information if he or she knows about the history of the game played so far while he or she is taking a particular decision or taking any particular action.

That means, say one player is moving after another then the one who is moving late so, the late mover if whether she or he can observe the all the actions of the first mover, ok. So, if the actions the previous actions are observable. So, that means, the history of the game played so far while taking a particular decision it is observable that is called a game of perfect information.

So, of course, a simple example of a game of perfect information is, you can understand its game of chess. Because you can see what your rival is doing. So, when you are taking a particular when you are deciding your next move you know you have seen all the previous moves of your rival. So, it is a game of perfect information. So, history of the game is known to the next mover, ok.

Same if you remember in a one of the classes we distinguish between different types of competitive or non-collusive oligopoly model. So, we found that the when two firms move one after another and they decide about output. So, sequential move output computation model is known as the Stackelberg model. So, there we assume that the late mover or the second mover can observe the first movers output decision. So, this is also the Stackelberg model is also an example of game of perfect information.

However, sometimes it may happen that all the previous actions may not be observable to the late movers. So, if players cannot observe some of their rival's action, then we say that such game is called game of imperfect information. So, history of the game played thus for is not observable by the late movers. So, if you remember we also classified two types of dynamic games, right.

Dynamic games are what which are played many times that is one type of dynamic game. So, this is called a simultaneous move repeated game. So, this class we took the example of this class, right. So, it is a simultaneous move repeated game and the other type of dynamic game is like a game of chess where one player is moving after another so, sequential move game.

So, in we took the example of chess and Stackelberg model and that is an example of perfect information. But you see if some of the earlier actions are not observable. So, it will become a game of imperfect information. Even you see the simultaneous move repeated game. In simultaneous move game at each stage both the players are taking decision simultaneously, right.

And if the game is repeated many times so, it becomes essentially a dynamic game. But at each stage you cannot observe the action of your rival what the other player is doing. So, the action of the rival is not observable when you are deciding about your move. I do not know whether you are going to attend this lecture or not right, because we are it is a simultaneous move, it is the same game this class is happening for a particular period of time. So, it is a dynamic game of repeated simultaneous move.

So, here both the players are taking decision at the same time; hence they cannot observe each other's choice. So, it becomes a game of imperfect information. Some of the sequential move game also can become imperfect information like the example I was giving the foreign firm is invading the home market. So, they have already taken that the

foreign firm has information better information regarding the home firm, but home firm does not know.

So, if the foreign firm decides the output how much to sell in the home market, but the home firm does not know about the foreign firm. So, it also can become a game of imperfect information, if the history of the game is not known. So, that means, you see one important point to note here is that whether the game is of perfect or imperfect information that is applicable to only dynamic games.

Because whether the game is information is perfect or imperfect, we classify that depending on the observability of actions, previous actions, past actions whether the history can be observed or not history is known or not. So, that means, history is relevant or observability becomes relevant when?

When the players are moving one after another, right so, that means, it has to be essentially a classification for the dynamic games right, either sequential move or repeated dynamic game. So, we can classify the dynamic games on in terms of perfect and imperfect information. You can also classify dynamic games based on whether it is a complete or incomplete information. But for static game we can only classify whether a static game is of complete or incomplete information, right.

So, whether the game is of perfect or imperfect information that will be applicable to dynamic games. Now, if some of the strategies or some of the means environment of the game is not common knowledge there is asymmetric information, there is some private information in a dynamic game. So, that can also become a game of incomplete information.

But on the other hand, for the static games only it is relevant whether it is game of complete or incomplete information, ok. So, the distinction of perfect and imperfect information is applicable or relevant only for dynamic games. So, for our purpose we will be just considering only on the simultaneous move game so and complete information static game. Now, how do we represent game?

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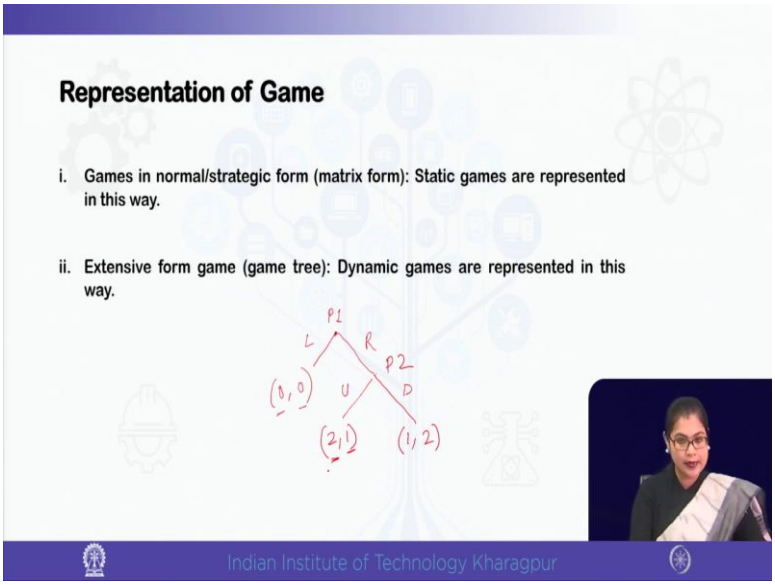

Now, games can be represented in different ways. So, as you can understand so, we have a static game and dynamic game. So, the representations will be different for static and dynamic game. So, the static game can be represented in what we call the normal form or strategic form or in a matrix form.

So, I will show you the matrix form in a next lecture we will be discussing with some example of how to represent game. So, we will be focusing for our purpose only on the static games. So, we will be using means it is a strategic form like a matrix, ok. So, if we representing the number of players list of strategies.

But the sequential move dynamic games cannot be represented in this way because at each stage the players are moving one after another. So, we are for our purpose we will be just considering the static game. But if you are interested, I can show you, how do we represent the dynamic games. So, this is called the representation of dynamic game is called the game tree. So, this these way we represent the dynamic game suppose this is the node this is called information node. So, here say player 1 moves.

So, player 1 suppose has 2 strategies say going to the left or going to the, right. Now, if player 1 goes to the left suppose the game ends and they get payoff of (0, 0). So, here the first one the left side this one correspond to the payoff of player 1 and this one correspond to the payoff of player 2. And in the next stage say player 2 goes this is the

information node for player 2, ok. So, if player 1 plays or goes right then player 2 gets a chance of moving.

So, suppose player 2 has 2 options of going up or going down. And you can write this is just an hypothetical example to show you how we represent sequential move dynamic game in terms of a game tree. So, you see it looks like a tree with the actions being the branches of the tree. So, that is why it is called game tree. So, you can assign some payoffs over here like this.

So, the first one correspond to the payoff of player 1 and the second one correspond to the payoff of player 2; so, that means, how do we interpret this 2, 1? So, given that player 1 goes to the right player 2 plays U and then player 1 gets payoff of 2 and player 2 gets payoff of 1. So, you see how much a particular player will get depends on the rivals' action, right.

Because if player 1 plays L player 2 does not get a chance of moving and both of them get (0, 0). So, how much player 2 is getting depends on what player 1 is playing. If player 1 plays L, player 2 is getting nothing, if player 1 plays R then player 2 can get either 1 or 2, ok. Similarly, you see if player 1 plays R then the player 2 gets a chance to take a decision and then how much player 1 gets that again depends on what will be played by player 2 in second stage.

You see, if player 2 plays U then player 1 gets a payoff of 2. If player 2 plays D player 1 gets a payoff of 1. So, you see now the importance. So, what your payoff will be that not only depends on your action or strategy. So, your payoff depends on the action or strategy of the other player. So, that is why you see the strategic interaction it is so important in game or even overall in oligopoly models, ok.

So, for this part the strategic form or the matrix form I will elaborate in the next class when we will be dealing with some examples we will be discussing about the very famous and important concept of Nash equilibrium, which is very relevant for our purpose to understand the cartel model, to understand the behavior of the OPEC firms, why we often see that the OPEC member countries end up producing more than what they agree to.

So, we will be seeing, we will be representing the strategic form or normal form game the static games in the next class. So, that is why I am not drawing here. So, this one this one we have drawn it is a game tree to represent the dynamic game or it is also called the extensive form game, ok.

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So, here you see what we have done in today's class. We have classified games depending on information. So, we see that depending on information we can either classify games as games of complete versus incomplete information or games of perfect versus imperfect information.

So, complete information occurs when the environment of the game is common knowledge and there is no asymmetric information or private information and perfect or imperfect information occurs it is regarding the observability of action. So, if the history of the game played is can be observed by the late mover so, that is called a game of perfect information. And if some of the earlier moves are not observable. So, that is called the game of imperfect information.

So, we can see that, we can easily understand that whether we can classify the games based on whether it is perfect or imperfect information. So, that is applicable only for the dynamic games. We can classify the static games only as either games of complete or incomplete information. Then we also discussed about a very interesting a new concept

of common knowledge, ok. And we distinguish between a fact is known to everyone is not just a common knowledge.

So, we already elaborated the importance of the assumption of common knowledge, definition of common knowledge and rationality and intelligence is they are also common knowledge in games of complete information. And then finally, we discussed how do we represent game.

So, we have shown an example of a game tree where how we represent the extensive form game to show a dynamic or sequential move game. But in the next class we will be showing you how to represent a static form game in a matrix form or what we call a normal or strategic form game. So, that is all for today's a class.

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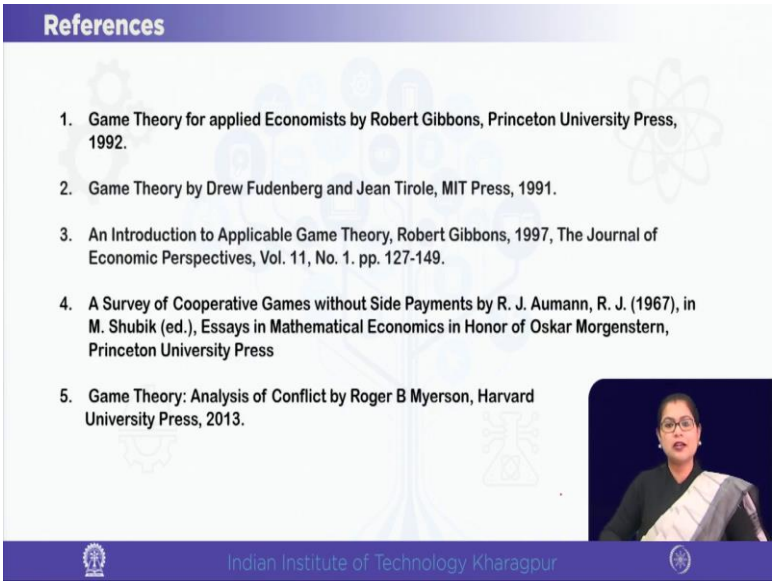

Now, you see there are so many references for game theory. So, it is a difficult to choose one other. So, the best book of Game Theory you can consider one of the best book and worldwide followed book is the book of Gibbons. And there is also one famous paper by Gibbons. So, I will be sharing with you the paper of Gibbons 1997 which includes very interesting examples that some of those we will be discussing in the upcoming lectures.

Then there is another simple book for the beginners Gibbons you may find bit a bit advanced. So, another simple book by for the beginners is by Fudenberg and Tirole. But there are mainly many other books also you can follow any standard book on Game

Theory. And of course, we have also noted down some of the original references like the book work of Myerson Aumann. So, those who are interested you can look at these original papers or the references.

So, thank you very much. See you in the next class.