

Petroleum Economics and Management
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Module - 08
Oligopoly and Game Theory
Lecture - 38
Introduction to Game Theory - I

Hi everyone, welcome to the NPTEL course Petroleum Economics and Management. I am your instructor Dr. Anwesha Aditya. So, we are in module 8 of our course where we are discussing Oligopoly and Game Theory. So, this is our lecture number 38 of the course where we will be introducing you to game theory concepts.

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Concepts Covered

- ❖ What is Game Theory?
- ❖ Environment of the Game
- ❖ Assumptions about Economic Agent
- ❖ Concept of Expected Utility
- ❖ Classification of Games

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Now, to give you a very brief justification about today's lecture as you already know that the way we have structured the course we are going to discuss the market structure of the global oil industry in depth and we have already studied whether OPEC is a cartel or not. So, for that we need to know the cartel model.

Now, in the previous two lectures of this particular module we have already classified different types of market. We have already devoted some time in understanding oligopoly model, but we have seen that oligopoly models nowadays are studied under a game theory framework because oligopoly is just like a game of chase where each firm

has to take into account its rivals' decision while deciding about its own profit maximizing behaviour.

That is why we need to know some basic concept of game to understand the cartel model, the price leadership dominant firm model which best resembles the global oil industry. And also it will help us the basic understanding of the some of the very famous games like the Prisoners' Dilemma game will help us to understand that often why the OPEC members also cheat from their agreement.

So, that is with this purpose in mind we have designed today's lecture. So, we will be devoting two lectures to give you a very brief understanding of game theory. Now, you see game theory it is a vast area. One can spend one's life in studying and applying game theory, but we do not have any time.

For that so, we will be just giving you an introduction to game theory – what is game theory? What are the different components in a game, how do you define environment of the game? What are the assumptions that we make about the economic agents and some related concept of expected utility and then we will be classifying game.

So, it is not a course on game theory at all, but I hope that you will find this course very interesting because game theory itself is a very interesting area. It has wide application not only in economics, but also in international relations. In many branches of social science it is very interesting you can get wide application of game theory in day to day life.

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What is Game Theory?

- ❖ It is a study of mathematical model of cooperation and conflict between rational and intelligent decision makers.
- ❖ Simple real life situations can be represented in terms of game theoretic models.
- ❖ It has wide application in Economics, International relations and other branches of social sciences.

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So, to start with, what is game theory? You may have heard about what is game theory, you may have heard about a lot about game theory, now you may ask, what is game theory? So, game theory in a very simple language it is a study of mathematical model of cooperation and conflict between rational and intelligent decision makers.

So, let me tell you in very simple language first and then I will be explaining you each of the term that I have noted down here. So, game theory in a very simple language is representing a simple real life scenario in terms of a mathematical model. So, when you represent a real life situation in terms of a mathematical model that is called a game theory. Ok.

Let us say let us say for example, this particular class. Ok. So, there are two parties in this class the instructor and the student or the audience, the learner's right in any class not only this class. So, what the instructor gets from giving a class. So, there are monetary as well as non-monetary benefits, right. So, monetary benefit correspond to the honorarium paid to the teacher and of course, the teachers also enjoy means get satisfaction from teaching, right. So, that is the non-monetary benefit.

So, that is the you can say that is the pay off or value addition of taking a class by the teacher and what about the students or the learners they learn something new, right. So, you can also represent the benefit or the gain of the students who are attending a

particular lecture. So, you see these simple real life scenario like this class can be represented in terms of a game theory framework.

So, two parties ok have two options either for the teacher the option is either to take the class or to not take the class; for the students the option is either to attend the class or to not attend the class. And corresponding to each of these they get some payoff or they have some associated gains, right.

So, if the teacher takes the class he or she gets the honorarium plus the satisfaction I mentioned and if he or she does not get the take the class. So, the payoff will be 0. Same for the students if a particular student attends the class will he or she will learn something new. So, you can we can write it in terms of a particular value. We can assign a value to that and if a student does not attend the class so, he or she is not learning that so, you can put a value say for example, 0.

So, this type of simple scenario can be represented in terms of a game theory model. So, there are very interesting games like we will also be discussing some examples of game where we will be finding out the equilibrium, we will be studying the application of Nash equilibrium. So, very simple scenario like a couple deciding where to go for a date. So, this type of single very simple real-life situation can be represented in terms of mathematical model by using game theory.

So, it is a study of mathematical model of cooperation and conflict because you see it is not always that the economic agents they conflict with each other they can also cooperate. Like you see in oligopoly behaviour we have already found out that the firms can also collude like the cartel model ok. So, it is mathematical model of cooperation and conflict between rational and intelligent decision makers.

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Assumptions about economic agent :

1. Rationality

A decision maker is *rational* if s/he makes decisions consistently in pursuit of his own objectives.

In particular, as Von Neumann and Morgenstern (1947) demonstrated a rational decision maker will always choose the option (amongst the various possible outcomes that he cares about) that maximizes his expected payoff/utility.

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So, I will come to that, what do we mean by rationality? A decision maker or the economic agent is said to be rational if he or she takes decision to maximize his or her objective or self interest. So, basically it is the self interested economic agent ok by rational individual we mean self interest economic agent, ok.

So, in particular if we follow Von Neumann and Morgenstern 1947 they demonstrated a rational decision maker as the one who will always choose the option among the possible various options that will maximize his or her expected payoff or utility ok. So, basically it is a self interested economic agent can be thought of as a rational one.

So, the game theory framework that we will be following we will be assuming rational economic agent. However, just to mention that nowadays we also see it is not always we behave rationally in reality. So, nowadays we also try to capture irrational behaviour and that is basically a part of behavioural economics nowadays it is a upcoming area ok so, where we try to capture irrationality.

However, for the time being since we will be just sticking to the Nash equilibrium for our purpose we will be just concentrating on rational economic agents. So, you see rationality as defined by Von Neumann and Morgenstern is the is a rational individual means the one who is maximizing his or her expected payoff or utility.

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Expected Utility:

- ❖ It is the weighted average of the utilities of each possible outcome under uncertainty.
- ❖ It is calculated by taking weighted average of all possible outcomes under different circumstances, where the weights are the probability assigned to an event.

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So, that means, we need to know what do we mean by expected payoff or utility. So, expected utility is calculated by weighted average of all possible outcomes under different circumstances, ok under different situations and what are the weights? The weights are the probabilities.

See in reality we have lot of uncertain scenarios, right. Let say for example, suppose you are going out. So, there is a chance of rain ok it may rain or it may not rain, right. So, if it rains then you have if you carry an umbrella if it rains. So, it will be better for you. Now, if you do not carry an umbrella and if it rains so, you will be drenched.

So, you see you have two options of either to carry an umbrella with you or to not carry ok. And, you have the associated payoffs of carrying the umbrella or associated benefits or utility of carrying an umbrella. So, the chance of rain may be 50-50 or if it is a rainy season so, the chance of rain will be very high. If it is summer the chance of rain can be less, right.

So, expected utility is the weighted average of the utilities of each possible outcome under uncertainty ok. So, by rational individual we mean the one who is maximizing his or her expected utility. Now, you see in defining game theory, we assumed the economic agent to be rational and intelligent.

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2. Intelligence

A decision maker is *intelligent* if he knows everything that we know about the game and can make inferences from the situation that we can make.

e.g., Perfectly competitive firms are rational but not intelligent players.

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So, what do we mean by intelligent agent? So, a decision maker is said to be intelligent if he or she knows everything that we know about the game and can make the correct decision or can make the correct inferences from the situation that we can make. So, that means, the this player the particular economic agent can also take the decision like the game theorist is doing.

So, the decision maker is intelligent ones who can understand the rule of the game. He or she knows everything about the game that we know about the game and can make the correct guesses from a particular situation just like we can do we means the game theorist can do, ok.

So, one typical example of an intelligent behaviour is say perfectly competitive firms who are rational, but not intelligent. See, we have already discussed a lot about market structure, we have discussed about perfectly competitive market also and we have seen that what happens in perfect competition? There are many numbers of large number of small firms. So, each firm is very small in size with respect to the total market share, right.

So, what happens there? So, you see they take the price as given they take the market price as given and they so, we have already studied that perfectly competitive firms are price takers. So, they decide about their how much to sell or how much to produce by with the by seeing the market price. So and how they decide how much to produce they

maximize their profit. So, you see perfectly competitive firms are rational because they are profit maximizers.

But, now you see instead of taking the market price as given so, what they could have done the perfectly competitive firms could restrict their output. Now, if all the firms individually they produce less what will happen? We know the total supply will be less and from our basic understanding of demand and supply we know that if total supply falls, price will increase.

So, you see individually the perfectly competitive firms are not in a position to influence the market price, but collectively they can do so, they can influence the market price. But, they are not able to understand that. So, that means, we can take the example of perfectly competitive firms as rational economic agents because they are maximizing the price, but they are not intelligent. They fail to understand that their collective action can influence the market price. Ok.

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Environment of the game:

- i. Number of players
- ii. Payoff: Value associated with a possible outcome.
- iii. List of Strategy: Rule or plan of action for playing a game.

Optimal strategy: Strategy that maximizes a player's expected payoff.

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So, before proceeding further on classifying the games, very important component of game is when we describe a game a very important concept is the environment of the game. So, what do we mean by environment of the game? There are three components in the environment of the game. 1st component is number of players. So, how many players are there in a game?

2nd component is payoff. So, what do we mean by payoff? As I told so, if I take the class I get some benefit, right. So, it is the value associated with a possible outcome for the firm it is the profit. Ok. So, the payoff will vary depending on the context. For a firm it is the profit for the consumer it is the utility, right. So, payoff is the value associated with a possible outcome.

3rd component in the environment of the game is list of strategy. That means, what are the strategies? So, just I took the example that the teacher has the option of taking or not taking any class. The student has the option of attending a class or not attending a class. Coming to the oligopoly example the firms have the option of colluding with each other or competing with each other. So, you see list of strategy means the strategy means basically the rules or plans of action for playing a game. Ok.

And another component which is I have not included, but you can think of a fourth dimension in environment of the game is the timing of move. Whether the firms are moving simultaneously or that means, not the firms of course, in case of oligopoly it is specific to firm, but overall. So, if the players or the decision makers if they are moving simultaneously or sequentially; that means, the timing of move, because that can also influence the outcome in a game.

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Nature and Characterization of Game

Games can be classified based on

1. Value of the game
2. Sequence of move
3. Information

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Now, you see we can classify games in different forms. So, there are different aspects which we can use to classify the game. So, first aspect is to classify the game in terms of the value of the game.

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Classification of Game: Based on value of game

Value of the game: Suppose there are two players $n=2$ with (pure) strategy space $S_i = s_1, \dots, s_n$ and payoff functions $u_i(s_k)$.

Zero sum game: One player's gain imply other's loss and these gains and losses match exactly. e.g. gamble

Positive sum game: Scopes for all players to be mutually better off. e.g., multilateral free trade

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So, what do we mean by value of the game? So, suppose we stick to only a two player model. So, there are say two players only who play some strategy with certainty, ok there is no uncertainty. So, so there are n number of strategy s_1 to s_n and the corresponding payoff is u_i . u_i means payoff of i -th player $u_i(s_k)$ is payoff of i -th player from playing k -th strategy. Ok.

So, depending on the value of the game, we classify game in two parts. Either games can be of two types, zero sum game or non-zero sum game. So, zero sum game is where the total value of the game is zero. So, that means, what? That means, one player's gain is exactly matched with other's loss, say gamble or game of poker. So, one player gains and that gain is exactly matched by the loss of the other player. So, value of the game is zero.

But, in reality we do not have always zero sum game; we can also have games where the total value of the game can be positive. So, that is called a positive sum game where there is a scope for all the players to be mutually better off. So, one typical example of a positive sum game is free trade followed by all the countries.

So, if all the countries follow free trade, there are theories which show that such multilateral free trade is welfare improving for all the countries. It is not that a particular country will gain at the cost of its partner. That will arise if they deviate from free trade and they go for some imposing trade policy.

So, if they impose some restriction, some trade policy so, then a particular country can further gain from following free trade, but that additional gain will come from the cost of the partner. However, if all the countries follow free trade so, there will be scope for all the countries to gain at the same time. So, we say that multilateral free trade is an example of positive sum game.

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The slide is titled "Classification of Game: Based on Sequence of Move" and has a subtitle "Static versus Dynamic game". It features a central tree diagram with various icons representing different game types. In the bottom right corner, there is a small video inset showing a woman speaking. The footer of the slide includes the Indian Institute of Technology Kharagpur logo and name.

Now, the other broad classification of game is based on the timing of move. As you can see you have already seen with oligopoly models also, we discussed that the oligopoly models can be of simultaneous move or sequential move. So, coming to game theory also so, games can be of static game and dynamic game.

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Static Games

- ❖ Static games are usually the **one shot (played once) simultaneous-move games**, that is, where the game is played once and the players make choices without observing other's choices.
- ❖ **One shot** (game is played only once) and **simultaneous move game** (players make choices without observing other's choice).
- ❖ The critical part of the static games is **unobservable actions**.

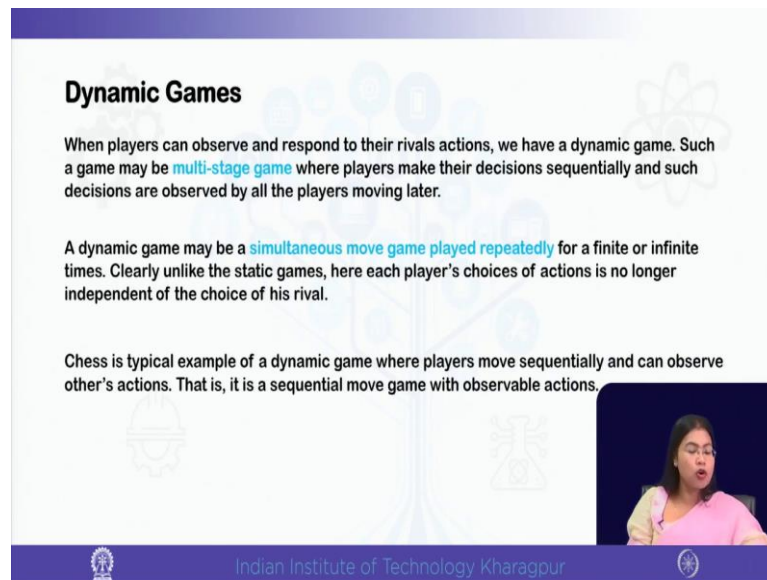
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So, what do we mean by static game? Static games are basically they are played only once. So, these are the one shot simultaneous move game; played only once, where the players are also taking decision at the same time. Ok. So, players make choices without observing or knowing others choices. So, this is important.

You see when you are deciding and the game is played only once and the two players suppose you have a two player game to keep it simple I am just taking the example two players. So, if a particular player is taking a decision, he or she is not able to know or observe what the other player is doing, the action of the other player.

So, static games are one shot simultaneous move game. Ok. So, both one shot simultaneous move game are examples of static game like this class. If it is happening only once, so, it is a static one shot game. Ok. So, the critical part of the static one shot game is unobservable action, but we see that often it is difficult to maintain a collusion if a game is played only once.

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Dynamic Games

When players can observe and respond to their rivals actions, we have a dynamic game. Such a game may be **multi-stage game** where players make their decisions sequentially and such decisions are observed by all the players moving later.

A dynamic game may be a **simultaneous move game played repeatedly** for a finite or infinite times. Clearly unlike the static games, here each player's choices of actions is no longer independent of the choice of his rival.

Chess is typical example of a dynamic game where players move sequentially and can observe other's actions. That is, it is a sequential move game with observable actions.

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So, more often in reality, we do see dynamic games, where dynamic games can be of two types. See, in the last lecture when we discussed about oligopoly models, we took the example of sequential move game. Sequential move game means one player is moving after the other.

So, if you remember, we talked about the Stackelberg model, which is an example of sequential move. So, the first from the leader form maximizes profit by choosing quantity and the follower form maximizes its own profit by taking into account the quantity or the output of the leader form. So, that is a sequential move. So, one player is moving after another. Ok. So, the sequential move it is important whether the second mover can observe the action of the first mover or not.

So, when players can observe and respond to their rivals action, we have a dynamic game. Ok. So, now dynamic game can be of two types also. One I have already told a sequential move game when the players are moving one after another. So, chess is a typical example of a sequential move game, where at each point of time your move will depend on the move of the rival. So, one player is moving after another, again the other player will move. So, it is a sequential move game.

But you see, there is the other type of dynamic game where the same game is repeated many times. Let say this class, right. This class is occurring at a particular time. So, same game is repeated many times. So, you see the players are same, the strategies are same,

the payoffs are same, but it is not a one-shot game, it is played many times. So, mostly in reality we see lot of dynamic games which are of means repeatedly played simultaneous move game.

The same game played repeatedly. So, dynamic game can be of simultaneous move repeatedly played game. This can be a finite time or it can be an infinite time also. Finite time means say this class; we know that we have 60 lectures. So, that is a finite game, right. But in reality, see we have lot of infinitely played repeated game. We do not know where that will be the last stage, right. Because suppose you think of business relations. So, a firm takes the raw material from some other firm, ok.

So, and this type of relation generally goes on for generations after generations. Now, we do not know where the at the point at which this relationship will be terminated, right. We are not immortal. So, this where it will stop, we do not know the last stage. So, many times this multi-stage games, we can think of them as infinite game because we do not know the last stage, ok.

So, in a nutshell what we can see here, the dynamic games can be a multi-stage game where the players are moving one after another or it can be a simultaneous move game where the players are taking decision at the same time, but the game is repeated many times, ok.

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Conclusion

- ❖ What is Game ?
- ❖ What are the components in Environment of the game?
- ❖ What is Expected Utility?
- ❖ Types of Game

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So, what we discussed in today's class, you see if we summarize. So, we started by defining game, what is a game. So, in a simple word game is a mathematical representation of a simple real life scenario. Then we discuss the application of game, it is not only limited to economics, but it can be applied to international relation and other branches of social science. And, then we discussed about the different components of game means the different components of environment of the game.

So, there are three components, number of players, list of strategies and the payoffs and the fourth component can also be considered as the timing of move. So, that is the environment of the game. Then you see we defined a game to be a mathematical model of cooperation and conflict between rational and intelligent decision makers. So, you also defined what do we mean by rational and intelligent economic agent.

So, rational economic agent is the one who is maximizing his or her self-interest or benefit or expected utility to be more specific and intelligent is the one who can understand the rule of the game, ok. And, then we came to a very interesting part where we classified the game based on a different aspect. So, if you remember we followed in today's class, we followed two aspects. So, we classified the game based on it is value and then we also classified the game based on the timing of move.

So, based on value we can classify games as zero sum or positive sum game and based on the timing of move, we can classify games as the static game and dynamic game. So, static games are the one-shot simultaneous move game and the dynamic games are the sequential move multi-stage game. So, that can be, so, I already just discussed that dynamic games can be of two types.

So, one is a sequential move game where the players are moving one after another just like game of chase or even in oligopoly model. So, Stackelberg model is an example of sequential move output competition and the other type of dynamic game is where the same game is played many times. Say the Cournot model, the simultaneous move output competition if it is played many times or repeated Prisoners' Dilemma same game is repeated many times.

And, we can see that often as I told that if a game is played only once, we may not be able to sustain collusion. But, if the game is played repeatedly, it is possible to sustain collusion because if I tell that we have any lateral incentive of deviation from agreements

or collusive behaviour. So, you may also think of, but we do see collusion like OPEC is also an example of collusive oligopoly.

So, how is that possible? That is possible you see because the OPEC members, OPEC is there for such a long time. So, OPEC members agree to output level and they know that they are dependent on oil; they have to rely on oil rents. So, that is why we see some kind of deviation by the prisoners' dilemma motive. But of course, they have maintained the cartel OPEC is there. It may not be that successful, it may not have increased the price so much, it may not have exerted the price power.

But, till OPEC is a successful cartel because it is there for long time whereas, in other markets the cartel are not that long lived. So, one reason for this is that it is not a one-shot game, ok it is a dynamic game, it is played. So, the OPEC members, they meet and decide. So, from 1970s almost it is going on like this, until today now OPEC has been expanded to OPEC Plus.

So, you can see the context why we are studying this game theory. So, that will give us a better understanding of studying of knowing the complexities of the global oil market. So, it is because of it is a dynamic game and it is also an infinite, right. Finite means this multi-stage game or the repeated simultaneous move game we just mentioned that it can be finite or it can be infinite.

So, like this course for example, it is an finite game because we know we have a last stage, we will be having a last class, right and then we will be having exams and all we will be having interactions. So, there will be and you get the certificate. So, that will be the end right. So, it is a finite game, repeatedly played simultaneous move finite game.

But, there are other examples like OPEC; these are examples of infinite game. It is not that OPEC will last forever, it may also get terminated, but we do not know the last stage. So, if you do not know the last stage that essentially becomes infinite game. So, often we see that we may not be able to sustain collusion if the game is played only one once.

So, you see you can think of a very simple example of say tourist spot. In the tourist spot you see we do not we generally we do not buy very costly expensive items because we know that we may be cheated because the shop owners also know that the tourist will not

come back. And, at the same time in the tourist spot you will not get anything if you cannot pay, right. You will no one no shopkeeper will give you anything in debt.

But, that will not happen in your locality. In locality often we take particular item and we pay later because your local shop owner knows you right, but that will not happen in the tourist spot. So, you see in the tourist spot that is a one shot game because the tourist may not be coming back. So, most of the tourists do not come back to the particular place. So, the shop owners will not give you anything in debt, right.

So, you see it cannot find collusion in one shot behaviour, but if the same game is repeatedly played many times. So, we may have the possibility of sustaining collusion. Now, there are models also to find out how we can sustain collusion, but that is beyond the scope of our course. So, we are not going to that direction. Those were interested they can offer better means more in depth courses on game theory or oligopoly models.

So, the we are for the game theory part we will be studying in the upcoming lectures, we will be studying the Nash equilibrium part and also will be classifying games based on information because information will be a very important component in game which we will see in the upcoming lectures.

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References

1. Game Theory by Drew Fudenberg and Jean Tirole, MIT Press, 1991.
2. An Introduction to Applicable Game Theory, Robert Gibbons, 1997, *The Journal of Economic Perspectives*, Vol. 11, No. 1. pp. 127-149.

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So, as a reference you can of course, follow the other microeconomics books that we have been following Pindyk and Rubenfield, but you can also follow some better books.

So, like the book of Tirole, Fudenberg and Tirole as I mentioned and book of Gibbons and even there will be a research paper of Gibbons which is very interesting which includes lot of interesting examples. So, I will be sharing with you the research paper of Gibbons to show the interesting examples, simple examples of game. So, that is all for today's lecture.

So, thank you and see you in the next class.