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Module - 02 Basics of Microeconomics Lecture - 11 Revenue and price elasticity of demand

Hi everyone. I am Dr. Anwesha Aditya your instructor for the course of Petroleum Economics and Management. So, we are in module 2, the last lecture of module 2 is Revenue and Price Elasticity of Demand.

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Concepts Co	vered		
* Concepts	of Total Revenue, Average Reve	nue and Marginal Reve	enue
* Relationsh	ip between revenue and price e	lasticity of demand	
* Derivation	of the revenue curves		
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In today's class we are going to talk about the very important concept of revenue. So, if we remember just in module 2 we are brushing up our knowledge of economics and those who are completely new to economics they are learning some new concepts of economics only those which are required for successful understanding and completion of the petroleum economics and management course.

So, we have already studied, how the market equilibrium outcome is determined. We have studied the two sides in a market. The demand side and the supply side, we have already studied the very important concept of elasticity, we have studied how the

equilibrium is arrived at and what happens to the equilibrium if there is some disturbance.

So, suppose we discussed many examples like oil price shock, how that changes the demand or the wars Russia Ukraine War, how that affects the world oil price. Now, in the last lecture of module 2 very important concept of economics will be discussed is the concept of revenue. So, we start today's class by defining what do we mean by revenue. There are three concepts of revenue total revenue, average revenue and marginal revenue.

Then we will discuss the shape of each of the revenue curve. In today's class we will be deriving the three revenue curves and we will relate it with elasticity ok. So, this is very important as we will proceed we will see that this class will be very important for understanding the tax policies and also understanding how petroleum is so important in our imports. Because India is a major importer of energy and in energy our major share goes for petroleum import.

And we have already discussed, we have motivated our course by saying that huge import of petroleum puts a lot of pressure on India's import bill and we will see in today's class even if price of oil falls, but India's import bill keeps on increasing why so, so these are very important for our purpose and that is why the relationship between revenue, price elasticity, marginal revenue total revenue these are so important for the petroleum economics and management course.

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So, first we start by defining, what do we mean by total revenue? So, a seller when; a seller is selling a product, the seller will receive a price out of that product, so price per unit into the total amount that a seller is selling is called the total revenue ok. So, market price multiplied by the amount sold by the seller is the total revenue; that means, it is in simple word it is nothing but the total cash flow or total value of the product, price into quantity.

So, there is a per unit price suppose the OPEC countries are selling oil at a per unit price and how much they are selling? So, price the per unit price into the total quantity sold is the total revenue earned from selling petrol. So, this is not the profit because profit will be obtained after subtracting the total cost from total revenue. So, total revenue is just the price into quantity or the total cash flow of the firm or the total value of the product ok.

Now, what is average revenue? Average as the name suggest to you know average means per unit. So, total revenue divided by the total quantity transacted. So, average revenue or AR is TR or total revenue divided by the quantity transacted.

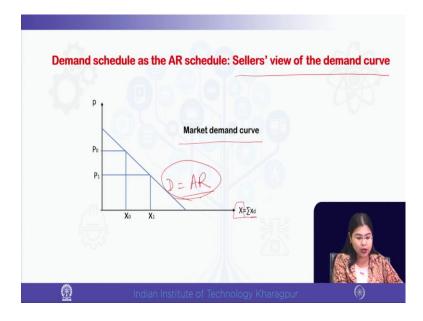
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TR, AR and MR	
Uniform pricing: TR(x <sub>0</sub> ) = p(x <sub>0</sub> ) x <sub>0</sub>	y = f(P)
$TR(\mathbf{x}_1) = \mathbf{p}(\mathbf{x}_1) \mathbf{x}_1$	$P_{a} = g(x)$
TR(x) = p(x)x for all $x > 0$	$g = f^{-1}$ exists
AR = TR/x=P(x) = per unit revenue= Market de	emand
The demand curve appears as the AR curve to	o the seller.
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So, we can see from here, this average revenue AR is TR by x, x is the total quantity transacted. So, is equal to p x, p x is what? p x is the per unit price. So, per unit price is a function of quantity demanded. What is that? We already know that we have interpreted the demand curve from the seller's point of view, right.

Where we have represented the demand curve as the inverse demand curve, so if you just remember our direct demand function is  $x_d$  as a function of p, but provided the inverse exist we actually work with the inverse form of the demand function provided g equals to f inverse exist, right. So, this; so  $p_d$  as a function of g (x), so g equals to f inverse should exist. So, this price as a function of quantity this is our inverse form of the demand function.

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So, this demand function is nothing but the average revenue curve. So, what we can see is that, from the seller's point of view the market demand curve appears as the average revenue curve. So, in this figure you see we have plotted the market demand ok. So, price on the vertical axis because we are working with the inverse form of the demand function the way we have written here and the quantity is the in horizontal axis.

Therefore, in quantity you see the capital X is as we have defined capital stands for total or aggregate. So, the total output in the market is the sum of individual demand, right. Total demand is sum of individual demand that is how we have derived the market demand. So, we add the individual demand to get the market demand.

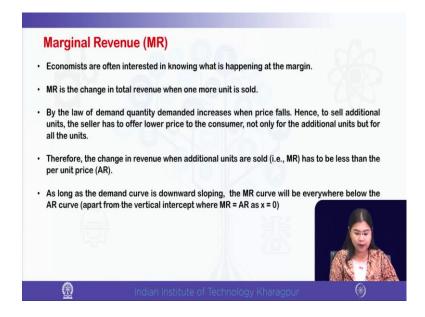
So, capital X is summation  $X_d$ , where suppose there are n number of individuals, so summation x i d, i running from 1 to n. So, the small value stands for small letter stands for the individual and capital stands for the market. So, we have represented the market demand curve, but downward sloping market demand curve provided that law of demand holds.

So, to the sale and of these demand curve market demand curve is obtained by adding up the individual demand, but what is the individual; this market demand curve to the seller. So, what we can say is that? This market demand curve appears as the average revenue curve to the seller. So, seller's point of view of the market demand is nothing but the average revenue curve because average revenue is the per unit revenue is nothing but the TR by x where TR is equal to P(x) into x.

So, when you divide TR by x it becomes nothing but AR is equal to P (x), which is nothing but the market demand. Because price as it is as if, the price is a function of quantity and that is the interpretation of the inverse form of the demand curve. So, the demand curve appears as the average revenue curve to the seller.

So, remember this is the market demand we are talking about because seller is not looking at the individual demand, right. The sellers are looking from the point of view of the total market. So, the market demand curve is the seller's average revenue curve, right. So, this is the first takeaway from today's class.

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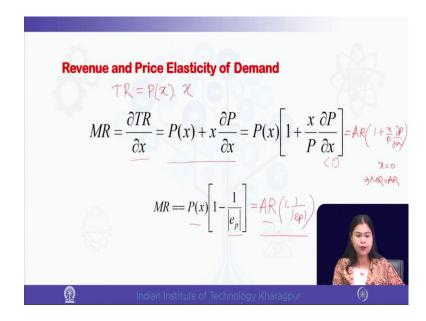


Now, we will derive the shape of the AR curve before that let me also elaborate on the concept of marginal revenue because in economics we are often interested to see what is happening at the margin. For very infinitesimally small change, what is happening? So, what is marginal revenue? Marginal revenue is addition to total revenue when one more unit is sold. So, that means, how total revenue will change, if the firm sells one more unit.

Suppose a firm for the time being is selling say 100 unit of a particular product. So, what will happen? How the total revenue or total cash flow of the firm will change, if the firm

sells the 100 first unit ok. So, that is MR, so MR is the change in total revenue, when one extra unit is sold.

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So, mathematically then we can write MR as del TR del x ok. When one unit is sold, how total revenue is changing? Now, we have already written that TR is equal to P (x) into x, so we can solve the mathematical expression of MR. So, TR is P (x) into x, so MR is equal to del TR del x. So, if we now solve what we get P (x) into del x by del x is equal to 1, so the first term is P (x) plus x del P del x, right?

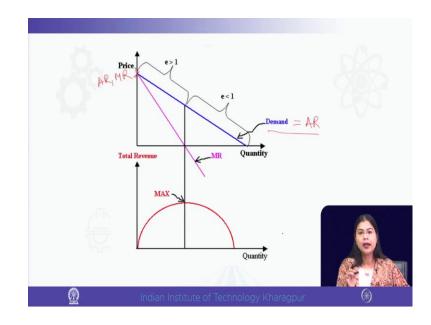
So, what we do? We do little bit of manipulation in the right hand side and we express MR in terms of elasticity. Now, in this right hand side expression, if we take P as common, so what we can write? MR is equal to P (x) into 1 plus x by P into del P del x. Now, what is this? We already know that this is 1 upon the elasticity. So, in the next line we are writing MR is equal to P into 1 minus 1 upon  $e_p$ .

Now, why is this a minus sign? Because we have written  $e_p$  the own price elasticity of demand in absolute term. If you remember as long as law of demand holds elasticity value is negative. We have classified the elasticity ranges in different values and we have seen that  $e_p$  will be positive only when law of demand is violated which is a very rare case. So, empirically mostly we see that law of demand holds. So,  $e_p$  is negative ok.

Now, within this negative  $e_p$ , we have also discussed about different cases of elasticity. So, we will relate it with marginal revenue also. Let us now just find out the expression of MR in terms of  $e_p$ . So, what we see is that? If we write, so MR is equal to P into 1 minus 1 upon absolute value of elasticity ok. So, this may also alternatively be written as MR is equal to AR into 1 minus 1 upon  $e_p$ .

Why? Because P is equal to AR we have already seen, that P (x) is nothing but the market demand curve is the average revenue curve. So, MR is equal to AR into 1 minus 1 upon  $e_p$ . So, this expression will be very helpful for us for deriving the revenue curves ok.

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Properties of MR curve	
$i)\frac{\partial P}{\partial x} < 0 \Rightarrow MR < AR \qquad \text{for all } x>0$	
That means, as long as demand curve is negatively sloped, MR <ar.< th=""><th></th></ar.<>	
If x=0, MR=AR.	
<i>ii</i> ) <i>MR</i> = <i>P</i> + $x \frac{\partial P}{\partial x} < 0 \Rightarrow \frac{\partial MR}{\partial x} = 2 \frac{\partial P}{\partial x} = 2 \frac{\partial AR}{\partial x}$ for all linear demand fn.	
That means, MR curve is linear, when demand curve is linear.	
Also, MR falls as x increases if law of demand holds.	
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So, now, with this knowledge let us first derive the total revenue, average revenue and marginal revenue functions the shape of the curves ok. So, first what we can see? What; let us start with the MR curve, because we have already seen that this demand curve is nothing but the AR curve. So, AR curve we have already derived ok, because from seller's point of view, the average revenue curve is nothing but the market demand. So, the demand curve is downward sloping AR is also downward sloping.

Now, first we will derive after this the marginal revenue curve and lastly we will derive the total revenue curve. So, what will be the MR? So, here we can see that MR is equal to AR into 1 minus 1 upon  $e_p$ , right? So, from this expression, if we now put the value of x equals to 0 ok. So, x is coming on the horizontal axis, so if x equals to 0, what can we conclude? If x equals to 0, then MR will have the same vertical intercept because from this expression MR is equal to AR into 1 plus x by p into del p del x ok.

So, if x equals to 0, we can see that MR is equal to AR, right? So, in the next slide we have shown, we have plotted the demand function. So, in the demand function we know that price is on the vertical axis and the quantity is on the horizontal axis. So, we have the market demand curve which is nothing but the average revenue curve. Now the vertical axis also measures the average revenue and marginal revenue. So, this is the average revenue curve.

Now, what about the marginal revenue curve? So, we just now proved that if x equals to 0; that means, x equals to 0 means along the vertical axis, MR is equal to AR. So, that means, the MR curve marginal revenue schedule will also start from the same vertical intercept of the demand function ok.

So, both of them will have the same vertical intercept. After that what will happen let us see the relationship, so if del P del x is negative, if we go back to this relation that del P del x is negative that means what del P del x is negative means, price and quantity demanded are negatively or inversely related which is nothing but the law of demand. So, that means as long as the law of demand holds, so that the demand curve is downward sloping that is the way we have drawn the market demand over here.

So, MR will be less than AR because this term is negative ok. So, x and P are positive ok. We started with x equals to 0, if x equals to 0, MR is equal to AR and then we consider for any stray x which is strictly greater than 0. For any positive amount of x what we see is that MR is strictly less than AR. So, that means the marginal revenue curve will lie everywhere below the market demand curve or average revenue curve for any positive amount of transaction.

When quantity is equal to 0, then marginal revenue is equal to average revenue. So, what we can conclude is that? As long as the demand curve is negatively sloped, MR is less than AR ok. So, this is the first point in deriving the shape of the MR curve. So, MR starts from the same vertical intercept of the AR curve or the demand curve and after that it will lie everywhere below the AR curve or the demand curve.

Second, so this is the expression we have got from this derivation of establishing the relationship between MR and elasticity, so this is how we can write? So, what we can see from here is that, if we now see how MR is changing when quantity demanded is changing. So, del MR del x is what? Del MR del x is equal to twice, del P del x because the first term on this right-hand side will be del P del x plus the second term will be del x by del x into del P del x.

So, del x by del x will be equal to 1, so the right hand side boils down to twice del P del x. So, del MR del x is twice, del P del x, right. Now what is P? P is nothing but AR, so we can write this twice del P del x is equal to twice del AR del x. So, this will hold for all linear demand functions, right? Therefore, what we can see is that? If MR is

negatively sloped, that means if AR is negatively sloped, MR will also be negatively sloped.

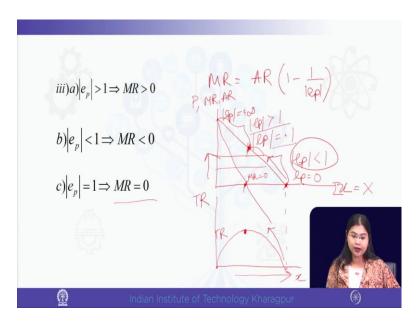
So, as long as law of demand holds del P del x is negative. So, del AR del x is also negative therefore, del MR del x will also be negative, right? So, MR curve; if the MR curve is linear, the demand curve will also be linear and not only that if the market demand is downward sloping that means, the law of demand holds AR is downward sloping, MR is also downward sloping.

So, MR will fall as x increases. And of course, this will be true for all linear demand function because if it is non-linear then in; when we are getting this expression of del MR del x. So, if del 2 P del x square is not equal to 0 for non-linear demand function. So, for linear demand function only, we can write del MR del x is equal to twice of del AR del x.

If it is non-linear then we have a del 2 P del x square term which is not equal to 0 for non-linear demand function. So, for linear demand function we can definitely say that MR curve is linear, demand curve is also linear and of course, if the market demand or the law of demand holds AR is downward sloping, MR is also downward sloping. And we can also see the slope we can see that the slope of the MR curve will be just twice the slope of the AR curve ok.

So, we can see from here. So, from the first property we see that MR curve starts from the same vertical intercept after that it is everywhere lying below the demand curve from the second property we see that MR curve will also be downward sloping as long as law of demand holds for a linear demand function MR will also be falling.

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Next we have to draw the MR curve, now we will use the concept of elasticity. How elasticity is useful in drawing the demand curve? We see because we have already got this relation that MR is equal to AR into 1 minus 1 upon  $e_p$ . So, let us write this relation. So, with this relation, now let us derive the MR curve depending on the value of  $e_p$ .

Now, if you remember in earlier classes we have already discussed that, in a linear downward-sloping demand function like this, the slope is the same but elasticity value varies from minus infinity to plus infinity, right? So, it is just from minus infinity to 0 sorry, the elasticity value varies from minus infinity to 0 and at the midpoint  $e_p$  is just equal to minus 1. So, I am writing in absolute value, so we do not need the negative sign ok.

Now, in the upper part of the demand curve, we can see that it corresponds to the elastic segment of the demand function  $e_p$  is greater than 1. So, you have price elastic demand in the lower segment of the demand curve  $e_p$  is less than 1 ok. So, here we are plotting the price and the marginal revenue and the average revenue and quantity on the vertical axis ok. This should be to be more specific this is the market quantity ok, so which is the sum of individual's demands.

Now, we see what happens to MR depending on different values of  $e_p$ . So, if  $e_p$  is greater than 1, we can definitely say that MR will be positive ok. So, corresponding to the upper region; the upper part of the demand curve above the midpoint of the demand curve this

linear downward sloping demand curve MR will be positive. Now, what happens when  $e_p$  is exactly equal to 1?

Now at the midpoint of this linear downward sloping demand curve  $e_p$  is equal to 1, so MR will be exactly equal to 0. So, for any elasticity greater than 1; that means, in absolute value I am talking in absolute term. So, as long as the good is price elastic or we are in the range of elastic demand for the good then MR will be positive. But of course, MR will be less than AR, for the reasons we have already discussed in the previous slide. So, MR will be less than AR, but MR value will be positive.

Now, when  $e_p$  is equal to one then MR is equal to 0 ok. And then in the third range when  $e_p$  is less than 1; that means, we come down along the demand curve when we enter the inelastic segment of the demand curve, we see that  $e_p$  is less than 1 in absolute value and MR becomes negative. So, addition to total revenue when more quantities are sold that becomes negative.

So, with this we can now draw the MR curve which is first positive falling, but positive and then when  $e_p$  equals to 1 then MR is equal to 0 and when  $e_p$  is less than 1, MR becomes negative. So, this is how we derive the marginal revenue. Now this is very important for firms perspective for a producers perspective because in economics we are often we consider the equilibrium outcome from the firms perspective by comparing the addition to revenue, when one more unit is sold with at the change in cost when one more unit is sold that is marginal cost.

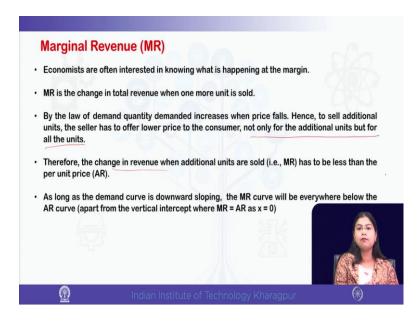
So, this will be useful; when suppose you will be studying the market structure of world oil market will be studying how much the OPEC produces theoretical models you will be studying and how much the non-OPEC countries produce. So, there will see that this equilibrium condition will be determined by using the marginal revenue curve. So, these concepts are very important for our purpose.

So, with these properties, we derive the marginal revenue curve which is also linear as long as the demand curve is downward sloping and MR is also downward sloping, MR will be less than AR and we draw the MR curve. So, corresponding to elastic segment of the demand curve, marginal revenue is positive, when elasticity is equal to 1, marginal revenue is equal to 0 and when elasticity is negative, marginal revenue becomes sorry

elasticity is less than 1, that is we enter the inelastic part of the demand curve marginal revenue becomes negative ok.

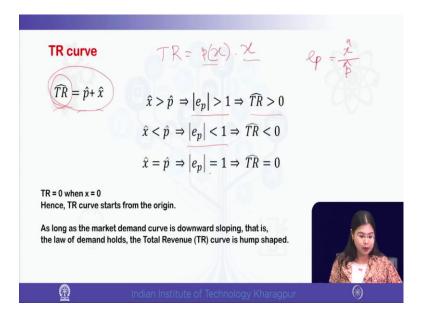
Now, one intuition we need to know that we mathematically and graphically we have already proved that MR is less than AR for any positive quantity transacted.

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Now, what is the intuition behind this intuition should be very clear.

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That why the addition to total revenue when one more unit is sold, why? That is falling. Now, we need to know because you see as law of demand holds; that means, when will the consumer buy more. See we are going along the if we look at any figure we have drawn; we are going along the horizontal axis means the consumer is consuming increasing the quantity of consumption, right.

Now, as long as law of demand is holding the consumer will buy more only if he or she is offered a lower price, right? Now see by law of demand quantity demanded will increase when price falls. Therefore, if the producer wants to sell more units right, the seller has to offer a lower price to the consumer not only for the last unit, but also for all the units. So, to sell more the producer should offer a lower price, otherwise the consumer will not buy more quantity of the good.

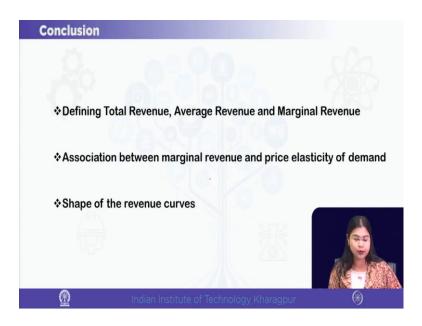
Why? Because we have law of demand in place which tells that the quantity demanded will increase if price falls. So, because of law of demand to sell more, the producer has to offer a lower price to the consumer not only for the additional unit, but also for all the units. Therefore, the rate at which total revenue is changing that is falling when more units are sold; that means, the addition to total revenue will be less than the per unit price. So, this intuition should be very clear to us ok.

So, a change in revenue means, the MR has to be less than the per unit price AR. So, this is intuition, it's not only important that we derive the relation mathematically and graphically, but intuition should be very clear to us ok. So, for law of demand only MR should be less than AR, so that means, the rate of change of total revenue when additional units are sold should be less than the per unit price.

Now, with this knowledge of the shape of the MR curve will do our last part in today's class that is deriving the total revenue curve ok. So, what is total revenue? See we have already defined total revenue is nothing but the total value of the product. So, price into quantity of the product, where price is a function of quantity demanded ok.

So, this is nothing but, the total value of the product or total cash flow earned by a firm. So, what is the change in total revenue? So, suppose we take a log and totally differentiate, so we have already discussed about hat operation. So, if we take a log and totally differentiate we get d of; T log of TR and then this will be, if we just take a log this will be log of TR is equal to log of p(x) plus log x. Now, if we now totally differentiate this will be TR hat is equal to p hat plus x hat, where TR hat is nothing but taking a log and totally differentiating. Because if we take a log of TR and then we totally differentiate it becomes d of log TR is 1 upon TR into d of TR which we can write as, TR hat ok. So, TR hat is equal to p hat plus x hat.

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Now, when we are moving along the horizontal axis if we look at this figure over here. So, in this below panel, say we are plotting quantity as usual on the horizontal axis and total revenue appears on the vertical axis ok. So, where we are drawing the TR curve we can see that, what will happen to TR, that depends on what?

As we are moving down along the demand curve, so quantity demanded here increases, price is falling. So, that means, if we are moving down moving along the horizontal axis x increases p falls. So, that means, in TR you have two terms, p and x which are moving in opposite direction, as we are going along the horizontal axis.

So, that means, what will happen to TR that means, whether TR hat will be positive negative or equal to 0 that depends on relative change of price and quantity. Now, we already know the measurement, how much quantity demanded will change when price changes that depends on what? That depends on the elasticity value. So, we have already defined the own price elasticity it is the percentage change in quantity demanded brought about by percentage change in price.

So, I am just writing in absolute term x hat by p hat, right? So, what happens? As long as we see that x hat is greater than p hat; that means, we are in the elastic part of the demand curve corresponding to the upper region; upper part of the demand curve, so TR hat is positive, right? As long as x hat is greater than p hat we are in the upper part, TR hat is positive.

Of course we can see it is very trivial to see that TR of course, starts from the origin because at the origin quantity is equal to 0. So, total revenue is also 0. So, TR curve starts from the origin, after that it is increasing if it is positive because as price increases, quantity demanded falls. But here corresponding to the elastic part of the demand curve  $e_p$  is greater than 0, so total revenue curve will be increasing.

After that we see that, we reach a point where x hat is exactly equal to p hat. Because if you are coming down along the demand curve  $e_p$  is first greater than 1 in absolute term and then  $e_p$  is equal to 1. So, at that point what happens? When  $e_p$  is equal to 1, then p hat is equal to x hat. So, TR hat will be equals to 0, right. If TR hat is equal to 0 means, the total revenue curve reaches maximum.

So, that is the point you see, corresponding to the upper panel of the figure we see that del TR del x is nothing but MR and corresponding to the point where  $e_p$  is equal to 1 in absolute value MR is equal to 0. So, that is the point where, TR is maximum. And after that when you enter the inelastic part of the demand curve segment then we see that as price increases, quantity demanded change will be less than proportionately.

Therefore  $e_p$  is less than 1, TR hat is negative, therefore we see that the total revenue will start falling from here, right? So, this is how we derived the total revenue curve. So, this is the MR ok. So, this is how it derived the shape of the total revenue curve. Now, you see, why this is so important for our purpose, we will be also studying this later when we will be discussing about tax policy or say import, bill how the oil price shock how they affect the economy, how they put a stress on economies import bill.

Because you see if we bring it to our context that petroleum is inelastic product we have already discussed. So, if elasticity is less than 1 what happens? Then even if price increases, quantity demanded falls. So, you are moving up along the demand curve if say price increases, quantity demanded falls but by less than proportionately. So, that means, what? p hat is positive, x hat is negative, but p hat is greater than x hat in absolute value.

So, TR hat will still increase, so that means, our import bill import bill is price of import into the amount of import.

So, petroleum being inelastic in nature, if price increases quantity demanded falls, but by less than proportionately. So, or if even if price falls what will happen? Quantity demanded will increase ok. So, total revenue will keep on increasing. So, even if oil price increases, our import bill will increase because total revenue will keep on increasing, why? Because quantity demanded of petroleum will fall by less than proportionately ok.

Therefore, that is why you see that this rising oil price puts huge pressure on a country's foreign exchange reserve because the country has to spend more to buy petroleum products which are inelastic in nature. So, it is very important you can see how the today's lecture is very relevant for our purpose. So, in a nutshell what we discussed in today's class? We discussed the very important concepts of total revenue, average revenue and marginal revenue and we derived the three revenue curve.

So, we saw that the average revenue curve is nothing but the market demand curve, then we derive the marginal revenue curve for a linear downward sloping demand curve and we also derived the TR curve which is a hump-shaped or inverted U shaped and we see that it is very important for our purpose of petroleum products when which are inelastic. So, even if price of petroleum product increases, our import bill keep on increasing because quantity demanded of such inelastic product fall less than proportionately.

Therefore, this topic is very relevant for our purpose. Even we will be also coming back to this topic when we will be discussing the tax policies ok. So, these concepts should be very clear to us.

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So, thank you very much and as we have already discussed we can follow any standard microeconomics book for studying this part ok.

So, thank you very much we will be meeting you for the next module.