Management of fixed Income Securities
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Module No # 02
Lecture No # 09
Bond Returns- IV

Welcome back. So we discussed about the horizon analysis as well as how the total return is different from the YTM whenever the interest rate will change and the horizon period of the investor is different from the maturity period that we have discussed in the previous class.

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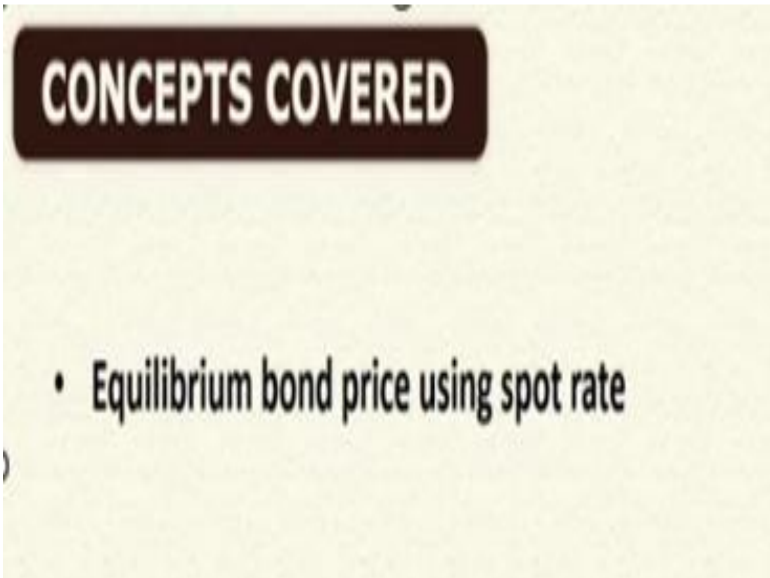

So, today in this particular session, we will be discussing about the concept of the spot rate and how the price of the bond is calculated with respect to the spot rate. So if there is a deviation from the price that we have calculated using the spot rate from the market price then there is a chance of arbitrage opportunity. Then what exactly that arbitrage opportunity and how that arbitrage opportunity is basically can be utilized by the investor. So those concepts are important from the investment point of view.

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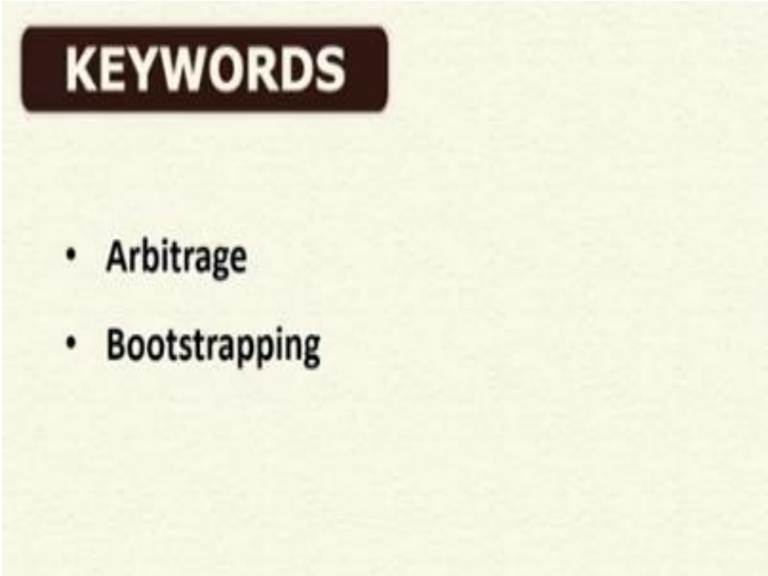

So let us see, that is why we will use these two keywords, you will understand that is your arbitrage and the second one is the bootstrapping. Bootstrapping is basically a method through which we calculate this spot rate. This spot rate is calculated using the method of the bootstrapping.

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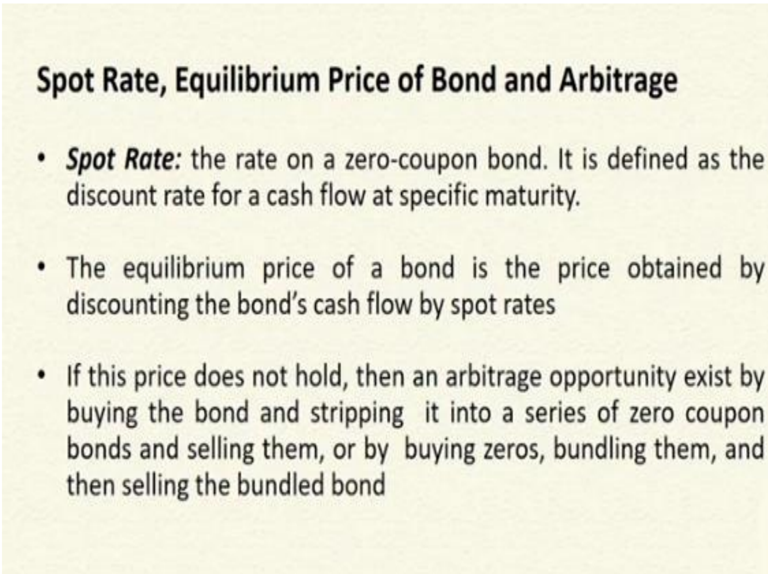

Let us see, what exactly the spot rate is? So three things you will understand for this discussion: one is your spot rate, equilibrium price of the bond and the concept of arbitrage. So, what is the spot rate? Generally, we consider the spot rate is nothing but the rate on a zero-coupon bond. And otherwise, it can be defined as the discount rate for a cash flow at the specific maturity. That basically, we can call the spot rate but it is basically a rate on a zero-coupon bond. So whenever we calculate the price of a bond; the intrinsic price or the equilibrium price of a bond is basically obtained by discounting the bond's cash flow by the spot rates. So we have to understand, we have to measure the spot rate already what we have taken: the spot rate is nothing but the rate on the zero-coupon bond.

And the equilibrium price of the bond is calculated on the basis of the spot rate which is prevailed in the market with respect to that particular cash flow. So if the price does not hold that means the bond is traded in the market at a different price and you have calculated the price of the bond on the basis of the spot rate which is different from that particular market or the market price. Then there is a chance of arbitrage opportunity and how that advantage in terms of that particular arbitrage opportunity can be created or how this arbitrage opportunity exists? That can be always carried out by buying the bond and stripping that particular bond into a series of zero-coupon bonds and selling them. Depending upon the particular price difference which one is more and which one is less you can adopt either of these two methods. One is buying the bond and strip that particular bond into a series of the zero coupon bond and selling them. Or buying the zero coupon bonds bundling them and then selling these bundles bonds. Either of these 2 ways it depends that which one is more and which one is less that basically, you have to keep in the mind. So therefore, using the spot rate first, you calculate this equilibrium price of the bond and then compare this particular price of the bond with the market price then see how that particular arbitrage opportunity can be created. Right!

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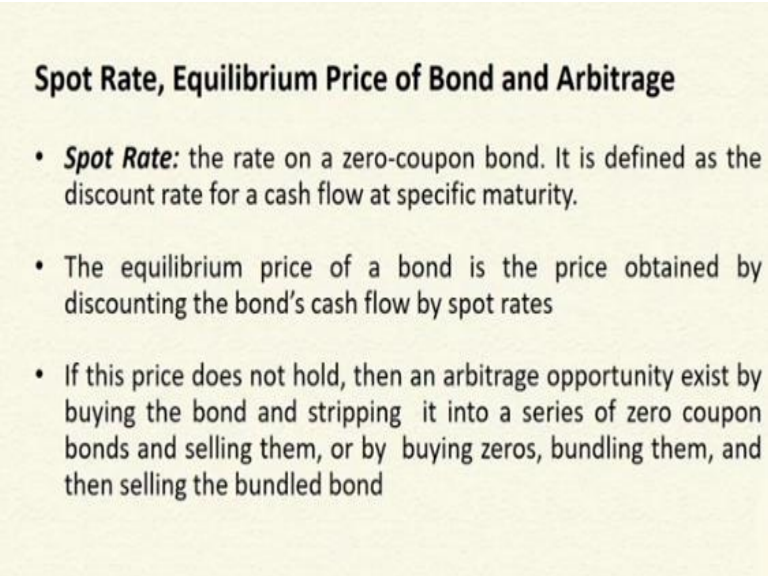

So let us see, how basically this works? Let me give you an example in this case, how basically you can do this? Let, there are 3 zero coupon bonds: each bond principal value each with a principle of rupees 100 right and they are trading at annualized spot rates. Let, the spot rate for one year that S1 is 7%, S2 is let 8% and S3 is let 9%. Right. And there is a coupon-bearing bond and this bond's maturity period is 3 years and the coupon is 8%.

Then what is the price of the bond? You can calculate that is your 8 divided by your 1.07 to the power 1 + 8 divided by 1.08 that is 7% that is 8% that is 9%1.08 to the power 2 + the last 1 you will get 108 divided by 1.09 to the power 3. We are discounting it with respect to the spot rates, then you get let 97.73 that is this one is basically your equilibrium price, right.

Price of the Bond(P_0)= $\frac{8}{(1.07)^1} + \frac{8}{(1.08)^2} + \frac{108}{(1.08)^3}$ = 97.73

Hence, equilibrium price of the bond is 97.73.

Suppose a bond of 3 years maturity with an 8% coupon is traded in the market at a price of 95 Rupees. Let, 3 year bond with 8% coupon is traded in the market at a price of 95 rupees but your equilibrium price is 97.73 what you have calculated using the spot rates. So then what is the situation here? The market price is different from the equilibrium price so there is a chance of arbitrage? Then, how the arbitrage can be created or how you can create the arbitrage? So what basically you can do, first of all, number 1, buy the bond from the market buy the bond for rupees 95 which is available. Right! Then, what basically you can do? You form 3 stripped zero coupon bonds 0 coupons and sell them. How we can make it? Number 1: one year zero coupon bond with your face value of Rupees 8 then, your selling price will be 8 by 1.07 that is 7.4766. Second: 2 year zero, I am using the short form zero, with your face value 8 again then what is your selling price? Selling price has become 8 by 1.08 to the power 2 that you get it 6.8587.

Number 3: your 3 years zero and here your face value will be 108 then your selling price will be 108 divided by 1.09 to the power 3 that will give you 83.3958. So now what will happen? This is basically second step so for simplicity let us write a this is a, b this is c then in the third step what basically you will do you sell these trip bonds sell of the strip bonds the total value if you add it then it will give you 97.73.

Then how much profit that is basically arbitrage means? It is basically risk free profit you have generated 97.73 - 95 that is 2.73.

The arbitrage profit can be calculated as:

Step1: Buy the bond for Rs 95

Step 2: Form three stripped zero coupon bonds and sell them

a) Case1- One year zero coupon bond with face value of Rs8, then here selling price will be = $\frac{8}{(1.07)^1}$ = 7.4766

b) Case2- two years zero coupon bond with face value of Rs8, then here selling price will be = $\frac{8}{(1.08)^2}$ = 6.8587

c) Case3- three years zero coupon bond with face value of Rs108, then here selling price will be = $\frac{108}{(1.09)^3}$ = 83.3958

Step 3: Sale of stripped bond at Rs (7.4766+6.8587+83.3958) = 97.73app.

Step 4: Due to arbitrage, the risk free profit = 97.73 – 95 = 2.73

You have bought the bond at a price of 95 strips them into 30 coupon bonds sell them then you get 97.73 then straight forward you get a profit of 2.73 rupees. This is the way basically you can create this opportunity and get some kind of riskless profit from this.

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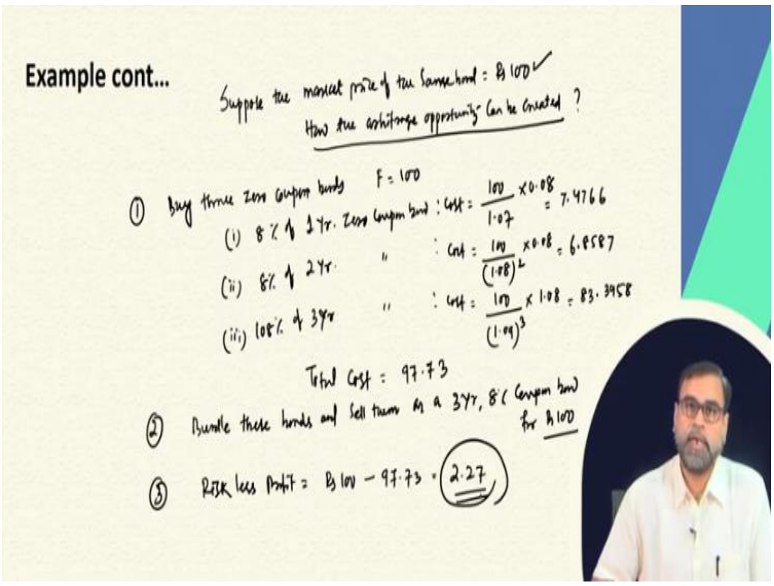

Suppose the market price of the 3 years 8% coupon bond is traded in the market at a price of 100. Suppose we will continue with this discussion. Suppose the market price of the same bond is 100 rupees then how the arbitrage opportunity can be created? This is the question, how it can be created? We will just do the reverse one, buy three zero coupon bonds, your F = 100 + value is equal to 100. So number one: 8% of one year zero coupon bond will be what is the cost? Cost will be 100 divided by 1.07 first yours 1 = 1.07 into your 0.00 8 that will give you 7.4766.

Then your 8% of 2 year zero coupon bond that cost will be 100 divided by 1.08 square into 0.08 that will give you exactly 6.8587 itself. Third it will be obviously 100 8% of 3 year zero coupon bond and what is the cost? That will be 100 divided by 1.09 to the power 3 into your 1.08 that will give you 83.3958. The total cost if you add it will be 97.73 then second what basically you can do bundle these bonds and sell them as a 3 year 8% coupon bond for face value of 100.

For 100 Rupees which is the market price then your profit will be your riskless profit you have generated that is your 100 rupees minus your 97.73 that is 2.27.

In this case, the market price of the same bond is Rs100. The arbitrage opportunity can be created in the following steps:

Step1: Buy three zero coupon bonds whose face values are Rs 100.

a) Case1- 8% of 1 year zero coupon bond and the cost will be = $\frac{100}{(1.08)^1} \times 0.08 =$ 7.4766
b) Case2- 8% of 2 year zero coupon bond and the cost will be = $\frac{100}{(1.08)^2} \times 0.08 =$ 6.8587
c) Case3- 8% of 3 year zero coupon bond and the cost will be = $\frac{100}{(1.08)^3} \times 0.08 =$ 83.3958

Step 2: Bundle these three bonds and sell them as a 3 year, 8%coupon bond for Rs100.

Step 3: Total purchase cost of three zero coupon bonds (7.4766+6.8587+83.3958) = 97.73app.

Step 4: Due to arbitrage, the risk less profit = 97.73 – 95 = 2.73

This is the profit you have created. The first case, we have stripped this particular coupon bond into 3 zero coupon bonds. So here in this case, you are buying three zero coupon bonds ,bundling that 3 zero coupon bonds to a coupon bond then sell it in the market. So, why basically you are doing it?

Here, in this case, the market price is different than the equilibrium; price previous case, the market price was less than the equilibrium price. And equilibrium price means whatever price basically we have charged on the basis of these spot rates. And the equilibrium price is always calculated using the spot rates which are prevailed in the market. So this is the way the arbitrage opportunity can be created and the investor can always earn some riskless profit through that particular process.

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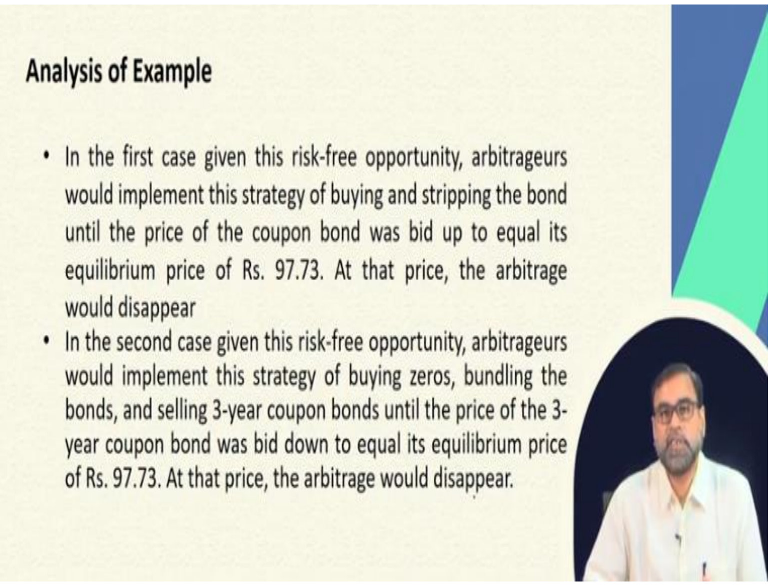

So here the next question is that what basically we have seen here, in the first case what we have seen? Given this risk-free opportunity, arbitragers should implement this strategy of buying and stripping the bond until the price of the coupon bond was bid up to equal its equilibrium price that is 97.73. And once it will reach 97.73, the arbitrage would disappear. So automatically, the price will change looking at the equilibrium price and the market price. Because, if many people will go ahead with this strategy automatically it will increase the price from 95 to 97.73 then in the end the arbitrage basically will disappear.

And in the second case, what we have seen? Arbitrageurs basically will implement this strategy of buying the zero coupon bonds or zeros, bundling these bonds and selling the 3 years coupon bond until the price of the 3 years coupon bond was bid down to equal its equilibrium price that is 97.73. Then the arbitrage would disappear. End of the day, the arbitraries will disappear but within that particular period, the profit riskless profit can be generated by the investors by taking this kind of position. That actually you can keep in the mind. In the end automatically this will create a certain kind of demand in the market and once the demand-supply, things will change the price will automatically come back to equilibrium. But it will take some time then within that particular time this investor can get the advantage from this particular arbitrage opportunity.

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So now the question is that how to estimate the spot rates? Why basically, the problem arise? Because you see whenever we are going for evaluation with the spot rates and creating the different types of the stripped securities. But in the true sense in a practical sense, there are not enough; longer-term zero coupon bonds available to determine the spot rates on higher maturities. Zero coupon bond may be available for one year period, 2 year period but for longer-term maturity zero-coupon bonds here we have taken for first year zero coupon bond rate 7% second year 8% third year 9%. But that is not always possible for the investor to find out a zero coupon bond whose maturity period is 15 years. And that can be utilized as a spot rate for the calculation of the equilibrium price. So because of this the long-term spot rates have to be estimated; we have to estimate the long-term spot rates and one of the estimation approaches that can be used, which is a basically sequential process is called boot strapping.

So, given the bond’s rate, a coupon bond with the next highest maturity is used to the implied spot rate. That means we are taking the help of the rates which is prevailed from the coupon rate to calculate the spot rates of the longer-term maturity zero coupon bonds. Then another coupon bond with the next highest maturity is then used to find the next spot rate like that it will go on.

So that is the process basically what we follow whenever we use this boot strapping method for calculation of the spot rate for the longer maturity zero coupon bonds and that thing again can be utilized to calculate the equilibrium price of the bond.

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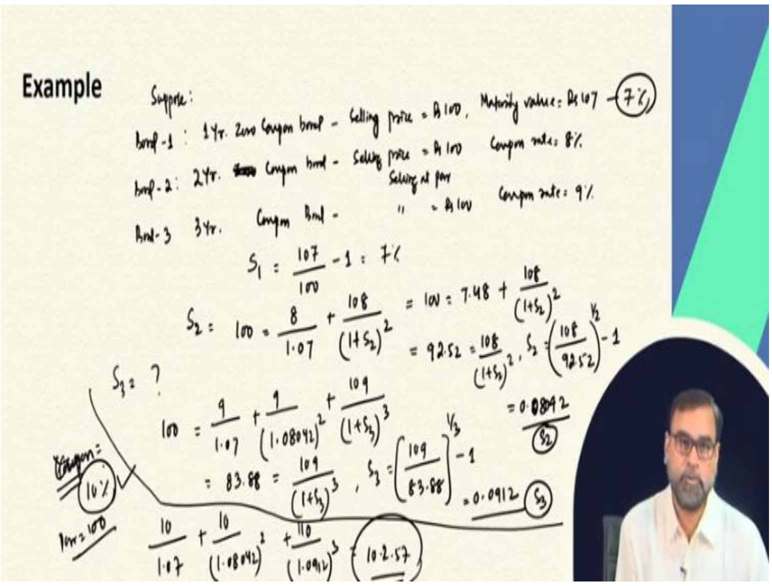

So let us see that how this particular bootstrapping method is utilized or is used to calculate the longer-term spot rate. Let I will give you certain things here let suppose bond 1: you assume suppose there is a bond one that is the one year zero coupon bond selling price, let Rs.100 and the maturity value is 107. Let there is a bond 2: there is a 2 year zero coupon bond your selling price selling at par that is rupees 100 and coupon rate is let 8% it is a coupon bond, it is a 2 year coupon bond, coupon rate is and it is selling price means Rs.100 means it is selling at par.

Then there is a bond 3: that is the 3 years coupon bond it is also selling at par that Rs.100 and the coupon rate is at 9%. These are 2 coupon bonds and the first one is basically zero coupon bond, it was bought at a price of 100 and the maturity value will be 107. Then what basically what is your S1 then? S1 will automatically your 107 divided by 100 - 1 right that is your 7%. First rate it is S1 is 7% this is your S1.

The spot rate (S1) can be calculated when we are going for 1 year zero coupon bond= $\frac{107}{100} - 1$ = 7% 0r 0.07.

Right, now what is S2? S2 is equal to your principal value is 100 selling price is 100 then that means it is 100 is equal to year 8 by coupon is 8%, 8 by 1.07 + 108 divided by 1 + S2 to the power 2 is it clear 1 + S2 to the power 2 but S2 we do not know then we have to find out the S2 then that will be is equal to 100 = your 7.48 by 1.07 that is 7.48 + 108 divided by 1 + 1 + S2 square. Then that is equal to that means it will be 92.52 = 108 divided by 1 + S2 square then yours 2 = 108 divided by your 92.52 to the power 1 by 2 - 1 that will be 0.08428042.

Similarly, the spot rate (S2) can be calculated when we are going for 2 year at 8% coupon bond as: $100 = \frac{8}{(1.07)^1} + \frac{108}{(1+S2)^2}$

- ⇨ $100 = 7.48 + \frac{108}{(1+S2)^2}$
- ⇨ $92.52 = \frac{108}{(1+S2)^2}$
- ⇨ S2 = 0.08042 or 8.042%

So this is the spot rate if you are going for a 2 year zero coupon bond then your spot rate will be 8.04% 0.08042. Now we go for the third one what is your S3? S3 will be then again you solve this equation 100 = 9 by 1.07 + your 9 by one point now we have to use this 108042 square plus your 109 divided by 1 + S3 to the power 3. So if you solve it then automatically this part will come 83.88 = your 109, 1 + S3 to the power 3 then your S3 = 109 divided by your 83.88 to the power 1 by 3 - 1 that will give you 0.0912. So this is your S3.

Similarly, the spot rate (S3) can be calculated when we are going for 3 year 9% coupon bond as: $100 = \frac{9}{(1.07)^1} + \frac{9}{(1.08042)^2} + \frac{109}{(1+S3)^3}$

- ⇨ $100 = 83.88 + \frac{109}{(1+S3)^3}$
- ⇨ $S3 = \left[\frac{109}{83.88}\right]^{1/3} - 1$
- ⇨ S3= 0.0912 or 9.12%

The equilibrium price of other 2 and 3 years bond can be obtained by the 3 year spot rates. So in this case if you want to calculate the equilibrium price how will you calculate the equilibrium price? Now you can calculate let you have a bond whose yield to maturity is 10% YTM. Then now you have got the S1, S2, S3 you can find out your equilibrium price that means your 10 by 1.07 that the par value is 100, then it will be 10 by 1.077,10 by 1.08042 to the power 2 + 110 divided by 1.0912 to the power 3 if you solve it you will get 102.57.

The equilibrium price of three year coupon bond can be calculated by using spot rates as discount rates. And here, S1, S2 and S3 are 0.07,0.08042 and 0.0912 respectively.

The equilibrium price (Po) = $\frac{Coupon}{(1+S1)^1} + \frac{Coupon}{(1+S2)^2} + \frac{Coupon+mv}{(1+S3)^3}$

$$= \frac{10}{(1.07)^1} + \frac{10}{(1.08042)^2} + \frac{110}{(1.0912)^3}$$

= 102.57

Hence, the equilibrium price is 102.57.

So this is your equilibrium price. We are using this spot rate for calculation of an equilibrium price of a bond whose par value is 100, your coupon is 10% and you are using this spot rate as the discount rate. So then you got your equilibrium price 102.57. So then you compare at that time what is the market price of this 10% coupon and 3 years maturity bonds, then accordingly your arbitrage can be created.

So this is what basically the bootstrapping method, like that for bond 4 you can take a 4 years coupon bond, you know this coupon rate, you know this par value or the selling price from that you can use this S1, S2, S3 to calculate your S4 then like that is 5. So that is the way basically this bootstrapping method works.

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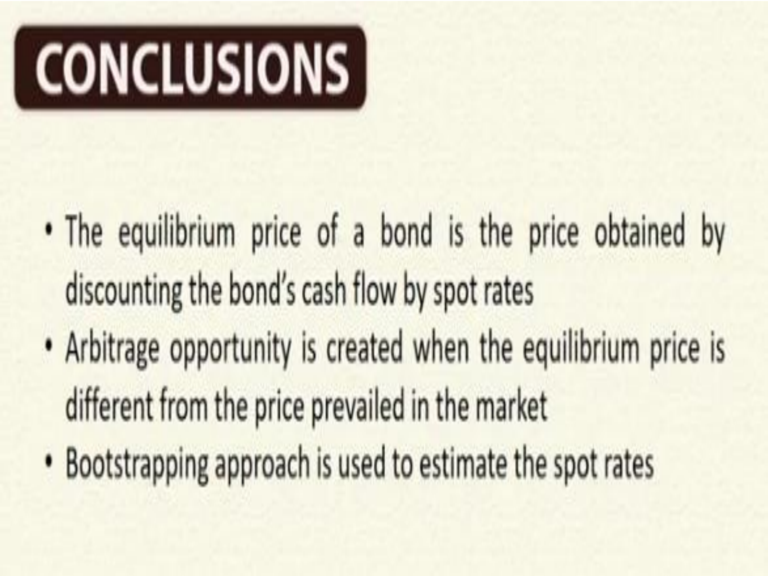

So what we have basically discussed here? The equilibrium price of a bond is nothing but, we can get this equilibrium price by discounting bonds cash flow with the spot rates. And the arbitrage opportunities created when the equilibrium price is different from the price which is prevailed in the market. And bootstrapping approach is generally used to estimate the spot rates, there we use the different maturity periods coupon bonds for calculation of this spot rates for the particular kind of analysis or the zero coupon rates.

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So this is the reference what you can follow for this. Thank you.