Management of Fixed Income Securities
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Module No # 02
Lecture No # 08
Bond Returns-III

Good morning. Welcome back, in the previous class, we discussed about certain concepts which are related to the calculation of the bond returns or the bond yield. There we have discussed about particularly the current yield, then we have yield to maturity then as well as we also discussed about the yield of the zero coupon bond. Then we discussed about the concept of the total return.

But, here what basically we have taken this example in the context of the yield to maturity is constant across the particular period. But, in the true sense, if you observe that the interest rates always varies in the market and whenever the interest rate will change automatically the yield of that particular bond also will change. So let us see that if these particular interest rates in the market will change then how the total return of the particular bond is calculated.

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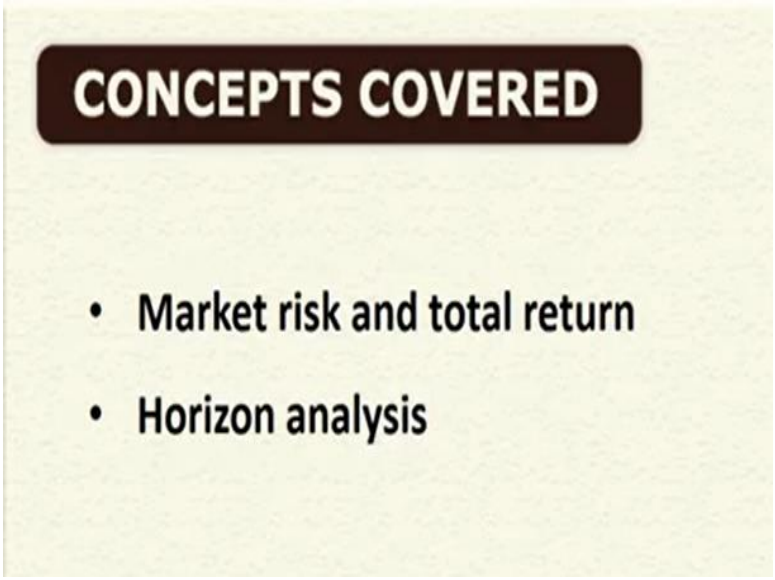

So, therefore, in today's context, we will discuss about the market risk and total return. So here market risk means fluctuations of the interest rate. So if there is a change in the interest rate how the total returns of the particular bond is going to be changed? And as well as another concept we will discuss about the horizon analysis. So here we will discuss about the different type of bonds which are available in the market and the investor will choose what kind of bond to minimize this particular interest rate risk what basically they are always exposed to.

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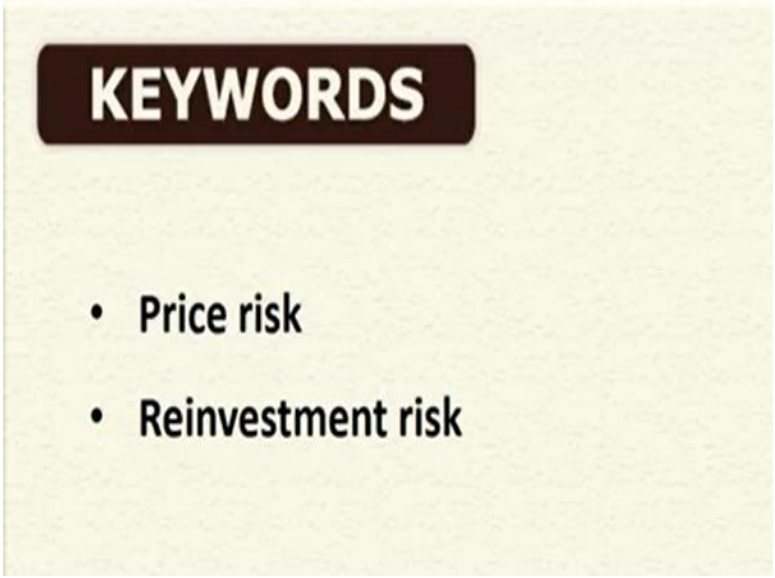

And after this, you will be knowing about the 2 types of risk and how this particular risk is playing a major role in terms of the bond return? That is basically your price risk and the reinvestment risk which are basically the components of the interest risk. If there is a change in interest rate then the bond price will be changed and as well as the reinvestment of the coupon that value is also going to be changed. So we will see that how exactly the price risk is and if there is a fluctuation of the interest rate then what kind of bond is suitable for investor for the investment.

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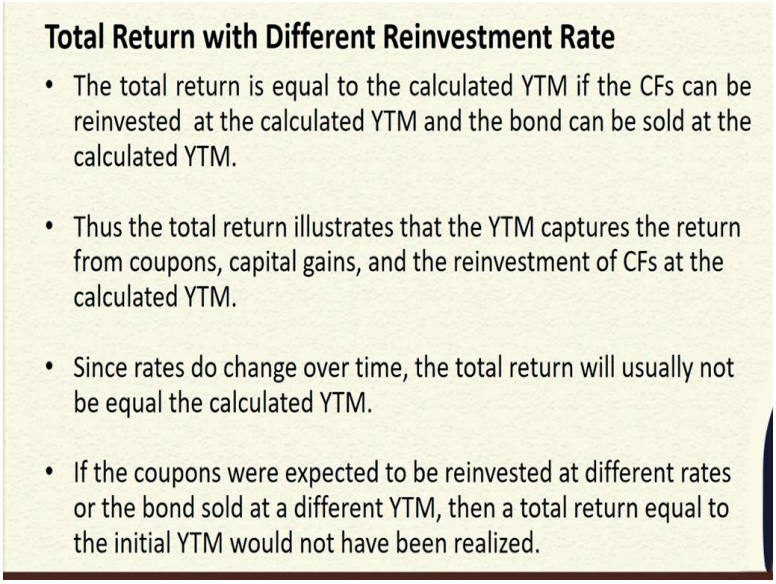

So here already what in the beginning I told you, we will be discussing about the total return with different reinvestment rate. Because, whenever we calculate the total return what exactly the total return is? Total return is generally always calculated on the basis of the yield to maturity and here we have assumed the cash flows are always reinvested at the calculated YTM and the bond also is sold at the calculated YTM.

So whatever yield to maturity is there if in between before the maturity the investor is going to sell the particular bond, they can sell the bond on the basis of the yield to maturity already what yield to maturity was there whenever they have bought the bond. So that is the basically way we calculate the total return. So, the total return basically what it basically then it explains? It explains that the YTM the yield to majority basically captures the return from the coupons, the capital gains and the reinvestment of the cash flows at the calculated YTM.

Because there are 3 components we have seen whenever we calculate the bond return, total return of the bond, we have seen there are 3 components. One is your coupons whatever you are getting then you have the capital gain. The capital gain is basically will be calculated on the basis of what price you have bought the bond and what price you have sold the bond. Accordingly, the capital gain can be calculated.

Then whatever coupons you are receiving over the period of time or in the periodical basis, whatever amount of interest on interest or how you are going to reinvest that particular coupon in the market that will give you the reinvestment of the cash flows. And all these

things are calculated on the basis of the calculated YTM. But the interest rates do change over the time. So, in that context, the total return is not exactly equal to the calculated YTM.

Whatever yield to maturity you are receiving or you have with the bond that is not exactly equal to yield to maturity of the particular bond whenever you have started investing in that particular bond. So, if the coupons are expected to be reinvested at the different rates of the bond or the bond sold at a different YTM: whenever. Let your bond maturity period is 10 years and you have sold the bond at the end of 7 years. Then in that particular year, the yield to maturity of the bond if you observe that basically will depend on the market interest rate. So, therefore, we cannot always use that YTM in the beginning whatever we have considered or what price we have basically bought the bond, that concept cannot be used for calculation of the total return. So, in that case if you see the total return may not be equal to the initial yield to maturity of that particular bond whenever you have bought it.

So there may be a variation. The return may be more or return may be less but that basically will depend upon that whether the 2 types of risk whatever we are talking about one is your price risk and the reinvestment, which particular effect: the price effect is dominating the reinvestment effect or the reinvestment effect is dominating price effect. That will decide that the whether we should get the more return or we will get the less return.

So that is basically the general concept always we should keep in the mind. So, let us take one example, it will be more clearer for you that what basically we are trying to explain in the context of the total return with the different reinvestment rate.

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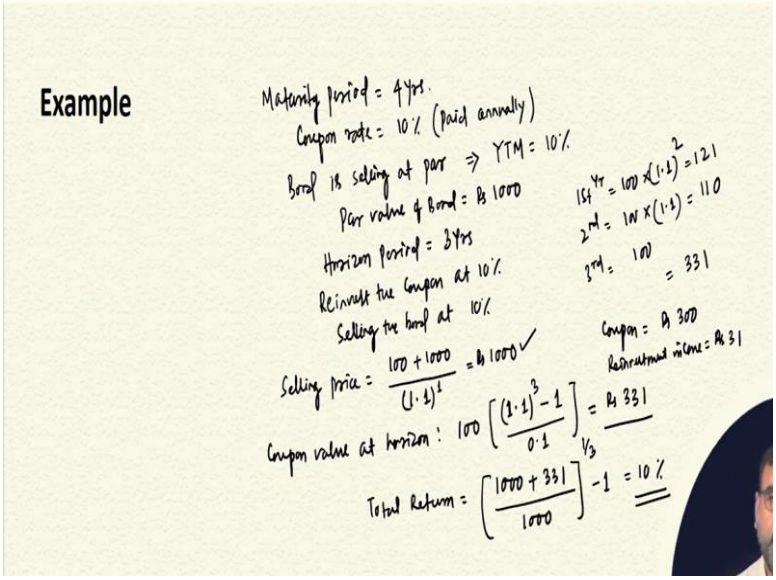

So, let us take this particular thing with a simple case then we can move towards the other thing. So let there is a bond, let this maturity period of the bond is 4 years, and let the coupon rate is 10% and coupon is paid annually. For simplicity, let we assume, bond is basically selling at par. What does it mean? It implies that your yield to maturity is also 10%. If the bond is selling at par that means the yield to maturity is also 10% because for if the particular bond which is traded at par that means the coupon is equal to the into maturity.

Let the par value of the bond is 1000 rupees, right! so let the horizon period of the investor horizon period is 3 years that means the investor is going to sell the bond after 3 years but actual bonds maturity period is 4 years. So here what does it mean? So if the horizon period is 3 years and the coupon is basically paid annually that means and the interest rate does not change. The interest rate will not change. For simplicity, we are taking that assumption then what this investor will do? They will reinvest the coupon at 10% because yield to maturity is not changed. So, they will reinvest the coupon at the rate of 10% and also selling the bond at the end of the 3 years at the rate of 10%. So here everything is basically at the 10%. So here the question is that how basically if you are selling the bond at the end of the 3 years then what is the selling price of the bond?

How much cash flow is remaining? You have 100 rupees coupon, only 1 year is remaining and plus 1000 what basically will be getting in the on the basis of the par value divided by your only 1 year is left then 1.1 to the power 1.

Selling price= $\frac{100+1000}{1.1^1}$ =1000

Hence, the selling price of bond is 1000.

So you will be getting exactly 1000 rupees. Why you are getting? Because all the rates we have kept it same.

So now what is the coupon value at the horizon period? The coupon value at horizon means after 3 years you get the coupon how many coupons you will be receiving 100 rupees at end of the 1 year, another 100 rupees end of the 2 years, another 100 rupees for the end of the 3 years. And that 100 rupees what you get at end of the 1 year that will be again reinvested for 2 years and for the second year it will be reinvested for one year and third year coupon basically is not going to be reinvested, immediately, you have just received the coupon.

So if directly you want to calculate, what is the value you will be getting? That is nothing but the coupon times the future value of the annuity, the coupon times the future value of the annuity that formula basically we can use. So, here 100 rupees is the coupon then what is the future value of annuity? That is your 1.1 to the power 3-1 divided by 0.1 that means at a time I am calculating the total amount of coupon and as well as the reinvestment income whatever you are getting from that particular coupon.

Coupon value at time horizon: $100\left[\frac{(1.1)^3}{0.1} - \frac{1}{0.1}\right] = 331$

So that will be: you are getting 331.Where basically you get this formula? You see, end of the first year, you will get 100 rupees and that 100 rupees will be reinvested. Then 1.1 to the power 2 that will be 121 and in the second year basically you get 100 rupees then that will be 1.1 again you have reinvested for 1 year. That you will be getting 110 rupees and in the third year you will be getting 100 rupees so that if you add all these three that will give you 331.

The value of the coupon if reinvested at ytm can be calculated as:

First year, the coupon is invested for two years. So the value of reinvested coupon at the end of three years will be: $100(1.1)^2=121$

Second year, the coupon is invested for two years. So the value of reinvested coupon at the end of three years will be: $100(1.1)^1=110$

Third, the coupon received is not invested because it is received at the end of third year. So the value of this coupon is 100.

When we add the three values, we get 331.

That means your coupon: total coupon you have received that is 300 and your reinvestment income reinvestment income that you are getting 31 rupees. And you have sold the bond the selling price of the bond is 1000 rupees. And now the total value of the coupon in the horizon period that is 331 rupees your reinvestment income is 31 and coupon is 300. So now your total return will be what will be your total return?

Already you know that then your 1000+331 divided by 1000, this 1000 rupees basically your buying price of the bond. Whenever you have started investing because par value of the bond is 1000 and the bond is selling at par. So you have bought the bond at a price of 1000 to the power 1 by 3-1.

Total return = $\left[\frac{1000+331}{1000}\right]^{1/3}$ - 1 = 10%

That will be giving you 10 %. Why we got 10% because we have kept the YTM unchanged.

We have also kept that the coupon is again reinvested at that particular rate that means almost all the rates are same. So, because of this, your total return will be equal to the YTM the initial YTM at which the bond was basically issued. Right! So this is what basically for simplicity we have taken these assumptions. Now, let us see, is there any change if the coupon is paid semi-annually? Let the coupon is paid semi-annually, if the coupon is paid semi-annually then is there any change in terms of the formula?

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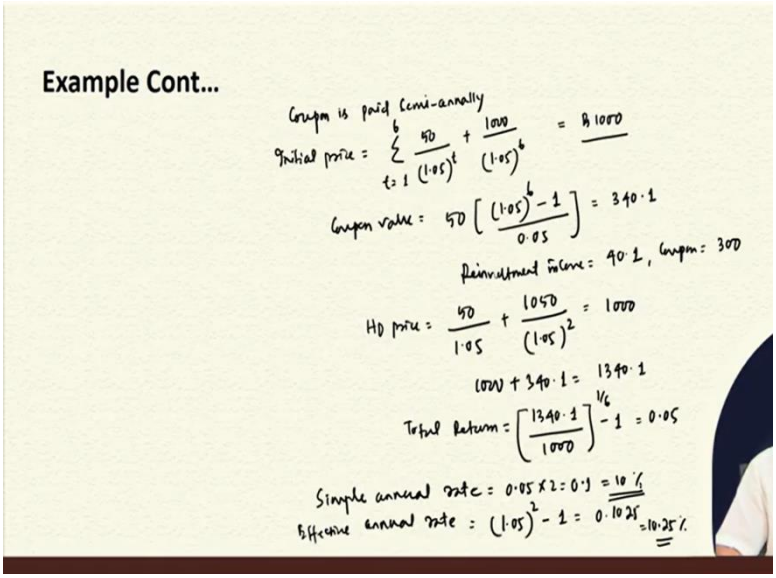

So let us see, for example, the coupon is basically paid semi-annually then what will be the initial price of the bond? Now with the same example, we have taken coupon is paid semi-

annually right. If coupon is paid semi-annually then what is the initial price of the bond? How will you calculate this? Already you know that that is your t = 1 to 6 50 divided by your 1.05, 10% of the coupon now divided by 2 to the power t + your 1000 divided by 1.05 to the power 6.

Initial Price= $\sum_{t=1}^{6} \frac{50}{(1.05)^1} + \frac{1000}{(1.05)^6}$ = 1000.

So this will also give you exactly your 1000 rupees. Now, what is the coupon value? I mean this price you are basically selling the bond then what is the coupon value? The same formula you can use 50 into 1.05 to the power 6 - 1 divided by 0.05, that will give you 340.1.

Coupon value = $50 \left[\frac{(1.05)^6-1}{0.05}\right]$ = 340.1

It will be little bit higher; the reason is basically the coupon is reinvested semi-annually. So now your investment income is how much?

Your reinvestment income will be 40.1 and your coupon is 300 that is fixed; coupon is 300 right. So now what is your HD price that basically will get 50 divided by your 1.05 + your in the end how much you are going to get? 100 rupees plus your coupon 1050 divided by 1.05 to the power 2 times it will be reinvested because 6 month species that will give you 1000 rupees.

HD price= $\frac{50}{1.05} + \frac{1050}{1.05}$ =1000.

So now the total value what basically you are getting? 1000+340.1, that will be 1340.1. Then your total return will 1340.1 divided by 1000 to the power 1 by 6 - 1, it will be also giving you 0.05.

Total return = $\left[\frac{1340.1}{1000}\right]^{1/6}$ - 1 = 0.05.

Then simple annual rate = 2×0.05=0.10 or 10%.

Effective annual rate = $(1.05)^2 - 1$= 0.1025 or 10.25%

So your simple annual rate will be 10% simple annual rate will be 0.05 into 2 that is 0.1 that means 10%. And if you go for the effective annual rate will be how much that will basically give you 1.05 power 2 - 1 that is 0.1025 that is 10.25%. But effective annual rate will be 10.25% but exactly you will be getting your10% only in this case. So therefore what basically

we have seen that whenever we are calculating this particular total return assuming that YTM remains constant across the period then your total return is equal to your YTM.

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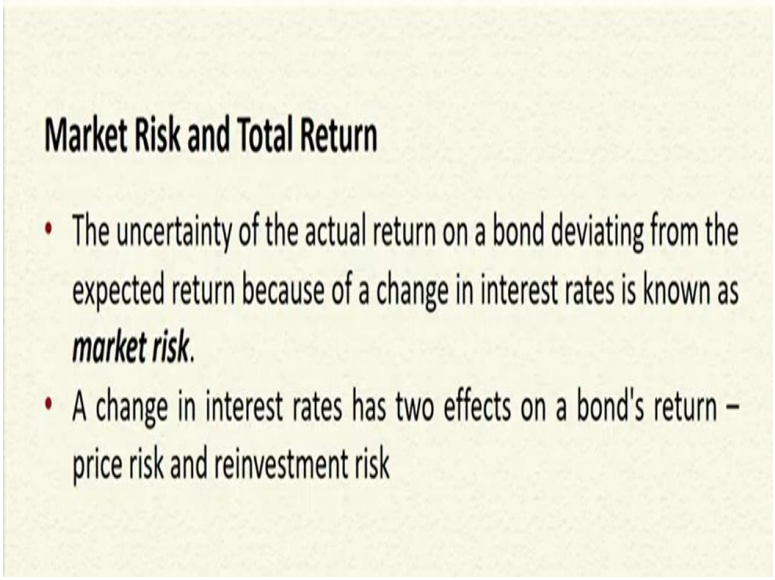

But in the true sense, there is uncertainty that the actual return of a bond which may be deviated from the expected return because of change in the interest rate which is basically known as the market risk. So the change in interest rates has 2 effects that already told you one is your price risk and the reinvestment risk. Let us see in the same example what will happen if the interest rate will change?

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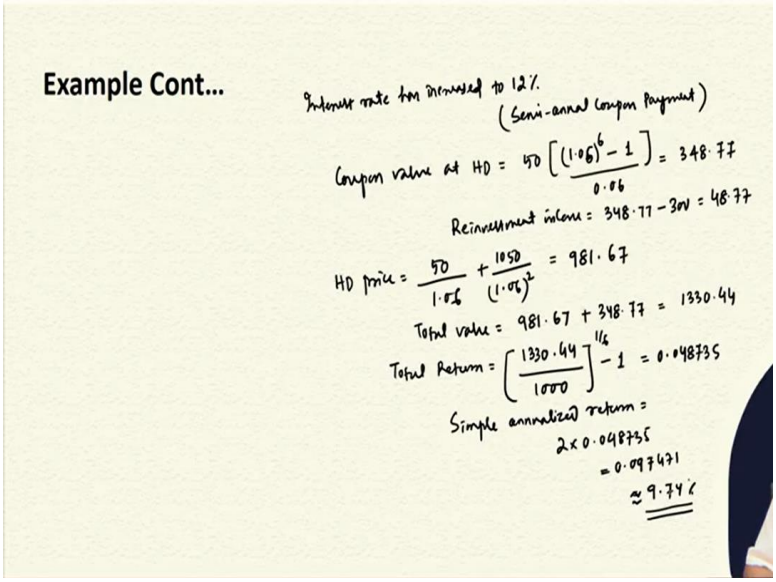

So if the interest rate will change, let the interest rate has gone up you have bought the bond. But after that the interest rate has increased to rate 12%, if the interest rate has increased to 12%. After you purchase the bond and let the coupon is paid semi-annually. Whenever you have bought the bond that time the yield was 10% but now after you bought the bond the price or the interest rate has increased to 12%, then what will be the coupon value at the horizon period is 3 years?

That we have taken then it will be 50 into 1.05 to the power 6 right it will be 1.05 or it will be 1.06 because now the interest rate has increased to 12% divided by 2 that is 6% then 1.06 to the power 6-1 divided by 0.06 so that will give you 348.77.

Coupon value at HD = $50\left[\frac{(1.06)^6-1}{0.06}\right] = 348.77$.

So the coupon value is 348.77.

Reinvestment income = 348.77 – 300 = 48.77

HD price = $\frac{50}{1.06} + \frac{1050}{1.06} = 981.67$.

The reinvestment income and HD price are 48.77 and 981.67 respectively.

So now your reinvestment income has become 348.77-300 that is 48.77. Now your what is your HD price? At the end of the 3 hours or what price you are going to sell the bond?

That is 50 divided by 1.06+1050 divided by 1.06 to the power 2 square of this now this will give you 981.67 or automatically the price will decline because the interest rate has gone up. So the price basically will be getting whenever you sell the bond that is 981.67.

Then total value how much is 981.67 + 348.77 that will give you 1330.44. So, now what is your total return.

Total return will be 1330.44 divided by 1000 at what price you have bought the bond to the power 1 by 6-1 that will give you 0.048735.

Total return= $\left[\frac{1330.44}{1000}\right]^{1/6} - 1$ =0.048735.

Simple annualised return = 2×0.048735 = 0.097471 or 9.74%

So if you make it analyzed return that; will be 2 into 0.048735 that will give you 0.097471.So that means 9 up 9.74% approximately. So now what you have observed here the yield to maturity in the beginning was 10% but whenever the interest rate has gone up from 10 to 12%, the total return if you are selling the bond at the end of the 3 years the total return has become 9.74%. What does it mean? It means that you get some better return in terms of reinvestment of the coupons that value has increased; you got 48.77 which was more than the previous value. But at the same time the price of the bond has come down whenever price of the bond has gone down. Now if what you can judge? You can judge that price effect is more than the reinvestment effect. The return what you got in terms of the investment of the coupons that is not sufficient enough to compensate the loss whatever you have made in terms of the change in the prices due to increase interest rate. So therefore, we have to understand that what exactly the price effect and the reinvestment effect are.

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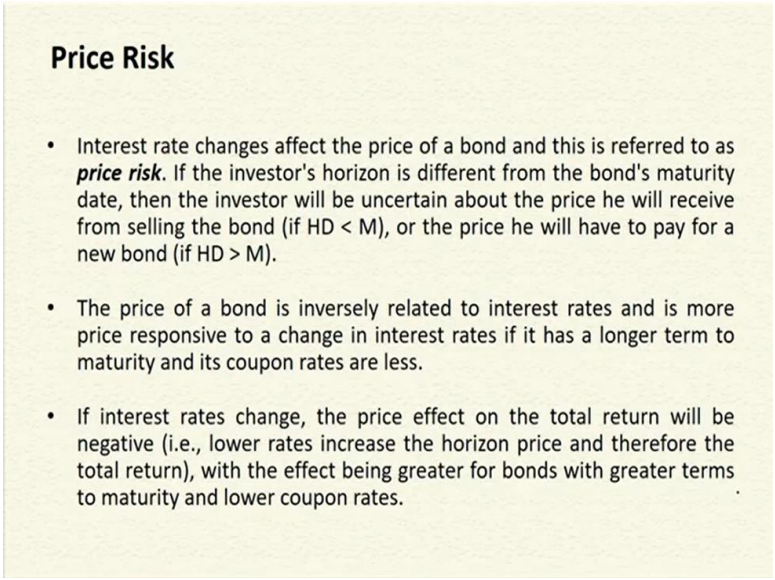

So whenever you talk about this price risk already what I told you, the interest rate changes basically will affect the price of a bond and generally this is called as the price risk. So here, the investor's horizon period which was 3years is different from the bonds maturity which was 4 years. So, then what will happen that the investor will uncertain about the price what he or she will receive from the selling the bond because the horizon period is less than the maturity period.

Or what price he or she has to pay for a new bond, if their horizon period is greater than the maturity period because they need some money maybe after 5 years. So they can go for another bond, so in that case, they are very much uncertain that what kind of yield or what kind of return they are going to get if the horizon period and the maturity period is not matching. So already you know that the price of a bond is inversely related to the interest rate and is more prices responsive to a change in interest rate if it has a longer term to maturity and its coupon rates are less. Already all we have discussed that part that the price responsiveness of the less coupon bond and long maturity bonds are more than the high coupon and the shorter maturity bonds. So the interest rate change then the price effect on the total return will be negative. That means the lower rates increase the horizon price and therefore the total return and with the effect being greater for bonds with greater terms of maturity and the lower coupon rate that actually you have to keep in mind.

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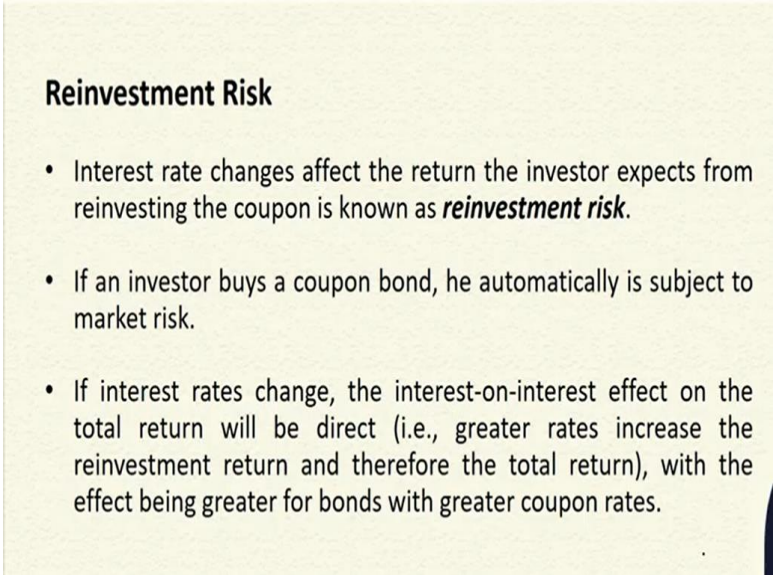

Then we have the reinvestment risk. So, in the reinvestment risk, the interest rate change basically affect the return; the investor expects from the reinvesting the coupon that is called the reinvestment risk. If the investor buys a coupon bond, he or she is automatically subject to the market risk. If the interest rate changes then the interest effect on the total return will be direct. That means greater rates increase the reinvestment in return and the total return may be increased because of that.

With the effect being greater for the bonds with greater coupon rates, the higher the coupons then automatically their investment return will be more that is called the reinvestment risk.

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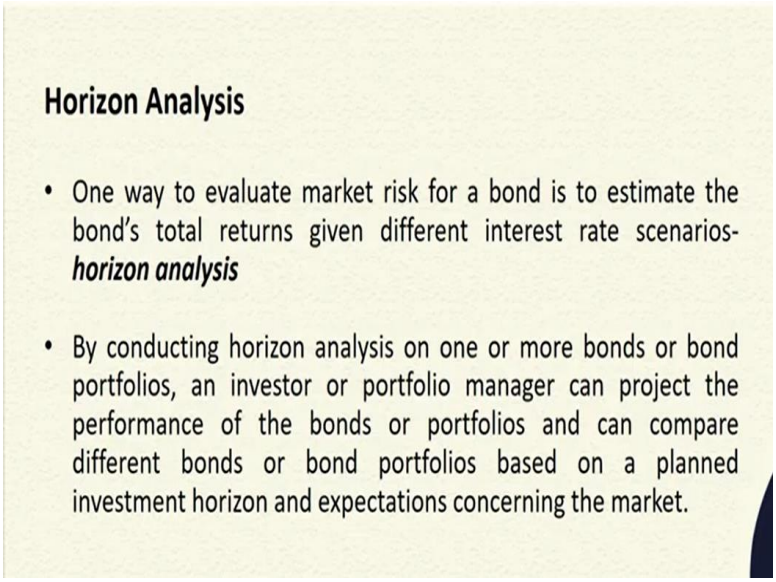

So one way to evaluate the market risk for a bond is to estimate the bonds total return in a given interest rate scenario that is called the horizon analysis. So in this case, what these investors do? Investors basically go for analysing one or more bonds or bond portfolios and they try to see that or try to compare the different bonds based on their planning or planned horizon period and the expectations what they have concerning about the market. Then accordingly they decide that which bond is better for them.

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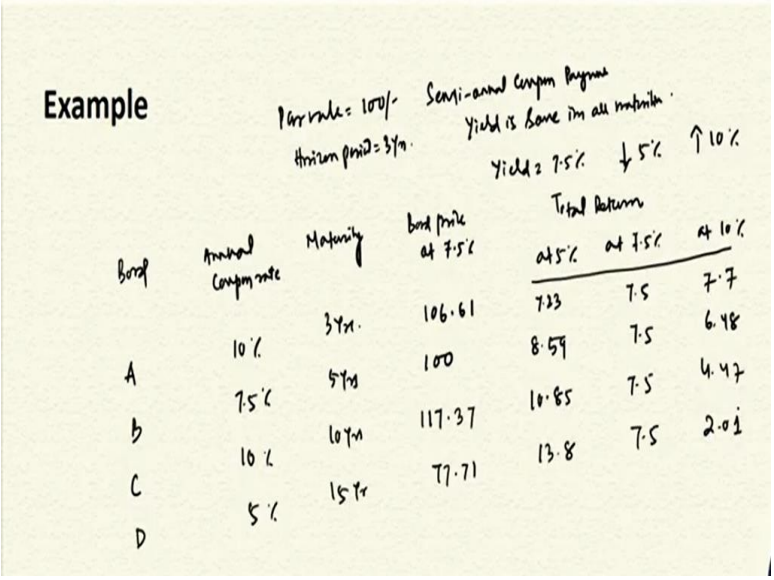

Let me give you one small example, in this case, let there is a bond, the par value is 100 rupees, horizon period is let 2 years and semi-annual coupon payments and yield are same in all maturities. And coupons are different and maturities are different. Let we have considered the yield is 7.5% and it has declined to 5% and it has increased to 10%. There are 3 scenarios that are called the scenario analysis so I will give you some figure then you can find out.

Scenario analysis of different bonds:

Bond	Annual coupon rate	maturity	Bond price at 7.5%	Total returns		
				At 5%	At 7.5%	At 10%
A	10%	3years	106.61	7.23	7.5	7.7
B	7.5%	5years	100	8.59	7.5	6.48
C	10%	10years	117.37	10.85	7.5	4.47
D	5%	15years	77.71	13.8	7.5	2.01

Let bond your annual coupon rate then you have the maturity then you have the bond price at 7.5% then you have the total return. Already we have calculated this. Let at 5% at 757.5%, then you have the R 10%. Let bond A, annual coupon rate is 10%, maturity period is 3 years then the bond price at 7.5% is 106.61, total return whenever we calculate at 5% we got 7.23 and at 7.5% automatic it will be same at 10% it is 7.77.

For bond B, 7.5% that maturity period is 5 years we got the price is 100 it is 8.59% 8.59%, 7.5% then you have 6.46,6.48. Then bond C, it is 10% that let it is 10 years price become 117.37 it is 10.85 this is 7.5, 4.47. Bond D: 5%. Let 15 years is the 10 maturity price becomes 77.71 it is 13.8 it is 7.5 and it is 2.01. So in this is a scenario of the different bonds with the different coupon and different maturity but the yield we have kept is at 7 point bond price is calculated on the basis of 7.5%.

So whenever the interest rate will change at 5% and it has gone down to 5%, 10%, this is the total return what already we have calculated. So in these case what basically observations we got?

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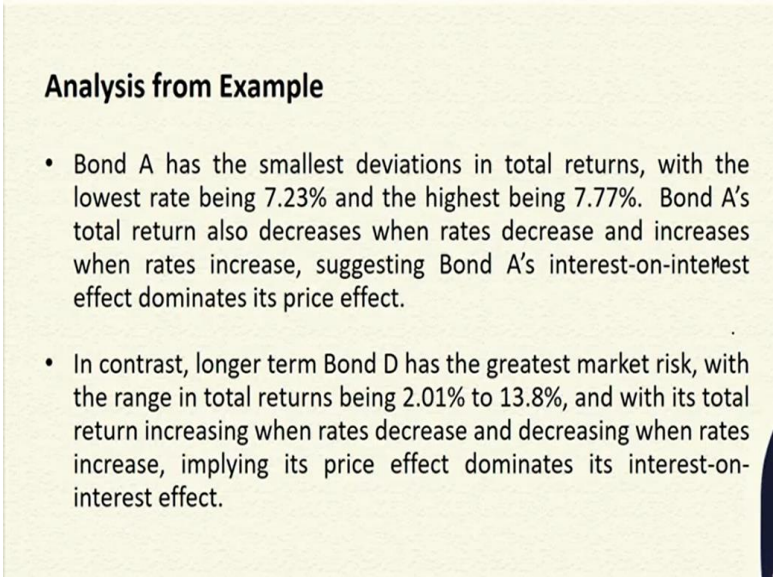

We got the bond A has the smallest deviation in total returns with the lowest rate being 7.23% and highest being 7.77%. So bond A's total return also decreases when rates decrease and increases when rates increase. What it gives? It basically suggests that the bonds interest on interest effect dominates the price effect. But in contrast, if you see the longer term bond with the last bond D, has the greatest market risk with the range of the total return varies from 2.01% to 13.8%.

And with its total return increasing when rates decrease and it is decreasing when rates increase which implies that the price effect dominates his interest on the interest effect. So here we have to understand, what this investor or one manager do? They go for a scenario building analysis, they try to find out that which one is less exposed to market risk or for which bond is beneficial for them for the investment. That is basically called the horizon analysis.

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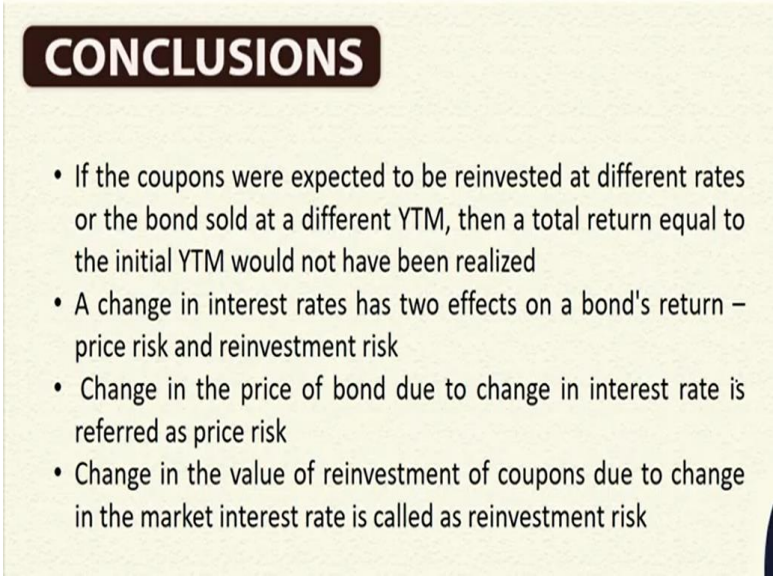

So what we have discussed today? If the coupons are expected to be invested at different rates, then the total return may not be equal to the YTM and the interest rate change has 2 effects one is price risk another investment risk. And the price risk is nothing but due to the interest rate how the price of the bonds is going to be changed. And the reinvestment effect is nothing but the due to the interest rate how the reinvestment return from the coupon is going to be change.

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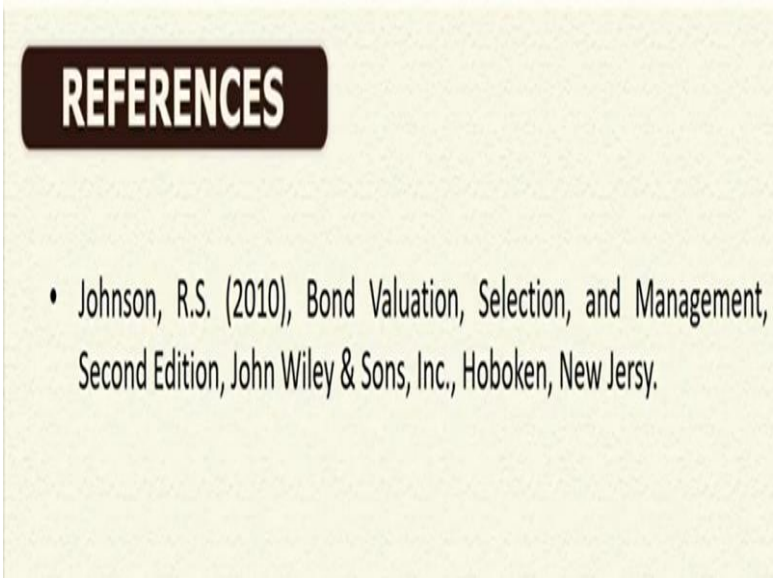

So, this is the reference what you can go through for the detailed discussion on this.

Thank you.