Management of Fixed Income Securities
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Module No # 02
Lecture No # 07
Bond Returns - II

Welcome back, we have discussed certain return concepts in the previous class particularly the bond return concepts. We discussed about the current yield, we discussed about the yield to maturity then we have the yield to call and we have discussed also yield to put. We will continue with our discussion.

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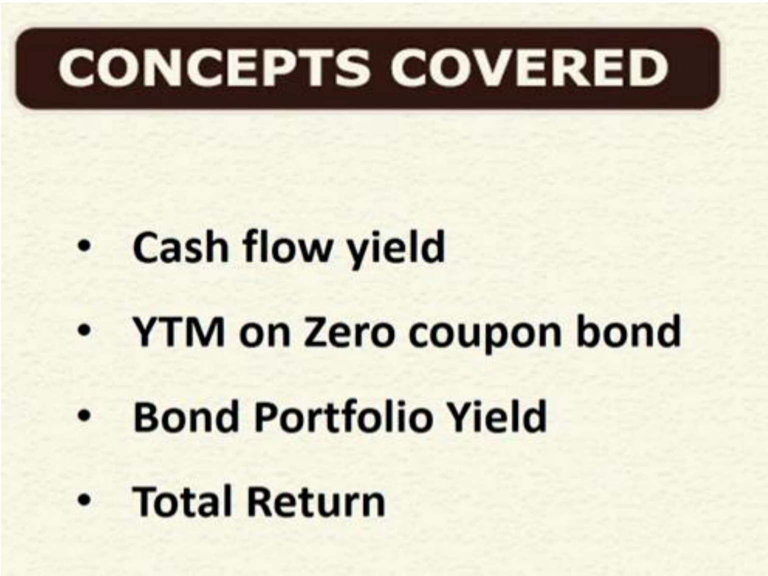

There are some other concepts what always we use in the bond analysis one is called the cash flow yield. And we have yield to maturity on the zero coupon bonds, we have the bond portfolio yield if you are holding a portfolio of the bonds then how the yield of that particular portfolio can be calculated. Then we have a concept of the total return. So these are the other concepts what we will be discussing today.

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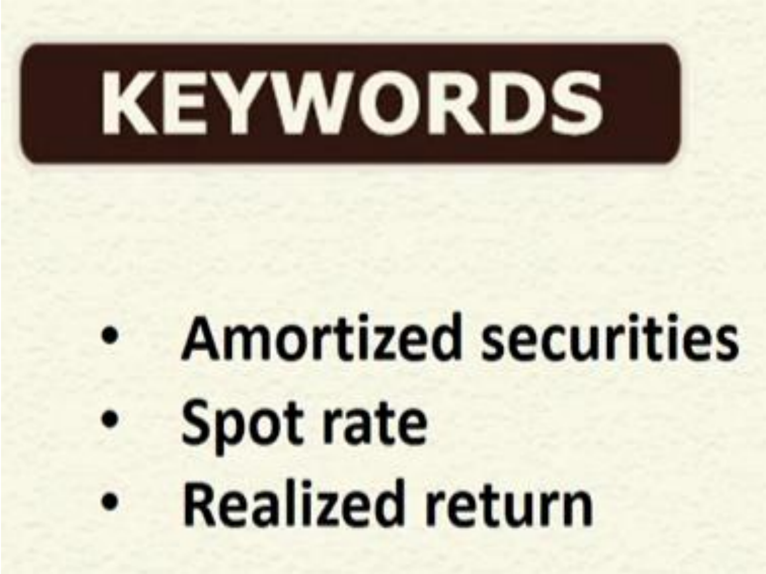

So there are certain keywords you will be learning in this particular session: we have the amortized securities, the concept of the spot rate and the concept of the realized return. So these are the different keywords what basically we will be learning in this particular discussion.

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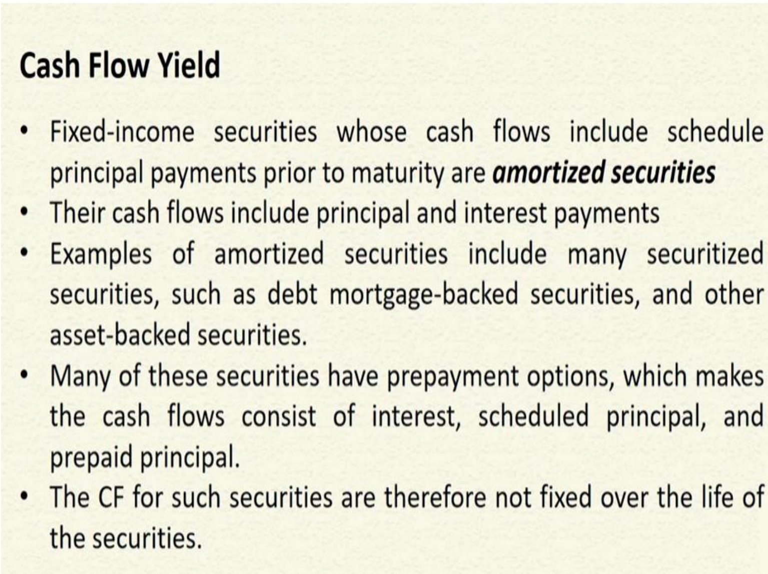

So let us see, what is cash flow yield? You know that there are different kind of fixed income securities always we see. So there are certain fixed income securities whose cash flow generally includes the scheduled principle payments prior to the maturity. Those kinds of securities are called the amortized securities. Generally, those cash flows are always include the principal and the interest payments. And which are those amortized securities? The amortized securities include generally the securitized securities like your mortgage backed securities or other assets backed securities. In the beginning of the session, I have discussed with you, there are certain kind of fixed income securities which are recently quite emerging these are called the mortgage backed securities and the asset backed securities. So, whenever, we calculate the yield of that particular kind of securities, generally, these yields are basically calculated with respect to the cash flows which are involved with respect to that kind of instruments.

And why generally we do that because many of the securities of the prepayment options. Because, let, one security is based upon the housing loans and may be this particular person who has taken the loan, they can go for the prepayments. So, whenever this kind of thing arises this basically makes the cash flow which consists of the interest schedule interest principal then we have some prepaid principal.

So therefore, the cash flows of this kind of securities are not fixed over the life of the securities. It is always constantly changing on the basis of nature of cash flow what we are receiving from the original instrument on which these particular securities are basically designed or constructed. So therefore, that particular concept is quite important with respect to the yield with respect to this mortgage backed securities cash flow. So that is called the cash flow yield.

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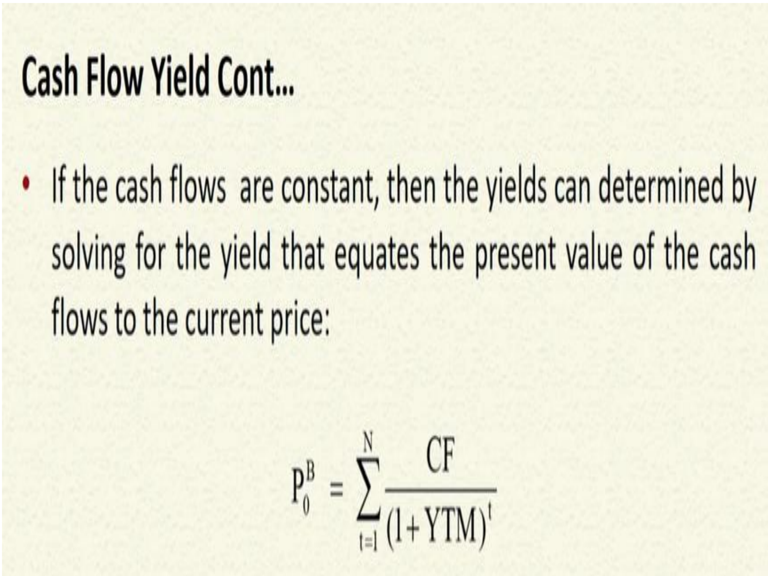

So let us see, what basically how it basically this works? If for example we assume that the cash flows are constant then the yield can be determined by solving for the yield that equates the present value of the cash flow to the current price that already you know. Your price of a particular security is nothing but it has to be discounted with respect to this yield to maturity.

Then if there is a series of cash flow then it is summation t = 1 to m CF by 1 + YTM to the power t.

$P^B_0 =$

$$\sum_{t=1}^{N} \frac{CF}{(1+YTM)^t}$$

So the cash flow may be your coupon as well as the principal in the generic bond but here in this case we have the different type of cash flow. The cash flow with respect to the interest payment, cash flow with respect to the principle repayments, then with prepayments, then you might have a cash flow with respect to the payment of the principal. So, all kinds of things are involved with respect to that.

So in this case, how basically the cash flow yield can be calculated? I will give a simplistic case then further we can go into the complexity of that. Because, the process of securitization, itself is a complex process. So we will discuss how basically this process works but today I will just certain brief idea about that.

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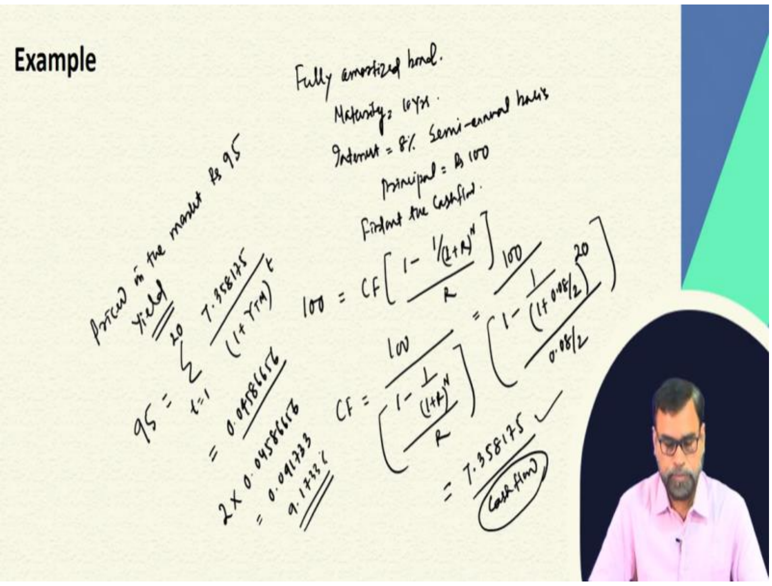

Let there is a fully amortized bond which is available then here let the maturity is 10 years. And interest against this particular kind of cash flow you are getting 8% and we are doing it on the semi-annual basis. Let, the principle is let 100 Rupees. So my question is that find out the cash flow. So if you get your cash flow from there you can calculate your yield. So now

100 Rupees which is basically your principle so if you go by our formula then this is nothing but the coupon into your 1 – generic bond wise but in case of we are using the cash flow this is 1 – 1 by 1 + R to the power N divided by R.

$$100 = CF\left[\frac{1-\frac{1}{(1+R)^N}}{R}\right]$$

$$CF = \frac{100}{\left[\frac{1-\frac{1}{(1+R)^N}}{R}\right]} = \frac{100}{\left[\frac{1-\frac{1}{(1+0.08/2)^{20}}}{0.08/2}\right]} = 7.358175$$

Hence, cash flow is 7.388175.

So here that means if you take cash flow this side then your cash flow will be 100 divided by this 1 – 1 by 1 + R to the power N divided by R. Now we have 100 which is available then 1 – 1 by 1 + R means it is 0.08 coupon is paid basically semi annual bases the interest is paid. When 0.08 by 2 to the power 20 maturity period is 10 means 6 months rate is number of period will be 20 divided by 0.08 by 2.

So if you calculate you will get this 7.358175 which are basically you cash flow because there is no coupon concept. So your cash flow is coming 7.358175. Let, you assume, this particular bond is priced in the market with a price of 95 in that particular point of time. Then now your job is to find out the yield from that particular bond. Particularly, we are talking about the semi-annual because the interest is paid semi-annually.

So in this case what basically we have to do we have to equate it with the price of the bond that is 95 your t = 1 to 20 then 7.358175 divided by your 1 + YTM to the power t. So if you solve it then you will find let 0.04586656.

Bond is priced in the market at Rs. 95.

$$\text{Price} = \sum_{t=1}^{N} \frac{CF}{(1+YTM)^t}$$

⇨ $95 = \sum_{t=1}^{20} \frac{7.358175}{(1+YTM)^t}$

⇨ YTM = 0.04586656

So, semi-annual yield of the bond is 0.0458665.

Hence, that is your semi-annual yield what you get it from that particular bond. Because, in the beginning case, for the generic bond, we know the coupon and accordingly, that coupon is directly used for calculation of the particular bond price. And then we try to equate it to the price of the bond at what interest rate which will equate the price of the bond with the present value of the cash flow. On that basis, we can call it YTM. But, in this case, what you are doing? First, we have to find out the cash flow involved with respect to that particular bond then after that you have to find out the yield. So if you make it annualized then you just multiply here 0.04586656 that will give you 0.091733, 9.1733%.

Annualised or Cash flow yield= 2 × 0.04586656 = 0.091733 or 9.1733%.

So, this is basically called the cash flow yield and that is generally related to the mortgage back securities. But, the complexity of the mortgage securities or asset backed securities and the pricing of that all these things will be discussing in the future sessions.

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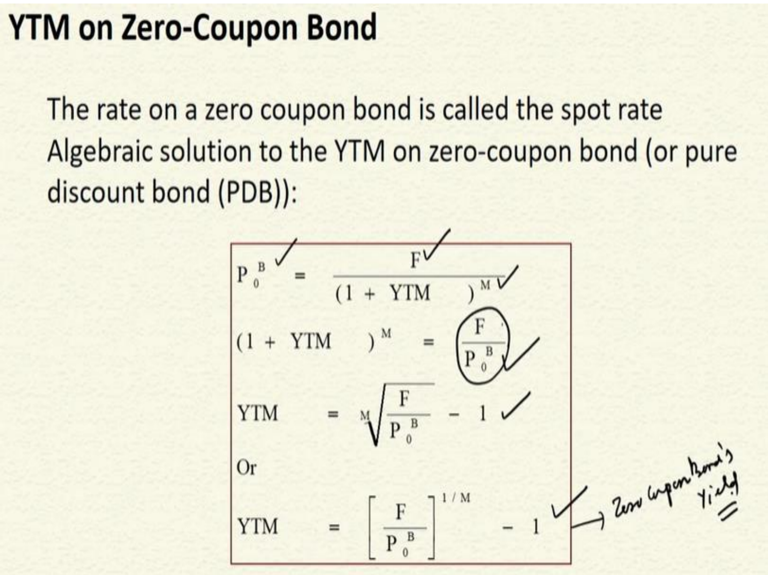

So, then we also discussed about the zero coupon bond; the valuation of the zero coupon bond that already we have discussed. And, one thing you remember, the rate on zero coupon bond generally called is spot rate. Then, the solution if you want to find out the YTM of the zero coupon bond; And zero coupon bonds also called pure discount bond that already we have discussed because there is no coupon involved in that. The bond is issued at a particular price at a discount and it will be redeemed at par so that is why it is called the pure discount bond. So, if you find out this particular yield of a zero coupon bond how basically we can go ahead? Already, you know this part, your price of a zero coupon bond is equal to your face value of the bond or the par value of the zero coupon bond which is the maturity value divided by 1 + YTM to the power M.

Then your 1 + YTM to the power M is basically F by P 0 B, I have just taken to that side and here it is coming this side this is F by P 0 B means is the price of the bond. Then you YTM will be automatically F by P 0 to the power 1 by M -1. So automatically F by P0 to the power 1 by M – 1 that will give you the yield to maturity of that particular zero coupon bond, it is the zero coupon bond shield. Right?

Algebraic solution to the YTM on ZCB will be:

$$P_0^B = \frac{F}{(1+YTM)^M}$$

⇨ $$(1+YTM)^M = \frac{F}{P_0^B}$$

⇨ $$YTM = \sqrt[m]{\frac{F}{P_0^B}} - 1$$

⇨ $$\text{YTM} = \left[\frac{F}{P_0^B}\right]^{1/M} - 1$$

Whatever you will get, that is known as zero coupon bond shield.

So, in this case,

Let, how basically; because there is no such coupon involved with respect to this in that case. So if you want to calculate the YTM on a zero coupon bond then how basically we can calculate this?

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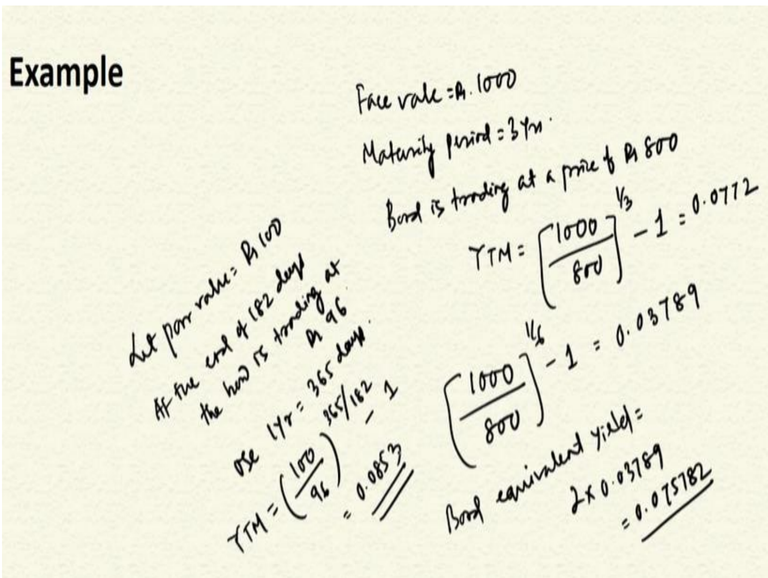

Let us take the maturity value of the face value of a zero coupon bond is 1000. Let the maturity period is 3 years, let this bond is trading the zero coupon bond is trading at a price of Rupees 800. That means that is the initial price at which the bond is basically purchased, then what is your YTM? Already, as per the formula, this is very simple 1000 divided by 800 to the power your 1 by 3 – 1 that will be 0.0772.

$$\text{YTM} = \left[\frac{F}{P_0^B}\right]^{1/M} - 1$$

$$\Rightarrow \text{YTM} = \left[\frac{100}{800}\right]^{1/3} - 1 = 0.0772$$

If you assuming semi-annual compound then what basically will happen? Then the simple annual or what we call it a bond equivalent rate, what would be? That will be what? That will be automatically your 1000 divided by 800 to the power 1 by 6 – 1 that will 0.03789 then your bond equivalent yield will be how much? 2 into this that is 0.075782.

If you assuming semi-annual compound then bound equivalent rate can be calculated as:

$$\text{YTM} = \left[\frac{F}{P_0^B}\right]^{1/M} - 1$$

$$\Rightarrow \text{YTM} = \left[\frac{1000}{800}\right]^{1/6} - 1 = 0.03789$$

Then annual bond equivalent rate will be 2×0.03789 = 0.075782.

But, there is also possibility the maturity period of that particular bond is less than 1 year. We have discussed that part before we have used this concept of day count conventions. Let, I will another examples. Let the par value of a zero coupon bond is 100 Rupees then at the end of 182 days the bond is trading at 96 Rupees. And use this 1 year = 365 days then what will be the YTM of that bond? Obviously it will be 100 divided by 96 to the power 365 by 182 – 1 that will give you 0.0853.

YTM of the bond will be = $\left[\frac{1000}{800}\right]^{365/182}$ -- 1 = 0.0853.

So, depending upon the takeout convention if the particular bond maturity period is less than 1 year on that basis we can calculate that what will be the yield of that particular bond.

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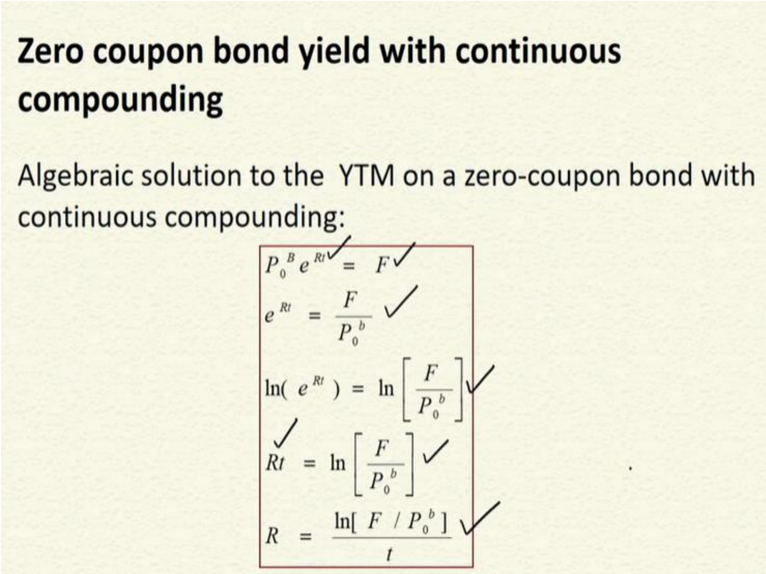

So, then let us see that sometime also the zero coupon bonds are continuously compounded. So, what is the solution? Algebraic solution to the YTM on a zero coupon bond with continuous compounding, this part already you know.

Your F is equal to this whenever we talk about the final value of that particular bond that generally we call it that then P 0 B e to the power Rt = F. Then e to the power Rt because our job is to find out R then e to the power Rt = F by P 0 then you take the log in both sides you take ln both sides.

Then ln e to the power Rt = ln F by P 0 now ln e to the power Rt = Rt = ln F by P 0. Then Rt if you find then R is equal to what? R = your ln F by P 0 b divided by t. Directly, you can calculate this yield to maturity of that particular 0 coupon bond.

Algebraic solution to the YTM on a zero coupon bond with continuous compounding:

$we\ know\ that,\ P_0^b\ e^{Rt}$ = F

- ⇨ $e^{Rt} = \frac{F}{P_0^b}$
- ⇨ $Rt = \ln\left[\frac{F}{P_0^b}\right]$
- ⇨ $R = \ln\left[\frac{F}{P_0^b}\right] \div t$

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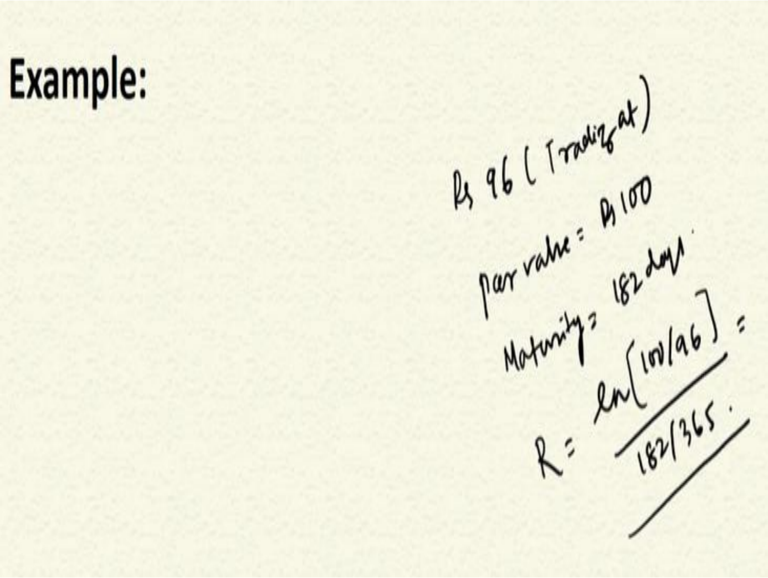

So, then in our example whatever we have taken let the bond is trading at the price of 96. You par value is already we have taken that is 100 Rupees, then let maturity we have taken that is 182 days. Then what is your R? R = your ln 100 by 96 divided by 182 by 365 because we are assuming 1 year = 365 days.

$$R = \frac{ln\left[\frac{100}{96}\right]}{\frac{182}{365}}$$

On that basis you can calculate your R you can calculate the R from this so this is what basically you can use or you can calculate whenever we are assuming that this particular zero coupon bond is continuously compounded.

So, depending upon this what we call it the compounding frequency, the yield also can be changed. So the concept of yield can be calculated in this way so that is about the yield from the zero coupon bond.

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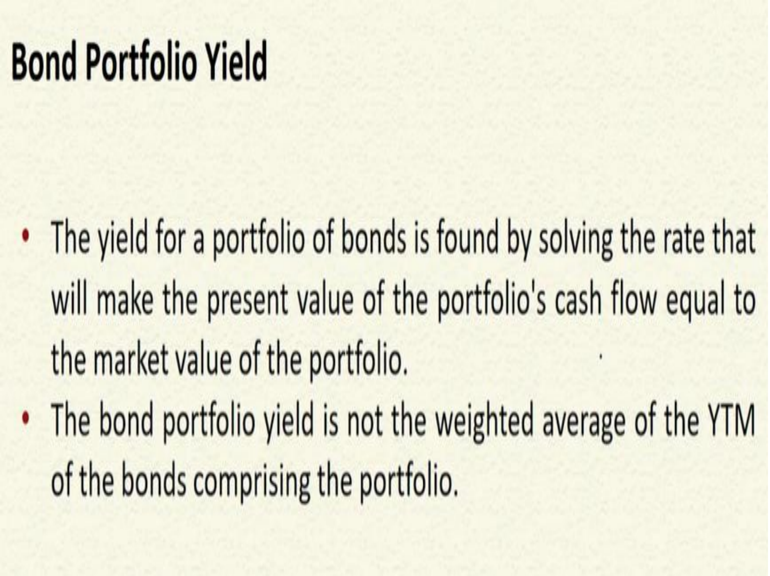

Then let us see about the portfolio yield. Let, somebody is holding a portfolio bond instead of holding a single bond somebody is holding a portfolio of the bonds. Then how the portfolio of the bonds yield can be calculated? Remember, the bond portfolio yield is not the weighted average of the yield to the maturity of the bonds comprising this portfolio what generally people do? People calculate the yield of each bond then accordingly they calculate the weights that how much fund they have invested against that particular bond. And giving the weights, they can calculate the weighted average but that is not basically the yield of we can say that bond portfolio yield. What is the bond portfolio yield? Then bond portfolio yield basically is a how basically we can solve it?

You can solve it by making the present value of the portfolio cash flow equal to the market value of the portfolio. So you have to find out what is the market value of the particular portfolio. And after that you can see that what is the cash flow you are going to receive against that particular portfolio whatever we have? Then accordingly the bond portfolio yield can be calculated.

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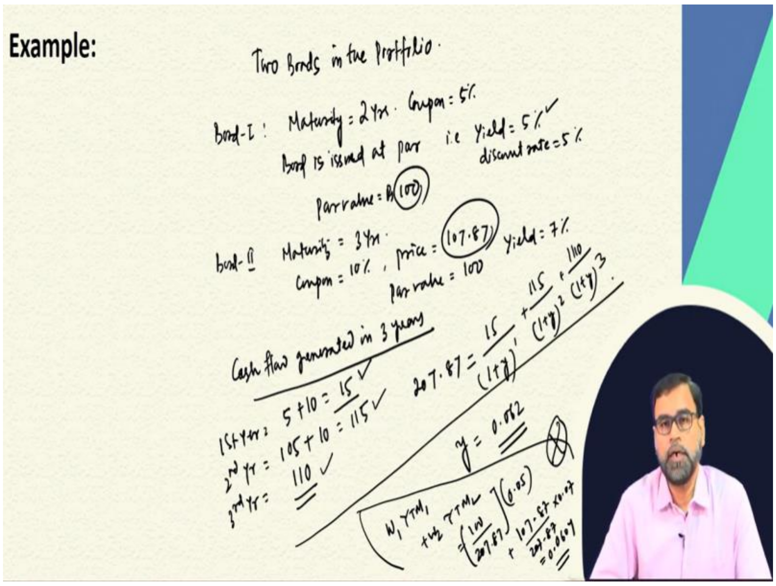

So let us see what basically the concept is let there is a portfolio which has two bonds in the portfolio. So what basically I told you how generally we do this? So let bond 1, I will give you certain features let the maturity is 2 years, coupon is 5% and this bond is issued at par. What does it mean? If the bond is issued at par means the coupon is equal to with the discount rate. So which means the yield is also 5% or the discount rate is 5%. Let, the par value is 100 Rupees.

Let, there is a bond 2, here the maturity is 3 years then your coupon is let 10%, let it is a premium bond, the price is 107.87 then your par value = 100. And obviously your yield is taken as 7% so what is the cash flow will be generated in 3 years from the bond. Cash flow generated in 3 years, tell me? Obviously, in the first bond, how much we are getting? 5% that means 5 first year from the bond 1. From the second bond, it is how much? It is, the coupon is 10%, then it is 10. That is 15 Rupees in the end of the first year you are getting this. In the second year how much you are getting first bond is matured then you will get 105 and from here you getting 10 that is 115. And in the third year, this bond 1 is matured; this bond is already matured then from this bond 2, how much you got 10 Rupees the coupon, 100 of the par value that is the 110.

So, the cash flow from the two bonds in the first year is: 15, second your: 115, third year: 110. Then, what is the market value of the portfolio in that maturity? That is 100 + your 107.87 that will be your 207.87. So this 207.87 = 15 divided by 1 + y to the power 1 + you 115 1 + y to the power 2 + 110 divided by 1 + y to the power 3. If you solve this y, then you will get your yield from that particular folio.

Portfolio yield will be calculated as:

$$207.87 = \frac{15}{(1+y)^1} + \frac{115}{(1+y)^2} + \frac{110}{(1+y)^3}$$

After solving this above equation, we get y= 0.062

So, that is basically called the portfolio. But, If you would have gone for weight then what basically you would have got? The W1 into YTM 1 + W2 into YTM 2 that means your 100 divided by 207.87 into 0.05 + your 107.87 divided by 207.87 into 0.07 it would have been given different result 0604 which is not the correct approach.

The portfolio yield can also be calculated frlm the weighted average approach:

$$Y = W_1.YTM_1 + W_2.YTM_2$$

$$= 0.05\left(\frac{100}{207.87}\right) + 0.07\left(\frac{107.87}{207.87}\right)$$

$$=0.064$$

So this approach is basically not the correct approach, so, we have to basically go by that way. So, this is the way basically the bond portfolio yield can be calculated.

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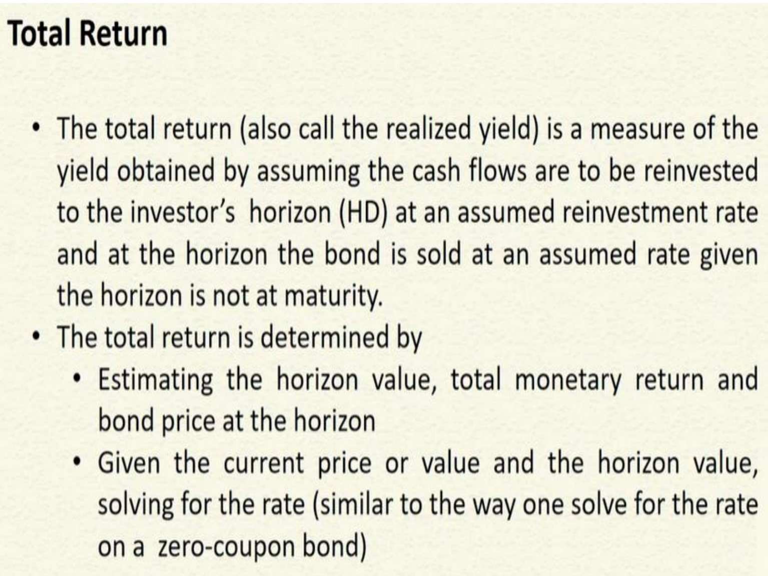

So, then the next one we have a concept of the total return. let me tell you, what the total return is?. In the simplistic way, let you are holding a bond, bond's maturity period is let a particular year. But, you want to sell your bond before that. If you want to sell your bond before that, how much extra return you can get it from there. So that is why it is called the realized yield, it is actual yield what you are getting.

It is basically a measure of the yield which is obtained by assuming the cash flow to be reinvested that means the coupon whatever you are getting receiving in between, that will be reinvested to the investor's horizon, that means up to what period you are holding the bond. And, at the horizon of the bond, you are selling that bond and you are not holding the bond up to that maturity.

So, that is why, the total return or realized return what you are getting from that particular bond. First of all, we have to calculate or estimate the horizon value; total monetary return and the bond price at the horizon. So, given the current price of value and the horizon value, solving for the rate, we can calculate the total rate. Let us, it will be clearer for you whenever I will take example. I will explain it through the example.

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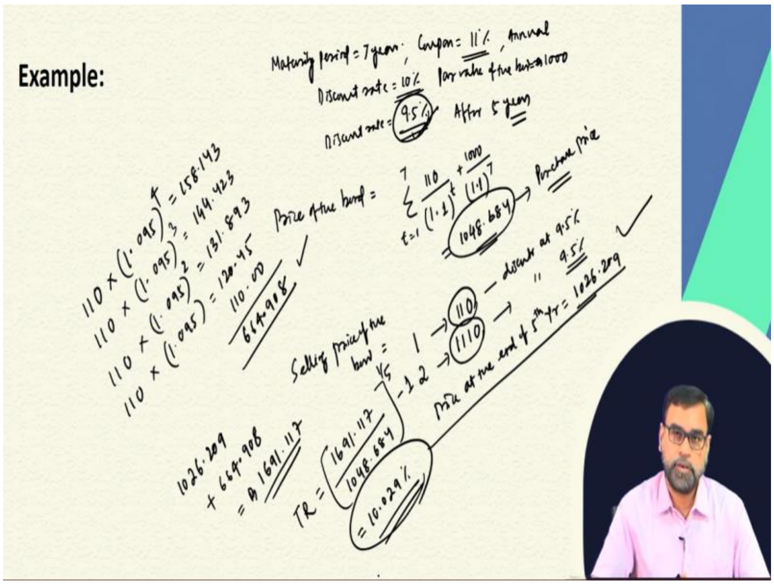

Let, as in investor, you buy by a bond: the maturity period is 7 years, coupon is let 11%. Coupon payment is annual, right! and the discount rate or market interest rate is let 10%, then the par value of the bond is 1000 Rupees. Assume that, immediately after you purchase the bond, the market interest rate has gone down; it has decreased to 9.5%. You have already bought the bond. The discount rate has gone down to 9.5% and it remains constant throughout the maturity until the investors basically sell the bond.

Let the investors need the money or is going to sell the bond, after 5 years you will sell the bond. And he is basically going to reinvest that coupon payment up to that particular 5 years on the basis of the 9.5%. So what is the price of the bond? At what price he has bought the bond whenever he has started investing? In that case, your coupon was 11% and market was discount rate was 10%.

So if you go by our formula, that t = 1 to 7 the coupon is how much 110 each year, 1.1 discount rate was 10% to the power t + your 1000 divided by 1.1 to the power how much? 7 years. So if you calculate, it will give a value of 1048.864.

Price of bond = $\sum_{t=1}^{7} \frac{100}{(1.1)^1} + \frac{1000}{(1.1)^7}$ = 1048.684

Hence, the price of the bond is 1048.684.

This is your purchase price of the bond. This is the market price of the bond at which you have basically bought the bond that time the interest rate was 10%.

So now you have sold the bond for 5 years. So, what is the selling price of the bond? How will you calculate the selling price of the bond, still another 2 years are remaining, coupon is 110 you are getting? So, the remaining years, in the first year, you getting a coupon of 110 and you have to discount it with respect to 9.5% and this discounted at 9.5%. Second year you will be getting you are getting back your 1000 + 110 that means 1110 you have also discount it at a rate of 9.5%.

So now if you discount it if you calculate the present value of cash flow of 110 and 1110 then the price at the end of the 5 year will be you get it is 1026.209. So, this is the price what you got then after that you got your coupons, the coupon you got after end of the first year, how much you got- 110 Rupees. That you have reinvested at a rate of 9.5% in the market then that is 110 into for how many years for 4 years?

For 4 years you have interested it, 1.095 to the power 4, you got let 158.143. Next year, you got coupon in the end of the second year you got a coupon, that you have interested it for 3 years to the power 3 that you got 144.423. Next year, you got another coupon that you have reinvested it for 2 years that you got 131.893. And at the end of 2 years you got another one 095 that will give your 120.45 and in the fifth year coupon you got 110 Rupees.

So if you add it up you get your coupon as well as you have the coupon is reinvested at the rate of 9.5% which is prevailed in the market. So in total you got your how much it is basically your 664.908.

The value of the coupon reinvested can be calculated as:

$$=110(1.095)^4+110(1.095)^3+110(1.095)^2+110(1.095)^1+110$$

$$= 158.14 + 144.423 + 131.89 + 120.45 + 110$$

$$= 664.908$$

So, now you have sold the bond at a price of this from the coupon and the reinvestment of the coupon you got this much. Then the total money what you got that is 1026.209 plus your 664.908.

Total money received (Horizon value) = 664.908 + 1026.209 = 1691.114.

So that will give you 1691.117, the total money what you got from this particular bond investment what you have sold after 5 years. And what price you have purchased? You have purchased it as 1048.684. So, now, if you calculate the total return, your total return will be your horizon value. How much you got? That is 1691.117 divided by your 1048.684 it is the ending value, this is the beginning value to the par your 1 by horizon period that is basically 5 years 5 – 1.

$$\text{Total return} = \left[\frac{1691.117}{1048.684}\right]^{1/5} - 1$$

$$=10.029\%$$

That basically will give you 10.029% so that means if even if you are holding the bond of the maturity you would have got the yield what basically is mentioned there and the par will be that 1000 Rupees would have got. But if you have sold the bond at the end of the 5 years then how much total return you have generated that is basically called the realized return, exactly how much return you have realized from that particular investment. So that is called the realized return.

So that thing is basically happening and that return will vary on the basis of the change in the interest rate in the market and also your horizon period when you have sold that particular bond.

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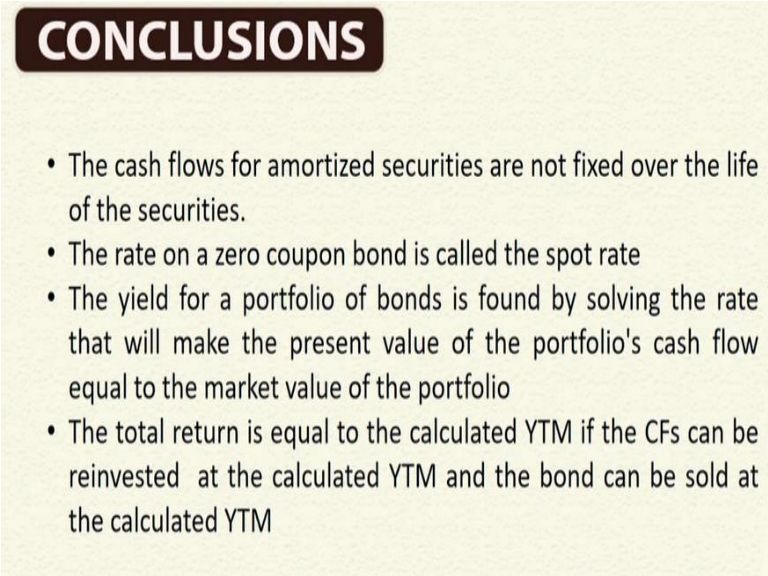

So, what we have discussed today? The cash flow for amortized securities are not fixed over the life of the securities. The rate on a zero coupon bond generally called this spot rate and the portfolio return or the portfolio yield generally is calculated by solving the rate that will make the present value of portfolio cash flow equal to the market value of the portfolio. And the total return is always equal to the calculated YTM if the cash flows can be reinvested or the calculated YTM and bond can be sold at the calculated YTM. The total return always calculated on the basis of 3 components: one is your selling price at the horizon rate and the coupons whatever you have received and the reinvestment if the coupons or the return from the reinvestments of the coupons. So these are the three things, three components have to be added whenever the realised return is calculated.

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This is the reference what basically you can follow. Thank you.