**Management of Fixed Income Securities**
**Prof. Jitendra Mahakud**
**Department of Humanities and Social Sciences**
**Indian Institute of Technology, Kharagpur**

**Module No # 02**
**Lecture No # 06**
**Bond Returns-I**

Welcome back, in the previous class we discussed about the different approaches or different methods which are used for the bond evaluation. And today, we will be discussing about certain concepts related to the bond returns. So, how the different types of yields are associated with the bond that is basically the basic objective of the today's discussion.

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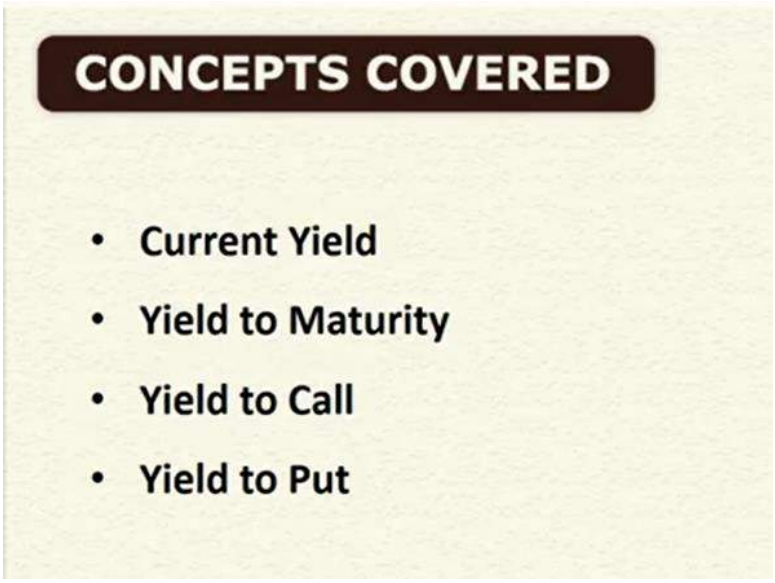


So here if you see that there are different concepts what we are going to discuss today one is your current yield. Then you have yield to maturity then you have yield to call then you have yield to put. Here the yield to call and yield to put means if any bond has any kind of call features already you must have the idea that there are certain bonds which are the call features. After certain point of time the bond can be called back by the issuer and generally the issuer always can go back for calling the bond whenever there is a low interest rate scenario in the market.

So that is why there is some risk involved in that particular context. And there is a call price already we know that is related to that particular bond and accordingly the yield or the return from that particular bond can be calculated. Then same thing also the put feature: it gives the kind of right to the bond investor to sell back that particular bond to the issuer. And already

there is a price involved in that in the beginning in the indenture provision that is mentioned and accordingly the yield of that particular bond also can be calculated. So these are the different concepts what we will be going to discuss today.

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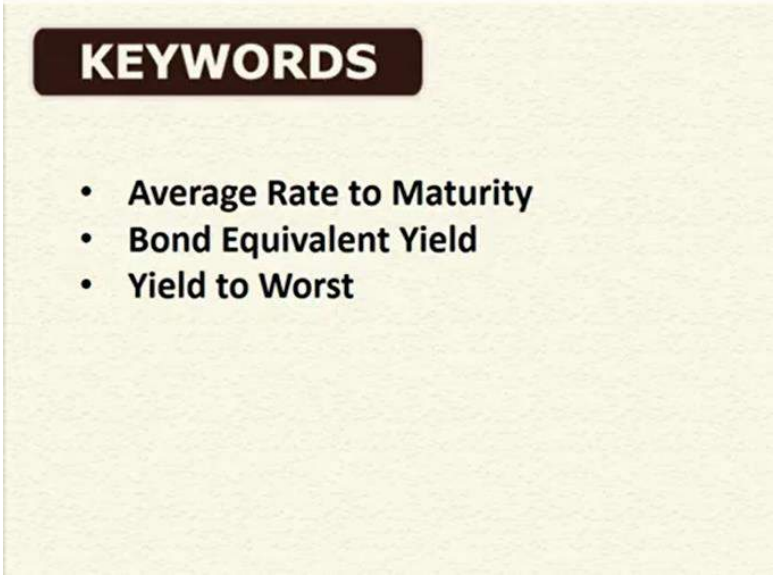


So then if you see that after this there are certain keywords or certain concepts you will learn in this discussion process. That is your average days to maturity which is used to calculate the yield to maturity of the bond that is the approximation formula that we will try to derive then the concept of the bond equivalent yield, then yield to worst. So these are the some of the things what you will earn after the discussion of this today's session.

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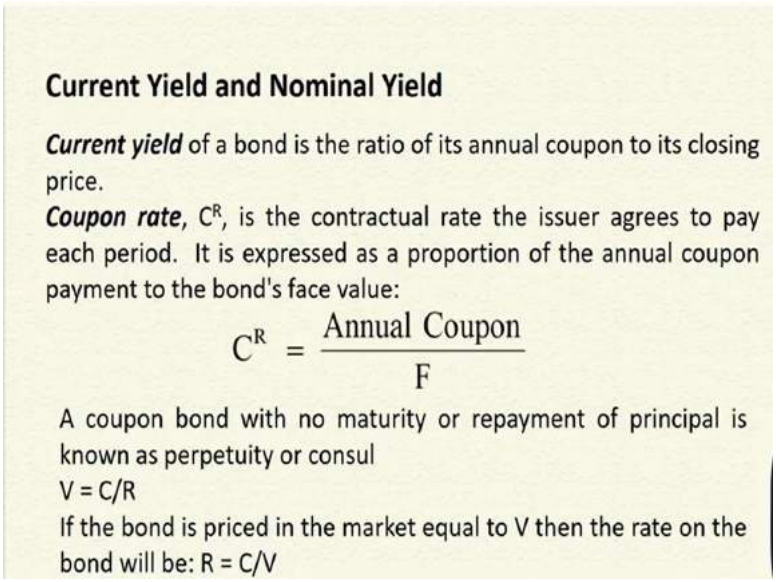


$$C^R = \frac{\text{Annual Coupon}}{F}$$

So let us see that what is current yield? The other name of that thing is also nominal yield. So you know whenever you talk about the bond the basic features of the bond if you look at. The bond has a par value, it has a coupon rate, it has a term to maturity and it has the discount rate and the discount rate basically depends upon the market interest rate. The price of the bond largely depends upon the fluctuations of the interest rates in the market. So here the question is that what is the basic difference between the coupon rate and the yield what we are receiving from this particular coupon? Are they different?

One thing you remember, although, we are little bit using this word in a similar way but the coupon rate whenever you talk about the coupon rate is calculated on the basis of the face value of the bond or the par value of the bond. For example, if you are getting 90 Rupees per annum with a par value of 1000 Rupees then we can say that the coupon rate is 9%. 90 divided by 1000.

So coupon rate is basically calculated with reference to the face value or the par value of the particular bond. But whenever, we talk about the current yield or yield from the coupon that is always calculated with respect to the market price of the bond. The coupon rate which is fixed in the beginning whenever the bond is issued that time, that thing is basically linked to the face value of the bond that is the basic difference between these 2 that actually you have to keep in the mind. So therefore, the current yield is the ratio of its annual coupon to its closing price. And whenever you talk about the coupon rate the coupon rate is the proportion of annual coupon payment to the bond face value already what example I have given to you.

If there is a bond with no maturity or the repayment of the principal then that is basically called the perpetuity or the consol. So in that case how basically we calculate that the value of that particular bond is nothing but it is C by R, R is the discount rate, C is the cash flow. So if the bond which is priced in the market equal to this value of that particular bond, the rate on that particular bond will be it is just R = C by V.

So in general if you look at or what is the basic objective of differencing between the yield and the coupon rate? One is with respect to the market value, other one is the with respect to the face value that is the basic difference what basically you can keep in mind.

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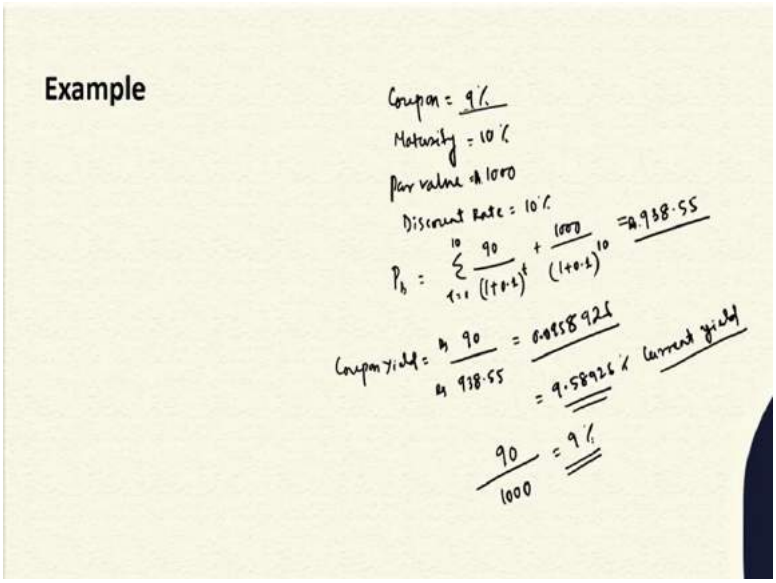


$$P_b = \sum_{t=1}^{10} \frac{90}{(1+0.1)^t} + \frac{1000}{(1+0.1)^{10}} = 938.55$$

$$\text{Coupon Yield} = \frac{\text{Rs. } 90}{\text{Rs. } 938.55} = 0.0958926$$

Let us take one example, how basically it is different. For example if you if you see that with a particular bond we can say that let the coupon is 9% then maturity is 10%, the par value is 1000 Rupees. And we have the discount rate that is basically your 10%. So in that case already we have taken this example in the previous sessions also what we have seen that if you find the price of the bond that you have to discount your T = 1 to 10. Your coupon which is 90 divided by 1+R to the power T that means 0.1 to the power T + your 1000 divided by 1 + 0.1 to the power 10. So in this case what we have seen that already we have calculated this that will give you 938.55 that means the price of the bond in the market at that particular point of time is 938.55. So now here the coupon rate is 9% but what is the coupon yield or current yield from the bond?

The yield is basically what your 90 divided by 938.55 because 90 Rupees is the annual coupon what you are getting so 90 divided by 938.55. Let they will be getting. 0.095898926. So basically you will get 9.58926%, that is basically your current yield but coupon already I told you that is 90 divided by your 1000 that is your 9%.

So that is the basic difference between current yield and the coupon rate that is what basically always keep in the mind because one is related to the face value other one is related to the market price.

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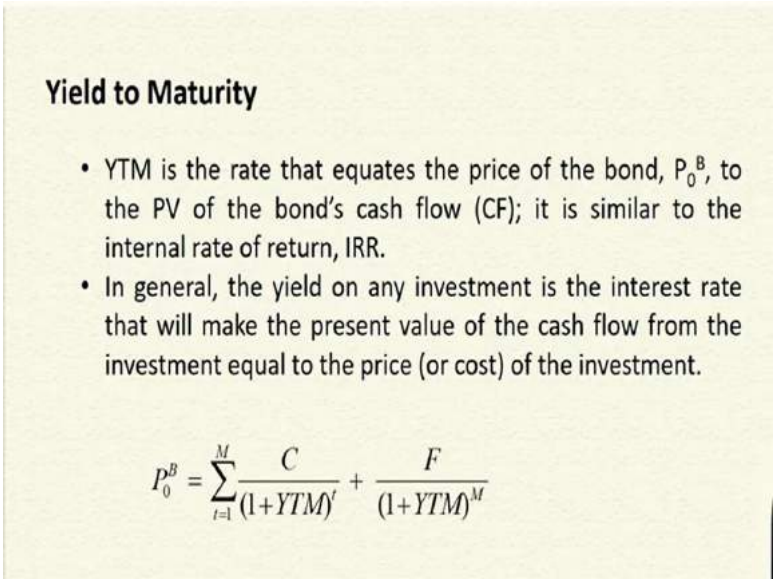


$$P_0^B = \sum_{t=1}^{M} \frac{C}{(1+YTM)^t} + \frac{F}{(1+YTM)^M}$$

So then the next is basically the most important concept in the bond basically we always use that is called the yield to maturity. Always we use this word, yield to maturity. What is basically yield to maturity? Yield to maturity or YTM in short what we call, it is a particular rate which basically equates the price of the bond to the present value of the bonds cash flow. The bonds cash flows are what?

The coupon payments what; you are getting and the first value of the bond in the end at the time of maturity. So which particular interest rate which make that particular price of the bond equal to the present value of the cash flow. That is basically called theYTM generally, it is similar to the internal rate of return what already you might have heard or you might have studied in the financial management or the corporate finance kind of papers.

So therefore, the yield on any investment basically is the interest rate which will make the present value of the cash flow from the investment equal to the price of the investment or the cost of the investment. So that is basically what we call it the yield and that is called the yield to maturity. So if you recall whenever we are going for this kind of for discussion.

We go back to our previous example or the equation what we have used for evaluation of the price of the bond that is your summation T = 1 to M C by 1 + YTM to the power T+F by 1+YTM to the power M. M is the maturity period, F is the first value, C is the cash flow or the couponand YTM is the yield to maturity. So now, if you know the price data, you know the coupon, you know the face value, you know the maturity period then you can calculate YTM.

From this particular formula, it will be easy for you to calculate this YTM if the market price is known to you, at what price the bond is traded in the market that is known to you and coupon rate is known to you and the face value is known to you then automatically your YTM can be calculated. So let us see that how this YPM basically it will be calculated and what is the concept what we are trying to derive through this YTM.

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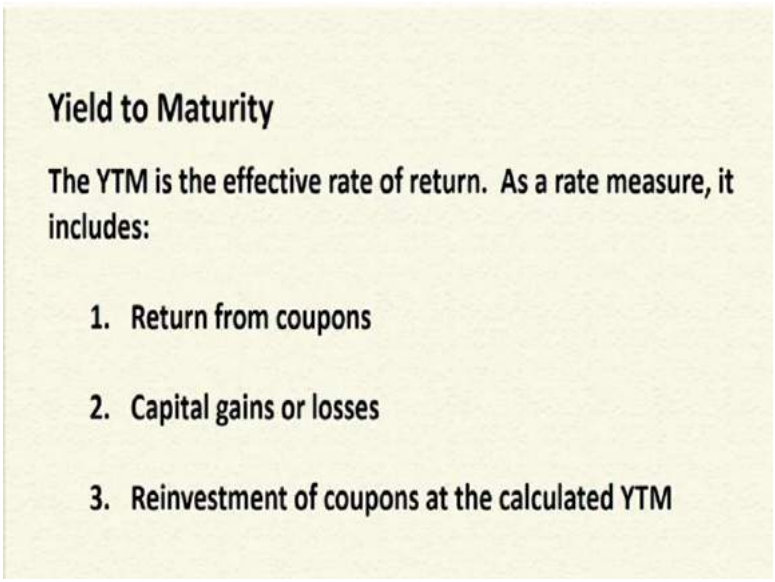


So YTM is basically a kind of effective rate of return, then what this YTM includes? Yield to maturity basically includes the return what we get it from the coupon and the capital gains or losses from that particular investment what we are making in that particular bond and the reinvestment of the coupons at the calculated YTM. So the particular kind of interest rate will combine all those aspects or the return or the output what we are going to receive from this particular investment.

To return from the coupon capital gain or losses and the reinvestment of the coupons are the calculated YTM. So these are the 3 things basically which is included for the measurement of the YTM that actually you should always keep in the mind.

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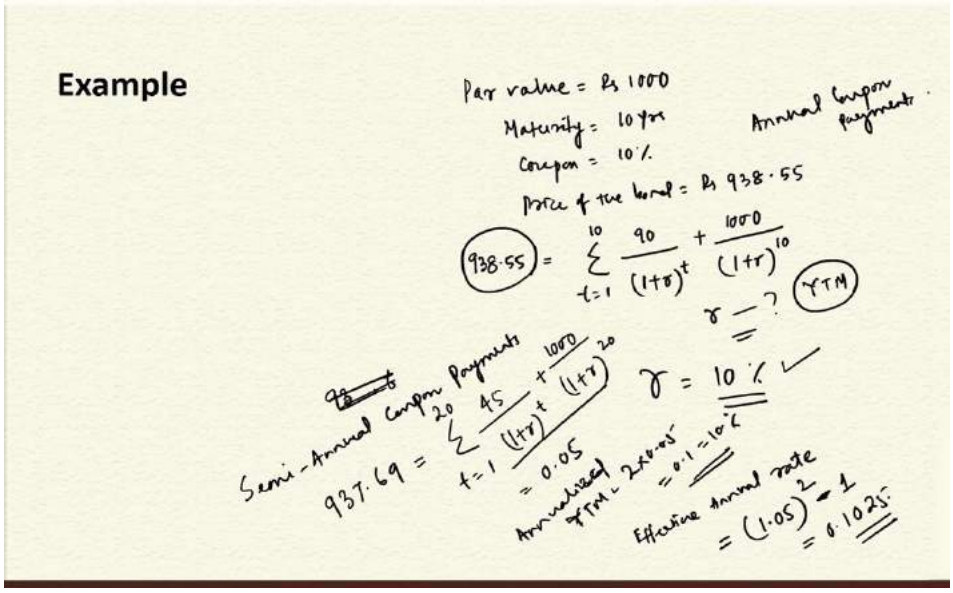


$$938.55 = \sum_{t=1}^{10} \frac{90}{(1+r)^t} + \frac{1000}{(1+r)^{10}}$$

$$937.69 = \sum_{t=1}^{20} \frac{45}{(1+r)^t} + \frac{1000}{(1+r)^{20}}$$

Semi-annual Coupon payments

Let us take one example we go ahead with the same data what basically we have used. Let the par value of the bond or the face value of the bond that is Rupees 1000 right then you have the maturity that is let 10 years then coupon is 10%.Then I have already told you the price of the bond that means the bond is traded in the market at a price of 938.55. Now let us put that thing in our bond price equation that 938.55 that means here the YTM is not given to you. But the bond price data is available to you, at what price the bond is traded in the market? So in this case if you see that then automatically if you go by our equation T = 1 to 10, 90 divided by 1+R to the power T+1000 divided by 1+R to the power 10. So then if you solve this equation whatever value of the R we calculate that is basically your YTM and that R value will you will be getting 10%.

So at the 10% rate whenever you are going for the valuation of the bond you got the price of 938.55. Now what has happened you get the 10% is the YTM which is basically giving you this particular present value of that particular bond using the other kind of data or the inputs. Same way, you can go for also the semi annual. This is basically the annual coupons the coupon is paid basically annually, annual coupon payments if semi-annual coupon payments let the price of the bond we have 937.69. Then that will be your equal to your other things you mentioned then automatically. It will become 20 it will be 45 divided by 1+R to the

power T+1000 divided by 1+R to the power 20. So if you solve it then you can get your semi annual price basis you can find out your yield you will get 0.05.

Then if you make it annualized then it will be annualized YTM that will be 2 into 0.05 that is your 0.1 that means 10%. But if you go for the effective annual rate if you recall what is the effective annual rate? The effective annual rate will be 1.05 to the power 2 minus 1 that will be basically giving you little bit higher rate that is 0.1025, 10.25%. So that effective rate concept already we have discussed in the previous session.

So if you go by this way, whatever way basically we calculate this, what we can call it the rate at which the price of the bond is equal to the present value of the cash flows that is basically called the yield to maturity.

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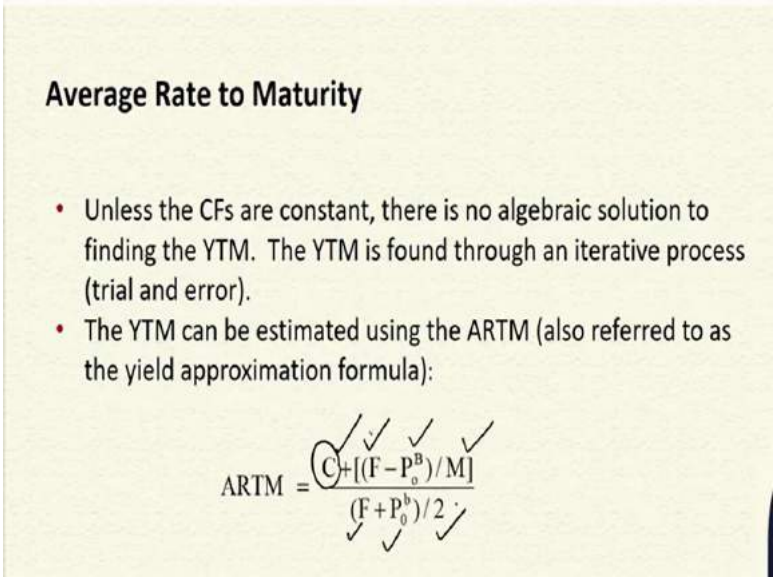


$$\text{ARTM} = \frac{C + [(F - P_0^B)/M]}{(F - P_0^b)/2}$$

Then how basically this is a trial error basis we calculate so to avoid that trial error calculations some approximation way basically you can calculate your average raise to maturity. Or we can say that the approximate yield how much you can receive it. Unless the cash flows are constant, there is no algebraic solution to finding the YTM. So the YTM is found basically through one iterative process like trial and error method.

But the ARTM, the average rate to maturity formula can give you some approximate yield to maturity of that particular bond. And what is this particular formula, ARTM formula? We

generally take the coupons the annual coupons what we got that is your what we call it the 90 Rupees in our case. Face value of the bond that is the price of the bond that is the term to maturity this is again same face value the priceof the bond divided by 2.

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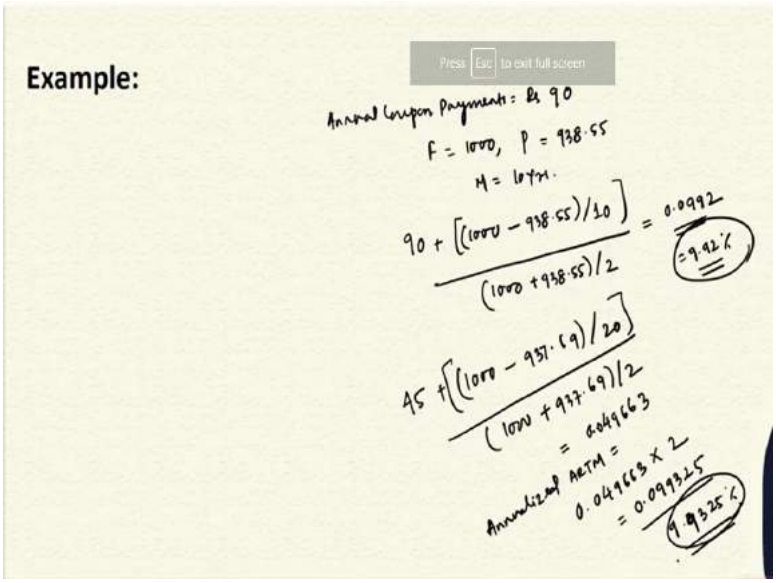


$$\frac{90 + [(1000 - 938.55)/10]}{(1000 + 938.55)/2}$$

$$\frac{45+[(1000-937.69)/20]}{(1000+937.69)/2}$$

$$\text{Annualized ARTM} = 0.049663 * 2 = 0.099325$$

So now in our example if you see let us check it using this ARTM what is the coupon payment annual you are getting the annual coupon payments is basically you are getting 90. F = 1000 your price of the bond is equal to your 938.55 and your M or term to maturity is 10 years. So if you put that particular formula here that is 90 which is basically our coupon then we have + your 1000 - your 938.55. So that is basically your difference between the face value and this market price divided by M. M means it is the 10. Then that will be divided by your 1000 + your 938.55 by 2. Approximately if you calculate this you will be getting 0.0992 that means 9.92 %. In our case generally we got how much 10%. So 9.92 % is more or less close to that. So use your ARTM formula, find out the approximate yield and after that you can use some trial error basis exact YTM you can calculate from that particular figure.

So if you go by your semi annual payment basis also you can calculate semi-annual coupon payment is 45 +1000-937.69 by 20. If you recall then divide by your 1000 + 937.69 divided by 2 that will give you a value of 0.049663. So if you go for annualisation of that, the annualized ARTM will give you 0.049663 into 2 that will give you 0.099325 which is 9.9325% which is more or less close to this 10% what basically we are calculating from our formula. So this is the way the ARTM formula is used to find out the approximate yield to maturity of that particular bond. So this is basically one of the way we can calculate the YT.

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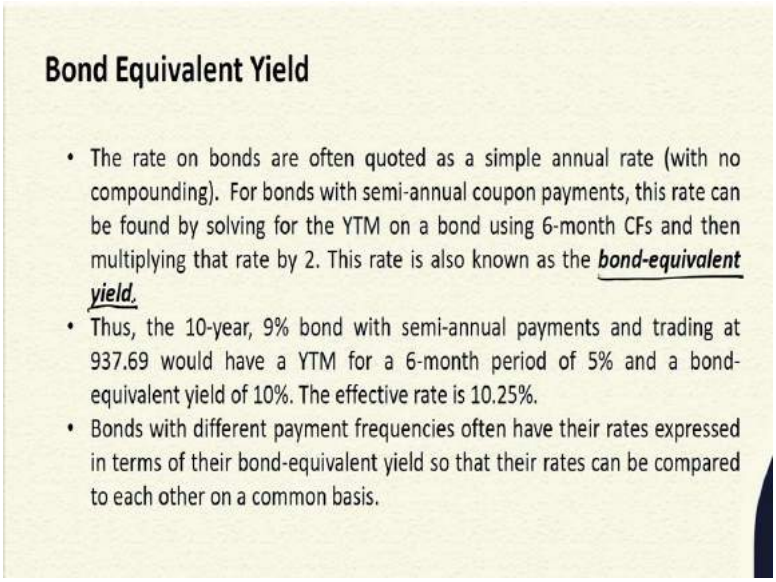


Then we come to the concept of bond equivalent yield. So, generally, the rates on the bond are often quoted as a simple annual rate with no compounding. For bonds with semi-annual coupon payments, the rate can be found by solving for the YTM and a bond using the 6 months cash flowsthen you can multiply that particular rate by 2 just now whatever we have done so that is called generally the bond equivalent yield.

So just now, whenever, we multiply that 2, because the coupons are paired semi-annually. So that concept basically is called the bond equivalent yield. If the 10 year 9% bond with semi annual coupon payments and trading at 937.69, it would have a YTM for a 6month period of 5% and bond equivalent yield of 10% but the effective rate is 10.25%.That already we have calculated that is 1.05 square -1 that already we have seen that is 10.25%.

So the bonds with different payment frequencies often have their rates expressed in terms of the bond equivalent yield, by that their rates can be compared with each other on a common basis that is the use of that bond equivalent yield. So first of all, we calculate this bond

equivalent yield even if the coupons are paid in the different frequency to compare it and on a common basis. By that we can choose that how much yield we are generating from the different type of the bonds so that is called the cons bond equivalent yield

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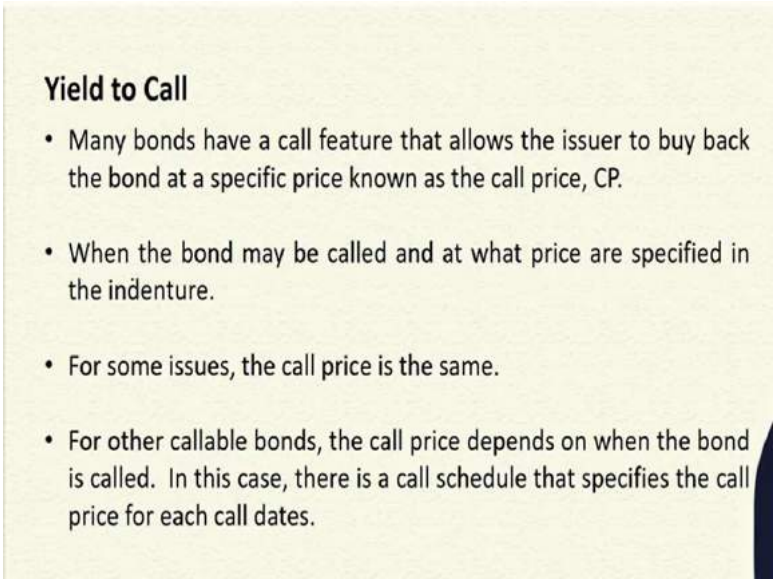


Then we have yield to call already I told you what do you mean by this callable bonds, many bonds generally have the call features .They have a call option if there is a call option available then it will allow the issuer to buy back that particular bond at a specific price which is known as the call price. And that call price is mentioned in the beginning whenever the bond was issued. That is why, when the bond may be called and at what price it will be called that will be specified inthe indenture or the agreement from the beginning part.

So for some issues, the call price is the same but generally the call price may also vary. For other callable bonds, the call price may depend on the time when the bond is called. Because you see the maturity period is 10 years the bond is called back after 5 years, minimum 5 years is mentioned, after 5 years at any time the bond can be called back or the issuer can buy back that particular bond. So in that case they will mention a price so if the bond will be called back after 6 years there is a different price, if the bond will be called back after 8 years there is a different price some cases. But some cases at any time the bond will be called back they will mention the same price whether it will be called back after 5 years or after 7 years, the price will be same; the call price will be same but there may be a schedule on the basis of the maturity period, on the basis of the time, may be the price will may also vary.

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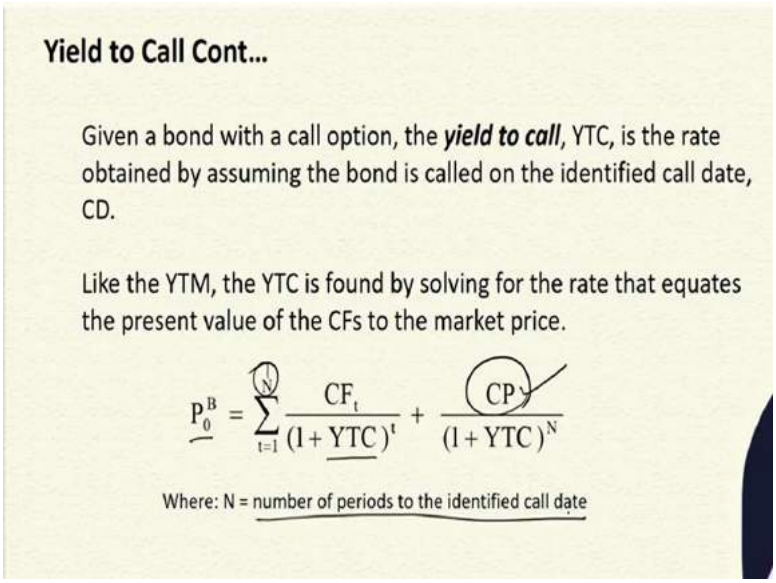


$$P_0^B = \sum_{t=1}^{N} \frac{CF_t}{(1+YTC)^t} + \frac{CP}{(1+YTC)^N}$$

So in this case how basically we calculate the yield to call? And the any yield which is related to that particular bond that is basically called the yield to call. So this is the rate where we can get by assuming that the bond basically is called back on an identified date, which is called the call date. So then we go for our original formula for the price of the bond that is P B = your t = 1 to N CF t divided by 1+YTC to the power t here we are using YTC.

And instead of face value we are using here basically the call price and what is the N year? The N is not the maturity period, the N is the number of periods to the identified call date so that is the difference basically what you can get it from here. So let us see that how this thing is basically varying.

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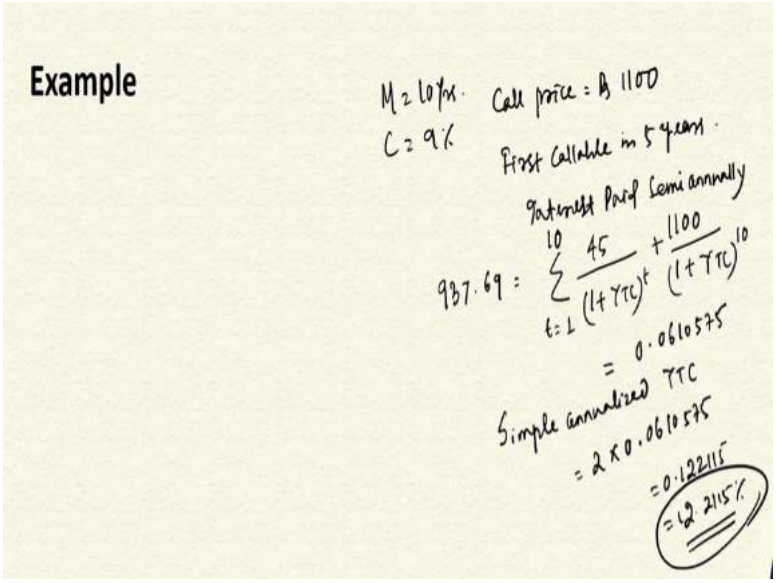


$$937.69 = \sum_{t=1}^{10} \frac{45}{(1+\text{YTC})^t} + \frac{1100}{(1+\text{YTC})^{10}}$$

$$\text{Simple annualized YTC} = 0.0610575 * 2 = 0.122115 = 12.2115\%$$

So in the same example, we have maturity period is 10 years, coupon is 9% then let the call price is mentioned. The call price is let mentioned 1100 and the bond can be fast callable in 5 years and let the interest is paid semi-annually. So in this case if you want to use this formula that 937.69 is equal to t = 1 to 1 How many periods are remaining? That is the 5 years that means 6 months that is you have the 10.

Then 45 divided by 1 YTC to the power t + here in the place of 1000 we will be using 1100 which is the call price 1+YTC to the power 10. If you solve it you will get 0.610575. Now if you make it simple analyzed YTC, then how much you will get? 2 into 0.0610575 that is 0.122115 that means 12.2115% so if the bond will be callable in 5 years then the actual yield from that particular bond will be 12.2115%. But the yield to maturity of that particular bond was 10% that we have seen because there is a call feature generally yield is relatively high.

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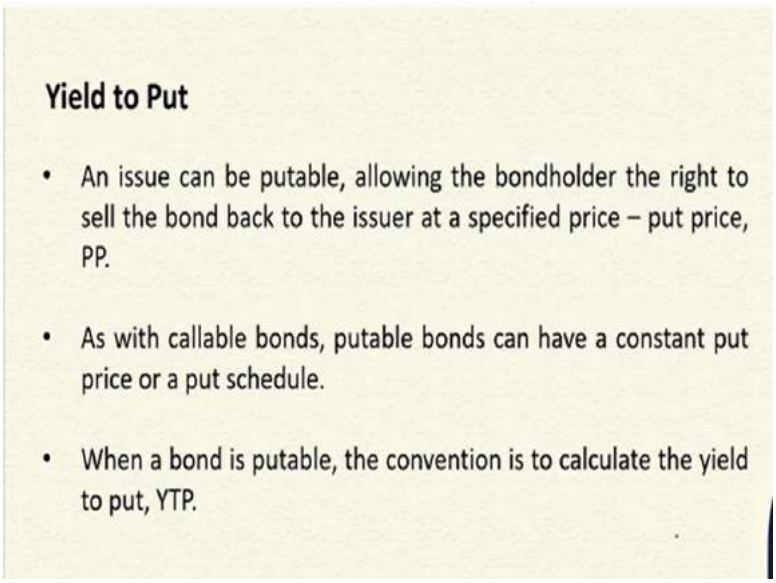


Same thing can be applicable for the yield to put also so any bond can be putable. If there is a put feature that means, it allows the bond holder the right to sell the bond back to the issuer at a specified price that is called the put price. Again like callable bonds, the putable bonds can have a constant put price or a put schedule depending upon the time period and whenever the bond is putable the convention is to always calculate the yield to put.

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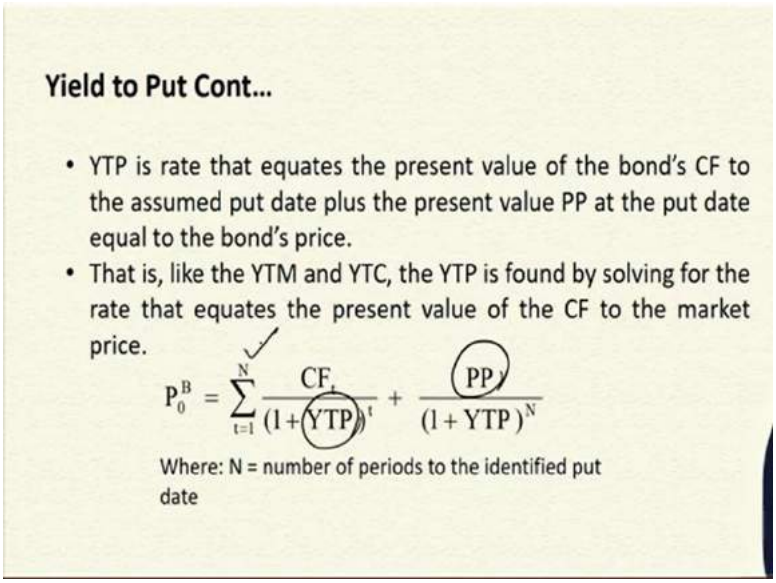


$$P_0^B = \sum_{t=1}^{N} \frac{CF_t}{(1+YTP)^t} + \frac{PP}{(1+YTP)^N}$$

So in this case, we can use this same formula but one difference is basically in the place of the call price we are using the put price in the price of YTC we are using the YTP. And the

number of period to the identifier put date that is basically your N. So we equate the present value of the bonds cash flow to the assumed put date plus the present value of the put price at the put date which should be equal to the price of the bond. So if you solve this equation then you can find out your yield to put.

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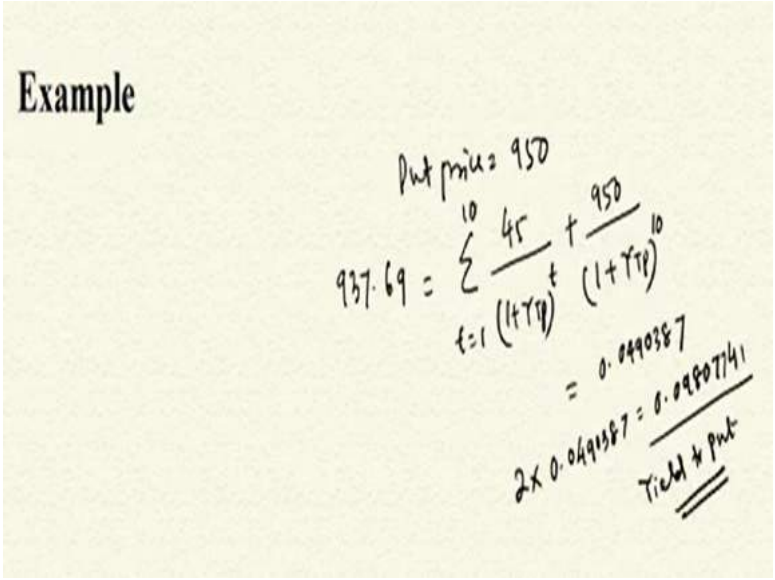


$$937.69 = \sum_{t=1}^{10} \frac{45}{(1+\text{YTP})^t} + \frac{950}{(1+\text{YTP})^{10}}$$

$$2 * 0.0491387 = 0.09807741$$

Let us see that how this basically works, let the put price is 950. So now it is semi annual coupon example we have taken 937.69 and let the put it is putable in 5 years that is the same thing we are taking then 5 into 2 that is 10. This is your 45 divided by 1+YTP to the power t + your 950 divided by 1+YT p to the power 10. So if you solve it you will find 0.0490387 make it if you make it analyze then 2 into this 0.0490387 that will be giving you 0.09807741.

So that is basically giving you this yield to put. So, now, the yield to put is lesser than the yield to maturity .That what we have taken because into maturity was 10 % now your yield to put is 9.8077%. So, that is the concept what we use for yield to put.

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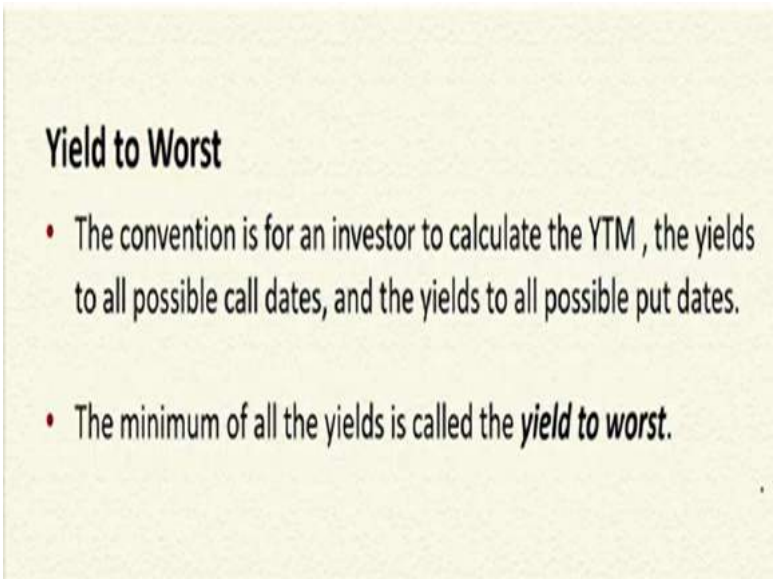


So what is yield to worst? The convention is for an investor to calculate the YTM, the yields to all possible call dates and the yields to all possible put dates. The minimum of all the yields is generally called the yield to worst. The minimum of all the yields, what basically we get it from the different possible call dates? If the prices are different then we call it the yield to worst. So that basically is a concept youhave to keep in the mind.

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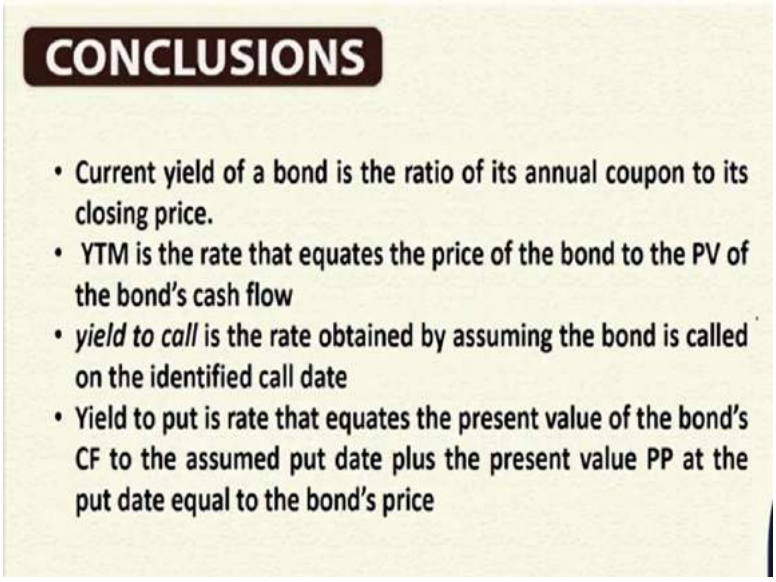


So what basically we have discussed that the current yield of a bond is nothing but the annual coupon to closing price. YTM is basically always equates the price of the bond to the present value of the cash flow. Yield to call is the rate which is obtained by assuming that the bond is called on an identified call date. And yield to put also the particular rate which equates the present value of the bonds cash flow to the assumed put date plus the present value of the put price at the put date equal to the bond price.

So these are the things what basically we have discussed and we may have a put schedule we might have a call schedule also. Accordingly also the each, we can calculate our yield with respect to that particular put option or yield with respect to that call option.

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So, this is the reference what basically you can follow for the detailed analysis on this.

Thank you.