Management of Fixed Income Securities
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Lecture - 56
Fixed Income Securities Derivatives - I

Welcome back, so in the previous class we discussed about the different type of mortgage-backed securities and today we will start the discussion on the fixed income securities and derivatives. So, all of you have fair idea about the derivatives.

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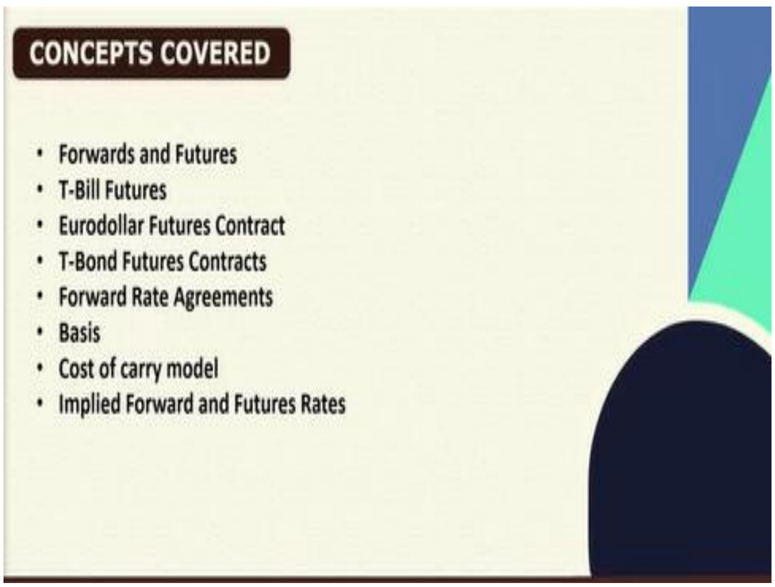

So, largely we will focus on the different type of fixed income derivatives instead of talking about the actual concepts of the fixed income derivatives or actual concept of the derivatives as a whole. So, largely we will discuss mostly on the different type of fixed income assets, which are used as underlying asset for the derivatives instrument or for have a derivatives contract. So, these are the different concepts we will be discussing.

Forwards and futures, T-Bill futures, Eurodollar future contract, T-bond future contract, forward rate agreements. These are basically the different type of the fixed income derivatives generally we always come across in the market. Then the concept of basisalready you must have the idea but we will just see that thing how that basis is basically measured,then the cost of carry model,then the concept of the implied forward and the future rates.

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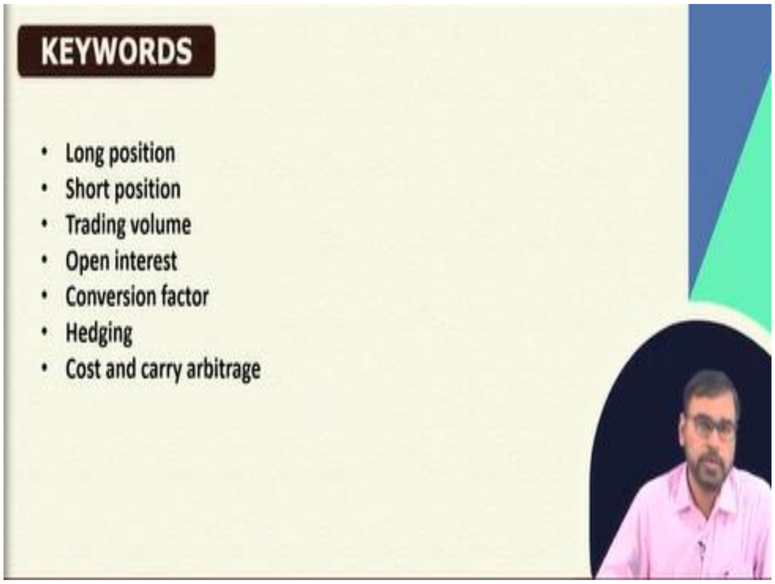

So, these are the different keywords we will come across like your long position, short position, trading volume, open interest, conversion factor, hedging, cost and carry arbitrage all kinds of things basically you will come across while discussing the different concepts for this particular session.

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So, let us see already I hope for you most of the fair idea about this but just for revision some of the basic things first let us see. What exactly the forward and futures are? The forward contract is basically a customized contract between the two entities, where the settlement generally takes place on a specific date in the future at today's pre-agreed price. The price has been fixed today,but the settlement or the transaction will take place in the future.And this particular agreement has been made or contract has been made between the two different parties.

Then we have the futures,the futures are basically a contract or in an agreement between again two parties to buy or sell an asset at a certain time in the future at a certain price. It is same with the forwards. But we can say that the future contracts are special type of forward contracts in the sense that the future contracts are highly standardized or they are basically the exchange-traded contracts.

So, mostly the futures are the concept is same but the future contracts are basically the standardized exchange-traded contracts, but the forwards are not. Forwards are basically between the two parties. So, that is the basic difference between forwards and the futures.

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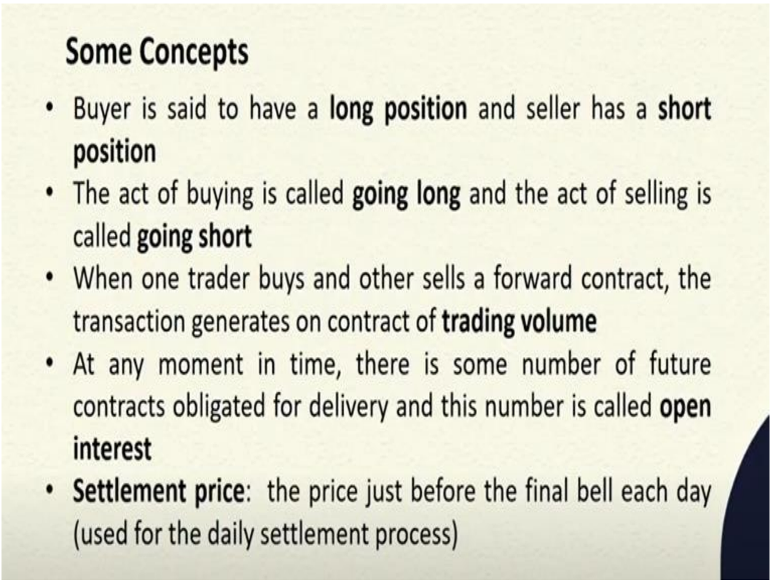

Then we will using certain kind of concepts while discussing about the derivatives instrument. I hope for you most have the idea about this. One is your concept of long position, concept of short position like if the buyer is somebody is buying then we can say that the buyer is taking a long position and seller has to take a short position. And the act of buying is basically called the going long and act of selling is called the going short.

And when one trader buys and other trader sells a forward contract, the transaction generates on contract of trading volume. At any moment in time, there is some number of future contracts which are obliged for delivery and that this number is basically called the open interest. And we have a concept called the settlement price it is the price just before the final bell each day generally used for the daily settlement process.

Then you have the concept of exercise price the exercise price is nothing but the period agreed price already which has been decided as per the contract on which the particular buyers and sellers are value or the payoff is going to be calculated. So, these are the different concepts over the discussion what we are going to use.

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So, if you say the basic difference between the forward and future already, we have discussed that forward contract is basically a private contract between the two parties what futures are exchange-traded. Forwards are not standardized but futures are standardized.Generally, forwards usually one specified delivery date but futures have range of delivery dates. And forwards are settled at maturity what futures are settled daily.

And forwards are generally delivery or the final cash settlement usually occurs for the forward contracts but in case of futures the contract usually closed out prior to the maturity. And forward contract there is some credit risk involved. But in futures virtually no credit risk because the exchange or the stock exchange basically comes into the picture. So, because of this there is some kind of guarantees involved with respect to the future contracts.But those kinds of things are not available with respect to the forward contracts.

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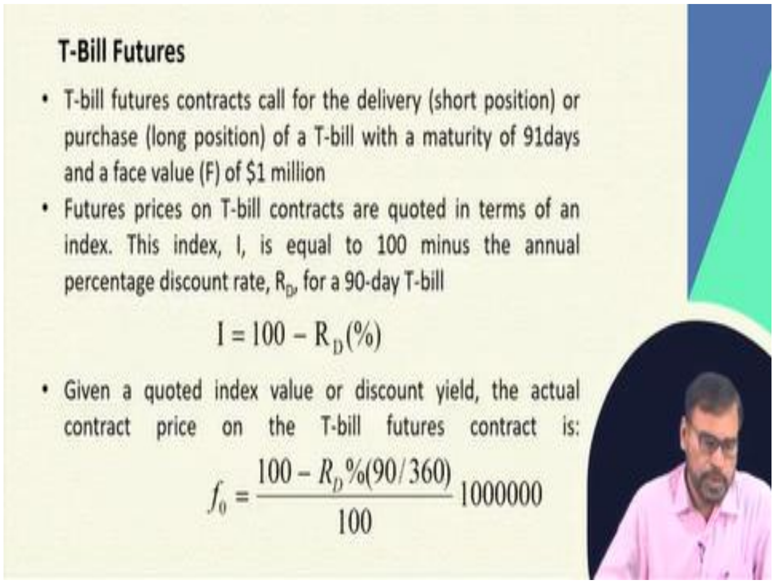

So, then coming back to specific instruments which are used as the fixed-income derivatives. One is your T-bill futures or Treasury bill futures. So, these treasury bill futures contracts generally what these are generally call for the delivery or the short position or going for a long position or the purchase of a T-bill within a maturity of 91 days and generally the face value f of the T-bill futures are 1 million dollars.

So, the future prices on T-bill contracts are quoted in terms of an index, already you know that. And this index is nothing but the 100 minus the annual percentage of the discount rate, for the 91- or 90-days T-bill. So, your index value will be 100 minus your percentage or discount rate whatever which is available.

Index can be calculated as:

$I= 100- R_D(\%)$

So, if you have an index value or the discount yield whichever is available to you, the actual contract price of the T-bill future can be calculated. That is your 100 minus your discount rate that is your R_D here we have taken into 90 by 360, because the treasury bills future, whenever we calculate or the valuation of treasury bill futures, we take most important thing is the day count conventions. And if you talk about the US treasury bills market, generally they consider, it is the your 30 by 360 conventions. But if you talk about India, we sometimes we use this actual way actual, that is maybe 91 by 365.

But in the case of US, it is 90 by 360, because they are used in the concept of 30 by 360 day count convention in that case. So, then the value of or this future price if you want to calculate that is 100 minus the percentage of this discount rate into 90 by 360 divided by 100 into 1 million. That is what basically we use it for calculation of the price of the T-bill futures.

Price of T-bill futures:

$F_0 = \frac{100 - R_D\%(\frac{90}{360})}{100}\ 1000000$

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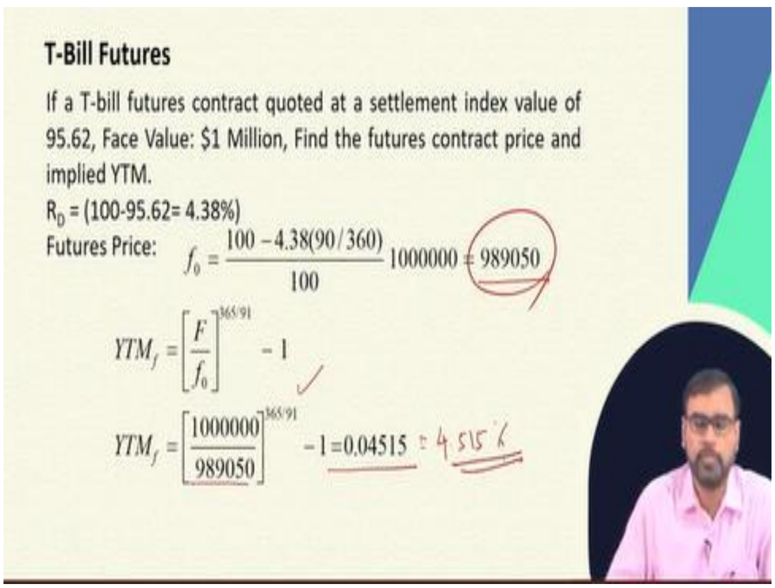

In this case futures contract price and Implied YTM will be:

R_0= (100-95.62= 4.38%)

Futures price: $f_0 = \frac{100 - 4.38(\frac{90}{360})}{100}\ 1000000 = 989050$

$YTM_f = \left[\frac{F}{f_0}\right]^{365.91} - 1$

$YTM_f = \left[\frac{1000000}{989050}\right]^{365.91} - 1$

= 0.04515

Therefore, in this case futures contract price and implied YTM are 989050 and 4.51% respectively.

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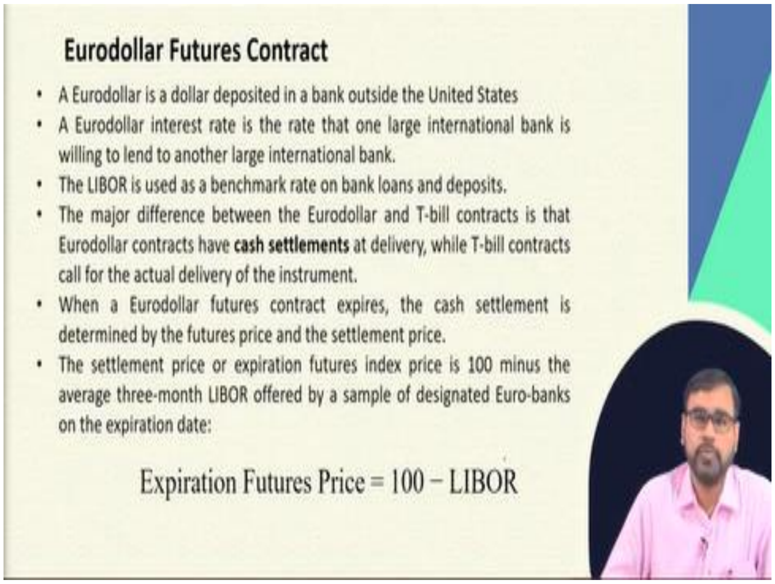

Then we have another type of fixed income derivatives commonly we look at that is called the Eurodollar future contract. So, Eurodollar is nothing but a dollar deposited in a bank outside the United States. So, the Eurodollar interest rate is again the rate that the one large international bank is willing to lend to another large international bank. So, generally LIBOR is used as a benchmark on the bank loans and deposits in the international market.

And the major difference between the Eurodollar and T-bill contract is that the Eurodollar contract have the cash settlement at the delivery. While the T-bill contracts generally call for the actual delivery of the instrument. So, in case of Eurodollar the cash settlement takes place and in case of T-bill contracts generally the actual delivery of the instrument takes place. So, when the Eurodollar future contract generally expires, the cash settlement is determined by the future price and the settlement price.

That happens with all type of derivatives that also happens with respect to the Eurodollar contracts. So, the settlement price or the future index price at expiration is nothing but the 100 minus the average three-months LIBOR offered by a sample of designated Euro-banks on the expiration date.

So, the Expiration future = 100 - LIBOR rate.

The average LIBOR rate is the rate which is prevailed in that particular point of time.

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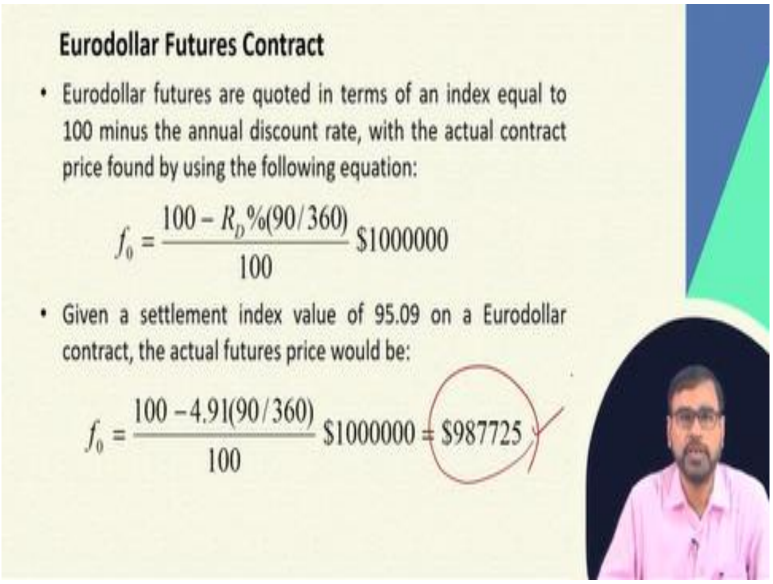

So, if generally the Eurodollar futures are quoted in terms of the index value which is equal to 100 minus the annual discount rate like your treasury bills.

The actual contract price can be calculated through the following equation:

$$F_0 = \frac{100 - R_D\%(\frac{90}{360})}{100}\ 1000000$$

Given a settlement index value, let 95.09 on a Eurodollar contract, the actual future price will be:

$$F_0 = \frac{100 - 4.91(\frac{90}{360})}{100}\ 1000000 = 987725$$

So, the actual contract price is 987725.

your 987725dollars. So, this is the formula what basically we are same formula we are using and we are calculating the future price of this Eurodollar contract.

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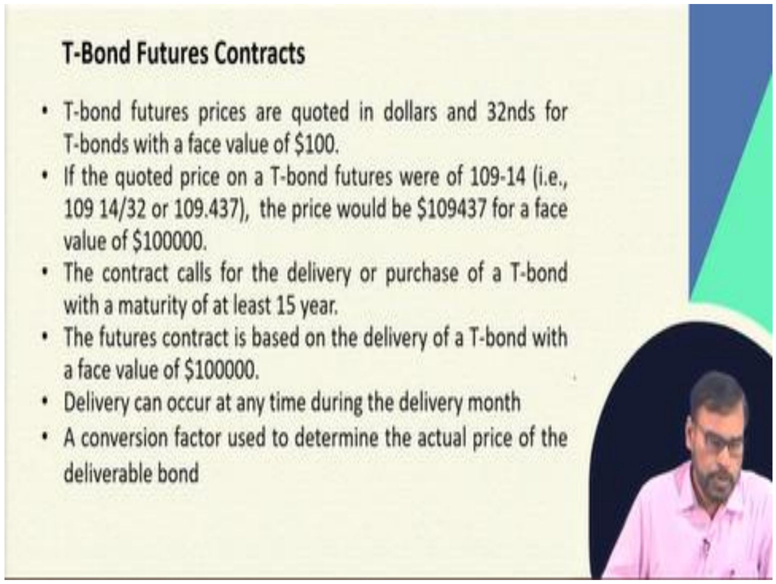

Then, we have another contract we call, the T-bond future contract. The T-bond future prices are generally quoted in dollars and generally 32nds of the T-bonds with a face value of 100 dollar.

If the quoted price on a T-bond future is let 109 14/32. That means it is 109-14 by 32 that means 109.437. And the price will be 109437 for a face value of let one lakh dollar.

The contract generally calls for the delivery or purchase of a T-bond with a maturity of at least 15 years.

And the future contract is based on the delivery of the T-bond with a face value of the100000 dollars. So, delivery can occur any time during the delivery month. And whenever we calculate this actual price of the deliverable bond, we generally use a concept called the conversion factor. So, a conversion factor is used to determine the actual price of the deliverable bond. And what exactly the conversion factor is?

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So, the conversion factor is basically what the actual price paid on the T-bond or the revenue received by the seller in delivering the bond on the contract generally is equal to the quoted future price times the conversion factor, on the delivered bond plus any accrued interest. So, the seller's revenue will be your quoted future price into your conversion factor plus the accrued interest.

For example, at the time of delivery if the delivered bond has a conversion factor of 1.3 and the accurate interest of 2 dollar and quoted future price is let 94-08. Then the cash received by the seller of the bond and paid by the future purchaser will be:

= (94.25) (1.3) +2

= 124.525

So, the T-note contracts generally are similar to the T-bond contracts.

Except that they call for the delivery of any T-note with maturity between 6.5 to 10 years. And T-bond contracts maturity period is generally 15 years, relatively longer in nature. But in terms of the T-bond, T-note contracts relatively the maturity period is less.

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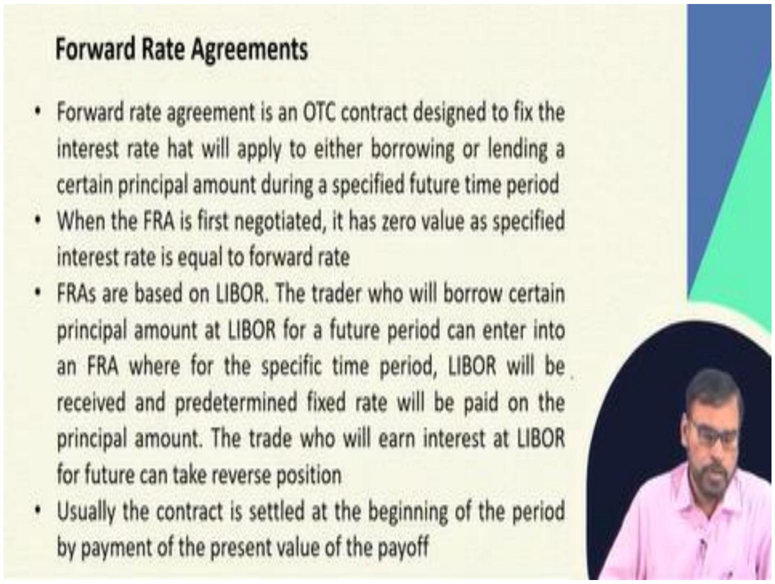

Then we have a concept of the forward rate agreements that already all of you know that. Forward rate agreement is basically an OTC contract over the counter contract, which is basically designed to fix the interest rate that will apply to either borrowing or lending a certain principal amount during a specified future time period. So, when the FRA is first negotiated it has the value is zero.

Because the specified interest rate is equal to the forward rate. And the forward rate agreements are generally based on the LIBOR. So, once you are going across the period over the time the value of this particular forwarded agreement changes depending upon the change in the LIBOR rate. So, the trader who will borrow certain principal amount or LIBOR for a future period can enter into one forward rate agreement for the specific time period.

The LIBOR will be received and a predetermined fixed rate will be paid on the principal amount. It is just like a swap contract. And the trader who will earn the interest at LIBOR for future can take the reverse position. Usually, the contract is settled at the beginning of the period by payment of the present value of the payoff. We will see that how basically it happens.

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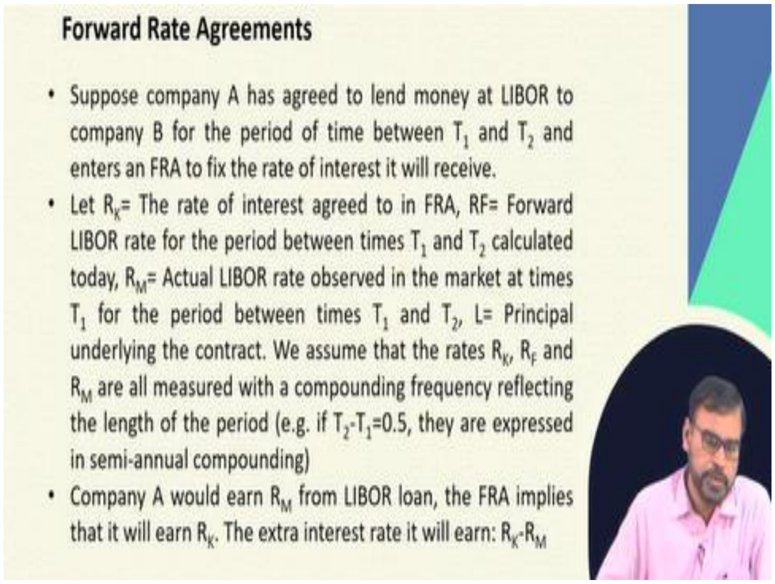

Suppose a company A has agreed to lend money at LIBOR to company B for the period of time between T_1 and T_2 and enters on forwarded agreement to fix the rate of interest it will receive. So, let R_K= the rate of interest agreed to in FRA, RF = forward LIBOR rate for the period between T 1 and T_2 calculated today. And R M= actual LIBOR rate observed in the market at times T_1 for the period between times T_1 and T_2 and L = the principal underlying the contract.

And we assume that the rates R_K, R_F and R_M are all measured with compounding frequency reflecting the length of the period. That means for example, if the T_2 -T_1= 0.5. That means they are expressed in semi-annual compounding if it is 0.25 then they are basically in the quarterly compounding. So, then in that case the company A would earn R_M from LIBOR loan. The FRA implies that it will on R_K. Then extra interest rate it will on basically R_K- R_M.

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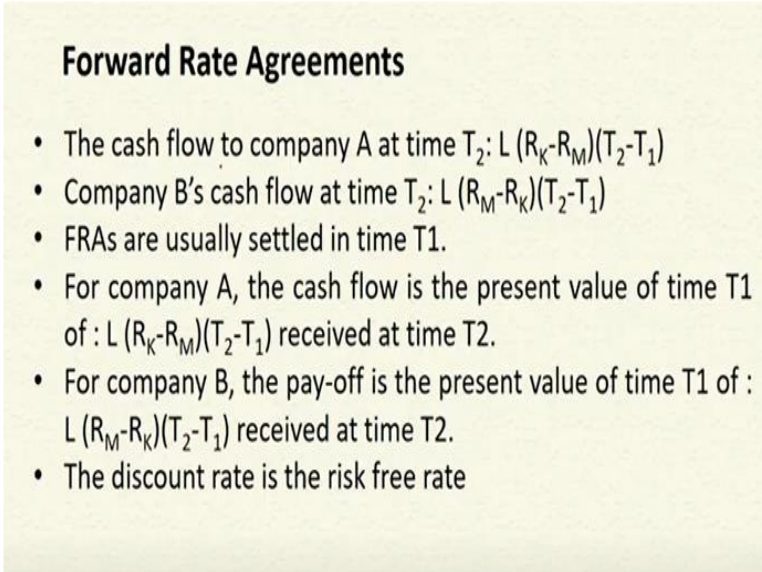

Then the cash flow to the company A at times T_2 will be how much it will be L into (R_K- R_M)into (T_2 -T_1). This is the time gap (T_2 -T_1). Then company B's cash flow will be at T_2 will be L into (R_M-R_K) into (T_2-T_1) it is just reverse. (R_M- R_K) in the first case it is (R_K-R_M). Then the forward rate agreements are generally settled in time T_1. So, for company A the cash flow is the present value of time T_1 of the L into (R_K-R_M) into (T_2–T_1) which is received at time T_2.

Because we are calculating at the time T_1 so because of that we have to calculate the present value of that. For company B the payoff will be the present value of time T_1 ofL into (R_M-R_K) into (T_2-T_1) which will be received at the time T_2. And whenever we discount it generally, we use the discount rate that is the risk free rate which is preferable in the market that will be used as the discount rate in the particular context.

So, this is the way the value of the cash flow with respect to the two different parties will be calculated whenever they have entered into any kind of forward rate agreements.

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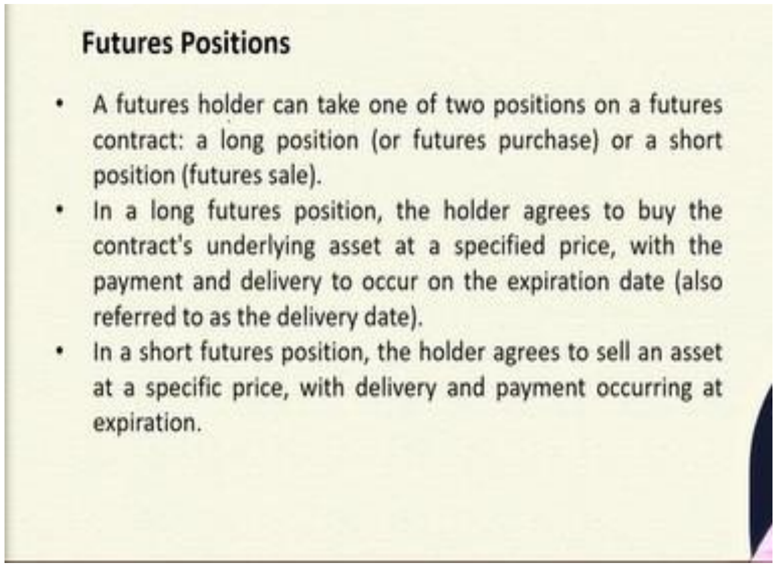

So, the future positions if you look at, the future holder can take one or two positions of a or on a future contract. They can either take long position or they can purchase it or they can take a short position for the future sale. So, whenever we are talking about the long future position here what happens the holder basically then agrees to buy the contract underlying asset at the specified price with the payment and delivery to occur on the expiration date also, we call the delivery date.

And in the short future position the holder basically agrees to sell an asset at the specific price with the delivery and payment basically occur at the expiration. So, these are the different general positions in the investor can take. Either they can take a long future position or they can take a short future position.

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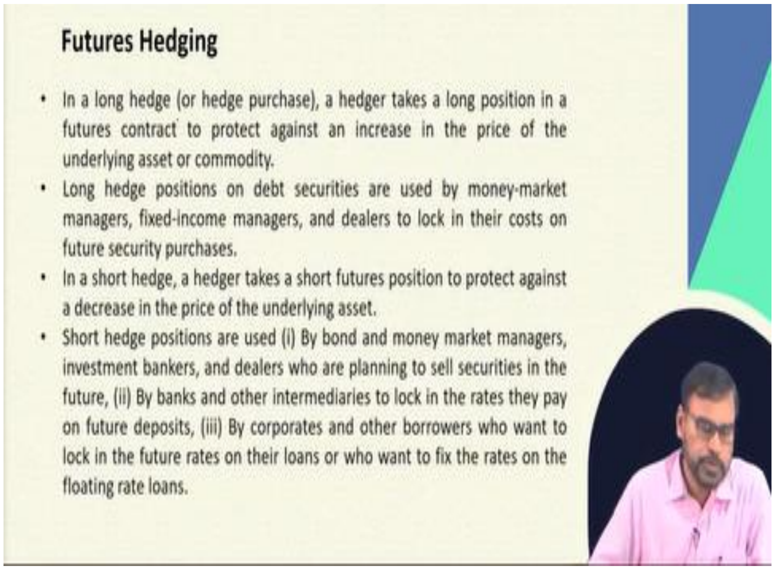

But in that context generally these positions are used for the hedging. We will discuss these things in elaborate how to use this derivatives instrument for the hedging. So, we have a long hedge and we have a short hedge. So, long hedge means hedger basically takes a long position in the future contract to protect against an increase in the price of the underlying asset or the commodity.

So, if you talk about the long hedge positions on the fixed income securities or the debt instruments. They are generally used by the money market managers, fixed income managers or dealers to lock their cost on future security purchase. If they have the plan to buy some security in the future they can lock the price, lock the cost by taking this long hedge positions.

In a short hedge, the hedger basically takes a short future position to protect against a decrease in the price of the underlying asset. If they want to sell then obviously, they want to lock that particular price today by that even if the rate or something is going to be change in the future or is going to decline in the future, they are not going to be affected by that. So, short hedge positions generally used by the bond and money market manager, investment bankers and dealers, who are basically planning to sell the securities in the future or by the banks and other intermediaries to lock the rates they pay on the future deposits or also it can be used by the corporates and the other borrowers who want to lock their future rates on their loans or who want to fix the rates on the floating rate loans. So, for the different objective for the different requirements different financial entities generally try to use this particular type of hedging positions.

Broadly there are two positions just now we have discussed; one is your long position another one is the short position or we can say that long hedge and another into shorthedge.

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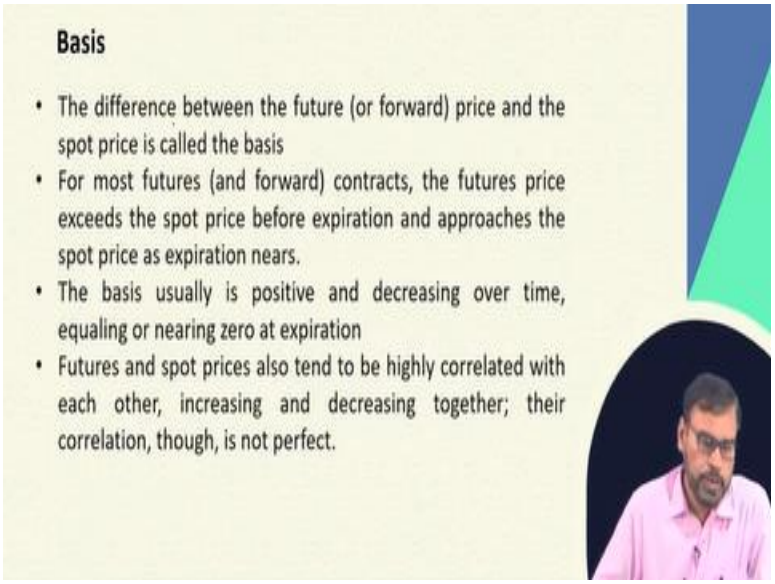

So, then another concept largely we use for pricing of the derivatives and as well as the investment strategy whatever we use for the using the derivatives contracts that is called the basis. The basis is basically nothing but it is the difference between the future or forward price and the spot price. And generally, if you look at in the market, the most future contracts the future prices generally more than the spot price before the expiration.

Before the expiration the future prices are more than the spot price. And generally, it approaches to the spot price or it will be equal to the spot price at the expiration nears. So, on the end of the maturity, basically it will be same or the basis risk will be 0. That is called the convergence. The future price and spot price will basically converge to each other at the time of the expiration or the time of the maturity.

So, the basis is usually positive and decreasing over time because once we go ahead towards this expiration date or delivery date you will find that this gap is going to be reducing. And it will be nearing zero at the time of expiration. And future and spot prices are generally highly correlated with each other either increasing or decreasing together. Although this correlation is not that perfect but that correlation is basically exist between these two rates.

So, there is some kind of relationship can be established between the future prices and the spot prices.

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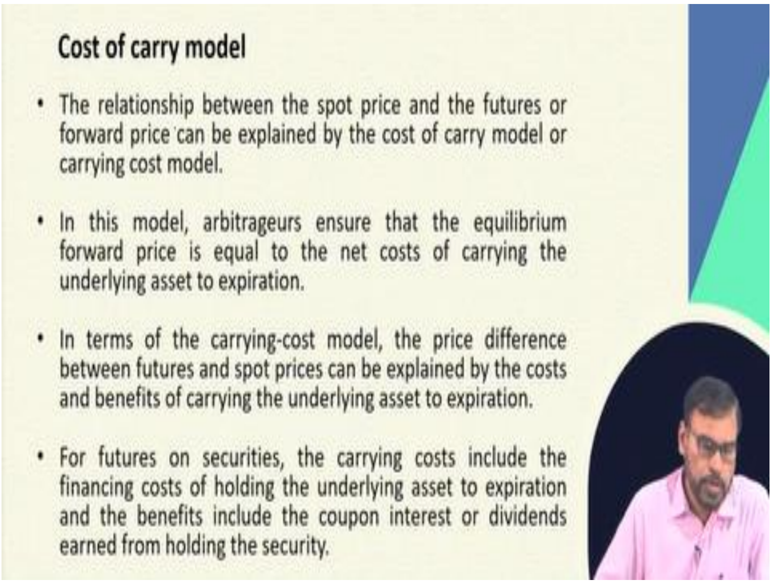

So, then you see that whenever we go for the valuation of the fixed income securities derivatives or any kind of derivatives, largely there are many models we try to use. And one of the popular models is basically your cost of carry model or carrying cost model whatever thing what you can use it for that. So, these are the different models which can be used. So, the relationship between the spot price and future price or forward price this can be explained by the cost of carry model or the carrying cost model.

Here what happens the arbitrageurs basically ensures that the equilibrium forward price should be equal to the net cost of the carrying the underlying asset to the expiration. So, the price difference between the future and spot prices can be explained by the cost and benefits of carrying the underlying asset to the expiration date. That is what the carrying cost model or the cost of carry model is trying to explain.

So, whenever we talk about the futures and securities, the carrying cost generally include the financing cost of holding the underlying asset to expiration and the benefits basically includes the coupon interest if you are talking about the debt instrument or if you are talking about the equity then it is the dividend what basically you are earned or you are earning from holding that security.

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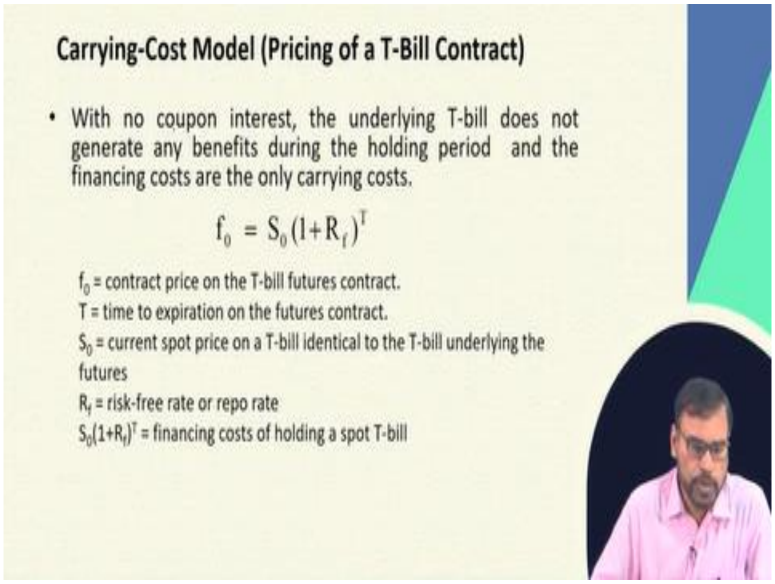

So, now if you talk about a carrying cost model for a pricing of a T-bill contract and T-bills do not pay any coupon. So, the underlying T-bill does not generate any benefit during the holding period and the financing cost are only the carrying cost. So, if you want to calculate this then this is the formula what you can use that is, your future price f_0= S_0 into 1 + R_f to the power T.

$F_0= S_0(1+R_f)^T$

Where, f_0 is the contract price of the T-bill future contract.

T is the time to expiration on the future contract,

S_0 is equal to current spot price,

R_f is equal to your risk free rate or repo rate and

T is the financing cost of holding the spot T-bill.

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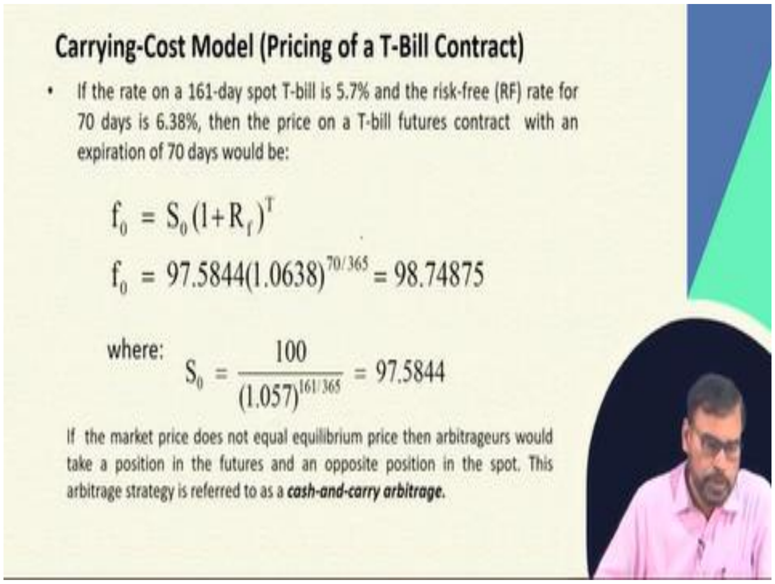

So, if you want for example, there is 161 days for T-bill, this rate is 5.7 %. Risk free rate for 70 days on T-bill is 6.38%. Then the price on a T-bill future contract with an expiration of 70 days would be:

$F_0= S_0(1+R_f)^T$

$F_0= 97.5844(1.0638)^{70/365}= 97.5844$

And here where you got this 97.5844 that is basically the spot price today that is S_0.

That how we have got we have basically got it by using that face value of that particular security will be:

$S_0= \frac{100}{(1.057)^{161/365}} = 97.5844$

So, if the market price does not equal equilibrium price, then the arbitrageurs would take a position in the futures and opposite position in the spot and this particular strategy is called as the cash and carry arbitrage.

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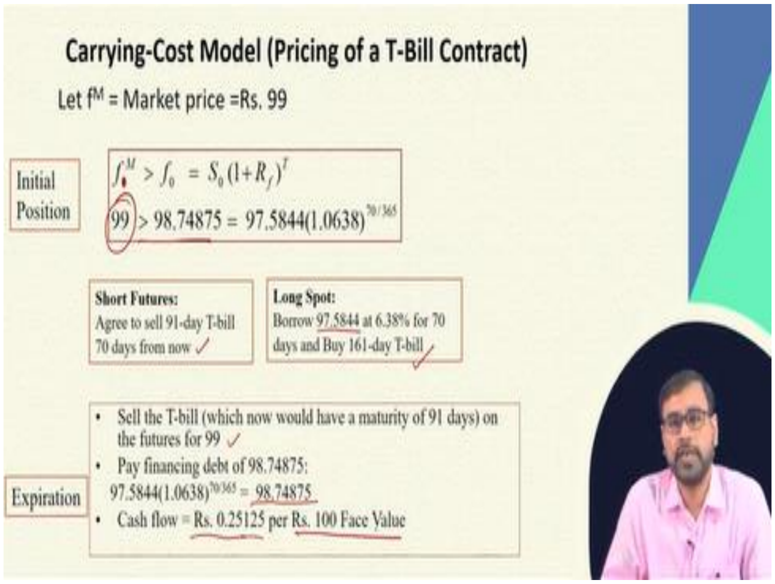

Then how this cash and carry arbitrage generally work? Let the market price is 99. Future market price is 99. Then obviously down your M your f^M is greater than f_0, we have calculated this is 98.74875. Then 99 is greater than this. So, in this case what we can do? We can short the futures that means agrees to sell the 91 day treasury bills 70 days from now.

Then you take a long position in the spot market. How you can take a long position in the spot market? Borrow this 97.5844 at 6.38% for 70 days and buy these 161 days treasury bills. Then in the expiration date what you can do; sell the treasury bills which would now have a maturity of 91 days on the futures for 99 because market price is 99. Pay the financing of debt, because you have borrowed this 97.5844.

Then you pay this 97.5844 into 1.0638 to the power 70 by 365. You have to pay this 98.74875. Then your cash flow will be your 99 minus this that will be 0.25125 per 100 rupees face value. So, depending upon the total face value that much profit basically you can generate. So that is called the cash and carry arbitraries.

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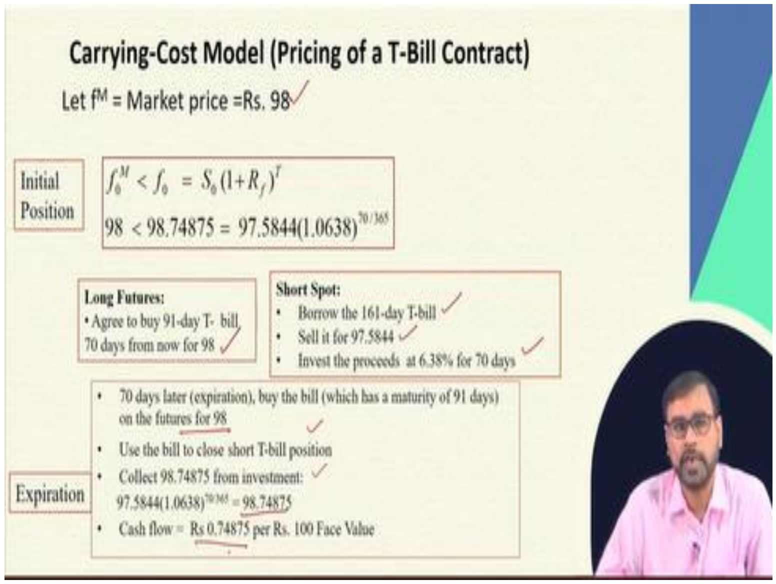

So, if the market price is less than that, let this is 98. Then what you could have done you can take a long position in the future market and short position in the spot market. So, in that case you are agreeing to buy the 91 day Treasury bill futures 70 days from now for the rupees 98. And in the spot market what you could have done borrow the 161 debt treasury bill sell it for price of 97.5844. And invest the profits at 6.38% for the 70 days.

So, at the time of expiration what you could have done 70 days later; buy the bill on the futures for 98. Use this bill to close the short T-bill positions. Collect this 98.74875 from the investment that is your 97.5844 into this that much you will get. Then your cash flow will be the difference between these two. That will be 0.74875 that means 98.74875 - 98. That will give you your 0.74875for 100 rupees face value like that you can generate your profit.

So, either if your market value is going down or going up depending upon that you can take your positions. By that you can create this constant carry arbitrage opportunity.

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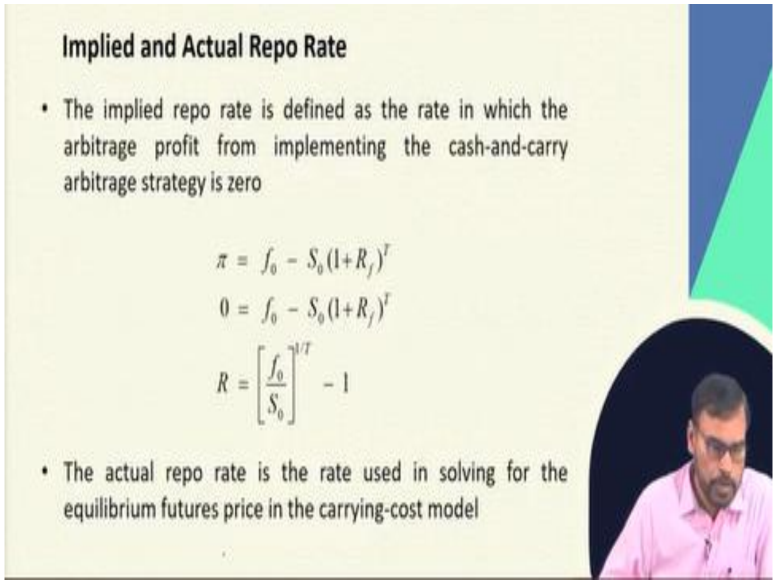

So, from this we can calculate our implied and actual repo rate. So, the implied repo rate is defined the rate at which the arbitrage profit from implementing the cash and carry arbitrage strategy is zero.

$\pi = f_0 - S_0(1 + R_f)^{T}$

$0 = f_0 - S_0(1 + R_f)^{T}$

$R = \left[\frac{f_0}{S_0}\right]^{1/T} - 1$

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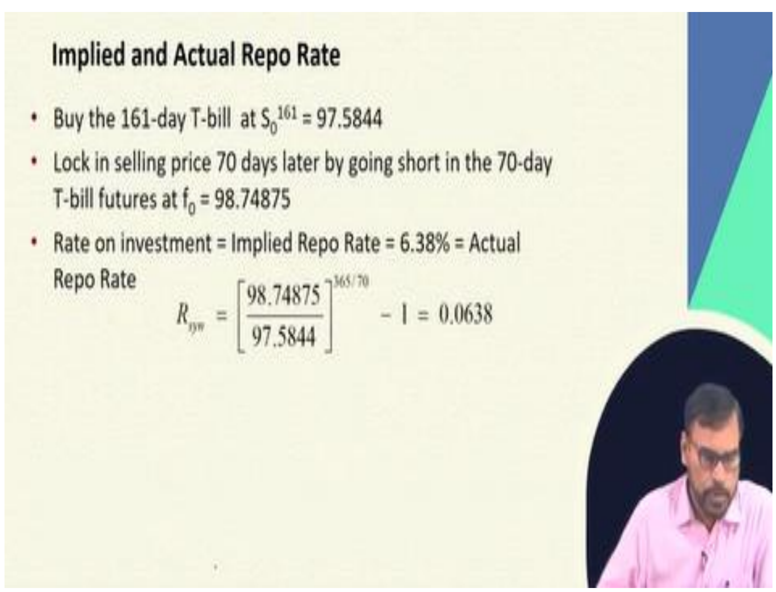

So, by the 161 day treasury bills at S_0161 that is 97.5844. Lock in selling price 70 days later by going short in the 70 day treasury bills future at $f_0$is equal to this 98.74875.Then rate on investment will be your implied repo rate that will be your 6.38%. That what we have assumed it is exactly you are getting the same. That means: $R_{syn}= \left[\frac{98.74875}{97.5844}\right]^{365/70}-1 = 0.0638$

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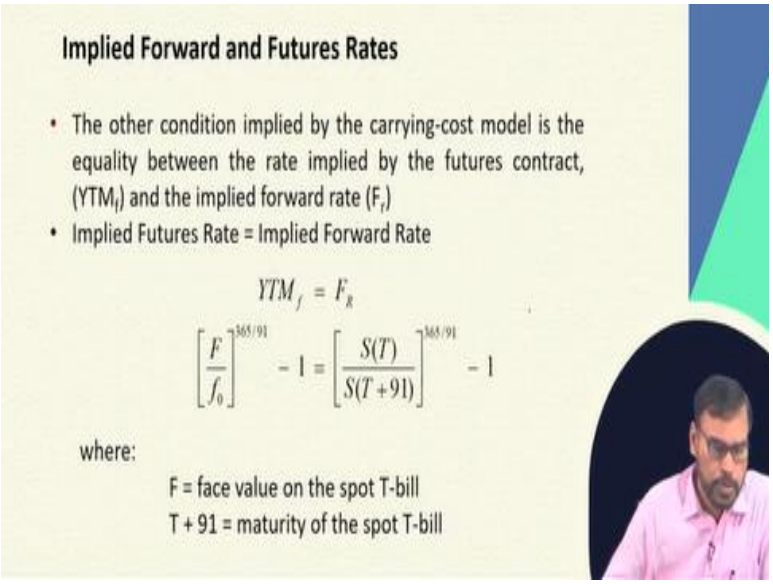

So, implied forward and future rates also you can calculate from this. The other condition implied by the carrying cost model is the equality between the rate implied by the future contract and the implied forward rate.

$$\left[\frac{F}{f_0}\right]^{365/91}-1 = \left[\frac{S(T)}{S(T+91)}\right]^{365/91}-1$$

So, F is the face value on the spot T-bill and T + 91is the maturity of the spot T-bills which are available in the market.

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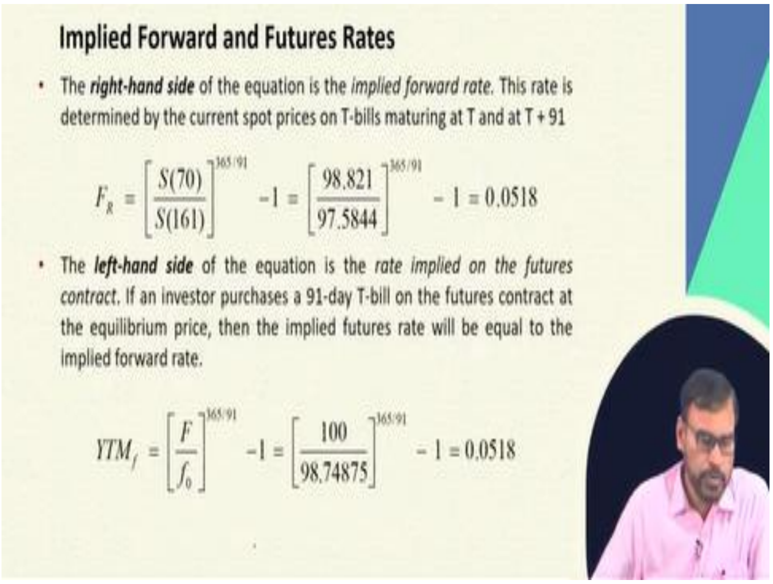

$$F_R = \left[\frac{S(70)}{S(161)}\right]^{365/91} - 1 = \left[\frac{98.821}{97.5844}\right]^{365/91} - 1 = 0.0518$$

$$YTM_j = \left[\frac{F}{f_0}\right]^{365/91} - 1 = \left[\frac{100}{98.74875}\right]^{365/91} - 1 = 0.0518$$

So, in that case in the right hand side of the equation generally the implied forward rate. The rate is determined by the current spot prices on T-bills maturing at between T and T + 91. So, therefore, if you can find this is 5.18% exactly. And in the left hand side equation generally the rate implied on the future contract. Then the investor if the investor purchase, the 91 days T-bill on the future contract at the equilibrium price.

Then the implied future rates will be equal to the implied forward rate. In this case also you will be getting exactly 5.18 %. So, this is the concept of implied forward and the future rates that you can calculate from that particular equation what we use it for the carrying cost model or the cost of carry modelling.

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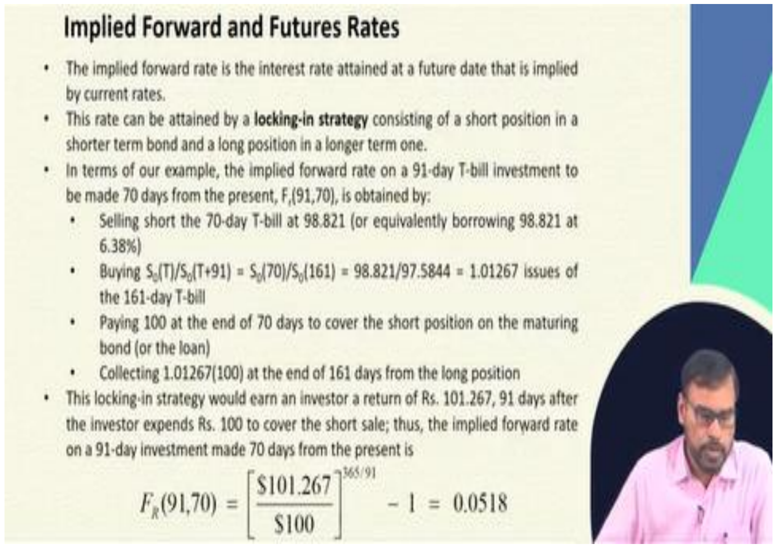

So, you can use it basically this the rate generally can be attained by a locking-in strategy. The locking in strategy generally consisting of a short position in a shorter-term bond and a long position in the longer-term bond. I think we have already discussed in one of the sessions about this. So, if you look at our example, the implied forward rate on a 91 day treasury bill investment to be made 70 days from the present.

$$F_R(91,70)= \left[\frac{101.267}{100}\right]^{365/91}-1= 0.0518$$

Generally, how you can obtain it? Selling the 70-day T-bills like 98.821 or you can borrow 98.821 at 6.38%. Buying this $S_0(T)$ divided by $S_0(T + 91)$. That is $S_0(70)$ by S_0 (161). That is 98.821 divided by 97.5844.That is 1.01267 issues of the 161 daytreasury billthen paying 100 at the end of 70 days to cover the short position on the maturing bond. Then collect this 1.01267 into 100 at the end of 161 days from the long positions.

So, this lock-in strategy will earn an investor return of 101.267, 91 days after the investor expends of 100 rupees to cover the short sale; the implied forward rate on the 91 days investment made 70 days from the present will be your 101.267 divided by 100 to the power 365 by 91- 1 that will be 5.18%. So, that is basically the concept of the locking-in strategy.

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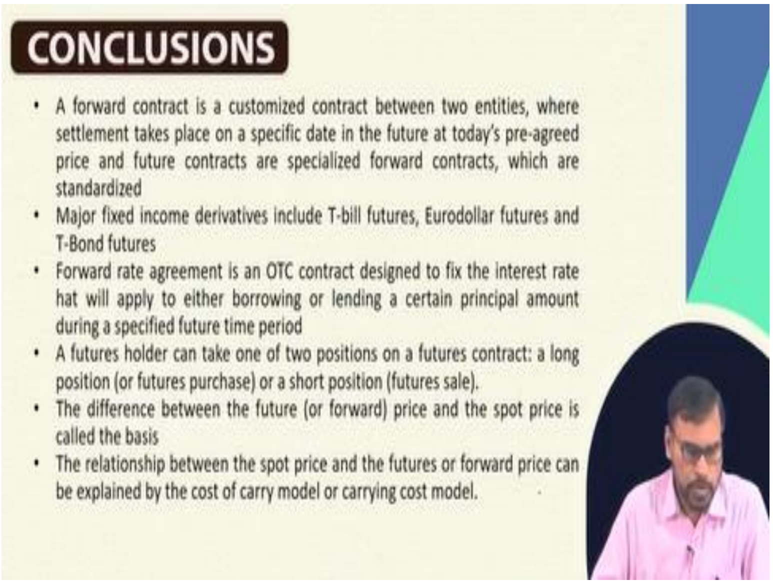

So, what basically we have discussed that a forward rate is a customized contract between the two parties, where the settlement takes place at a specified rate in the future with a today's pre-agreed price. But the forward contracts are standardized. And the major fixed income derivatives generally include: the T-bill futures, Eurodollar futures, and T-bond futures. And forward rate agreement is an OTC agreement OTC contract which is designed to fix the interest rate.

That will apply to either borrowing or lending a certain principal amount during a specified future time period. A future holder can take of two positions on the future contract: either a long position or a short position. And the difference between the future price and spot price is called the basis. And the relationship between the spot price and future price or the forward price can be explained by the cost of carry model or the carrying cost model.

So, these models also can be used for measuring the implied forward rate andfrom that particular kind of contractsor the implied repair it also can be calculated from this.

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These are the references you can you can go through. Thank you.