Management of Fixed Income Securities

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Module No # 01
Lecture No # 05
Bond Price Relations

Good morning, so, we have discussed about the bond valuations or how the valuation of the bond is carried out on the basis of the different characteristics of the bond and as well as on the basis of the coupon payments. But one thing to remember is that the price of the bond will get change on the basis of the change in interest rate, change in the coupon, and as well as change in the maturity.

And you will always find the different kinds of relationships whenever you find that keeping other things remain constant if you change other variables, then how the price of the bond basically behaves differently which we basically call, the bond price relationships.

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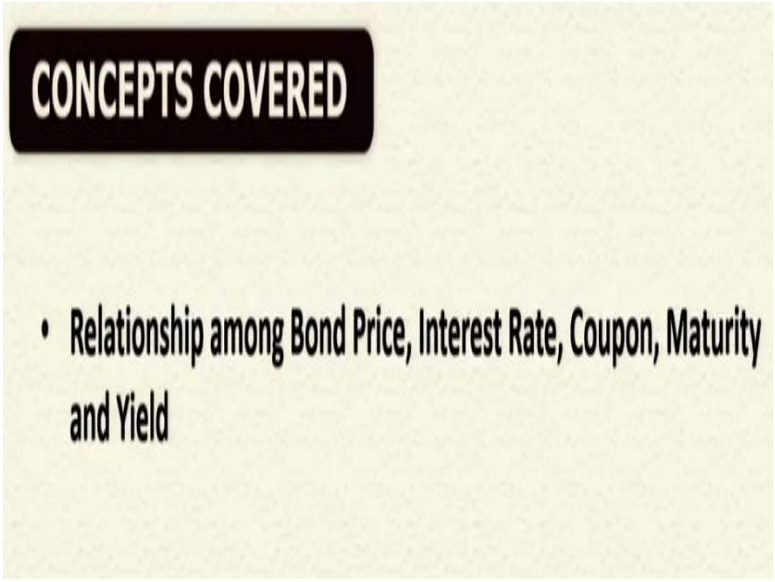

So, in today's discussion we will cover of that relationship among the bond price, interest rate coupon, maturity and yield. So, there are different kinds of relationship we can observe or the characteristics wise the price of the bond or the price change of the bond which basically will be different.

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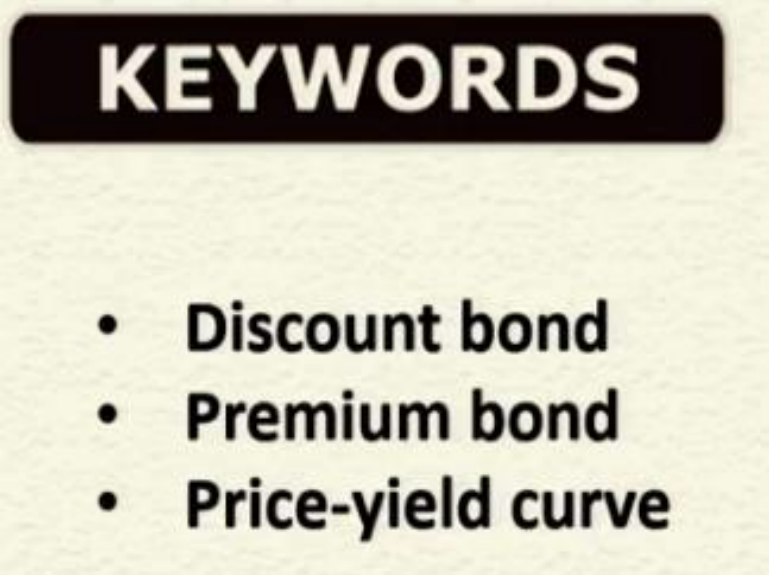

So in this context, you will come to know about certain keywords like discount bond, premium bond, price sealed curve so those things basically will be discussed in today's session.

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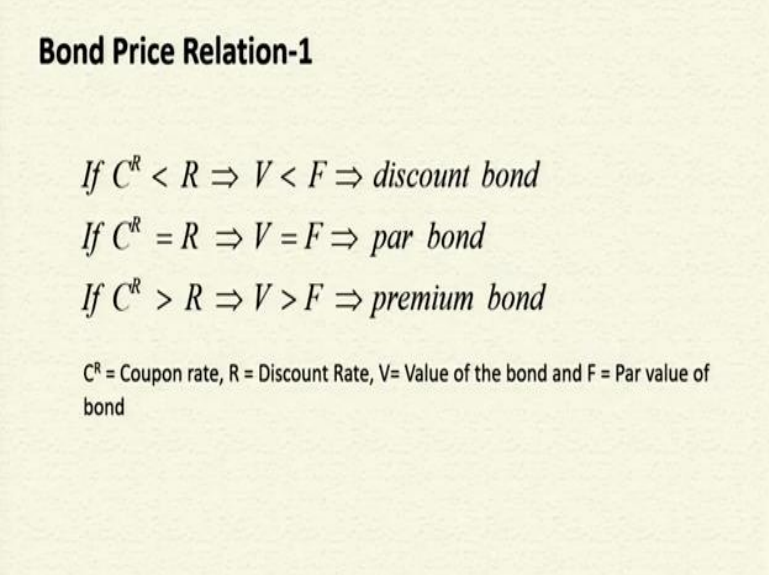

So, let us see that the first relationship you know that in the examples whatever we have taken the previous class if you recall. We have taken your coupon was 9% discount rate was 10% and the par value of the bond was Rs.1000 and what we got? Whenever we are paying your coupon annually, we got the price somewhere around 938.55 or something and whenever we are paying the coupon semi-annually we got 937.6889.

So, that means the value of the bond is less than the par value of the bond.The value of the bond what we got that is basically lesser than the par value of the bond, why we get that?

Because, your coupon was less than the discount rate, your coupon was 9% and your interest rate or the discount rate (R) was 10%. Because the coupon was less than the discount rate that means your V is less than F, we got the value is of the bond is less than the par value.

So in this case, we call it the discount bond. But, if your coupon is equal to the par value, let coupon is let 9% let the discount rate is 9% in that case, the market price of the bond will be equal to the par value of the bond that is called the par bond. If you are writing the bond is issued at par that means by default your coupon is equal to your discount rate then if your coupon rate is greater than the discount rate, then you will find the market value of the bond will be more than the par value of the bond which is called the premium bond. So, C R is the coupon rate, R is the discount rate, V is the value of the bond and F is the par value of the bond. So, accordingly on the basis of the relationship between the coupon and the discount rate whether coupon is more than or less than the discount rate.

Accordingly, the particular bonds are defined as whether it is a discount bond or whether it is a par bond or whether it is a premium bond. Already we have seen that the bond we have taken in the example is basically a discount bond. Because the coupon is less than the discount rate that is why; the market value of the bond is less than the par value of the bond.

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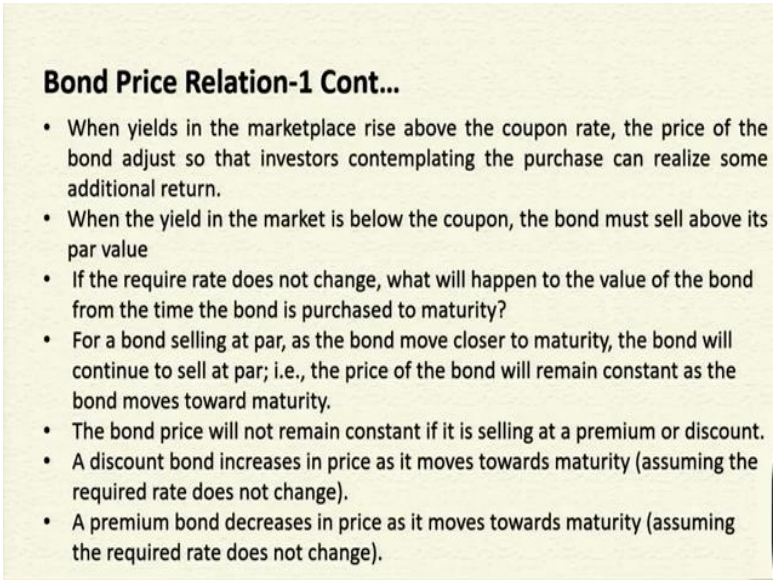

So, therefore, whatever thing we have discussed so when the yield in the marketplace rises above the coupon rate the price of the bond basically adjusts in such a way that investors are trying to realize that purchase with some additional return and when the yield in the market

rate is below the coupon the bond basically always sell above its par value. So, if the required rate does not change then what will happen to the value of the bond from time to time?

From the time the bond is purchased to the maturity for a bond which is selling at par, as the bond will move closer to the maturity, the bond will continue to sell at par. That means the price of the bond will remain constant as the bond move towards the maturity. So, whatever interest rate will be there if you are holding the bond up to the maturity then what will happen that the bond will always continue to sell at the par closer to the maturity period.

And the bond price will not remain constant if it is selling at a premium or the discount. So, a discount bond increases its price as it moves toward the maturity assuming that the required rate does not change and a premium bond decreases its price as it moves towards the maturity assuming that the required does not change. So, these are the different characteristics that basically you have to keep in the mind.

So, that means the bond which is selling at par if; you are going towards the maturity always the bond will sell at the par that means the price of the bond will remain constant as the bond moves towards the maturity. But a discount bond, it will increase the price as it moves towards the maturity assuming the required rate does not change and the premium bond decreases the price as it moves towards the maturity (assuming the required return does not change). That 3 things basically you have to keep in the mind. So, that is basically the first bond price relationship what we can get.

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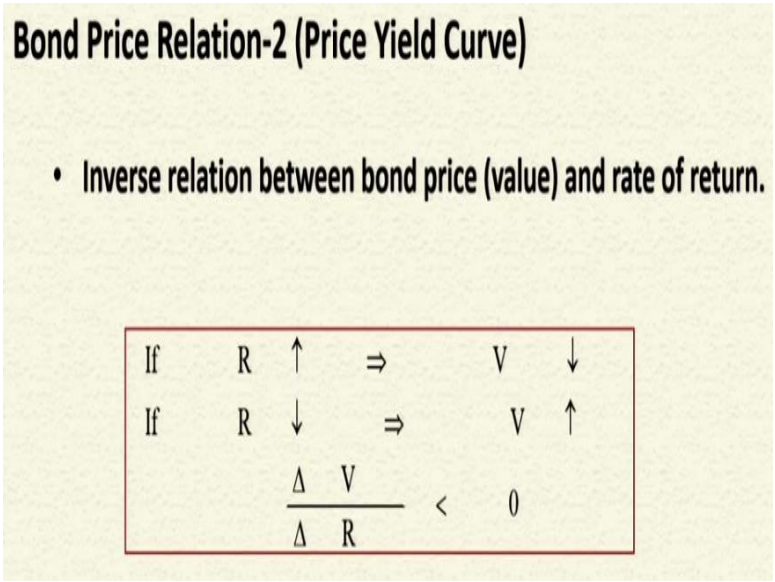

The second one is already we have observed that. There is an inverse relationship between the bond price and the rate of return. If your discount rate will increase, then, the value of the bond will go down keeping your coupon constant. Obviously, coupon is constant which is mentioned at the beginning and if your discount rate will decrease, then the value of the bond will increase.

That means change in the value of the bond with respect to change in the discount rate is less than 0. Delta V by Delta R is less than 0.

$\frac{\Delta V}{\Delta R} < 0$,

Where, ΔV = change in the value of the bond

ΔR = change in the discount rate

So, if R is increasing, the value is declining. If R is declining, the value is increasing. So, in that case, we can have an inverse relationship between the price of the bond and the rate of return.

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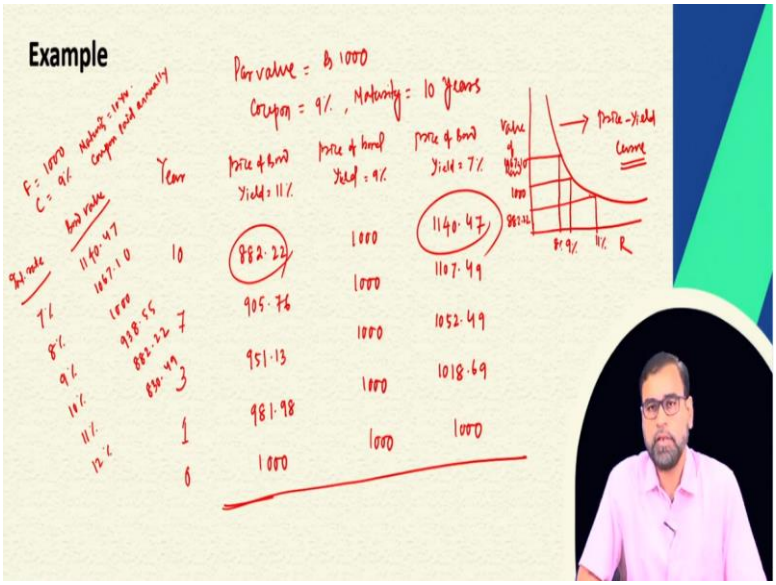

I will show you that in our same example, if you take that how this thing is basically is going to be changed. For example, we have taken the par value that is let 1000 then your coupon is 9% and maturity is 10 years. Right? Let with the different yield we will calculate the particular bonds price. Let I will take year. Let it the price of bond where the yield is 11%. Here your price of the bond when the yield is 9% and then your price of the bond when your yield is let 7 %.

If you see let years to maturity is 10 then if you go by the same way, whatever way we have calculated the value of the bond, here only R I am changing that is 11% you will find if your coupon is 9% your yield is 9 % then the value of the bond will be 1000. So, if you calculate your value of the bond whenever yield is 11% you will get 882.22. But, whenever it was 7% you will get 1140.47.

That means if your yield has become 7 which is lesser than the coupon then the value of the bond has gone up above the par value which is 1000 Rupees. Let if its maturity is 7 years, it will become 905.76 this will not change; It will be 1107.49. If it is let 3 years you will get 951.13. this will remain the same but this will become 1052.49. let it is 1 year then it is 981.98 but it will become same 1000 and it is 1018.69.

If it is a 0 coupon bond let 0 maturity period is not 0 coupon bond let the maturity period is 0 and that time everywhere the price of the bond will become 1000. So what we have seen? If your yield is increasing, then, the value of the bond is going down. But whenever your yield is decreasing, the value of the bond is going up. So, that is why there is an inverse relationship between the yield and the value of the bond.

So, if you basically see this thing that is called the price yield relationship or that is called the price yield curve. So the yield and value of the bond has a kind of inverse relationship always we can we can observe. OK? So, here what basically we have seen whenever your interest rate is changing let us see that how this price yield curve basically looks like.

See price yield curve basically looks like this way it will basically looks like this.OK? So here, wherever your let this is your R and this is your value of the bond it is basically the convex to the origin. So, let if it is 1000 then it will be definitely 9% which is equal to the coupon rate. Let it has become 11% then, you will find that this particular value will become how much? 882.22.

So, now, how basically we can draw this? We can draw this in this way what basically you can do you can take the interest rate, you can calculate the bond value, you can take the interest rate and the bond value. And here, we have taken your face value is equal to 1000 coupon is equal to 9% let maturity is 10 years and coupon basically paid annually. So, then at 7%, it will be already you have calculated 1140.47. If you take 8%, it will be 1067.10, 9% 1000, 10%, it will be 938.55.

If you recall we have already calculated this 11%, we have 882.22, 12%, 830.49. So, accordingly, if you draw this, then, you will find a curve like this. Let this is at 9% let this is 8% then this value will basically at rate of 8%, it will be 1067.10 like that. So, that is basically called the price yield curve. So, the price yield curve basically shows that there is an inverse relationship. It shows basically the relationship between the yield and the value of the bond and that relationship is basically inverse the convex curve basically proves that.OK.

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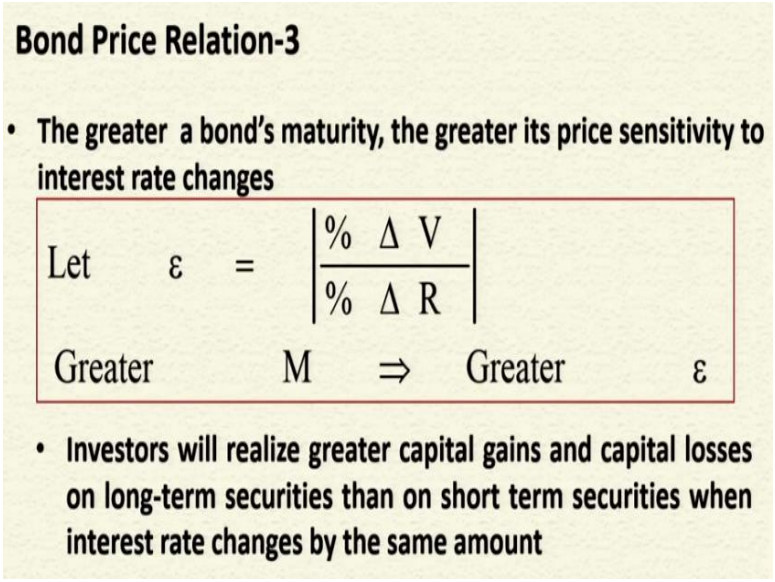

So, then, another relation we can also see the greater the bonds maturity if the bonds maturity is increasing, then, greater its price sensitivity to the interest rate change. Greater will be the aim. greater the epsilon. Epsilon is the percentage change in the value of the bond divided by percentage change in the interest rate. So, here, the investor will realize the greater capital gains and capital losses on the long-term securities than the short-term securities when the interest rate changes by the same amount. OK?

So, that means the sensitivity of the longer-term maturity bonds with respect to change in the interest rate is always more than the shorter modes.

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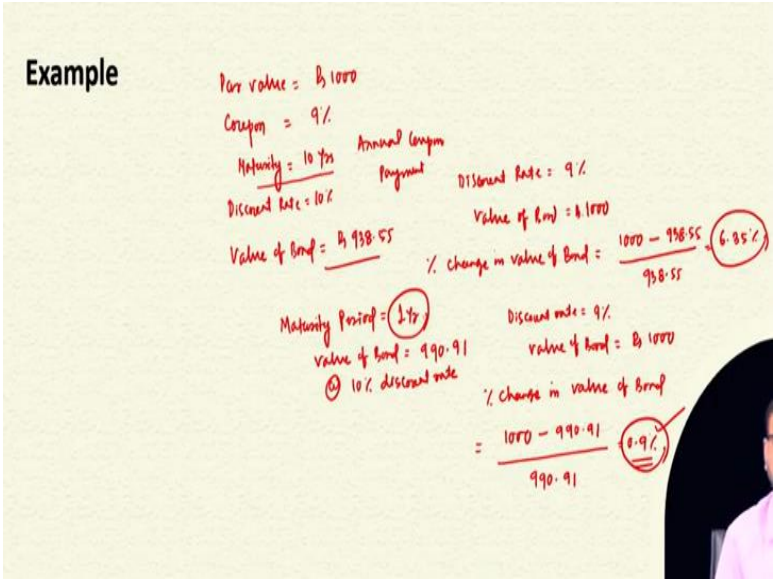

So, let us take one example in this case also we will continue with that. Our par value is let 1000 or the face value is 1000, coupon is let 9 %, maturity is 10 years and discount rate is 10%. So, in this case, what is the value of the bond? Already we have calculated this value of the bond we have calculated that is rupees 938.55 coupons is paid annually annual coupon payment. Annual coupon payment.

Let your discount rate has become 9% from 10% it has come down to 9% then what will be the value of the bond? For simplicity, I am taking this. The value of the bond become 1000 because now the coupon is equal to the discount rate then, what is the percentage change in the value of the bond?

Percentage change in the value of the bond= $\frac{1000-938.55}{938.55} = 6.85\%$

Let another bond you take another bond you take for simplicity, let let the maturity period is 1 year. If the maturity period will be one year, then what is the value of the bond? It will be 990.91 at the rate of 10% discount rate. Now, the discount rate has changed to 9% then the value of the bond will become 1000? value of the bond become 1000? Then, what is the percentage change?

Percentage change in the value of the bond = $\frac{1000-990.91}{990.91} = 0.9\%$

So, whenever the bonds maturity was 10 years due to one percent interest rate change, the value of the bond has changed by 6.35%. But, whenever the bonds maturity period was 1 year, the percentage change in the value of the bond has become 0.9%. So, that means greater the maturity, greater the sensitivity of the change in the value of the bond or the price of the

bond basically will be more sensitive. With the same amount of interest rate change, the sensitivity is more for the longer-term maturity bond than the short-term maturity bond. OK.

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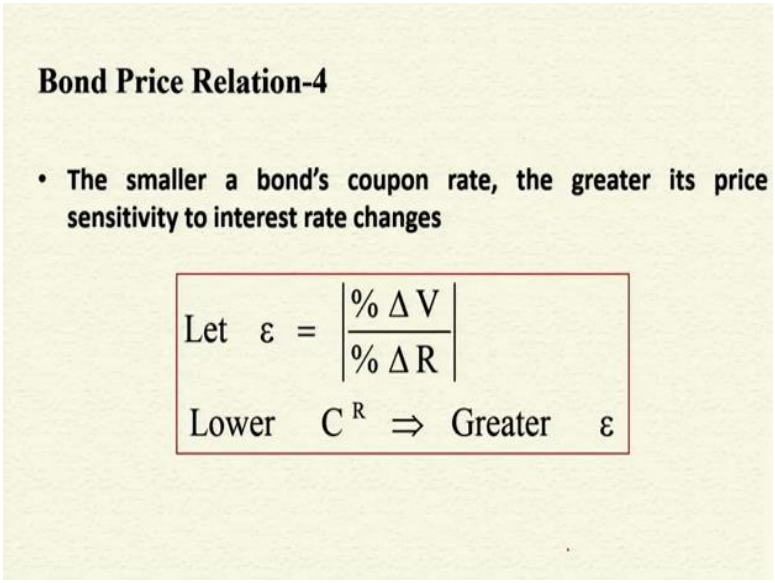

Then, another thing is the smaller the bonds coupon rate, greater its price sensitivity to interest rate changes. Bonds having low coupons are more sensitive to change in the interest rate in comparison to the bonds having high coupon rates. Lower the coupon rate, greater will be the sensitivity. Lower the coupon rate, greater will be the sensitivity. How basically it happens?

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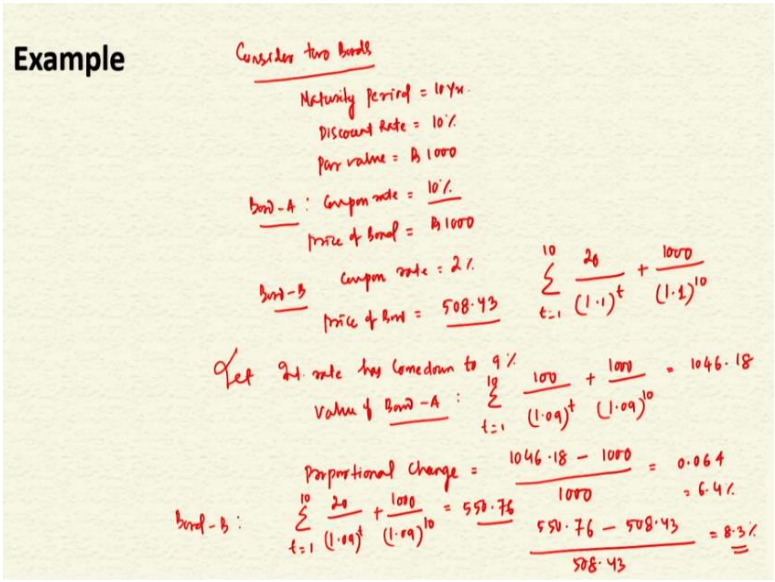

Let us take one example you consider 2 bonds. Right? Then, here, the maturity period is let 10 years, discount rate is 10% & par value let Rupees 1000. Let bond A has a coupon rate 10% then automatically the price of the bond will be same with the par value that is

1000.OK? Fine? Let the bond B the coupon is coupon rate is 2%. OK? Then, what is the price of the bond? Will be automatically you can if you calculate then it will be, let you will get 508.43.

How you got this?

$$\sum_{t=1}^{10} \frac{20}{(1.1)^t} + \frac{1000}{(1.1)^{10}} = 508.43$$

Let interest rate has come down to 9% then, what is the value of the bond-A?

$$\sum_{t=1}^{10} \frac{100}{(1.09)^t} + \frac{1000}{(1.09)^{10}} = 1046.18$$

Then what is the proportional change?

The proportional change = $\frac{1046.18-1000}{1000} = 0.064 \; or \; 6.4$

Now, you come to bond B. It is for the bond A. Now if you go to bond B, what is the value will be at a interest rate change to 9%?

$$\sum_{t=1}^{10} \frac{20}{(1.09)^t} + \frac{1000}{(1.09)^{10}} = 550.76$$

Then the proportional change = $\frac{550.76-508.43}{508.43} = 8.3\%$

So, that basically will give you 8.3%. That means the lower coupon bonds price is more responsive to a given interest rate change than the price of the higher coupon bonds. So, responsiveness is more for the lower coupon bonds than the higher coupon bonds.

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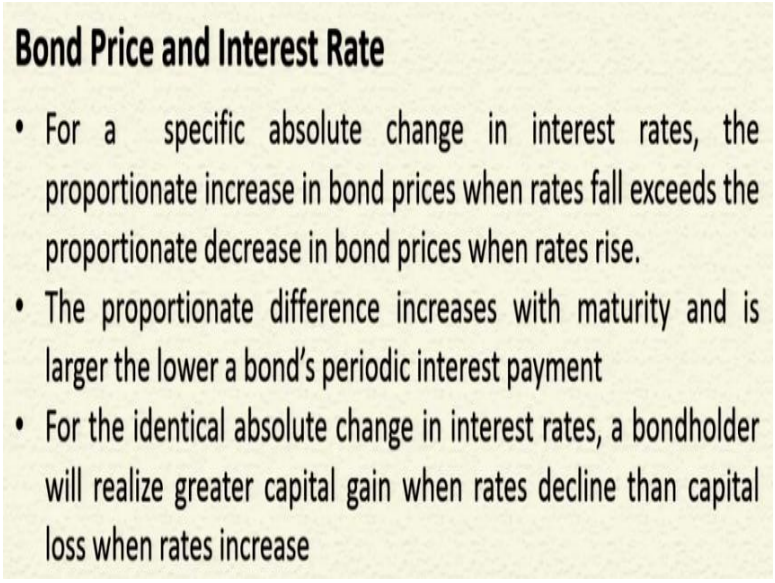

Then, we have the bond price and the interest rate relationship whatever we have seen for a specific absolute change in interest rates, the proportionate increase in the bond prices when rates fall exceeds the proportionate decrease in the bond prices when; in interest rise. So, the proportionate difference increases with maturity and is larger the lower bonds periodic interest payments. So, for the identical absolute change ininterest rates, a bond holder will realize greater capital gain when rates decline and capital loss when rates increase.

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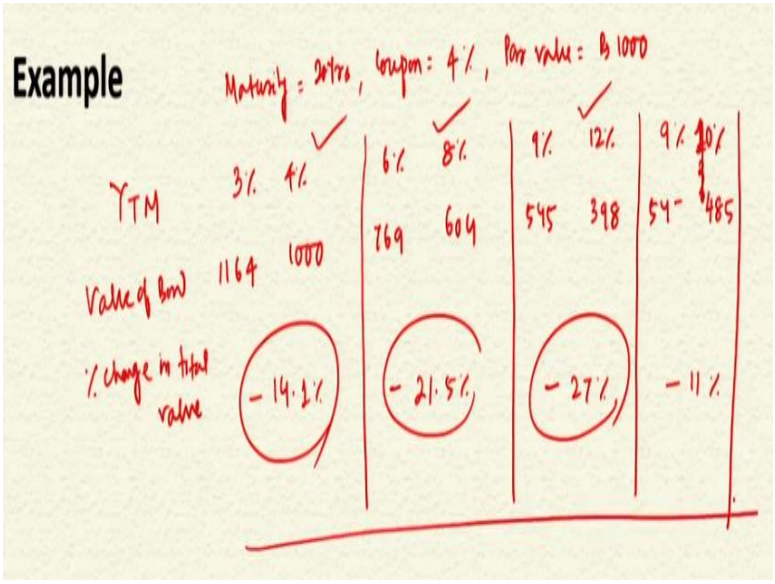

So, if you see that I will just give you one small example here. Let there is bond the maturity is 20 years, coupon is 4%, the par value is 1000.Right? Let I will give you the YTM(the yield to maturity), the value of the bond & percentage change in total value. So, let it was 3% it has changed to 4%. Let YTM was 6% it has changed to 8% ,let YTM was 9 % it has changed to 12% or let it was again 9% to 10%. 9% to 10%.

If you see here, in this particular bond at a rate of 3%, coupon is 4% that is fixed, we get 1164, at 4 % this is automatically 1000. At 6%, it is 769.At 8%, it will be 604. At 9%, it is 545. at 12%, it is 398 at 9% it is your 545 and at 10% it is 485. Then, if you change in the total value if you see, it is -14.1%, this is -21.5%, this is -27% this is -11 %. what basically it says? Its if you observe here, in the first 3 cases, this one the YTM percentage change is same 33.33%.

3 to 4% 4 - 3 by 3, 6 8 - 6 by 2, 8 - 6 divided by 6 so, if you see, the percentage change is basically 33.3%. Here, it is 100 basis point, it is 200 basis point, it is a 300 basis point. But, the percentage change is same. So, now, when the rates change by constant percentage, the change in the bond price is larger when the rates are at higher level. You see whenever it was 9% to 12%, the change was -27 but, here, it is -21.5.

But, here, it is -14.1%. But, percentage change in the interest rate is same. Right? The YTM basically has changed by constant 33.3% but now let you compare between; 3% to 4% & 9% to 10%. If you assume a constant basis point change in yield, you get the opposite results. Right? From 9 % to 10%, the change in the value of the percentage change in the total value of the bond is -11% but from 3% to 4% it is -14.1%. It is reverse.OK?

So, that is basically another interesting relationship what we can observe whenever we go for the one price relationship with respect to the change in the interest rate that is another thing basically what you can keep in the mind.

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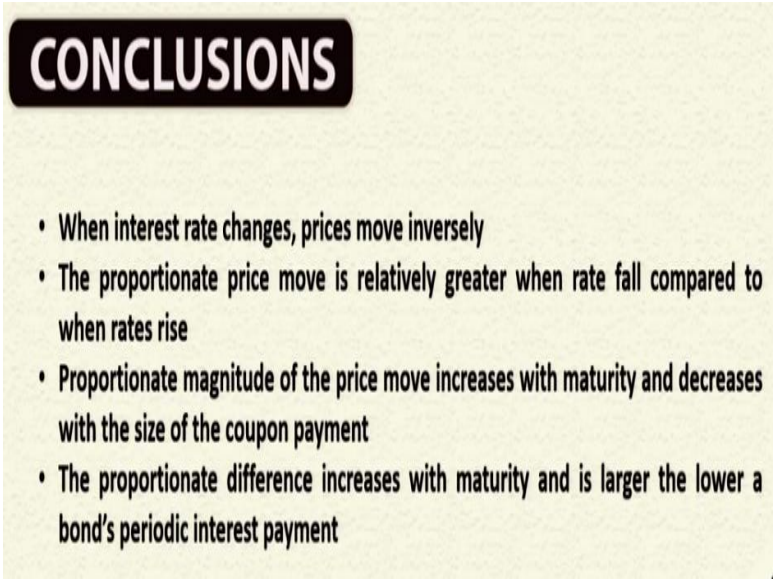

So, what basically we have discussed today? If you go for a conclusion, then, when the interest rate changes, price moves inversely and the proportionate price move is relatively greater when the rate fall compared to rates rise and the proportionate magnitude of the price move increases with maturity and decreases with the size of the coupon payments that we have observed and the proportionate difference increases with maturity and is larger the lower a bonds periodic interest payment that is nothing but the coupon.

So, that is what basically what we have observed that on the basis of the characteristics of the bond in terms of the coupon into maturity, if the interest rate will change, then, the bond percentage in the change in the bond prices will be different that is what basically you have to keep in the mind.

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So, for the references, you can follow this book you can get the broader idea about this particular concepts. OK? And the next class will be talking about the yield concepts or the return concepts of the bond and the other results related to that . Thank you.