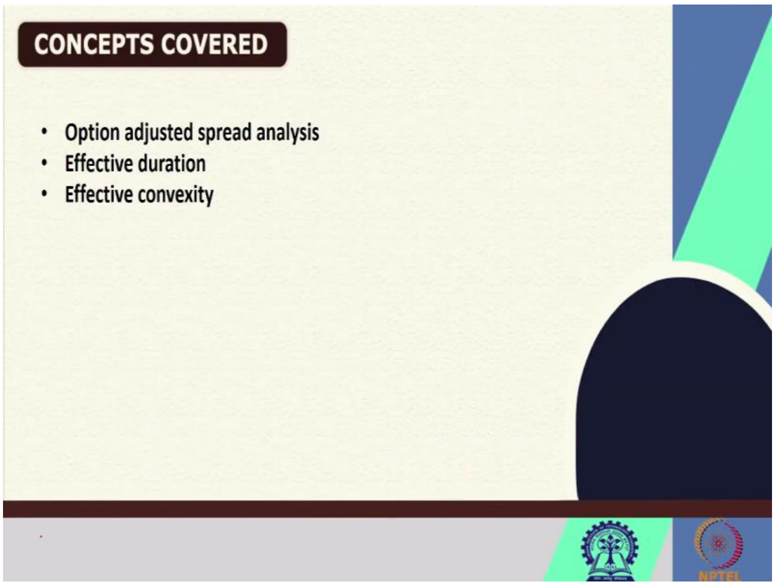
Management of Fixed Income Securities
Prof. Jitendra Mahakud
Department of Humanities and Social Sciences
Indian Institute of Technology – Kharagpur

Lecture – 45
Estimation of Binomial Trees – V

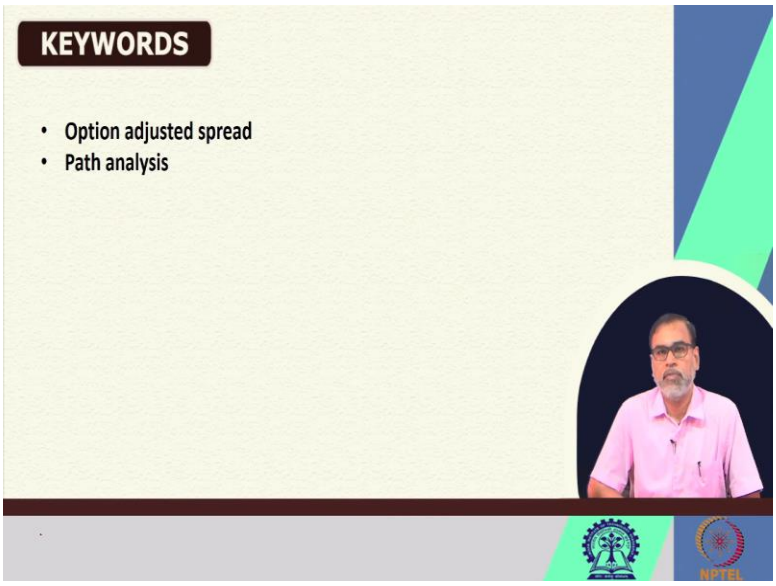
Welcome back. So, in the previous class we discussed about how we can estimate the binomial trees for the zero coupon bonds having the multiple periods and as well as the binomial tree estimation for the coupon bearing bonds and as well as the bonds having the call features. The coupon bearing bonds, but they are having the call feature. So, these are the three things we have discussed using the calibration model how the binomial interest rate trees can be formulated or can be estimated.

(Refer Slide Time: 00:54)



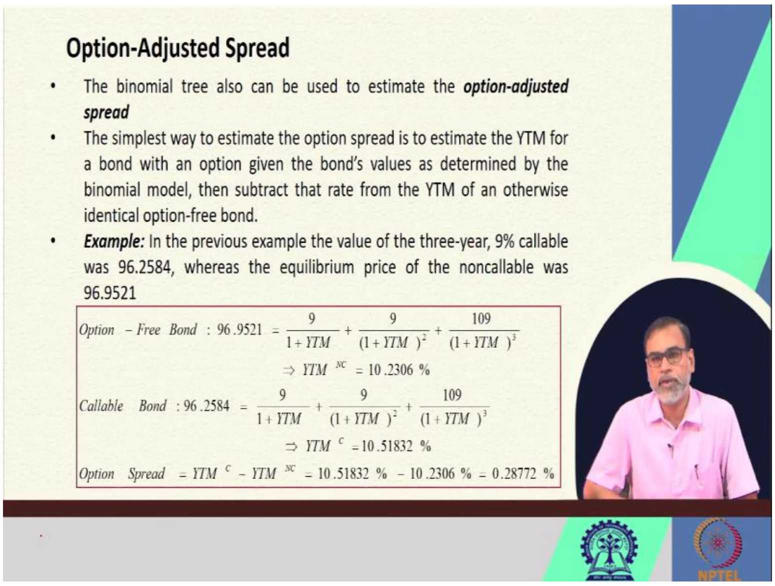
So, in this session we will be discussing about the concept called the option adjusted spread; the option adjusted spread analysis, the concept of effective duration and effective convexity. So, these are the different concepts what we are going to discuss in today's session.

(Refer Slide Time: 01:20)



And then we will come across the words like already I told you option adjusted spread or the path analysis. So, these kinds of words you will find in this particular class.

(Refer Slide Time: 01:38)



So, let us see that what do we mean by this option adjusted spread. Already we discussed that particular concept before also, but here what we are trying to see that the binomial interest rate tree also can be used to estimate this option adjusted spread. So, option spread is nothing, but the yield to maturity of a bonds having a option feature let call option feature minus the yield to maturity of a bonds having no options features or option features.

So, that is basically we call it the option adjusted or option spread generally we call it the option spread. So, whenever we try to calculate this option spread how we calculate? The simplest way of calculating the option spread is first to calculate this yield to maturity for a bond with an option given the bonds value as determined by the binomial tree and then subtract the particular yield from the yield to maturity otherwise identical option free bond.

So, using the binomial tree for a callable bonds or bonds having the option feature we find out the yield to maturity then you find out the yield to maturity of the bonds having no call feature or no option feature then take a difference between them you will find out the option spread. So, in our previous example whatever we have used if you see that we have three year 9% coupon bearing bonds which has a call feature.

And the call price was 98 if you recall so there what we have seen that whenever you find out the price of that option free bond we find the price is 96.9521. So, if 96.9521 is the price of that particular option free bond then if you want to solve this equation that price is equal to your cash flow divided by 1 + discount rate then you can find out your YTM that already extensively we have discussed whenever we are trying to calculate this yield into maturity.

Option-Free Bond: $96.9521= \frac{9}{1+YTM} + \frac{9}{(1+YTM)^2} + \frac{109}{(1+YTM)^3}$

$= YTM^{NC} = 10.2306\%$

Callable Bond: $96.2584= \frac{9}{1+YTM} + \frac{9}{(1+YTM)^2} + \frac{109}{(1+YTM)^3}$

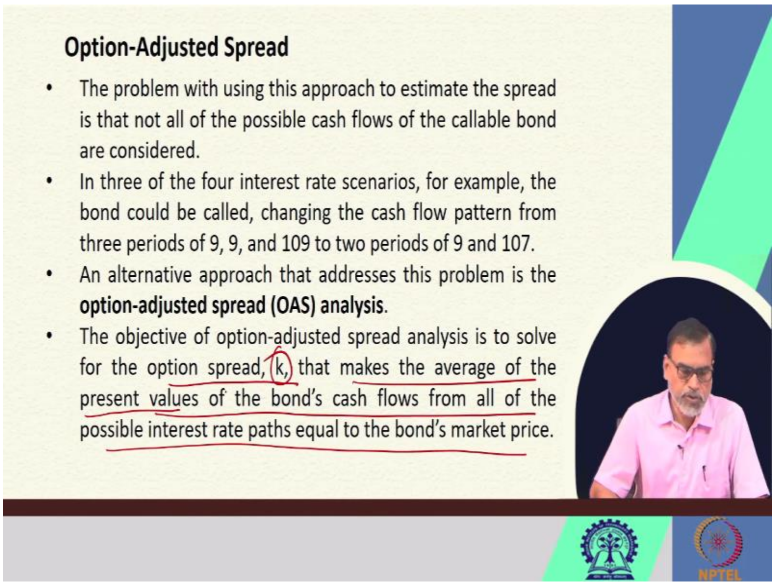
$= YTM^{C} = 10.51832\%$

Option spread= $YTM^{c} - YTM^{NC} = 10.51832-10.2306= 0.28772\%$

That is the option spread; that is the simplest way to calculate the option spread, but there is some issues here. There are some kind of difficulties, some kind of complexity always we can face whenever we are trying to find out the option spread in this way.

So, in that context we have find out what is that complexity and why that particular kind of difficulty we face whenever we calculate the option spread in this particular way.

(Refer Slide Time: 06:07)



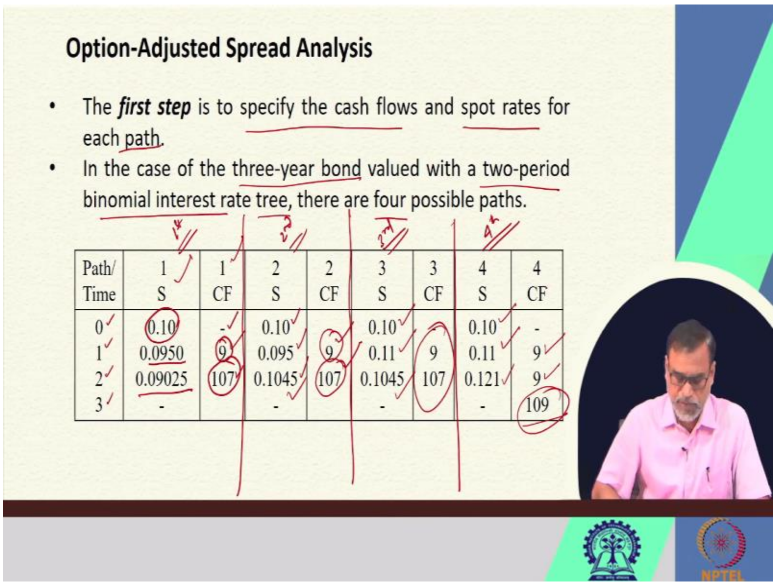
So, here what is the problem? The problem is using this approach to estimate the spread is that not all of the possible cash flows of the callable bonds are considered. Just now we have seen that we have taken the cash flow if you go back to our previous slide we have taken 99109 where the cash flow whatever we have considered, but in true sense if you see in three of the four interest rate scenarios, for example, the bond could be called, but the cash flow patterns can be 99109 or it can be also 9 and 107 because whenever the bond will be exercised or bond will not be exercised accordingly your cash flow will get **(()) (07:14)** because the bond call price was 98 bond coupon is 9.

So, wherever it will be exercised the cash flow will be 98 + 9 that will be 107 and some cases it will be 109. So, here in this case if you observe because the cash flows are varying the straight forward way of discounting the bonds using those cash flows are not feasible, are not practically applicable in this particular context. So, therefore to address that particular problem we have an alternative approach that is called the option adjusted spread analysis option adjusted spread that actually you have to keep in the mind.

So, the basic objective of the option adjusted spread analysis is what? We have to solve for the option spread let we are assuming the option spread is k then we have to find out that k which makes the average of the present value of the bonds cash flow from all of the possible interest rate paths equal to bond's market price and the same concept with the yield to maturity.

But the extra k we have to find out this option spread k which makes the average of the present value of the bonds cash flow from all of the possible interest rate path equal to the bonds market price. So, k basically is the option spread we have to find out which spread basically will satisfy this condition that is what is our objective. So, how we do this or how we can do this?

(Refer Slide Time: 09:22)



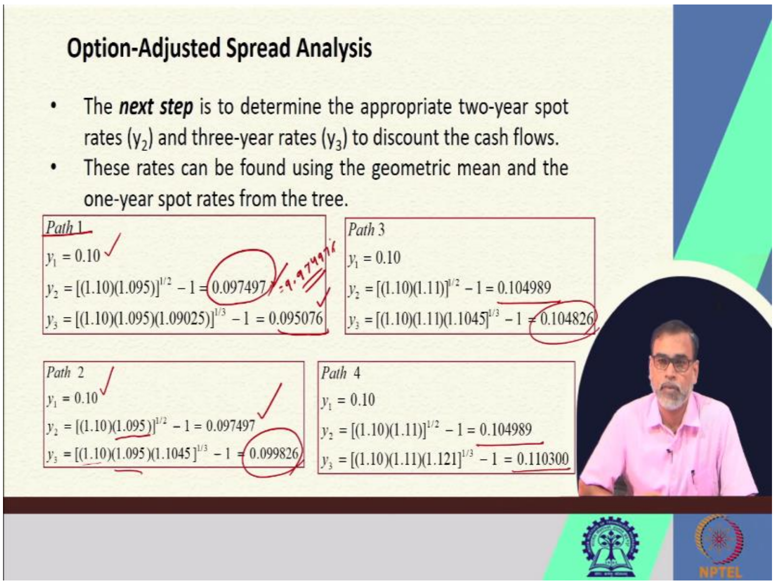
So, you see there are different steps we have to follow. So, in the first step of the option spread analysis or option adjusted spread analysis what we do specify the cash flows and spots rates for the each path. So, whatever example we have taken that is the three year bond valued with a two period binomial interest rate tree and there are four possible paths three year bond, 2 period binomial interest rate tree and four possible paths.

So, in these case what basically we have seen here. So, your time period is 0, 1, 2, 3. So, here if you see this there are cash flows in the different periods 10% was the initial discount rate obviously there is no cash flow then the discount rate has become either 9.5 or 9.025% and the cash flow either it will be 9, it will be 107 that is 98 + 9. So, this is for 1 period. Another period the rates can be 10% or another path we can say these are the different paths basically.

So, this is the first, this is the second, this is the third, this is the fourth; so these are the different paths. So, in the first path the cash flows are 9, 107 and the discount rates are 10%, 9.5%, 9.025%. Second path if you see your cash flow is 9 or 107 and discount rates are 10%, 9.5%, 10.45% which is in the node if you see that your 10% either become 9.5% or it can become 10.45 so this is the second path.

Third it can be 10%, 11%, 10.45% so this is another path and cash flow is 9 or 107 and in the fourth path if you observe the discount rates are 10%, 11% or 12.1% and the cash flows can be 9, can be 9 can be 109. So, first step you specify the cash flows, rates and path. So, these are the four paths what we have seen. So, after finding these paths what is the next step what we are going to do.

(Refer Slide Time: 13:09)



The next step is basically find out or determine the appropriate two year spot rates and three year spot rates to discount the cash flows. So, how we can find out the two year spot rate and three year spot rate go back to the term structure interest rate theory. So, according to that these are nothing, but the geometric mean of the different spot rates what basically we have estimated from this particular tree.

Path 1

$Y_1 = 0.10$

$Y_2 = [(1.10)(1.095)]^{1/2} - 1 = 0.097497$

$Y_3 = [(1.10)(1.095)(1.09025)]^{1/3} - 1 = 0.095076$

Path 2

$Y_1 = 0.10$

$Y_2 = [(1.10)(1.095)]^{1/2} - 1 = 0.097497$

$Y_3 = [(1.10)(1.095)(1.1045)]^{1/3} - 1 = 0.099826$

Path 3

$Y_1 = 0.10$

$Y_2 = [(1.10)\ (1.11)]^{1/2} - 1 = 0.104989$

$Y_3 = [(1.10)\ (1.11)\ (1.1045)]^{1/3} - 1 = 0.104826$

Path 4

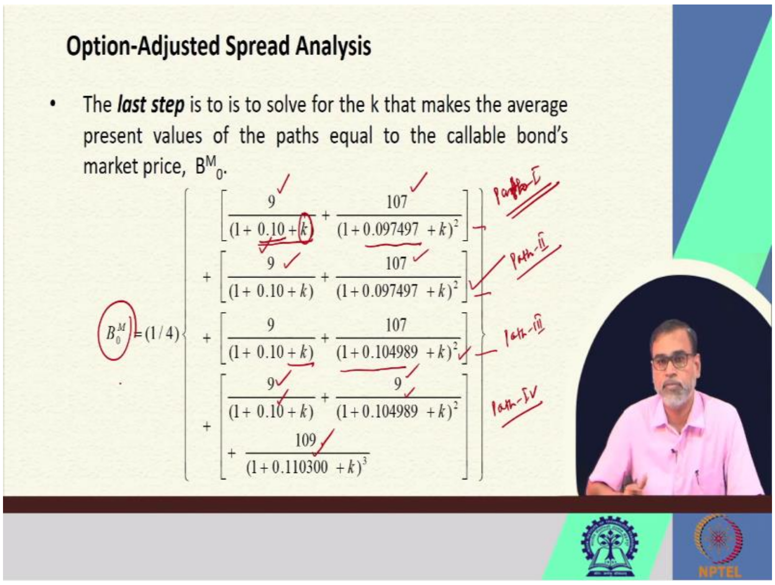
$Y_1 = 0.10$

$Y_2 = [(1.10)\ (1.095)]^{1/2} - 1 = 0.097497$

$Y_3 = [(1.10)\ (1.095)\ (1.121)]^{1/3} - 1 = 0.110300$

So, now in the second step we got the appropriate two year, three year spot rates which will be used to discount the cash flows. Now, what is the next step we are going to do.

(Refer Slide Time: 17:45)



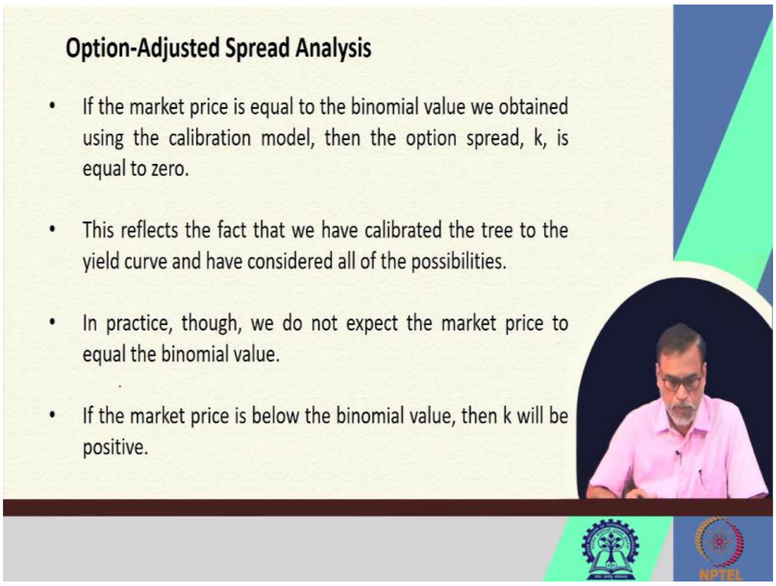
In the next step basically what we do we have to solve the k. In the beginning we have taken that assumption. We have to solve for the k with basic which makes the average present value of the path equal to the callable bond market price. So, now the rates are available with you, cash flow is available with you then you can find out the market price, how we can find out the market price in the different path.

$$B_0^M = (1/4)\left\{ \begin{aligned} &\left[\frac{9}{1+0.10+k} + \frac{107}{(1+0.097497+k)^2}\right] \\ &+ \left[\frac{9}{1+0.10+k} + \frac{107}{(1+0.097497+k)^2}\right] \\ &+ \left[\frac{9}{1+0.10+k} + \frac{107}{(1+0.104989+k)^2}\right] \\ &+ \left[\frac{9}{1+0.10+k} + \frac{107}{(1+0.104989+k)^2}\right. \\ &\qquad \left. + \frac{109}{(1+0.110300+k)^3}\right] \end{aligned} \right\}$$

So, you find out the values in 4 paths take a average of that; that will give you the market value of that particular bond which will be equal to we have to understand that where the if you solve this which makes the average present value of the path equal to the callable bond market price; market price already you have. So, you solve that particular equation which k basically will equalize this cash flow with the market price of the callable bond that is 96 point something whatever we have estimated before.

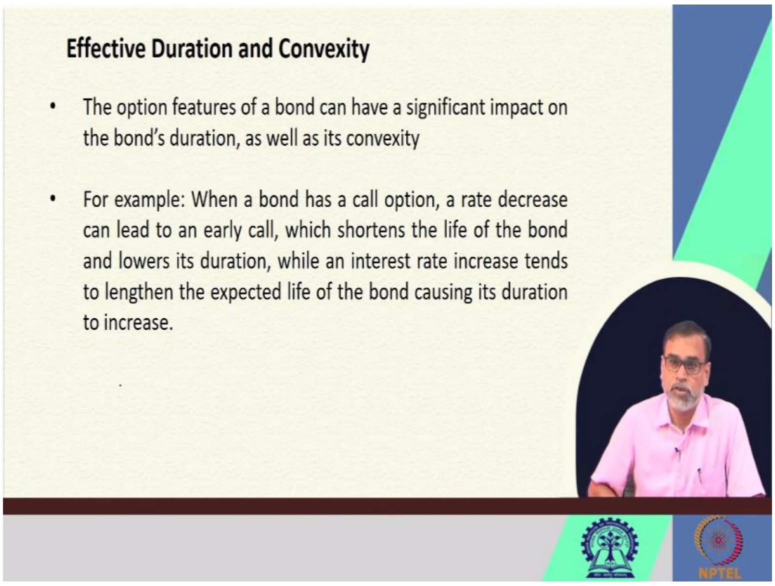
So, from there whatever k basically we will find that is the option spread. So, that is more realistic and practical way of calculation of the option spread instead of directly using that particular cash flows in finding out that particular path. So, that is basically called the option adjusted spread analysis that can be also used for valuation of other types of bonds like mortgage backed securities and other things that we will discuss further.

(Refer Slide Time: 22:58)



So, if the market price is equal to the binomial value we obtained using the calibration model then the option spread k = 0. So, this reflects that we have calibrated the tree to the yield curve and have considered all of the possibilities, but in practice we do not expect the market price always equal to the binomial value. So, if the market price is below the binomial value then the k will be positive that is basically what we call it the option adjusted spread analysis.

(Refer Slide Time: 23:46)

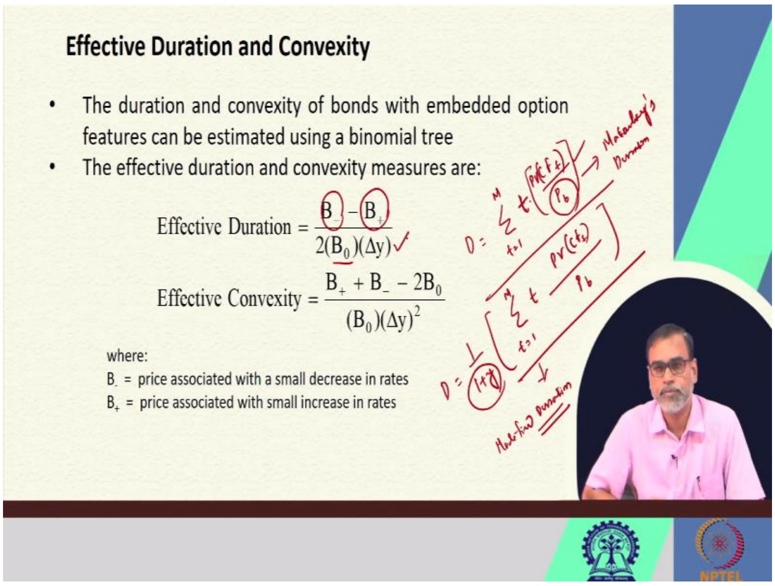


Then the option features of a bond can also have significant impact on the bonds duration and as well as convexity. The option features of a bond may also have the impact on the bonds duration and as well as the complexity, how? For example if you see when a bond has a call option if the bond has a call option then let the interest rate has declined. So, in that case it can lead to an early call the call option feature can be exercised.

Then what will happen which will reduce the life of the bond, let the maturity period of the bond is 10 years and the bond has been called back up to 6 years then automatically the bonds life will reduce the term to maturity of the bond has been reduced, if the maturity period will reduce then it will basically decline the value of the division it will lower the duration value, but if the interest rates increase then it will lengthen the expected life of the bond then causing its duration to increase.

So, that is why the call feature can have the impact or any kind of option feature can have the impact on the duration of the particular bond and as well as the convexity of the particular bond. So, that generally we call it the effective duration.

(Refer Slide Time: 25:41)



So, what is duration? We remember what do you mean by the duration? Duration is again a time measure and that generally is calculated by giving certain weights then how the weights are given so if you recall your duration of a bond is nothing, but let summation t = 1 to M your t into your cash flow t divided by the price of the bond if you recall these things. So, we find out this and this we basically giving the weight on the basis of the cash flow whatever we consider in that particular case.

So, there are certain other properties of the duration you might have recall it is basically your present value of the cash flow divided by the price of the bond and this is your t time. So, this is basically the weighted average of the bonds time period and the average is basically generally given on the basis of the proportion of the present value of the cash flow in that period with respect to the value of the bond.

So, in that case it is basically the weighted average maturity of the bonds cash flow on the present value basis that is the way we define the duration and you recall also the duration can be used as a price sensitivity measure. So, if you are using the duration as a price sensitivity measure generally we calculate this is what we call it the Macaulay's duration and if you are using this duration as a price sensitivity measure.

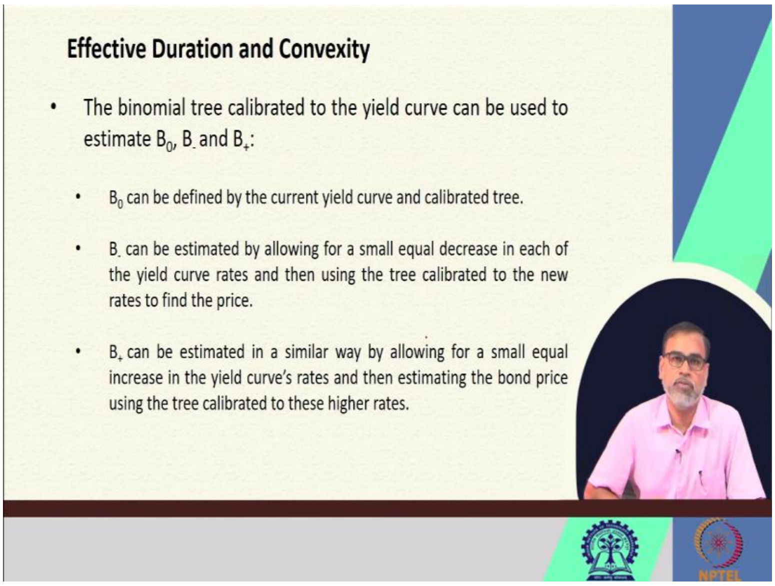
Effective Duration = $\frac{B_{-} - B_{+}}{2(B_0)\,(\Delta y)}$

Effective Convexity= $\frac{B_{-} - B_{+} - 2B_0}{(B_0)\,(\Delta y)^2}$

So, in that case if you recall we also discussed this part the effective variation is nothing, but you can find out the B – you can find B + then you have the B 0 then delta y **(()) (29:00)** change in the yield then you can find out the effective duration so B – means it is a price which is associated with small decrease in the rates and B + means price basically which is associated with the small increase in the interest rates.

So delta y means change in the interest rate. So, that is the way basically the effective duration can be calculated. So, if there is a features then how the bond price is going to be changed due to the increase in the interest rate or due to the decrease in the interest rate accordingly the effective duration can be estimated. These are the approximate formula for finding out the duration also that part we have discussed already.

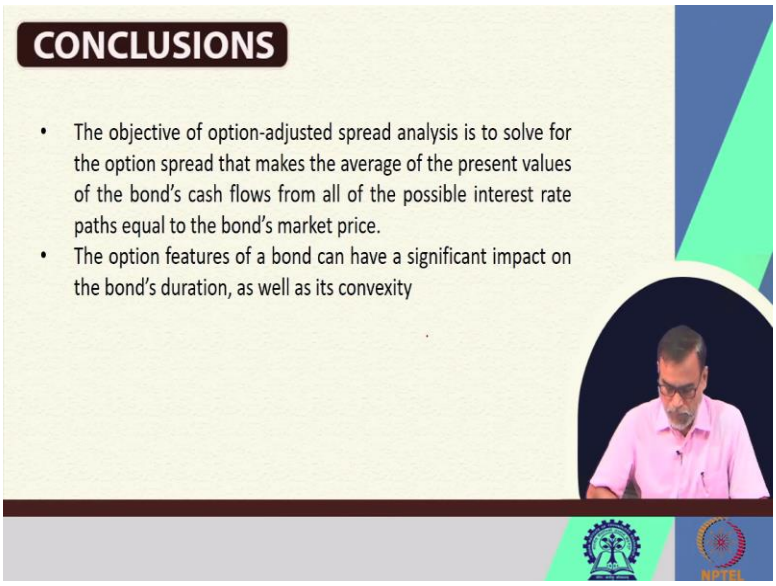
(Refer Slide Time: 30:28)



So, the binomial tree calibrated to the yield curve can used to estimate your B 0, B – and B +. B 0 can be defined by the current yield curve and the calibrated tree, B – can be estimated by allowing for a small equal decrease in each of the yield curve rates then using the tree calibrated to the new rates to find the price and B + can be estimated by a similar way by allowing for a small equal increase in the yield curve states.

And then estimating the bond price using the tree calibrated to these higher rates. So, the binomial tree model can be used to find out the B 0 B – and B + and then accordingly your effective duration and convexity can be calculated.

(Refer Slide Time: 31:19)



So, what basically we have discussed in this session the objective of the option adjusted spread is basically to solve for the option spread which makes the average of the present value of the bond cash flow from all of the possible interest rate paths will be equal to the bonds market price and the option features of a bond can have a significant impact on the bond duration as well as its convexity. So, these are the things what basically we have discussed in today's session.

(Refer Slide Time: 31:52)

So, these are the references. Thank you.