Management of Fixed Income Securities

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Module No # 08
Lecture No # 40
Binomial Valuation of Convertible Bonds

Good morning and welcome back. So, we were discussing about the valuation of the different bonds which have the embedded options. And we discussed about the bonds having the call feature, bonds having the put features, and as well as the sinking fund provisions. And there we have seen that how the call features or the put features or the sinking fund provisions are going to change the value of that particular bond.

Mostly we have used the binomial interest rate tree model for the valuation of this kind of securities. So today we will be discussing about another type of bonds which have some embedded options like the convertible features. So that is why we will see that how the binomial evaluation model is going to help us for valuation of the convertible bonds.

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So, we will be discussing about 2 things here valuation of the convertible bonds and as well as valuation of the callable convertible bonds. So, if the convertible bond also has the call feature, then we will see that how the valuation of those bonds generally carried out.

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So, you will come across certain keywords during this particular session. You have the conversion ratio, you have the conversion value, straight debt value, conversion price, and all these things. So, these are the some of the keywords that you will come across while discussing about this particular topic.

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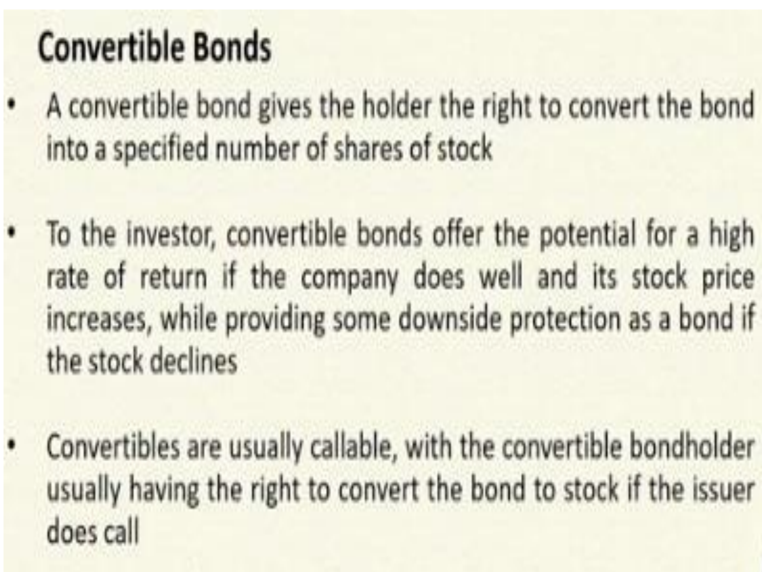

So let us understand first what do we mean by a convertible bond; the convertible bond is basically nothing but it particularly kind of a bond where the bond can be converted into equity after a specified point of time. So that's why we can say that it gives the holder the right to convert the bond into a specified number of stocks. The specified number of stocks or specified number of shares of stock can be converted from the bond.

So, if you are holding a bond instrument after certain point of time you have the right to convert that particular bond into equity, so that is basically what we call it the convertible bond. From the investor point of view the convertible bonds basically offers the potential for a high rate of return, if the company does well and its stock price basically increases. And also, there are some downside risks involved in the convertible bond and there are some kinds of we can say that downside protection also you can get if you are holding a convertible bond.

And what kind of protection basically at what time the protection you can get? The protection basically you can get whenever the stock price declines. So, if you look at the convertible bonds, they are usually callable in nature, where the convertible bondholder has the right to convert the bond to the stock if the issuer does the call. If the issuer is going for utilizing this call option, then the convertible bondholder has the right to convert that particular bond into the stock. So, from the investor point of view, it is a kind of advantage what the investor can get.

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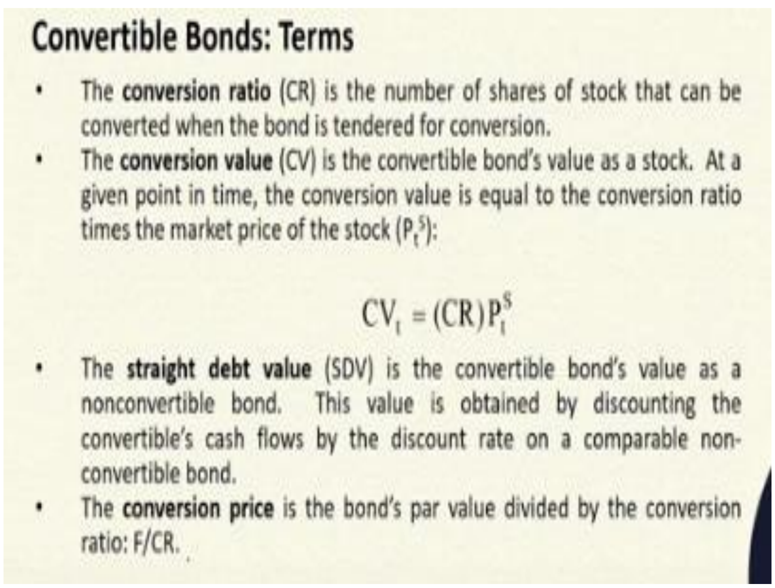

So, there are certain terms generally we come across while discussing about the convertible bond. The first one is the conversion ratio, so what the conversion ratio means? it is basically nothing but the number of stocks or number of shares of the stock which can be converted when the bond is tender for the conversion. Then we have the conversion value, the conversion value is nothing but it is the conversion or convertible bonds value as a stock.

So, at a point of time the conversion value is nothing but it is the conversion ratio times the market price of the stock. So, whatever the conversion ratio is there multiplied by the price of the stock at that particular point of time that will give you the conversion value.

$CV_t = (CR)P_t^S$

Then also we can use another term while discussing about the convertible bond that is called the straight debt value, it is basically nothing but the convertible bonds value as a non-convertible bond. So, this value is generally obtained by discounting the convertibles cash flow or convertible bonds cash flow, by the discount rate on a comparable non-convertible bond. Then we have the conversion price; the conversion price is nothing but it is the bonds par value divided by the conversion ratio. Let the conversion ratio is 10 the par value is 1000, then your conversion price will be 100. So, these are the different types of terms generally we come across whenever we discuss about the convertible bonds.

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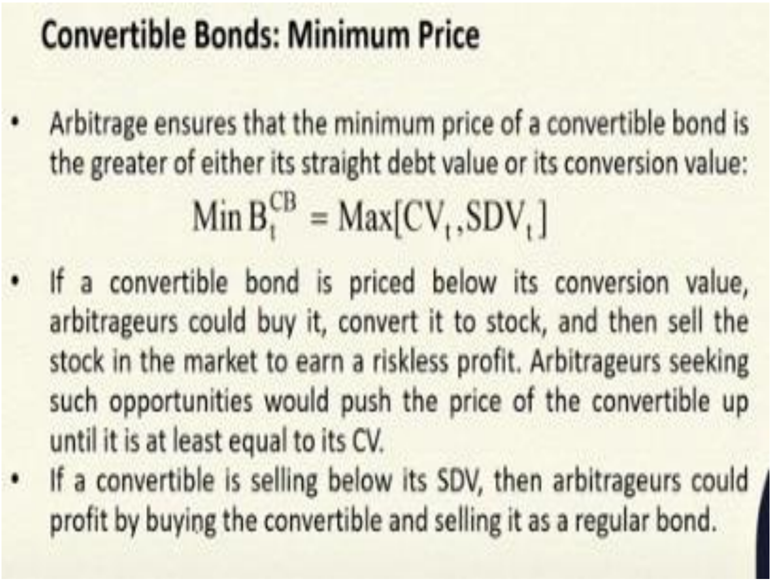

So then, whenever you talk about the convertible bond, generally whenever we look at the market the arbitrators generally always ensure that the minimum price of a convertible bond is the greater of either of its the straight debt value or its conversion value. So that means it is you have to look at the maximum of the conversion value or the straight debt value that basically we have to consider in that particular case.

$\text{Min } B_t^{CB} = \text{Max } [CV_t , SDV_t]$

So that means if a convertible bond is priced below its conversion value, then what the arbitrageurs can do; the arbitrageurs can buy it that particular bond, converts that particular bond into the stock then sell the stock in the market to earn a risk less profit. And generally, the arbitrageurs seeking such opportunities whenever the arbitrageur will go for this kind of operations that will push the prices of the convertible up until it is equal to its conversion value right.

So, if the convertible bond is priced below the conversion value, then arbitrageur can buy this convert that particular bond, into the stock and sell it in the stock market. And in that process, he can create the arbitrage opportunity or he can generate some riskless profit and, in this process, whenever the process will go on then obviously the demand for these activities will push the prices of the convertibles, and it will reach particular level which will be equal to the value of the equal to the conversion value.

So, if a convertible is selling below this SDV, then the arbitrageur could profit by buying the convertibles and selling it as a regular bond instead of converting into the stock they can create the profit by selling it as a regular bond. So, this is the way the mechanism generally works in the market.

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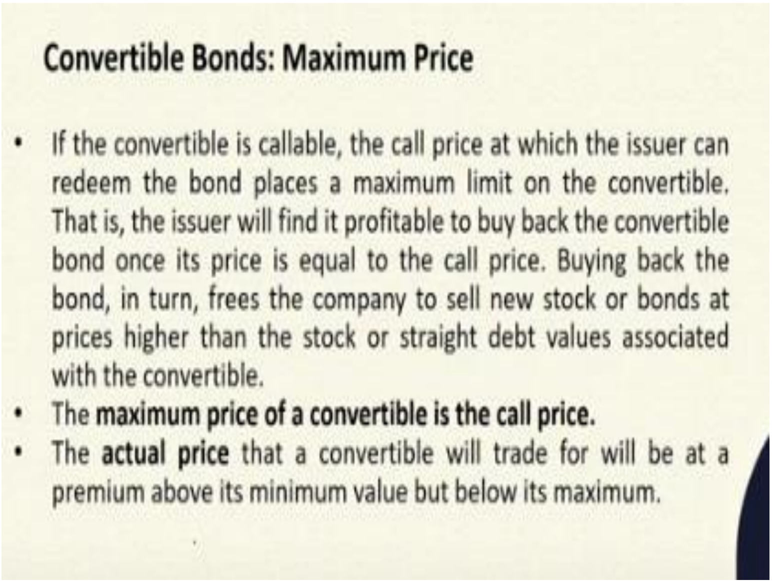

Then if the convertible is callable, let the convertible bond is callable, then the call price at which the issuer can redeem that particular bond generally places a maximum limit on the convertibles, or the convertible bonds. What does it mean, means the issuer will find it profitable to buy back the convertible bond once its price is equal to the call price.

So, buying back the bond generally what will happen once they will go for buy backing the bond it will basically freeze the company to sell the new stock or the bonds at prices higher than the stock, or the straight debt values which are generally involved or associated with these convertibles. So, the maximum price of a convertible bond is the call price.

And the actual price that a convertible will trade generally will be at a premium above its minimum value what is below its maximum value. That basically is the general observations always we see whenever we are dealing with the convertible bonds.

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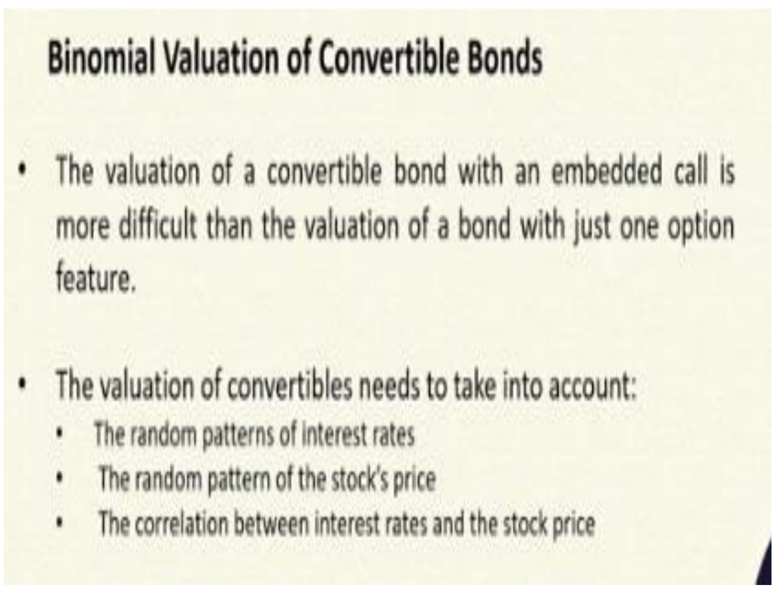

Let us see that how the convertible bonds valuation is taking place through this binomial models. The valuation of a convertible bond which has some embedded call option is more difficult than the valuation of a bond with just one option feature, either it is a convertible feature or it is a call feature. In comparison to the valuation of those kind of bonds, the bonds having a call feature and as well as a convertible feature is relatively difficult, the valuation of these kind of bonds is relatively difficult in nature.

Then whenever we are going for a valuation of the convertible bonds what generally we should look at; first of all, we have to look at the interested patterns. How this interest rate is going to move number one. And because the stock is involved in this particular process it is very much important to understand what is the random pattern of the stock prices, how the stock prices is moving, how the fluctuation of the prices is happening, that also we have to look at.

Then obviously the correlation between the interest rate and the stock prices that also has to be considered, we have to consider that thing also. So, these particular considerations or items are very much required whenever we are going for the valuation of the convertible bonds.

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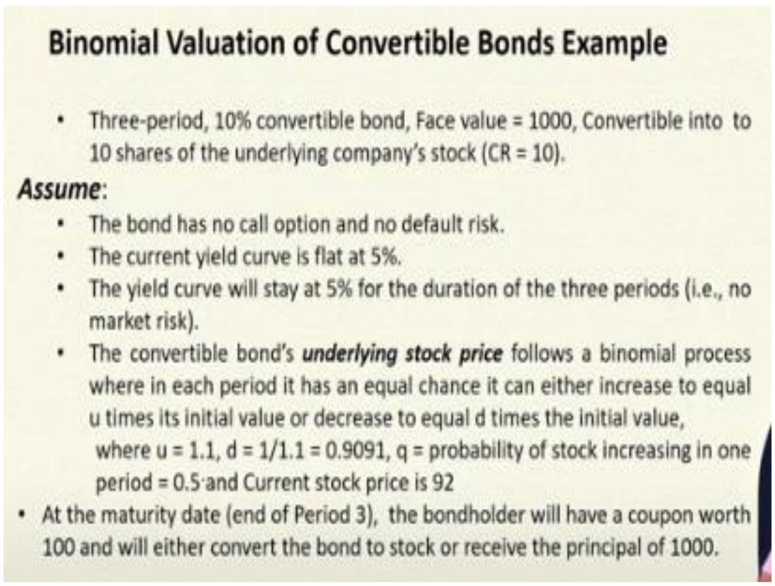

Now let us see that how this particular evaluation is done let us start with an example. Let there is a three-period 10% convertible bond, 10% means it is the coupon is 10%, face value is 1000 rupees and the conversion ratio is 10. That means the convertible into 10 shares of the underlying company stock the conversion ratio is 10. So here you are taking certain assumptions, then what are those assumptions you are taking: the bond has no call option and no default risk.

And current yield curve is flat and it is remaining at rate 5% and the yield curve is not going to change it will remain at 5% in this particular 3 years that means we are assuming no market risk involved. So, the convertible bonds underlying stock price let we are assuming follows a binomial process. So where in each period it has an equal chance, it can either increase which is the value will be equal to u times its initial value, or it is decreased to d times the initial value, that u and d concept that already we have discussed extensively.

And in continuation with that value whatever we have considered while discussing about the binomial interest rating model, we have assumed that u = 1.1 and d = 1/1.1. It is generally happens for the large samples that will see further it will be 0.9091. And q that we are assuming the probability of stock increasing in one period let that is 0.5 there is a 50% chance it may increase there is a 50% it may decreasing let the current stock price is 92 rupees ok.

So, at the maturity that means the end of the period of 3 or at end of the 3 years, the bond holder will have a coupon which will be 100 rupees and will either convert the bond into stock or receive the principal of 1000 rupees which is the par value of the bond right. So, these are the data or the assumption what basically we have considered to understand the binomial valuation of the convertible bond. Using this particular data or using this particular assumption, let us see how that particular valuation basically goes on.

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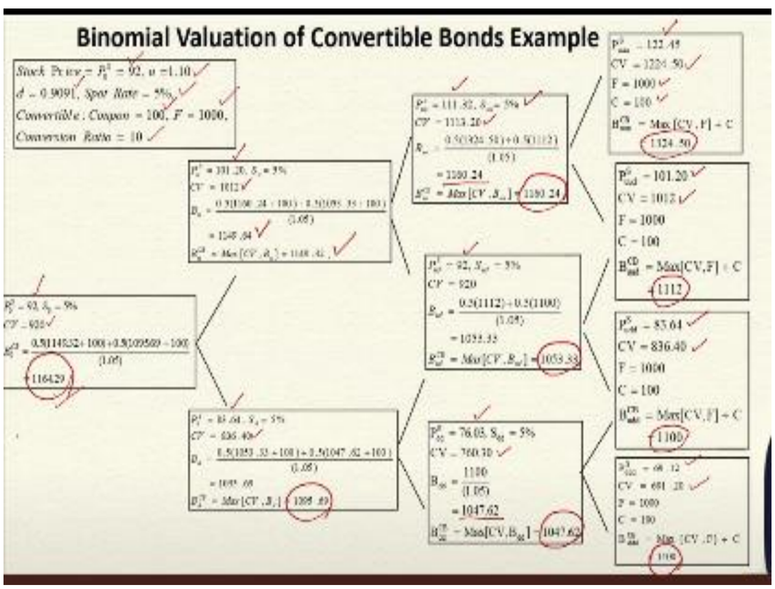

If you see this particular example whatever basically we have taken here, what we have taken the stock price is 92, u is 1.1 d is 0.9091, spot rate is 5%, convertible bonds coupon is 100 and par value or the face value is 1000, conversion ratio is 10 that we have taken. Now what basically will happen whenever it was basically consider in this particular context, how this interest rate or the price can move.

Let the price was 92 then the conversion value is 92 into 10 that is 920, then either it can increase up to a multiplier of 1.1 or it can decline up to a multiplier of 0.9091. So, in that case the price can become 101.2 or it can come down to 83.64 further. So obviously the conversion value will change that will be multiplied by 10 that will be 1012 it will be 836.40. Further it can go up to 111.32, again this(101.2) multiplied by 1.1 or it can decline also from this to 92 or again here it can reach up to 92 or it can decline to 76.03.

Further if you are considering again there is a chance to increase, then it can go to next period in the third period it can reach here the conversion value will become this, or it can reach here conversion value become this, or it can reach here conversion value will become this, or it can reach here the conversion value will become this right. So then here we will see that what basically happens here, so let us see that here the conversion value has become 1244.50 your face value of the bond is 1000 coupon is 100.

So now if you are calculating this the convertible bonds value it should be maximum of the CV or F + C, that means it will be 1000 + 100 that is 1100 or it will be 1244.50, whichever is the maximum. It is either maximum of the face value or the conversion value: the conversion value is 1244.50, face value is 1000. Then it has become basically you have to consider 1244.50 + your coupon what you are getting so that means 1224.50 + 100 so this value basically we have to consider.

$P_{uuu}^{S} = 122.45$ CV = 1224.50 F = 1000 C = 100

$B_{uuu}^{CB} = \text{Max}\,[CV, F] + C = \text{Max}\,[1224.50, 1000] + 100 = 1324.50$

Second case also we have seen that it is more than this face value that's why this value of the bond will be this, in this case if you see compare the conversion value with the face value, the face value is higher that is why we will get 1100 in this case also will be getting 1100, whichever is the maximum that basically we have already discussed.

$P_{uud}^{S} = 101.20$ CV = 1012 F = 1000 C = 100

$B_{uud}^{CB} = \text{Max}\,[CV, F] + C = \text{Max}\,[1012, 1000] + 100 = 1112$

$P_{udd}^{S} = 83.64$ CV = 836.4 F = 1000 C = 100

B_{udd}^{CB} = Max [CV, F] + C = Max [836.4, 1000] + 100 = 1100

P_{ddd}^{S} = 69.12 CV = 691.2 F = 1000 C = 100

B_{ddd}^{CB} = Max [CV, F] + C = Max [691.2, 1000] + 100 = 1100

Then we will see that how this particular price of the bond is calculated in this node in the second period. In the second period if you see already, we have taken that probability is 50 % the q = 0.5 that basically we have considered, then it will be 0.5 into 1324.5 + 0.5 into 1112 and you have to discount it at a rate of 5 % that is 1.05. So, in the first node in the second period the value of the bond can be 1160.24. So, if you consider between this conversion value and this this value whatever you are calculating then you will get your conversion value is 1113.20 that is 113.2 into 10. So now this one is the more than this; that's why the value we have to consider this.

P_{uu}^{S} = 111.32 CV = 1113.20 S_{uu} = 5%

$$B_{uu} = \frac{0.5(1324.50) + 0.5(1112)}{1.05} = 1160.24$$

B_{uu}^{CB} = Max [CV, B_{uu}] = Max [1113.20, 1160.24] = 1160.24

Second case if you look at it is 920 and it is 1053.33 that basically this value and this value we have considered and accordingly we have calculated this then we have to consider this.

P_{ud}^{S} = 92 CV = 920 S_{ud} = 5%

$$B_{ud} = \frac{0.5(1112) + 0.5(1100)}{1.05} = 1053.33$$

B_{ud}^{CB} = Max [CV, B_{ud}] = Max [920, 1053.33] = 1053.33

Third case again the if you look at your conversion value is this but your bond value in this case is this that's why we have to consider this then value will be this.

P_{dd}^{S} = 76.03 CV = 760.30 S_{dd} = 5%

$$B_{dd} = \frac{1100}{1.05} = 1047.62$$

B_{dd}^{CB} = Max [CV, B_{dd}] = Max [760.30, 1047.62] = 1047.62

So now you consider these values to calculate the price of the bond in the first period, so we have taken 0.5 into 1160.24 + 100 + 0.5 into 1053.33 then you discount it you get this value. Then you get this one is the maximum of this then you consider this.

P_u^S = 101.20 CV = 1012 S_u = 5%

$$B_u = \frac{0.5(1160.24 + 100) + 0.5(1053.33 + 100)}{1.05} = 1149.32$$

B_u^{CB} = Max [CV, B_u] = Max [1012, 1149.32] = 1149.32

Then again, the same thing you can get this value 1095.69.

P_d^S = 83.64 CV = 836.4 S_d = 5%

$$B_d = \frac{0.5(1053.33 + 100) + 0.5(1047.62 + 100)}{1.05} = 1095.69$$

B_d^{CB} = Max [CV, B_d] = Max [836.4, 1095.69] = 1095.69

Then finally you use this particular thing here you get this total value of the bond is 1164.29. So here what basically we have seen that the particular value of this convertible bond has become 1164.29, considering these assumptions whatever we have taken.

P_0^S = 92 CV = 920 S_0 = 5%

$$B_0 = \frac{0.5(1149.32 + 100) + 0.5(1095.69 + 100)}{1.05} = 1164.29$$

B_0^{CB} = Max [CV, B_0] = Max [920, 1164.29] = 1164.29

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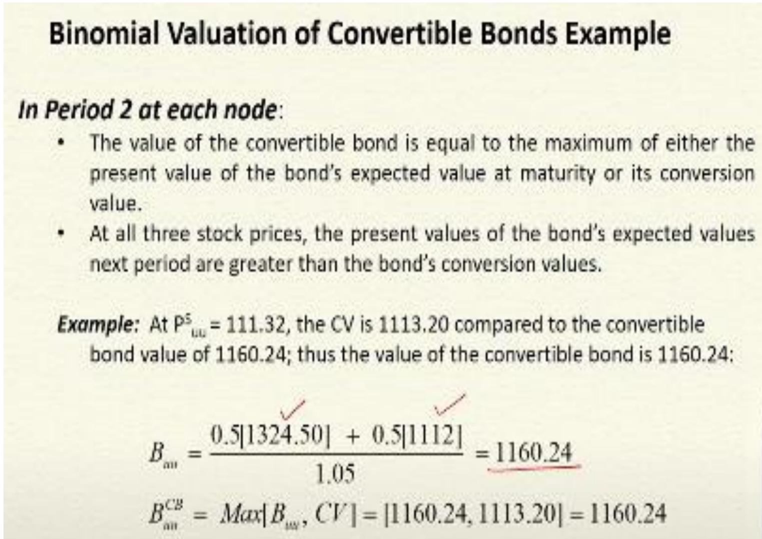

Let us already we have explained this I will just give you some cases in period 3 whenever the stock price is 122.45 this value has become this 1324.5 whenever in the period 3 at price of this(101.20) the value has become this(1120) and period 3 this(83.64 & 69,12) value has become this(1100) like that. Already we have explained this particular process in the exhibit whatever we have just now explained. So, like that in period 2 the value of the convertible bond is equal to the maximum for either the present value of the bonds expected value at maturity or its conversion value.

So here the at all three stock prices the present value of the bonds expected values next period are greater than the bonds conversion value. So that's why we have considered this 1324.50 then 1112 then we get this(1160.24) then finally we have to consider this(1160.24) value.

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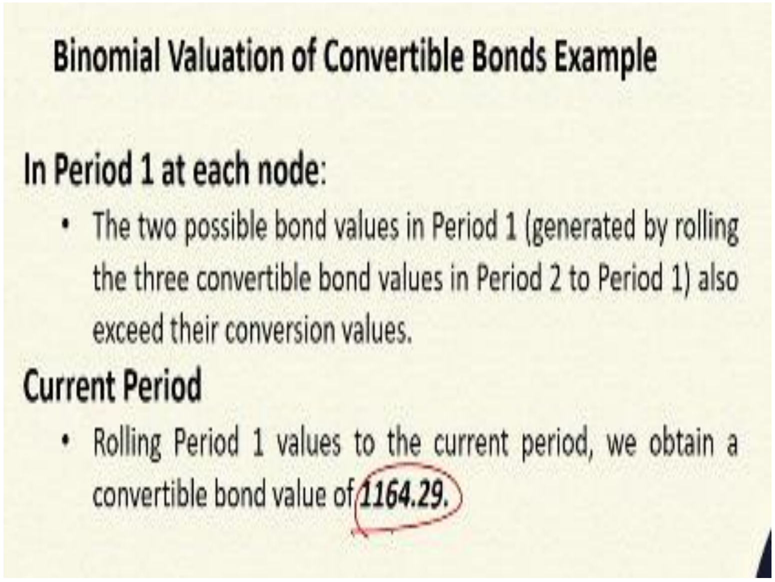

Then it will go on then in the period 1 at each bond the possible bond values in period 1 generally generated by rolling the 3 convertible bonds value in period 2 to the period 1, also exceed their conversion values. So finally, we obtain a convertible bond value that is 1164.29 that already we have discussed.

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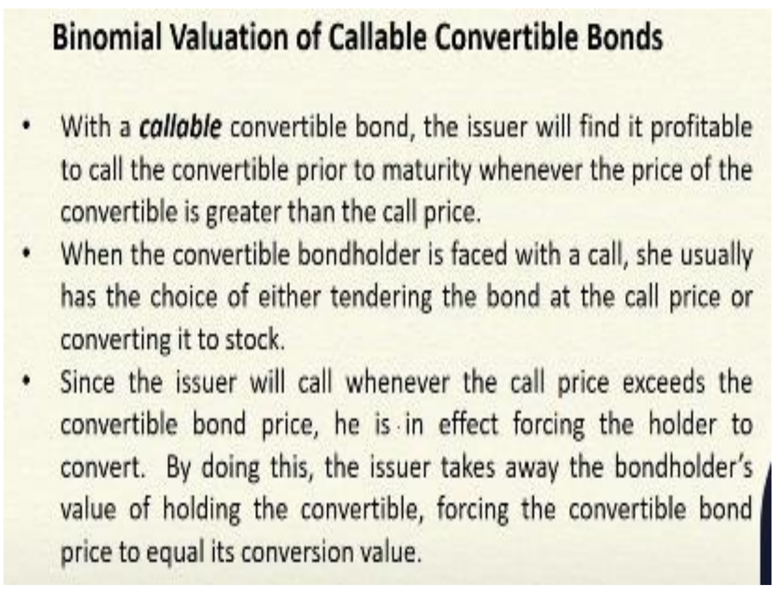

So now what basically we can do we have to see if the particular convertible bond has a call feature, then what will happen. So, with a callable convertible bond the issuer will find it profitable to the call the convertible prior to the maturity, whenever the price of the convertible is greater than the call price that already you know. That if the price of the convertible bond is greater than the call price then the issuer will find it profitable to call back the bond or utilize this call option.

So, when the convertible bondholder is faced with a call then what he or she usually do she has the choice either tendering the bond at the call price or converting into the stock, that only we have explained or we have discussed before. So, since the issuer will call whenever the call price basically exceeds the convertible bonds price in effect forcing the holder to convert it will force the bond investor to convert.

So, by doing this the issuer basically takes away the bond holders value of the holding the convertible and forcing the convertible bond price equal to its the conversion value, that basically can happen. So, in that process we will see that how that particular mechanism works.

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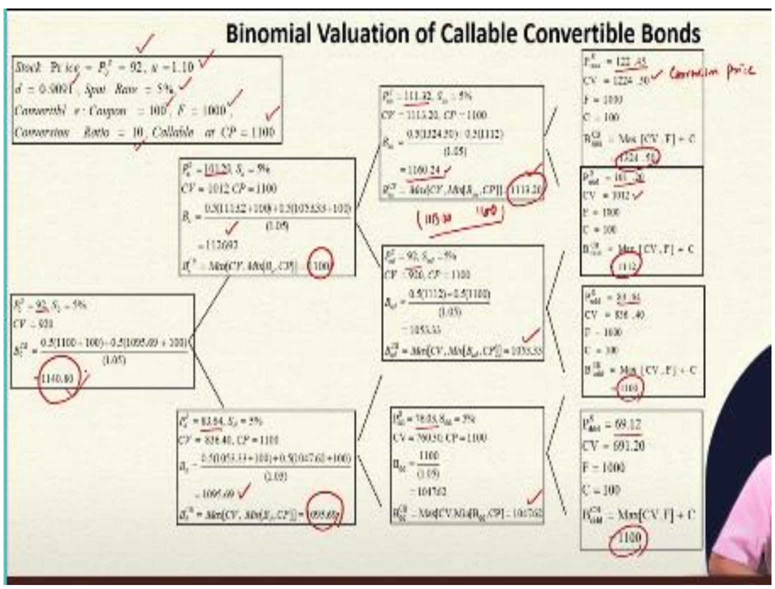

Let us take the example in this case we have already considered this values stock price is 92, u = 1.1, d = 0.9091, spot rate 5%, coupon is 100, face value is 1000, conversion ratio 10 and we have taken the callable at call price of 1100; that means the call price we have considered 1100. Already I have discussed with you that the price can reach here from this(92) to this(101.20) it can reach here(83.64) or it can reach in the second period, it can reach these 3 figures(111.32, 92, 76.03).

Then in the third period it can reach the four figures(122.45, 101.20, 83.64, 69.12). The conversion price is mentioned(1224.50), here is the conversion price(1012), so this is the conversion price(836.4)(691.2), so all the conversion prices are mentioned here. So, the maximum price is here(1324.50), this is here(1112), it is here(1100), it is here(1100). Now what will happen that, if you are calculating this, then how basically the value of the convertible bond with a call feature can be calculated.

So here the value is 1160.24. But here whenever we have said that it is more than the call price then the call option will be exercised. So, if the call option will be exercised, then the value will become that 1113.20 ok, whichever is the maximum between the conversion value and the minimum of the call price and the value of the call convertible bond. So, the value of the minimum of this means it is basically the it is the 1100 this part conversion value is 1113.20. Then if you take the maximum of this 2 then it is 1113.20. Like that here will find that it will be 1053.33, here it will be 1047.62.

CV = 1113.20 CP = 1100

$$B_{uu} = \frac{0.5(1324.50) + 0.5(1112)}{1.05} = 1160.24$$

$$B_{uu}^{CB} = \text{Max}\,[CV, \text{Min}[B_{uu}, CP]] = 1113.20$$

CV = 920 CP = 1100

$$B_{ud} = \frac{0.5(1112) + 0.5(1100)}{1.05} = 1053.33$$

$$B_{ud}^{CB} = \text{Max}\,[CV, \text{Min}[B_{ud}, CP]] = 1053.33$$

CV = 760 CP = 1100

$$B_{dd} = \frac{1100}{1.05} = 1047.62$$

$$B_{dd}^{CB} = \text{Max}\,[CV, \text{Min}[B_{dd}, CP]] = 1047.62$$

Then again if you discount it using this, particular values then it will become 1126.29, it will be 1095.69. Then finally we have considered here it will be 1100, it will be 1095.69. Then if you go for utilizing these values for the valuation then you will get 1140.80 right. So now the value of the bonds having the convertible bonds having the call feature the value has been changed, in comparison to only the bonds having the convertible features.

$P_u^S = 101.20$ $\quad$ $CV = 1012$ $\quad$ $S_u = 5\%$ $\quad$ $CP=1100$

$$B_u = \frac{0.5(1113.20 + 100) + 0.5(1053.33 + 100)}{1.05} = 1126.92$$

$B_u^{CB} = Max\ [CV, Min[B_u\ , CP]] = 1100$

$P_d^S = 83.64$ $\quad$ $CV = 836.4$ $\quad$ $S_d = 5\%$ $\quad$ $CP=1100$

$$B_d = \frac{0.5(1053.33 + 100) + 0.5(1047.62 + 100)}{1.05} = 1095.69$$

$B_d^{CB} = Max\ [CV, Min[B_d\ , CP]] = 1095.69$

$P_0^S = 92$ $\quad$ $CV = 920$ $\quad$ $S_0 = 5\%$

$$B_0^{CB} = \frac{0.5(1100 + 100) + 0.5(1095.69 + 100)}{1.05} = 1140.80$$

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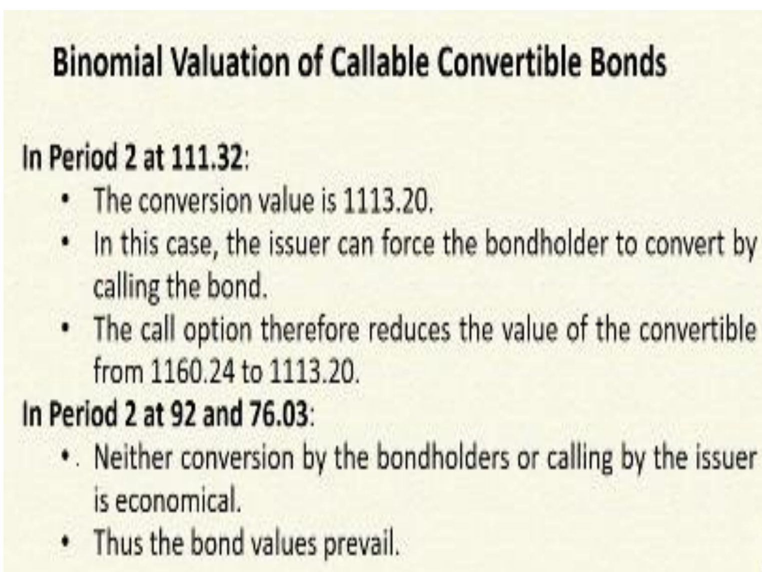

So, this is the explanation that already I have explained with you or we have discussed just now; so, in the different nodes how this particular value is going to be changed.

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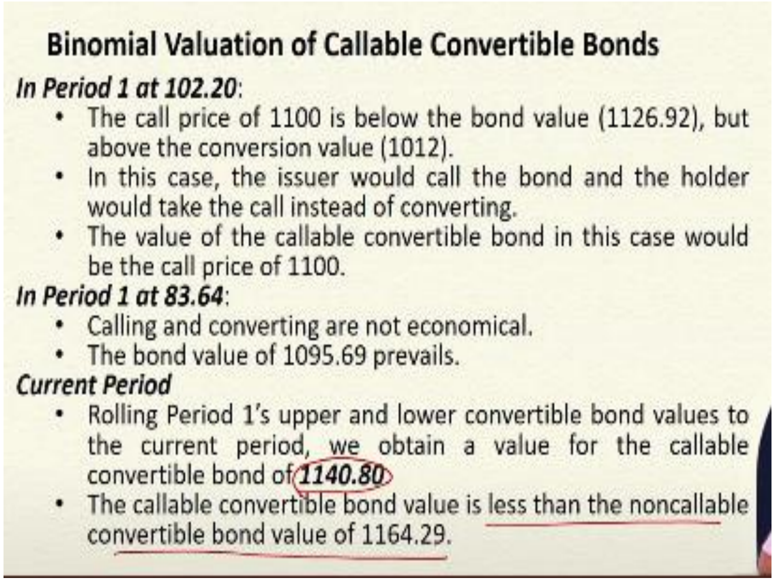

And the different period at the different nodes, then finally we have we find that it is 1140.80 and the callable convertible bond value is less than the non-callable convertible bond value of 1164.29.

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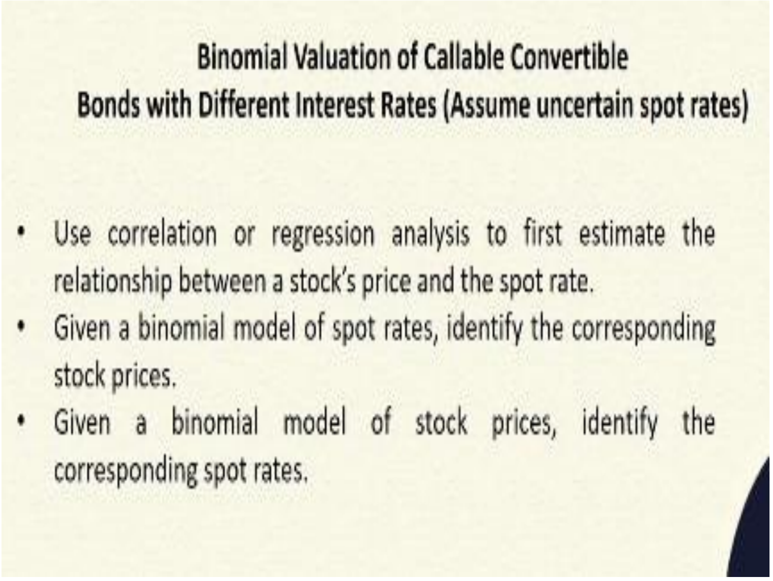

Then we have another kind of options here the binomial valuation of callable convertible bonds with the different interest rates, assume this interest rate is changing here we have taken 5%. So let you assume that the interest rate is changing so in that case what will happen. So first we use the correlation or the regression analysis to estimate the relationship between the stock price and the spot rate.

And given the binomial model of the spot rates identify the corresponding stock prices. Then given a binomial model of the stock prices identify the corresponding spot rates.

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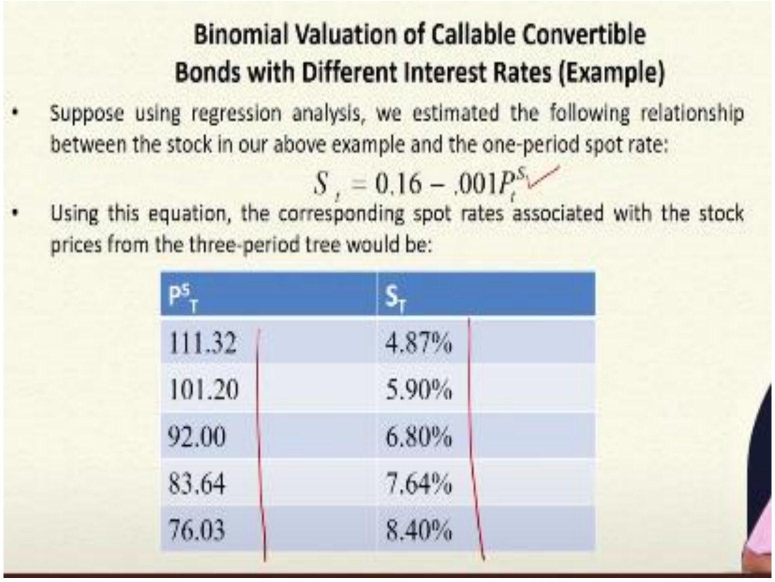

So, for example let you have run a regression, and you find your equation like this ok, between the stock and the spot rate. Then let using this equation the spot rate associated with the stock prices from the 3-period tree will be this. These are the stock prices and these are the spot rates you have estimated from this particular equation.

$S_t = 0.16 – 0.001P_t^S$

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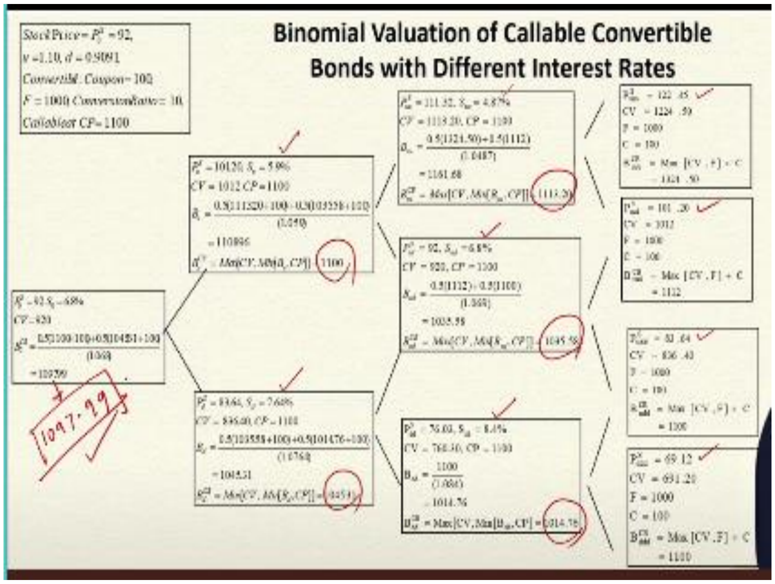

Then what you can do you can use that particular interest rates in the different stages other things basically, you are keeping the same if you see this one. But here what has happened we have our interest rate which was there previously 5% in this period it is 4.87 and this node it is 6.8, it is 8.4, it is 5.9, it is 7.64. So now your discount rate has been changed. So, if your discount rate has been changed, then automatically you will find that the prices also are going to be changed, so all these prices are going to be changed.

CV = 1113.20 CP = 1100 S_{uu} = 4.87%

$$B_{uu} = \frac{0.5(1324.50) + 0.5(1112)}{1.0487} = 1161.68$$

B_{uu}^{CB} = Max [CV, Min[B_{uu} , CP]] = 1113.20

CV = 920 CP = 1100 S_{ud} = 6.8%

$$B_{ud} = \frac{0.5(1112) + 0.5(1100)}{1.068} = 1035.58$$

B_{ud}^{CB} = Max [CV, Min[B_{ud} , CP]] = 1035.58

CV = 760.30 CP = 1100 S_{dd} = 8.4%

$$B_{dd} = \frac{1100}{1.084} = 1014.76$$

B_{dd}^{CB} = Max [CV, Min[B_{dd} , CP]] = 1014.76

P_u^S = 101.20 CV = 1012 S_u = 5.9% CP=1100

$$B_u = \frac{0.5(1113.20 + 100) + 0.5(1035.58 + 100)}{1.059} = 1108.96$$

B_u^{CB} = Max [CV, Min[B_u , CP]] = 1100

P_d^S = 83.64 CV = 836.4 S_d = 7.64% CP=1100

$$B_d = \frac{0.5(1035.58 + 100) + 0.5(1014.76 + 100)}{1.0764} = 1045.31$$

B_d^{CB} = Max [CV, Min[B_d , CP]] = 1045.31

$P_0^S = 92$ $CV = 920$ $S_0 = 6.8\%$

$$B_0^{CB} = \frac{0.5(1100 + 100) + 0.5(1045.31 + 100)}{1.068} = 1097.99$$

Then finally you will find that this price here is 1097.99, this is the basically the value. So now what basically we have observed that whenever the discount rate will change automatically the value of the bond is also going to be changed.

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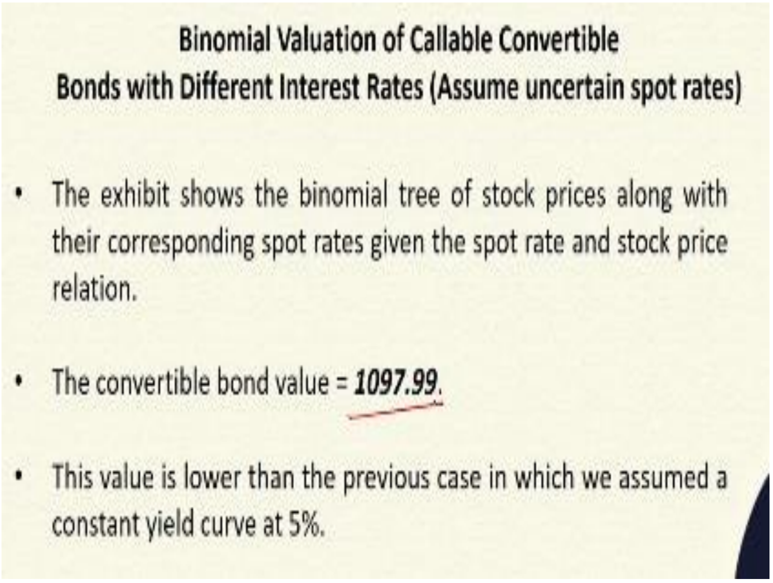

So, what this exhibit basically shows the binomial tree of a stock prices along with their current spending spot rates given the spot rate and the stock price relation and the convertible bond value has changed to 1097.99. So, the value is lower than the previous case in which we assumed a constant yield curve of the 5%. So now on our interest rate also we are changing automatically your value of the bond is also going to be changed.

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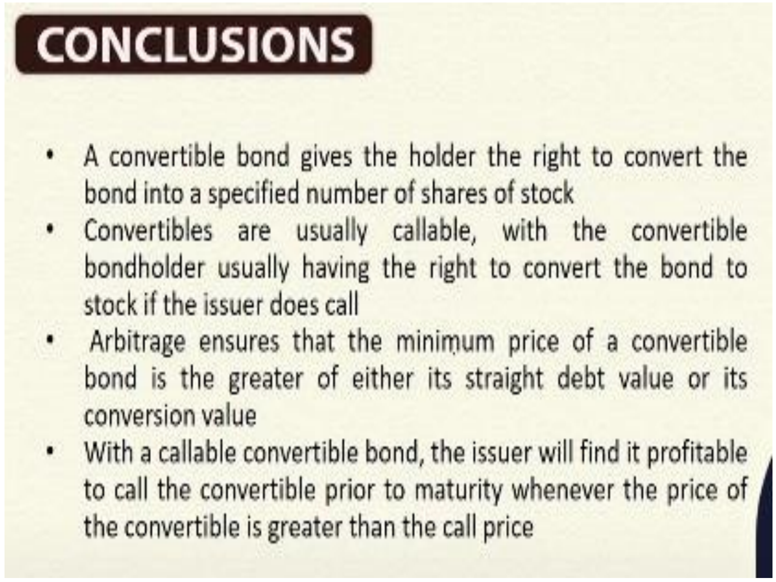

So, what we have discussed that a convertible bond gives the holder the right to convert the bond into a specified number of stocks. Convertible bonds are usually callable and the convertible bond holder usually have the right to convert the bond to stock. If the issuer is going for a call, then the arbitrageur always ensures that the minimum price of a convertible bond is greater of either its straight debt value or its conversion value.

So, with a callable convertible bond the issuer will find it profitable to call the convertible prior to the maturity whenever the price of the convertible bond is greater than the call price. And whenever you have the different features either call features or the differential interest rate features. Then you will find that the value of the bonds are going to be changed.

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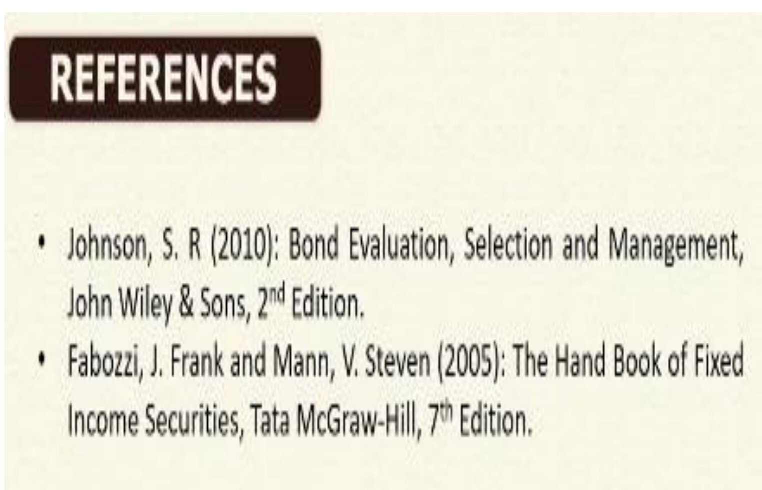

So, these are the references you can go through thank you.