Management of fixed Income Securities
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Module No # 01
Lecture No # 04
Bond Valuation-II

Welcome back. In the previous class, we discussed about the valuation of the bond and there we have seen that the value of the bond gets changed whenever the frequencies of the coupon payments get changed. And as well as basically the compounding frequency gets changed so accordingly the value of the bond always will vary.

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So today we will be discussing about the certain things that how to do the valuation of a zero coupon bond if the bond does not pay any coupon and also the valuation of the bond as non coupon dates. That means sometimes also we go and invest in a bond where already this particular coupon debt is over or the coupon date and the next coupon date is basically always you are buying the bond between these 2 dates. And how the bond prices are quoted and the concept of the discount yield?

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KEYWORDS

- Zero coupon bond
- Day count convention
- Clean price
- Accrued interest
- Full price

So these are the things what basically will be discussing, that is why after that you will get to know what is zero coupon bond? That is the concept of the day count convention what is day count convention the concept of a clean price concept of the accrued interest and the full price. So these are the certain concepts or certain keywords what basically we will come to know after the discussion of this particular class.

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Valuation of Zero-Coupon Bond

- These type of bonds do not make any periodic coupon payments.
- The investor realizes interest as the difference between the maturity value and the purchase price.
- These bonds are called zero-discount bonds, zero coupon bonds (also called pure discount bonds).

$$V_0^B = \frac{F}{(1+R)^M}$$

Where, F = Maturing value, R = Required rate and M= Maturity period

So let us see that what do we mean by zero coupon bond so this type of bonds do not make any periodic coupon payments, no coupons are paid against this zero coupon bond. The bond is basically issued at a particular price and after end of the maturity you will get your face value of the bond or the par value of the bond. So that means many a times or mostly the bond is issued at discount and redeemed at par.

So the investor basically will realize the interest as the difference between the maturity value and the purchase price so there is a purchase price and there is a maturity value. So the investor will get the difference only so these bonds are also called the zero discount bonds. And the zero coupon bonds also called the pure discount bonds the other name but popularly it is known as the zero coupon bond.

So if you go for a zero coupon bond how the value of a zero coupon bond is calculated the value of a zero coupon bond is basically F by 1 + R to the power M it is very simple. In comparison to the coupon basic bonds, the value of a zero coupon bond is relatively simpler so here F is the maturity value and your R is the required rate or the discount rate and M is the maturity period.

So, if you only get this face value of the bond then you get this required rate of return or this discount rate or you get your maturity period. Then this particular value of the bond zero coupon bond can be calculated. So comparatively this is easier to calculate.

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So let us take 1 small example let I will give you that let M = 10 years and your discount rate discount rate let 10% then what is the value of the bond? Then value of the bond and also the face value has to be given to you that is 1000. If maturity period is 10 years and it is a zero coupon bond then what is the value? That is 1000 divided by 1 + R to the power M so this is basically your R that means your 1000 divided by 1.1 to the power 10 that will give you let 385.54.

So the value of the zero coupon bond will be Rupees 385.54 in comparison to the coupon bearing bonds, and valuation of a zero coupon bond is much simpler. So that actually you have to keep in the mind.

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Valuation of Zero-Coupon Bond having Maturity of Less than One Year

- Assume on March 1 a zero coupon bond promising to pay ₹1000 on September 1 and trading at an annual rate of 8%, What is the value of bond?
- Maturity period: 184 days

$$\text{Vale of bond} = ₹1000 / (1.08)^{(184/365)} = ₹96.19$$

- The day count convention is defined as the way in which the ratio of the number of days to maturity (or days between dates) to the number of days in the reference period (e.g., year) is calculated.
  - A day count convention of actual days to maturity to actual days in the year (actual/actual)
  - A day count convention of 30-day months to maturity to a 360 days in the year (30/360)

Then already I told you that if any bond zero coupon bond which is having maturity less than 1 year then what basically we will do? So in that case we use a concept called the day count convention. Assume on march 1 a zero coupon bond is promising to pay 1000 Rupees on September 1 and trading at an annual rate of 8% then what is the value of that particular bond? So in that case you can go for then what is the maturity period?

From March 1 to the September 1 right so then that will give you that 184 days so then the value of the bond is F by 1 + R to the power m and what is the m here the m is basically 184 days then that is why we have second 184 divided by 365. Then it is 1.08 to the power 184 divided by 365 then whatever value we will get that will be the value of the bond. So the day count convention is defined as the way in which the ratio of the number of days to maturity or days between the dates to the number of days in the reference period that is basically year is calculated.

That concept is called the day count convention so day count convention is actual days to the maturity to the actual days in a year that is, actual by actual that is what we have done here that is 184 divided by 365. And day count convention also can be considered assuming 30 days in a month that also can be considered in this particular case. So that means for year it is 360 days so here we have to consider a convention of 30 by 360.

So there are certain bonds where basically this 30 days month concept the 30 by 360 days concept is used like corporate bonds in US. They use this particular kind of day count convention. We will discuss all these things in detail further. But you have to keep in the mind that the day count convention all will also have the impact on the value of that particular bond. In this case if you going for a recount convention it may not be coming 184 days it may be coming some different dates.

So because we are assuming every month is 30 days so March 1 to March 31st instead of then April we can consider 30 days may instead of 31 days we consider 30 days like that. So in this case we might have taken 180 days instead of taking 184 days. So the day count convention will have the impact on the value of that particular bond that actually you have to keep in the mind.

Valuation of Bonds at Non-coupon Dates

- Investor who purchases the bond between coupon payments must compensate the seller for the coupon interest earned from the time of the last coupon payment to the settlement date of the bond. This amount is known as accrued interest.

$$\text{Accrued Interest} = \frac{\text{Days from last coupon date}}{\text{Total days between coupon dates}} \text{ (Coupon)}$$

- In trading bonds on a non-coupon date, the amount the buyer pays to the seller is the agreed-upon price plus the accrued interest. This is called the full price or dirty price.
- Full price = Clean price + Accrued interest

So that's why this has lot of implication in that particular case and this part is more important you have to keep in the mind. Because the investor who basically buys the bond between the coupon payments. So in that case he should compensate the seller of the bond for the coupon interest and from the time of the last coupon to the settlement date of the bond. Because let every 6 months basis coupon is paid and coupon debts are fixed so if you are buying the bond for example March 1 is the coupon date and September 1 is the coupon date.

Let you have bond in bought the bond in July then from March 1 he has received the coupon but from March 1 to July let 15 in that particular period the coupon he has not received the coupon basically received in the September 1. But in that period his or she should be

compensated against that because the coupon he has not realized. So that particular thing is called the accrual interest which is basically nothing but the days from the last coupon date is divided by the total days between the coupon dates * the coupon.

So whenever we are basically or the investor is buying the coupon between these 2 coupon dates or in trading of the bonds on the non-coupon date the amount the buyer basically pays to the seller that is the basically has 2 components. One is your agreed upon price plus the accrued interest and if you take the total that is called the full price or the other name of that is the dirty price.

So the full price has 2 components 1 is clean price plus the accrued interest right very minutely you observe that that full price has 2 components 1 is clean price other 1 is the accrued interest.

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Method-1 for Valuation of Bonds at Non-coupon Dates

- Determine the number of days between the settlement and the next coupon date.
- Determine the number of days in the coupon period
- Compute the following:

$$v = \frac{\text{number of days between settlement and the next coupon}}{\text{number of days in the coupon period}}$$

- For a bond with N semi-annual coupon payment remaining, the value is

$$V_0 = \sum_{t=1}^{N} \frac{C}{(1+R)^{t-1+v}} + \frac{F}{(1+R)^{N-1+v}}$$

$$V_0 = \frac{C}{(1+R)^v} + \frac{C}{(1+R)^{1+v}} + \frac{C}{(1+R)^{2+v}} + \cdots + \frac{C}{(1+R)^{N-1+v}} + \frac{F}{(1+R)^{N-1+v}}$$

So here how basically we calculate these valuations of the bonds at the non coupon dates how the price is calculated? At the coupon date price is easier because coupon is realized you have to discount it with respect to discount rate. And the maturity is basically known the period is known. But now what is happening here our coupon is not yet realized next coupon. And we have basically trying to sell the bond between that coupon dates.

So then what we should do? There are 2 methods for calculations of that particular price of the bond what is those 2 methods? First of all, determine the number of days between the settlement and the next coupon date. Let July was the coupon date and next coupon date is September. And in between how many dates how many days are there? Determine the

number of days in the coupon period let coupon is paid annually or semi-annually whatever then you compute that V which is nothing but the number of days between the settlement and the next coupon divided by the number of days in the coupon period.

For example if a bond is paying the semi-annual coupons then what is the value of that particular bond in this case? It will be summation t = 1 to N C by 1 + r to the power t - 1 + v + F by 1 + R to the power N - 1 + v and if you expand it then you will get c by 1 + R to the power V + C by 11 + R to the power 1 + V + C by 1 + R to the power 2 + V and so on up to your C by 1 + R to the power N - 1 + p + f y 1 + R to the power N -1 + v. So here in this case how basically we can we can calculate this value of that particular bond.

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Let us take 1 example I will take certain things here let the last coupon date was first march right next coupon date is first September. I already told you let you assume that this is a corporate bond that means your day count convention is 30 by 360. It will be the day count convention is 30 by 360 ratio let the settlement date. I will give you let the settlement debt is seventeenth July 2020 maturity is let 2026 6 years period basically. Already you have to consider 30 by 360 ratio that means 30 days in a month.
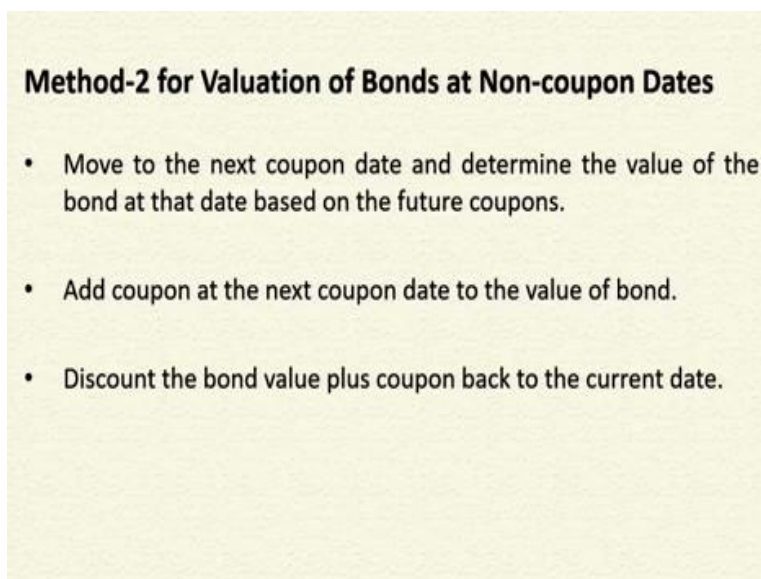
That basically we are considering right so that means you have how many days? In the July how many month days are remaining your 30 - 17. 13 days august we have 30 days then we have September 1 that is 1 day. So effectively we have 44 days from settlement date to next coupon date is 44 days. Then what is your V? Your V will be 44 by 180 coupon payment

period is 180 that that will be 0.2444. Let I will give you the par value of the bond, par value is let 100 Rupees. Right? let yield is let 10%.

Then how you can calculate the value of the bond then what is the value of the bond. So how the value of the bond will be calculated so what you need? You have a period which is t we need t - 1 then cash flow then we have the present value of the cash flow. So, period 1 t - 1 0 then this will be your period will be 0.24444, cash flow will be your cash flow will be 4 then present value of the cash flow. You can calculate 4 by this that will be 3.952578 for period 2, t - 1 will be 0 then this will be 1.24444, cash flow will be 4 then this will be your 3.764360.

Like that, you can go up to your period 12. This will be your 11.24444 your last cash flow will be Rs.104. 100 Rs is your face value. And 4 Rs is your coupon. That will give you that will give you your 60.085757. If you add up this it will give you ninety 4.559097. So, that will basically will give you the value of the bond on that particular day if the settlement is basically done between the 2 different coupon dates.

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Method-2 for Valuation of Bonds at Non-coupon Dates

- Move to the next coupon date and determine the value of the bond at that date based on the future coupons.

- Add coupon at the next coupon date to the value of bond.

- Discount the bond value plus coupon back to the current date.

And we have also another method what is called the method 2. Here what we do we go to the move to the next coupon date and determine the value of the bond at that date based on the future coupons, add the coupon at the next coupon rate to the value of the bond and discount the bond, loss the coupon back to the current rate backward. You can also do that here what basically you can do.

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Method-2: Example

Value of the bond at next Coupon date in 44 days:

$$\sum_{t=1}^{11} \frac{4}{(1.05)^t} + \frac{100}{(1.05)^{11}} = 91.693586$$

Value of the bond at next Coupon date + Coupon Paid at that date

$$91.693586 + 4 = 95.693586$$

Current value of the bond : Full price

$$180 - 44 = 136 \qquad \frac{95.693586}{(1.05)^{0.24444}} = 94.559 \checkmark$$

$$AI = 4\left(\frac{136}{180}\right) = 3.0222$$

Clean price: $94.559 - 3.0222 = 91.5368$

Let us go by the same example we have the certain things, we have to see then we have to find out the value of the bond at next coupon date in 44 days. Then what is the value? That is your t = 1 to 11 4 by 1.05 to the power t+ your 100 divided by 1.05 to the power 11 that will give you 91.693586. Then what will be the value of the bond? Value of the bond at next coupon date plus your coupon paid at that date you can add your coupon here that is your 91.693586 plus your 4 that will give you 95.693586.

Now what basically you have to do you have to find out your current value of the bond current value of the bond, which is nothing but the full price. That full price will become your 95.693586 divided by 1.05 to the power what 0.24444. That will give you same value your 94.559 right, so that is basically your current value of that particular bond so either of these 2 methods whatever method you can use you will get the same value then, you can also calculate your accrued interest from this. Your accrued interest also can be calculated from here.

That is your 4 into what? your number of days from the last coupon number of days from the last coupon means it is 180 - 44 that is 136 that means 4 into 136 divided by 180 that is 3.0222 so, that is basically your accrued interest value what you get and your this is your full price or dirty price then your clean price will become 94.559 - 3.0222 it will be 91.5368. But in US, the bond prices generally quoted with the clean price but the accrued interest, the buyer has to pay to the seller that actually you have to keep in the mind.

So that is the way basically this valuation of this bonds between the 2 coupon dates are calculated.

Bond Price Quotes

- Many traders quote bond prices as a percentage of their par value.

- For example, if a bond is selling at par, it would be quoted at 100 (100% of par).

- A bond with a face value of Rs.1000 and quoted at 80-1/8 would be selling at (0.80125)(Rs.1000) = Rs. 801.25

Then the question is how the bond price are quoted many traders if you see they quote the bond prices as a percentage of their par value. For example, if a bond is selling at par it would be quoted at 100 that means 100% of the par. If a bond with a face value of 1000, which if it is quoted like 81 by 8 it is not 80 - 1 by 8 it is 81 by 8, that means it will be selling at 0.80125. So, that means the selling price will be 0.80125 into 1000 that is 801.25.

That means at that time the bond is traded in the market at a price of 801.25 that is the way basically you have to interpret this.

Basis Points

- Fractions on yields are often quoted in terms of basis points (bp).
- A bp is equal to 1/100 of a percentage point.
  - 6.5% may be quoted as 6% plus 50 bp or 650 bp
  - An increase in yield from 6.5% to 6.55% would represent an increase of 5 bp

Generally they are quoted in the basis point all of you know what is basis point? Basis point is nothing but it is 1 by 100 of a percentage point so, 6.5% means it is 6% + 50 basis point or

650 basis point so, increase in the yield from 6.5% to 6.55% means it is an increase of the 5 basis point so many a times, the basis point concept is used instead of using the percentage things that actually also you have to keep in the mind.

Bid and Ask Price / Yield

- The **bid price** is the price the dealer is willing to pay for the bond.
- The **ask price** is the price the dealer is willing to sell the bond.
- The **bid yield** is the return expressed as a percent of the par value that the dealer wants if she buys the bond; this yield is often annualized.
- The **ask yield** is the rate that the dealer is offering to sell bonds.
- For Treasury bills and some other securities, bid and ask yields are quoted as a **discount yield**.

$$\text{Annual Discount Yield} = R_D = \frac{F - P_0}{F} \frac{360}{\text{Days to Maturity}}$$

Then there is a whenever it is quoted, there is a bid price that is ask price so the bid price is the whatever price the dealer is willing to pay for the bond. And the ask price is the price that the dealer is willing to sell the bond and the bid yield is the return which is expressed as a percentage of the par value. That the dealer wants if she buys the bond and many a case the yield is generally annualized.

And the ask yield basically the rate at that the dealer is offering to sell the bond. They want to sell the bond so then for treasury bills and some other securities bid and ask yields are quoted as the discount yield. We will discuss more on this whenever you discuss about the markets so the annual discount yield is basically your F - P 0 divided by F into 360 by days to maturity. We are using the 30 by 360 ratio in this case.

F is the final value and P is the purchase price and days to maturity if you know then the annual discount yield basically is calculated from that. So this is the way the bond prices are quoted and this is the way basically we can assume or we can define the bid price, ask price, bid yield, ask yield discount yield all these things.

Bid and Ask Price / Yield

- Given the dealer's discount yield, the bid or ask price can be obtained by solving the yield equation for the bond's price, P_0. Doing this yields:

$$P_0 = F\left[(1 - R_D(\text{Days to Maturity}/360)\right]$$

So if you know the discount yield then price also can be calculated from there. You can solve that equation for the bond price $P_0 = F$ into $1 - R_D$ into days to maturity divided by 360. Just now whatever formula I have shown you the same formula can be used also if you know the yield then from there, the price can be calculated. If you know the face value, if you know the yield, then the price of the bond can be calculated accordingly. So that is basically the relationship between the price and yield.
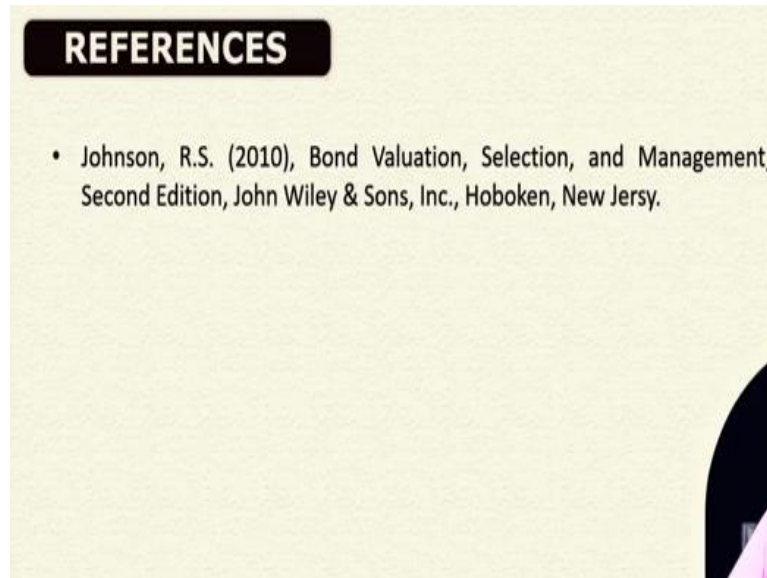
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CONCLUSIONS

- The Present value of zero coupon bond is the present value of the face value of the bond only.
- In the context of valuation of bonds at non-coupon dates, the price the buyer pays the seller is the full price.
- The buyer must compensate the seller for the portion of the next coupon interest payment the seller has earned but will not receive because the issuer will send the next coupon payment to the buyer. This amount is the accrued interest.

So whatever we have discussed that the present value of a zero coupon bond is the present value of the face value of the bond only. So, in the context of the valuation of the bonds at non coupon dates, the price the buyer pays the seller is called the full price which is a comprised of 2 :- 1 is your accrued interest. And we have the clean price and the buyer must compensate the seller for the portion of the next interest payment the seller has earned but

will not receive because the issuer will send the next coupon payment to the buyer. So that is why the amount is called the accrued interest that particular amount is defined as the accrued interest. And there are 2 formulas or 2 methods whatever we have discussed through which the price of the bond between the 2 coupon dates can be calculated.

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REFERENCES

- Johnson, R.S. (2010), Bond Valuation, Selection, and Management, Second Edition, John Wiley & Sons, Inc., Hoboken, New Jersy.

So this is the reference what basically you can follow for the detailed discussion on this. Thank you.