Management of Fixed Income Securities
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Module No # 08
Lecture No # 39
Binomial Valuation of Puttable and Sinking-Fund Bonds

Welcome back, so in the previous class we discussed the valuation of a callable bond which has used the 3 period binomial tree models. Today we will be discussing the other embedded options which are available that is one of the things is that there can be a put feature or there can be sinking fund provisions.

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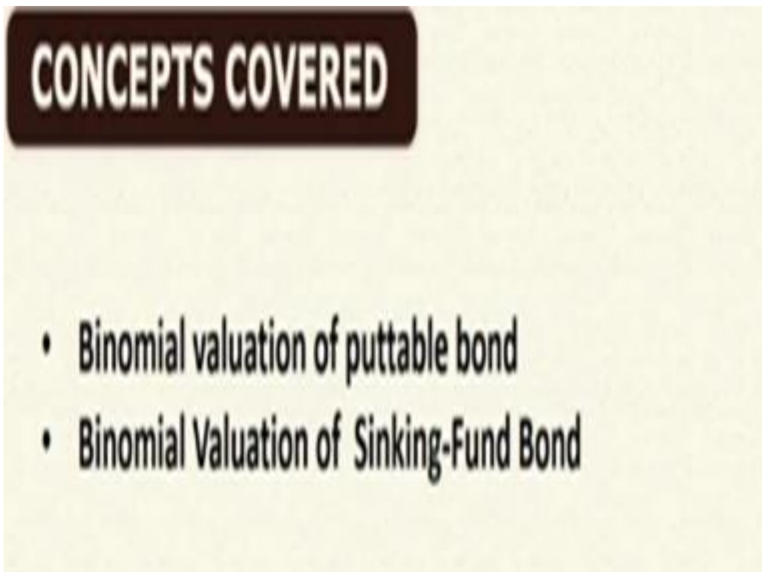

So, let us see what those things what we are going to discuss in today's class that is your binomial valuation of a puttable bond and the binomial valuation of a sinking fund bond. The bonds having the sinking fund provisions. How that valuation can be made? How the valuation of those kinds of bonds also can be made? so these are the 2 things that we are going to do in today's session.

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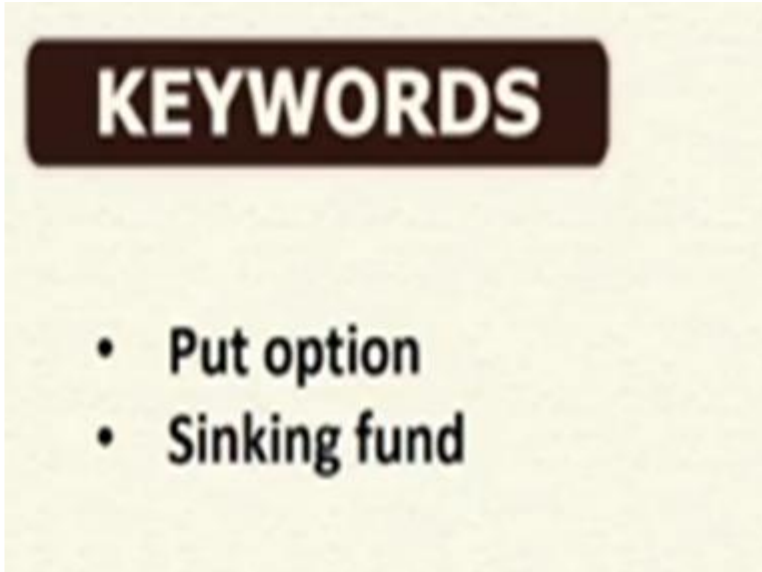

So, the keywords mostly will be using the major keywords that are the put option and the sinking fund.

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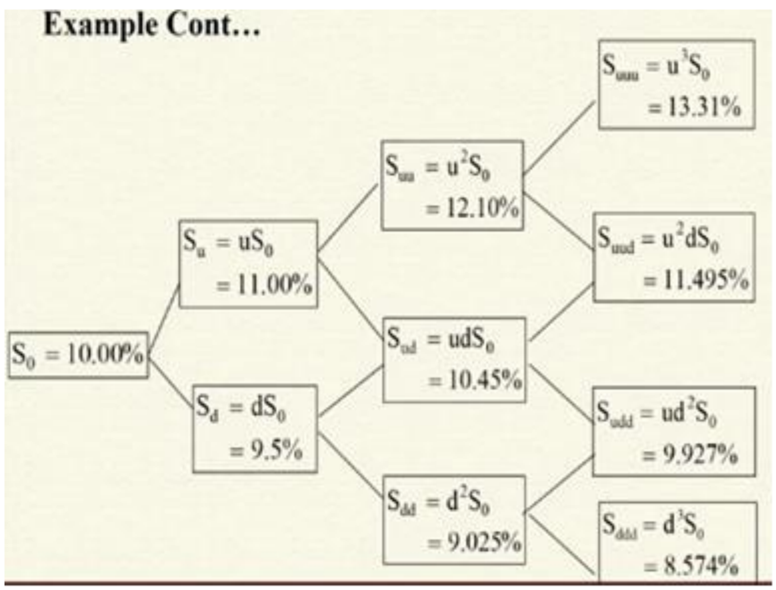

So, again will be using the same interest rate tree model the binomial interest rate model starts with the 10%. It can go up to the first period 11% or 9.5% or it can go up to 12.1, 10.45, 9.025 further it can go up to 13.31 or go down to 11.49 or it can reach to 9.927 or 8.574. So, these are the different interesting scenarios that we have built up in the beginning. That same example or same tree we are going to use for valuation of the bonds having the put options or the put features and the sinking fund provisions.

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Putable Bond

- A putable bond, or put bond, gives the holder the right to sell the bond back to the issuer at a specified exercise price (or put price)
- In contrast to callable bonds, putable bonds benefit the holder
- If the price of the bond decreases below the exercise price, then the bondholder can sell the bond back to the issuer at the exercise price
- From the bondholder's perspective, a put option provides a hedge against a decrease in the bond price.
- If rates decrease in the market, then the bondholder benefits from the resulting higher bond prices.
- If rates increase, then the bondholder can exercise, giving her downside protection.

You know what do you mean by a puttable bond. In the case of callable bond what we have seen there is a call feature involved and the investor whenever they are holding these callable bonds if the issuer wants at any point of time they can buy back or call back the bond that is called the call features and the call price is decided from the beginning. Wherever you go for a puttable bond the puttable bond means it gives the bond investor or the bond holder the right to sell the bond back to the issuer at a specified exercise price or we call it the put price.

Generally, the price put price is less than the face value of the bond. So, in contrast to the callable bonds the puttable bonds basically benefit the holder the risk is not with the holder the risk is with the issuer. So, in this case if you observe if the price of the bond decreases below the exercise price; that means whenever the interest rate is quite high then the bond holder can sell the bond back to the issuer at the exercise price exercise by means that the pre agreed price what already decided between the issuer and the bond holder.

So, if the price will go down below then it is profitable for the bond holder to sell back that particular bond to the issuer. From the bondholder's perspective generally the put option gives a hedge against the decrease in the bond's price. So, if the rate is decreasing in the market, then the bond holder benefits from the resulting higher bond prices. If the rate is increasing then the bond holder can exercise that option giving some kind of downward protection to them.

It gives a kind of protection to them that, below that if the price is going down in the market, they can exercise that particular option and that agreed upon price they can use it to sell the bond back to the issuer so that is the basic concept of the puttable bond.

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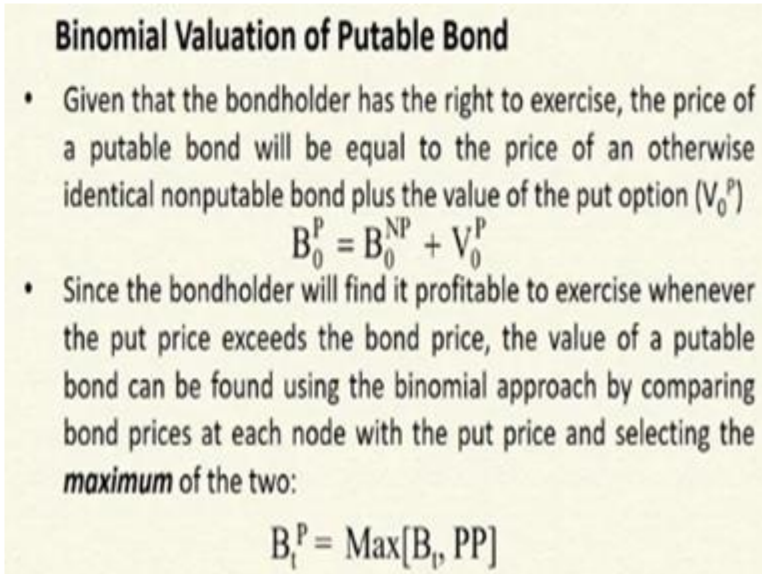

So, now if you see that whenever you go for the binomial valuation of the puttable bond it already, we have seen that it gives the bondholder the right to exercise. So, the price of a puttable bond is nothing but it is equal to the price of the identical non-puttable bond plus the value of the put option. In the previous case what we have seen is the value of the identical non-callable bond minus the call option.

$B_0^P = B_0^{NP} + V_0^P$

Here in this case what we have seen is the price of the non-puttable bond plus the value of the put option. As the bond holder generally or will find it profitable to exercise whenever the put price basically exceeds the bond price. It is profitable for them whenever the price of the bond in the market is lower and the put option price is basically higher. Then the value of a put option can be found using the binomial approach by comparing the bond price at each node with the put price and selecting the maximum of these 2 which one is the maximum.

So, the B_t^P, P means it is the put that can be either the bond price or the time t or it is the put price which one is the maximum that actually we have to see. So, if the price of the bond in the market is very low or below this put price then the bondholder can exercise that particular option so because of that the B_t^P is equal to the maximum of the B_t or the PP.

B_t^P = Max [B_t, PP]

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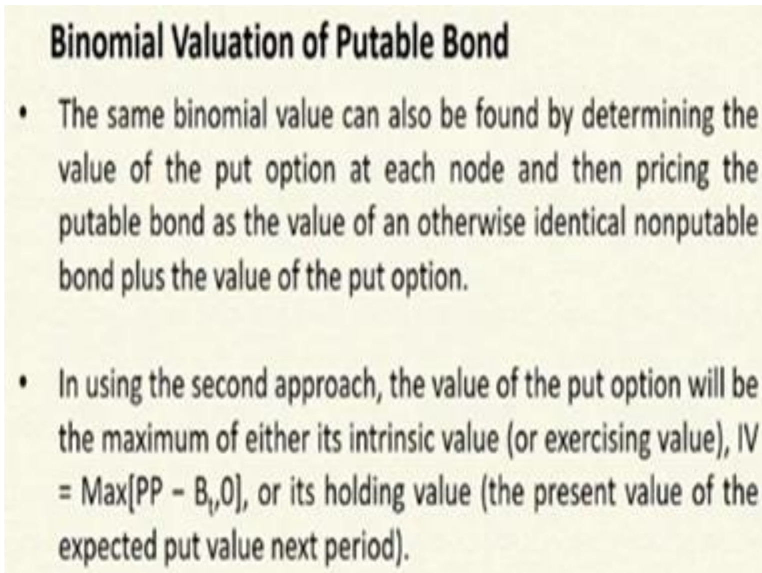

So now the same binomial value also can be found by determining the value of the put option at each node. And, then pricing this puttable bond as the value of the identical non puttable bonds plus the value of the put option. If you are using the second approach then the value of the put option also will be the maximum of either its intrinsic value or exercise value or it is the holding value; the holding value concept that we have already discussed in the previous session.

IV = Max [PP – B_t, 0]

It is nothing but the present value of the expected put values in the next period that also you can use. Because the option may not be exercise, they can hold that option that a particular option is not necessary either they will exercise or it will not be exercise there is option of holding that particular option also for the next period. So that’s why there is holding value, we are trying to use it in this particular case.

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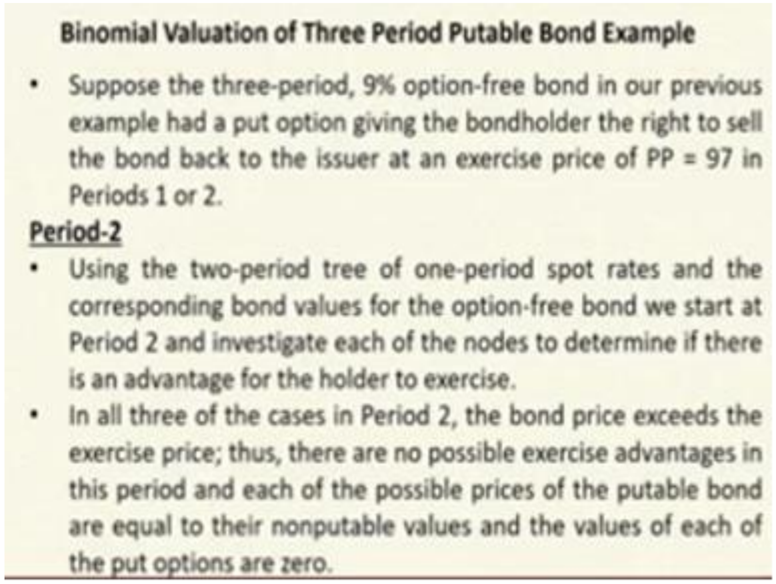

So, now suppose let us take the example suppose there is a 3 period 9% option free bond. And here the put price we have fixed it 97 in the period 1 or 2. So here we have 2 periods 1 is period one another 1 is period two that already we have seen in the previous example also for the valuation of a 3-period bond callable or puttable we need at least the 2 periods data or 2 period information by that we can roll back that particular information and find out the value of the current period.

So, in the period 2 what can be possible using the 2 period trees of 1 period spot rate and the corresponding bond values for the option free bond. We can start at period 2 and investigates each of the nodes to determine if there is an advantage for the holder to exercise that particular option or not. First, we have to see whether the bond holder will exercise that particular option or it will not exercise. If in all 3 of the cases in period 2 the bond price exceeds this exercise price.

So, there is no possible exercise advantage in this period and each of the possible prices of the puttable bond are equal to their non-puttable values and the values of each of the put options are 0.

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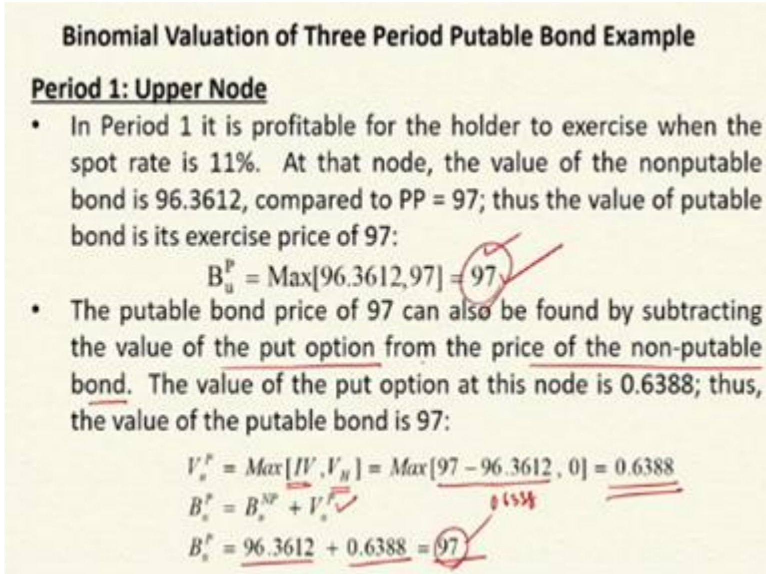

How it is possible we will see that thing because already we have calculated the value of the without any options accordingly, we are concluding that thing. So let us come to the period 1 period 1 we have 2 nodes one is upper node one is lower node. In period 1 it is profitable for the holder to exercise when the spot rate is 11%. In the first case it is 10% to 11%. It can so at that node the value of the non-puttable bond is 96.3612 and the put price is 97 that we have considered.

$B_u^P = \text{Max}\ [96.3612, 97] = 97$

So, the value of the puttable bond is its exercise price of 97. It is the maximum of these 2 that is 96.3612 and 97 so ninety-seven is the maximum so we will consider that. So, the puttable bond price of 97 also you can find or you can calculate by subtracting the value of the put option from the price of the non-puttable bond. So, the value of the put option at this node is how much, it is the maximum of the intrinsic value or the exercise value and the V_H.

$V_u^P = \text{Max}\ [IV, V_H] = \text{Max}\ [97 – 96.3612, 0] = 0.6388$

$B_u^P = B_u^{NP} + V_u^P = 96.3612 + 0.6388 = 97$

So, its intrinsic value is basically between 97- 96.3612 and 0. And here obviously this value will be zero 0.6388 so it will be 0.6388. So, then the value of the put option will be the value of the non-put options bond divided by the value of the put option so that is 96.3612 plus this that will give you 97. So, either you can use or this one or you can use this one both the cases you are getting that the bond will be exercised and the put price is 97 that will be considered in both the cases.

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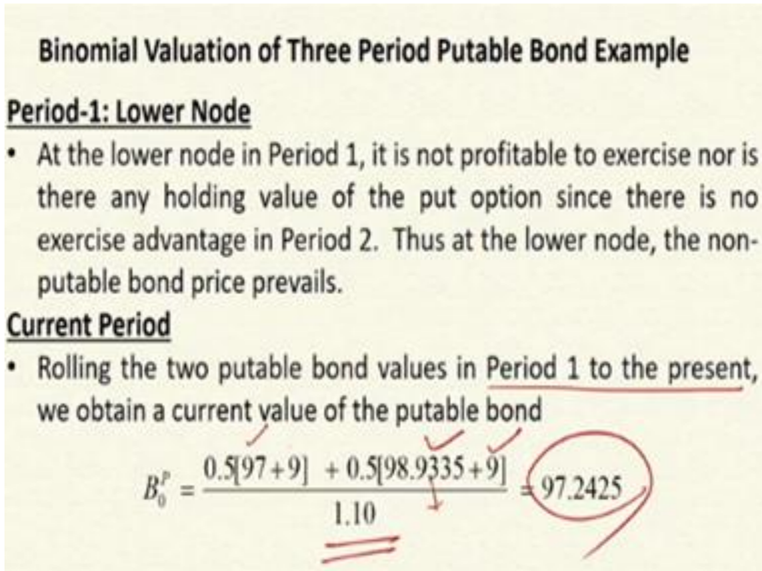

So, in the lower node it is not profitable to exercise nor is there any holding value of the put option since there is no exercise advantage in the period 2. So, in the lower node the non-puttable bond price prevails so then current period if you want to calculate rolling the 2 puttable bond values in the period 1 to the present. So, then it will be the first case it is 97 + 9 second case it is 98.9338 + 9 it will not be exercised because the value is more than 97. So, the same price will prevent + 9 and you discount it with respect to 10% you get this value of the bond that is 97.2425.

$$B_0^P = \frac{0.5\,(97 + 9) + 0.5\,(98.9335 + 9)}{1.10} = 97.2425$$

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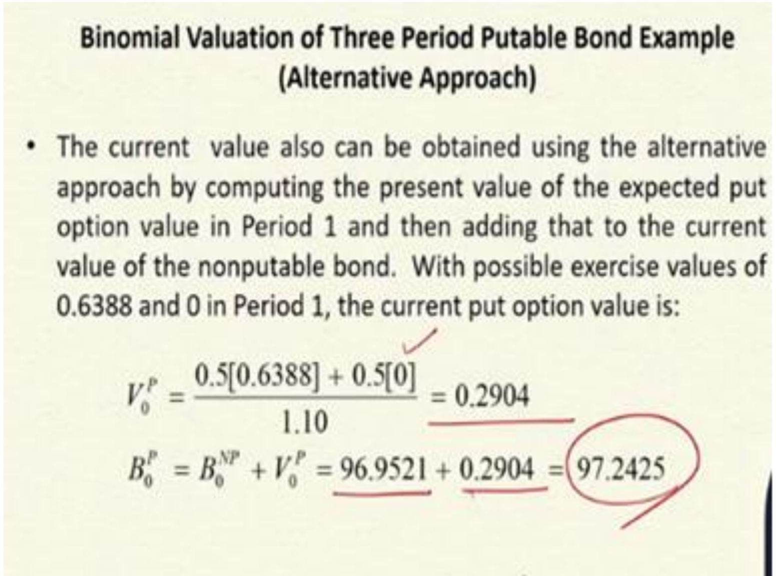

Let us see the alternative approach. What is the alternative approach? The current value also can be obtained using the alternative approach by computing the present value of the expected put option values in the period 1. Then, adding that thing to the current value of the non-puttable bond. With possible exercise values of 0.6388 and 0 in period 1 the current value of the put option will be 50-50% chance 0.5 into 0.6388+ 0.5 into 0 divided by this that will give you 0.2904. So, in that period the value of the non-puttable bonds is 96.9521+ your 0.2904 that will give you also the value of the bonds having the put options.

$$V_0^P = \frac{0.5\,(0.6388) + 0.5\,(0)}{1.10} = 0.2904$$

$$B_0^P = B_0^{NP} + V_0^P = 96.9521 + 0.2904 = 97.2424$$

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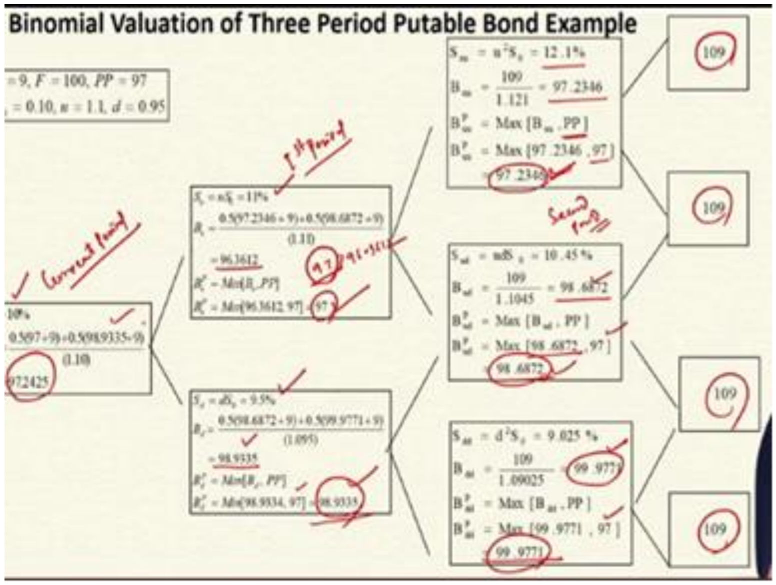

So, let us see these things. This particular figure will give you the clear idea we are rolling back from the second period to the first period. This is basically your second period. This is second period data and this is your first period and this is your current period. So basically, here we start with 109 every case 109 divided by 1.121 that is 12.1% if the interest rate got 97.2346 then your B_{uu}^{P} will be the maximum of this B_u and the put price that is 97.2346 and 97 then it is 97.2346 that price will prevail because it is more than the put price it will not be exercised so this price will prevail. Second case if you see also, you discount it with 10.45% you got this value then if you see between this and this; this value is higher then again, this value will prevail that's why we have seen that, in the second period in all the nodes the bond will not be exercised the put option will not be going to be exercised by the bondholder.

$S_{uu} = u^2, S_0 = 12.1\%$

$B_{uu} = 109/1.121 = 97.2346$

$B_{uu}^{P} = \text{Max}\ [B_{uu}, PP] = \text{Max}\ [97.2346, 97] = 97.2346$

$S_{ud} = ud, S_0 = 10.45\%$

$B_{ud} = 109/1.1045 = 98.6872$

$B_{ud}^{P} = \text{Max}\ [B_{ud}, PP] = \text{Max}\ [98.6872, 97] = 98.6872$

$S_{dd} = d^2, S_0 = 9.025\%$

$B_{dd} = 109/1.09025 = 99.9771$

$B_{dd}^P = \text{Max}\ [B_{dd}, PP] = \text{Max}\ [99.9771, 97] = 99.9771$

Third case third node you see the price of the bond has become 99.9771 compare it with 97 obviously this is higher so this price will prevail. Come to the first period, in the first period what is basically happening we got this there is a 50% chance it can become this, 50% chance it can become this. So, we calculate this expected value of that 0.5 into this + 9 + 0.5 into this + 9 that is the coupon the cash flow.

$S_u = u, S_0 = 11\%$

$$B_u = \frac{0.5(97.2346 + 9) + 0.5(98.6872 + 9)}{1.11} = 96.3612$$

$B_u^P = \text{Max}\ [B_u, PP] = \text{Max}\ [96.3612, 97] = 97$

This cash flow and this cash flow +9 obviously is the coupon what you are going to receive then if you discount it with respect to that period's discount rate that is 11% you got 96.3612 now if you compare it with your put option price that is 97, then 97 is more than this. So now 97 is more than 96.3612 so then this price will prevail. That is 97. Second case if you see now your cash flow is 98.6872 and your 99.9771.

$S_d = d, S_0 = 9.5\%$

$$B_d = \frac{0.5(98.6872 + 9) + 0.5(99.9771 + 9)}{1.095} = 98.9335$$

$B_d^P = \text{Max}\ [B_d, PP] = \text{Max}\ [98.9335, 97] = 98.9335$

So, 0.5 into 98.6872 + 9 + 0.5 into 99.9771 + 9 divided by discount rate is 9.5% you got 98.9335. And if you compare this and this, this is higher so we retain this 98.9335. So now you got the cash flow of 97 another one is 98.9335 now you come back to the current period so it is discount rate is 10%; 0.5 into 97 here in this case we have considered the 97 + 9 + 0.5 into 98.9335 +9 divided by 1.1 that will give you 97.2425.

$S_0 = 910\%$

$$B_0 = \frac{0.5(97 + 9) + 0.5(98.9335 + 9)}{1.10} = 97.2425$$

That is what the value of the current period price of that particular bond is, if you are considering this the put option features into the considerations. So, this is for a binomial valuation of a 3-period puttable bond.

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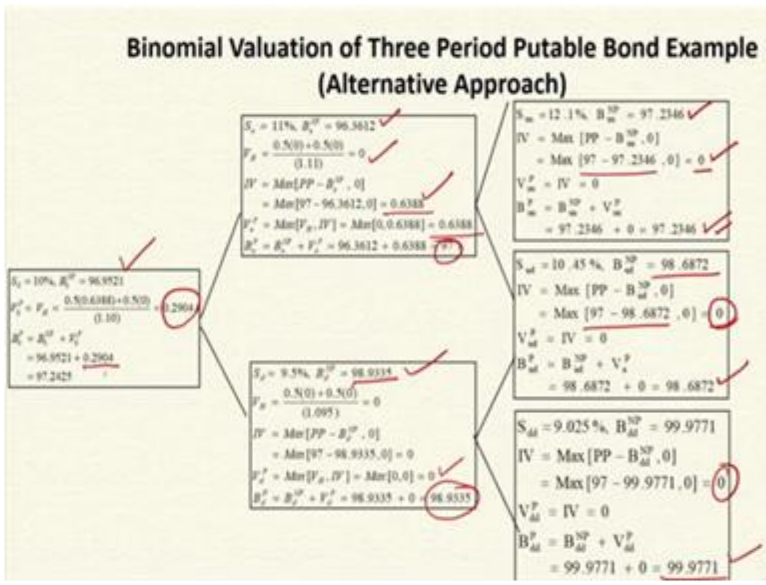

Then if you see that alternative case, what is going to happen? In the alternative case, what we have seen we have to find out the intrinsic value. The intrinsic value you see if you minutely observe here, we got this value is 97.2346 and the intrinsic value become this maximum of the put price minus this and 0 then 97 - 97.2346 that value is minus and 0 then that's why the value will be 0 then obviously the value of the B_{uu}^{P} that value of the bond having the put option that become 97.2346 + 0 that will become this.

$S_{uu} = 12.1\%$, $B_{uu}^{NP} = 97.2346$

$IV = Max\ [PP - B_{uu}^{NP},0] = Max\ [97 - 97.2346,\ 0] = 0$

$V_{uu}^{P} = IV = 0$

$B_{uu}^{P} = B_{uu}^{NP} + V_{uu}^{P} = 97.2346 + 0 = 97.2346$

S_{ud} = 10.45%, B_{ud}^{NP} = 98.6872

IV = Max [PP - B_{ud}^{NP},0] = Max [97 – 98.6872, 0] = 0

V_{ud}^{P} = IV = 0

$B_{ud}^{P} = B_{ud}^{NP} + V_{ud}^{P}$ = 98.6872 + 0 = 98.6872

S_{dd} = 9.025%, B_{dd}^{NP} = 99.9771

IV = Max [PP – B_{dd}^{NP},0] = Max [97 – 99.9771, 0] = 0

V_{dd}^{P} = IV = 0

$B_{dd}^{P} = B_{dd}^{NP} + V_{dd}^{P}$ = 99.9771 + 0 = 99.9771

Second case, also if you see it is 98.6872 so 97 minus this, 0 this will also 0 then the value will remain the same 98.6872. Third case also 0 the value will remain the same 99.9771; now what has happened that come back to here, the intrinsic value is 0 here it is 0 then it becomes 0 then you are between this 0 and this 97 - 96.3612 if you get the value; this value 0.6388. So that's why this value of the put option becomes 0.6388 and this is 96.633612 + 0.6388 that 97 is the value of the bond having the put option.

S_u = 11%, B_u^{NP} = 96.3612

$$V_H = \frac{0.5(0) + 0.5(0)}{1.11} = 0$$

IV = Max [PP - B_u^{NP},0] = Max [97 – 96.3612, 0] = 0.6388

V_u^{P} = Max [V_H, IV] = Max [0, 0.6388] = 0.6388

$B_u^{P} = B_u^{NP} + V_u^{P}$ = 96.3612 + 0.6388 = 97

S_d = 9.5%, B_d^{NP} = 98.9335

$$V_H = \frac{0.5(0) + 0.5(0)}{1.095} = 0$$

$IV = Max\,[PP - B_d^{NP}, 0] = Max\,[97 - 98.9335, 0] = 0$

$V_d^P = Max\,[V_H, IV] = Max\,[0, 0] = 0$

$B_d^P = B_d^{NP} + V_d^P = 98.9335 + 0 = 98.9335$

Second case, also you have seen that the intrinsic value becomes 0 so because of this the value of the bond of having the put option will remain same with this. Then in this case what we have considered that is here it is 0.6388 and here also it is basically 0 then we have discounted it with respect to the 10% then you got 0.2904 then this value of the option bond having the put option will become your 96.9521 + 0.2904 then it will become 97.2425 this is also another approach what you can use for valuation of this.

$S_0 = 10\%,\ B_0^{NP} = 96.9521$

$$V_0^P = \frac{0.5(0.6388) + 0.5(0)}{1.10} = 0.2904$$

$B_0^P = B_0^{NP} + V_0^P = 96.9521 + 0.2904 = 97.2425$

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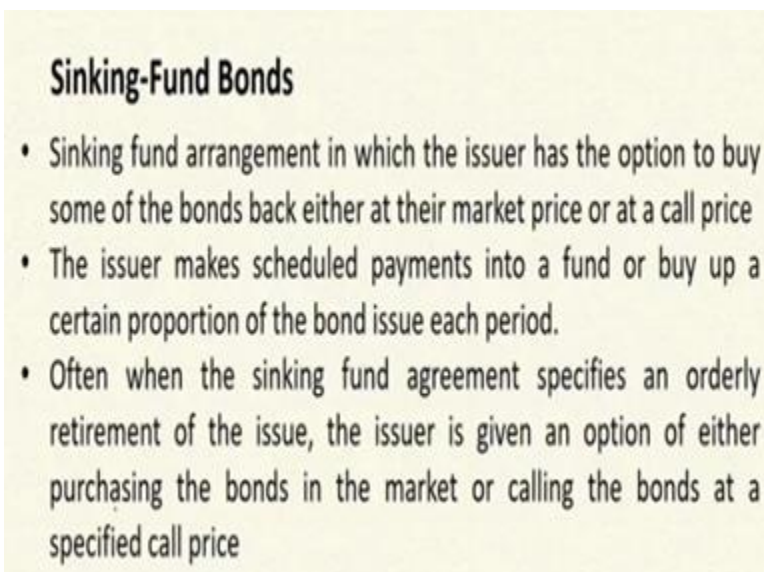

Then we have another feature also the bond can have that is called the sinking fund bonds. So, the sinking fund arrangements generally what, here the issuer has the option to buy some of the bonds back either at their market price or at a call price. So, in this case the issuer basically makes the scheduled payments into a fund or buys off a certain proportion of the bond issue in each period.

Often when the sinking fund agreement specifies the orderly retirement of the issue, the issuer is given an option of either purchasing the bonds in the market or calling the bonds at a specified call price. So that is basically what we call the sinking-fund provisions of the bond so that can be also another embedded option can be prevailed with respect to certain types of bonds.

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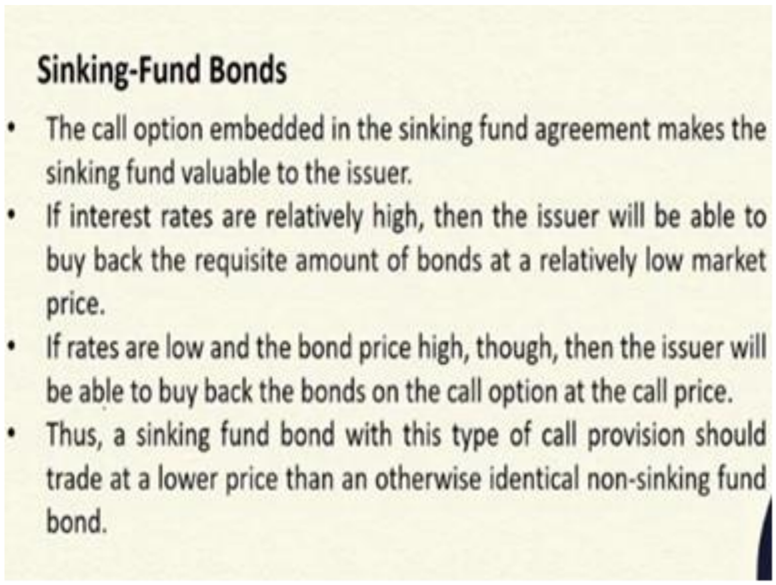

So, the call option embedded in the sinking fund agreement makes the sinking fund valuable to the issuer. So, if the interest rates are relatively high then the issuer will be able to buy back the requisite amount of the bond at a relatively low market price. If rates are low and the bond price high though also, the issuer will be able to buy back the bond on the call option at the call price.

So, the sinking fund bonds or bonds having the sinking fund provisions with this type of call provision should trade at a lower price than the identical non-sinking fund bonds. Lot of risk is involved from the investor point of view if you are having the sinking fund provisions.

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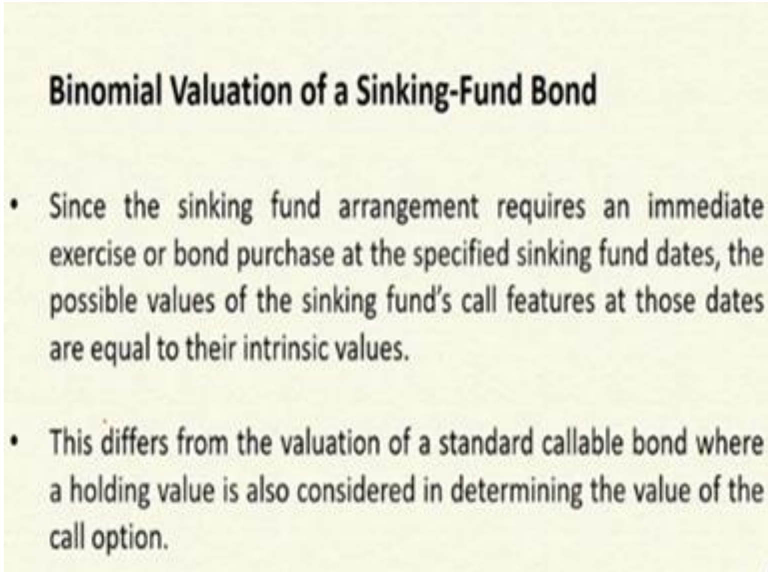

So, in this case how we can use our binomial valuation model for a sinking fund bond; bonds having the sinking fund provisions. The sinking fund arrangement generally requires an immediate exercise or bond purchase at a specified sinking fund date. So, the possible values of the sinking fund's call feature at those rates trades are equal to their intrinsic values. Generally, this differs from the valuation of a standard callable bond where a holding value is also considered in determining the value of the call options that we have already discussed in the previous class.

So, the callable bonds with the sinking fund provision are relatively difficult for the valuation in comparison to the bonds having only the call features or only the call options.

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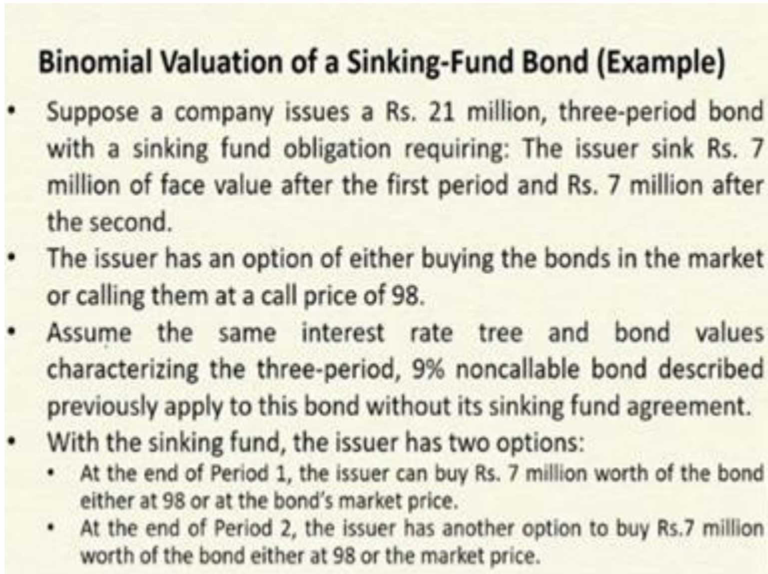

Suppose a company issues 21 million rupees 3 period bonds with the sinking fund obligations requiring: The issuer sink rupees 7 million of the face value after the first period and 7 million after the second period assume. And the issuer has the option of either buying the bonds in the market or calling them at a call price of 98. Assume the same interest rate tree and one value which is characterizing the 3 period valuations.

Whatever we have discussed coupon is 9% what already we have discussed previously without its sinking fund agreement. But now whenever you are bringing the sinking fund agreement the issuer has 2 options at the end of the period 1 the issuer can buy this 7 million worth of the bond either at rupees 98 or at the bonds market rise. At the end of the period 2 the issuer has another option to buy 7 million worth of the bond either at 98 or the market price.

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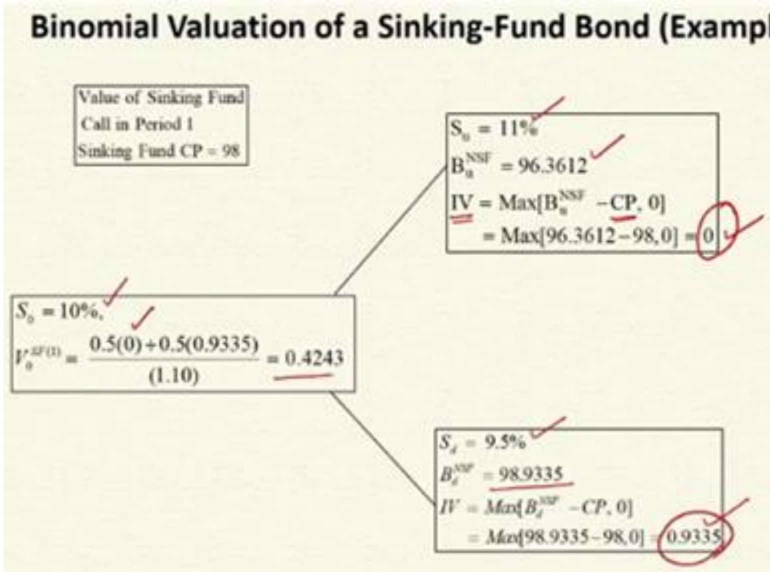

So, then what basically will happen in this case. So, already we know that 10% has become 11% or 9.5%. So, the value of a non-sinking fund in this case will be 96.3612; so, what is the intrinsic value, the maximum of this minus call price and 0 or otherwise 0. So, this is 96.3612 – 98, 0 then obviously this will be minus then this will be 0. Second case the price becomes 98.9335 which is obviously more than the call price. So that's why it will be 98 call prices of the 90 so that's why this will become the 0.9335.

$S_u = 11\%$

$B_u^{NSF} = 96.3612$

$IV = Max\ [B_u^{NSF} – CP, 0] = Max\ [96.3612 – 98, 0] = 0$

$S_d = 9.5\%$

$B_d^{NSF} = 98.9335$

$IV = Max\ [B_d^{NSF} – CP, 0] = Max\ [98.9335 – 98, 0] = 0.9335$

So, now what you have to do here; the intrinsic value is 0, here the intrinsic value is 0.9335. So now we are discounting it here 0.5 into 0 + 0.5 into 0.9335 divided by 1.1 that will give you your 0.4243.

$S_0 = 10\%$

$$V_0^{SF(1)} = \frac{0.5(0) + 0.5(0.9335)}{1.10} = 0.4243$$

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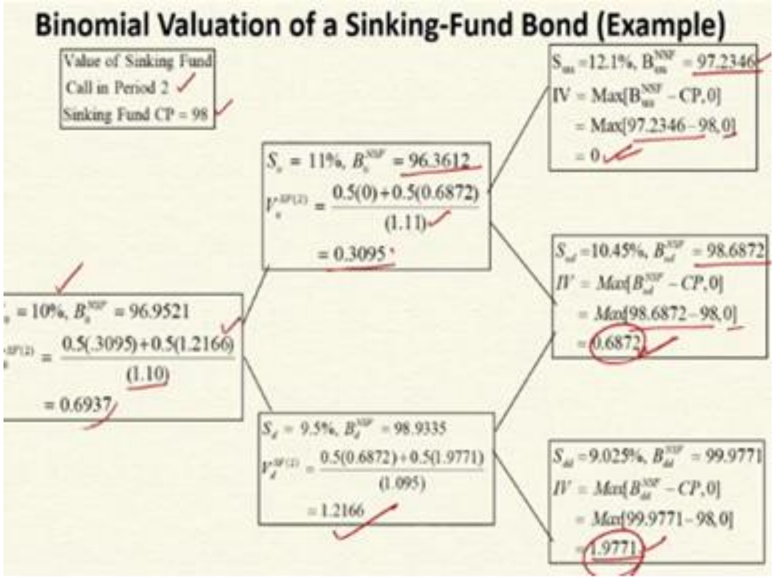

Then what basically here we have seen, second case whenever you go for let you are calling back the bond in the period 2 and sinking fund call price is 98. So again, you already know this value of the bond without any sinking fund provisions NSF non-sinking fund that is 97.2346 then the maximum of this is 0 between this and this here it is 98.6872 between this and this, it is 0.6872 is the maximum. Here in this case, it is 1.9771 is the maximum.

$S_{uu} = 12.1\%$, $B_{uu}^{NSF} = 97.2346$

$IV = Max\ [B_{uu}^{NSF} – Cp, 0] = Max\ [97.2346 – 98, 0] = 0$

$S_{ud} = 10.45\%$, $B_{ud}^{NSF} = 98.6872$

$IV = Max\ [B_{ud}^{NSF} – Cp, 0] = Max\ [98.6872 – 98, 0] = 0.6872$

$S_{dd} = 9.025\%$, $B_{uu}^{NSF} = 99.9771$

$IV = Max\ [B_{dd}^{NSF} – Cp, 0] = Max\ [99.9771 – 98, 0] = 1.9771$

Then you take these values here it is the value of the non-sinking front bonds has already we have calculated that is 96.3612 then you take 0 here 0.6872 here multiply this discount it with respect to 1.11 you got 0.3095. Here in this case, you take this, take this you got 1.2166 then take these 2 values here and again discount it with respect to 1.1or 10%. So, 0.5 into 0.3095 + 0.5 into 1.2166 that will give you 0.6937 in the second period.

$S_u = 11\%,\ B_u^{NSF} = 96.3612$

$$V_u^{SF(2)} = \frac{0.5(0) + 0.5(0.6872)}{1.11} = 0.3095$$

$S_d = 9.5\%,\ B_d^{NSF} = 98.9335$

$$V_d^{SF(2)} = \frac{0.5(0.6872) + 0.5(1.9771)}{1.095} = 1.2166$$

$S_0 = 10\%,\ B_0^{NSF} = 96.9521$

$$V_0^{SF(2)} = \frac{0.5(0.3095) + 0.5(1.2166)}{1.10} = 0.6937$$

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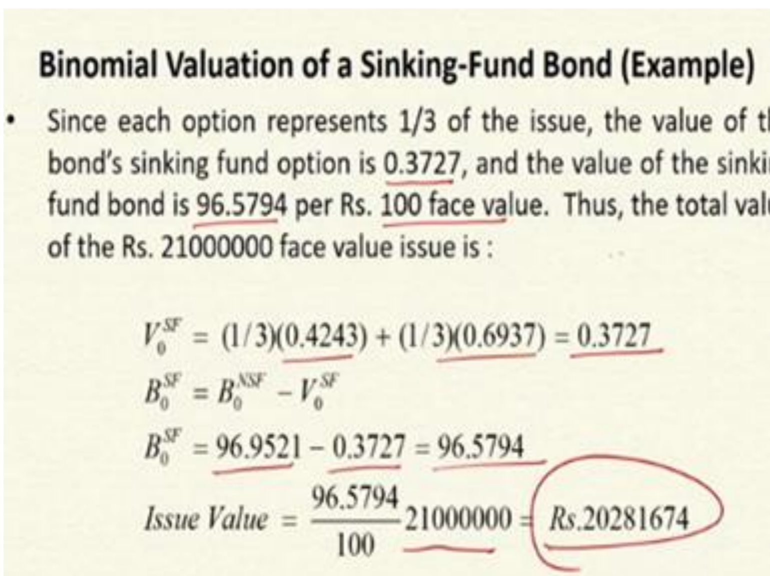

So, in this case what we have seen is that since each option represents one by third of the issue then the value of the bond's sinking fund option is 0.3727 and the value of the sinking fund bond is 96.5794 for 100 rupees face value. So how you got it that basically here 1 by 3 into 0.4243 + 1 by 3 into 0.6937 that becomes 0.3727. So, your 96.9521 - 0.3727 that would give you 96.5794.

$V_0^{SF} = (1/3)(0.4243) + (1/3)(0.6937) = 0.3727$

$B_0^{SF} = B_0^{NSF} - V_0^{SF} = 96.9521 – 0.3727 = 96.5794$

And the issue value will be your 96.5794 divided by 100 into your 21 million was the value of the face value of the particular bond if you multiply this you got this. So, the total value of this particular bond will be this much. That is the way basically we can use the valuation of the binomial tree for the bond having the sinking fund provisions.

$$\text{Issue Value} = \frac{96.5794}{100} * 2100000 = \text{Rs. } 2028674$$

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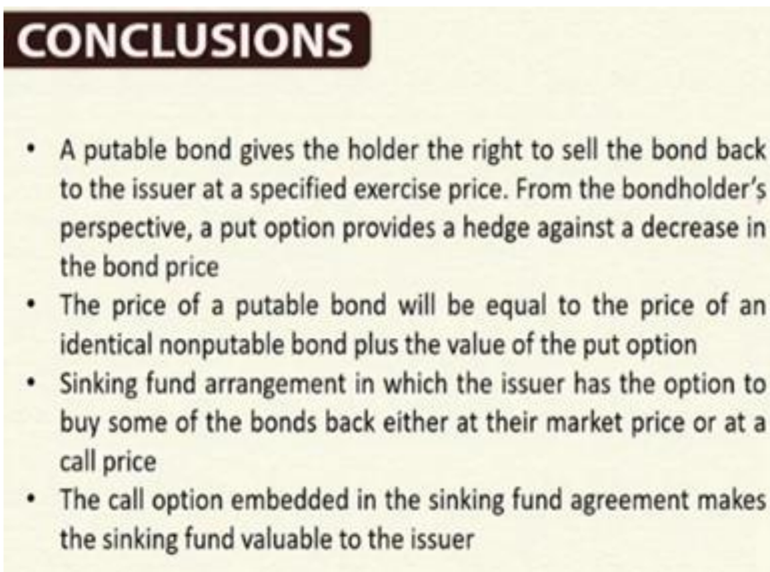

So, what basically we have discussed; the puttable bond basically gives the holder the right to sell back the bond at a particular price. And generally, it is considered as a hedge against the decrease in the bond prices and the price of a puttable bond is equal to the price of an identical non puttable bond plus the value of the put option. Sinking fund arrangements generally in which the issuer has the option to buy some of the bond back either at their market price or at a call price.

And the call option embedded in the sinking fund agreement makes the sinking fund valuable to the issuer perspective.

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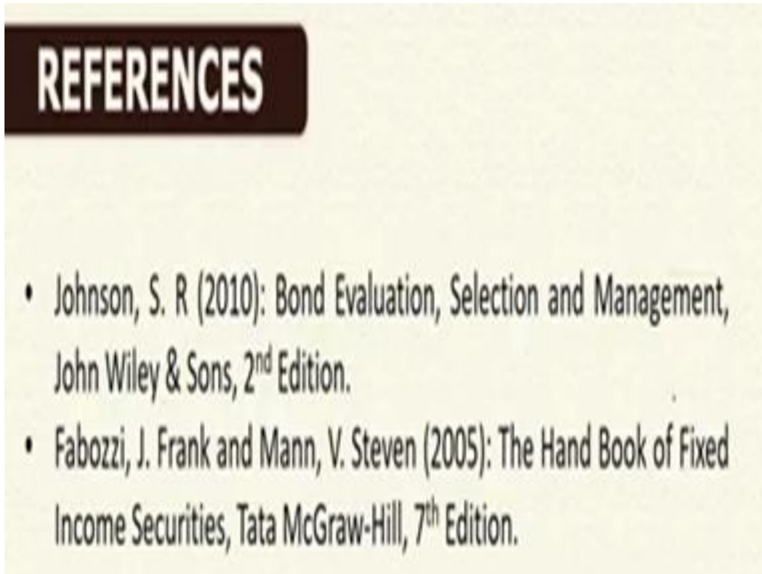

So, these are the references what you can see, thank you.