Management of Fixed Income Securities

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Module No # 07
Lecture No # 38
Binomial Valuation of Callable Bonds – II

So, in the previous session, we discussed the value of a callable bond, particularly the 2-period case we have considered using the Binomial tree model.

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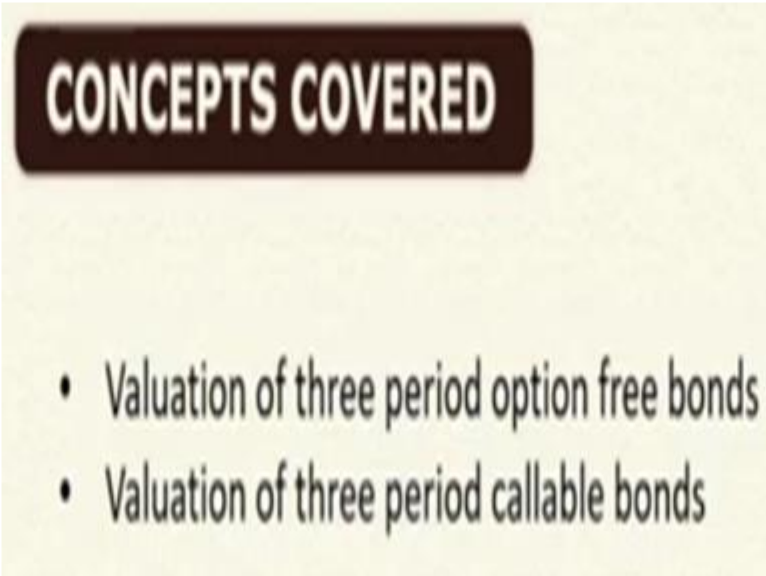

And today we will continue with this particular discussion and we will be considering or we can extend this particular example whatever we have considered in the previous session to a 3-period model. So, today we will see how we can use this binomial tree approach for the valuation of the 3-period option free bonds and the valuation of 3 period callable bonds. Conceptual understanding of whatever we have discussed in the previous class and the example of whatever we are taking, the same example we are going to continue for the next period.

And we will see that how it is different a little bit different. This 3-period model is different from the 2-period model.

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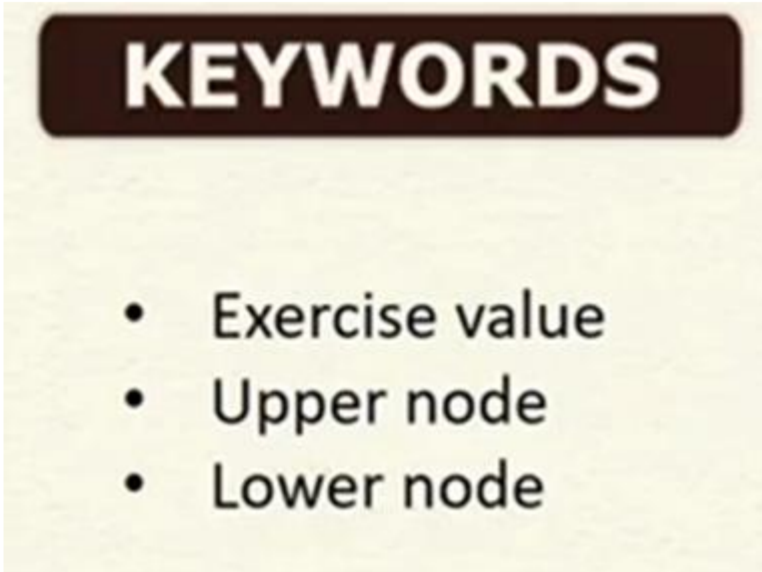

And we will come across certain kinds of keywords like upper node, lower node, exercise value and all these things for this particular discussion. And that thing is applicable for any kind of callable options or any kind of portable options whenever you are considering.

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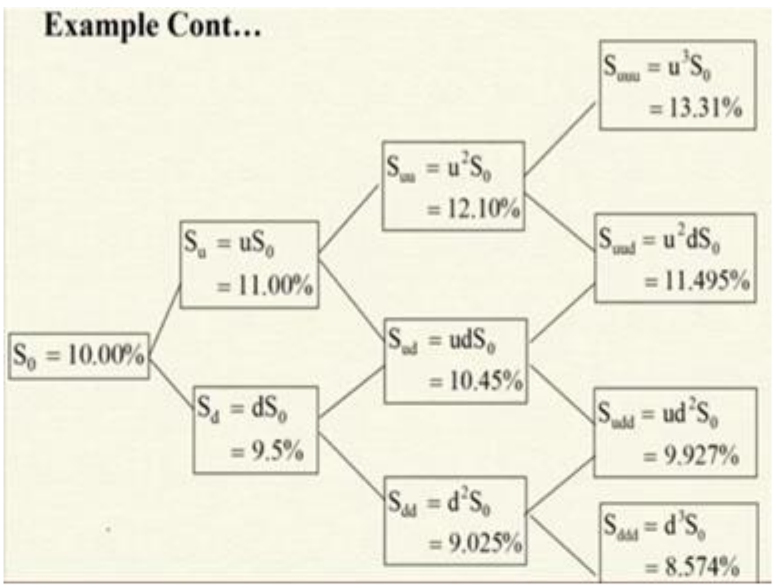

So, again the same example we are considering; so, keep that particularly interesting scenario in your mind, which we have started with 10% it can go up to 11 and go down to 9.5. 11 can go up to 12.1 or can go down to 10.45 or 9.5 can go up to 10.45 or it can go down to 9.025. So, all these scenarios it is the continuation of the example, whatever we have considered before that same example we are continuing.

So, this is the interest rate tree that we have considered the same interest rate tree we are considering for the 3-period model. Because of the 2-period model we have already discussed in the previous class. So, we keep this particular tree in your mind.

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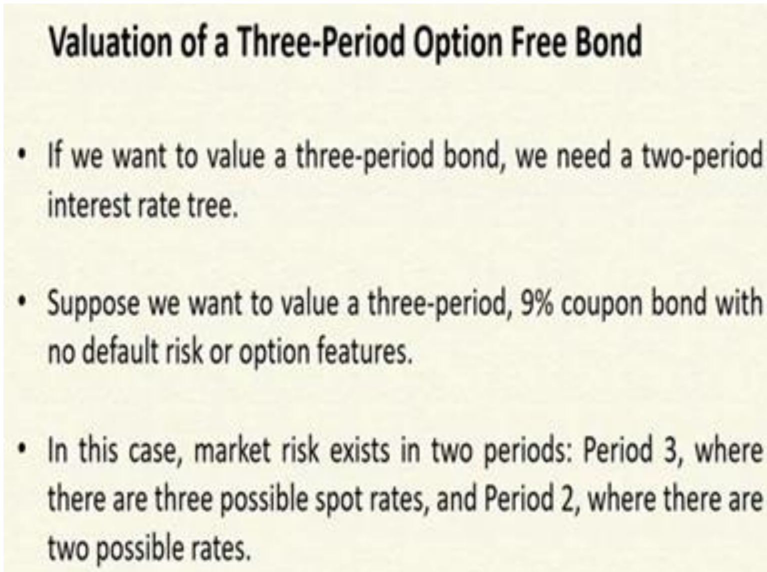

Let us see that what we need? Whenever we go for a valuation of a 3-period option-free bond, then we need a 2-period interest rate tree. If we want to value a 3-period bond then we need a 2-period interest rate tree and suppose we want to value a 3 period 9% coupon bond with no default risk or option features. This bond has no option feature and there is no default risk involved in that only the market risk is taken into consideration.

So, market risk basically can exist in the 2 periods. Period 3, where there are 3 possible spot rates, and period 2 where there are 2 possible spot rates, that is basically, what you can consider.

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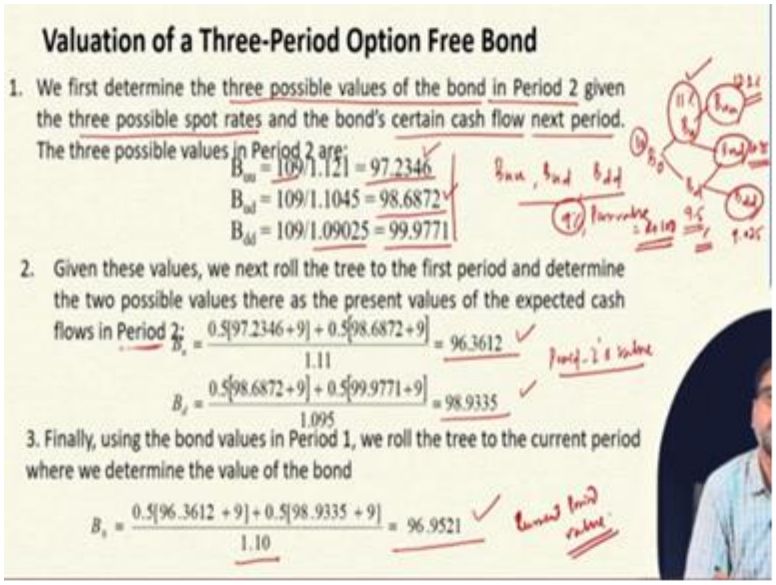

Let us see, what happens in that case? What are the steps you have to follow for the valuation of a 3-period option-free bond? First, determine the 3 possible values of the bond in period 2, given the 3 possible spot rates and the bond's certain cash flow next period. You have to consider 3 possible values of the bond in period 2, given the 3 possible spot rates and bond certain cash flows which are the next period.

So, what are those 3 possible values? It can be B_{uu} or it can be B_{ud} or it can be B_{dd}. It starts from the B_0 it can be it can go B_u then it can go to B_{uu} or here it is B_d. So, from B_u to it can come down also B_{ud} and further also it can come down to B_{dd}. So, these are the 3 values what we need in period number 2. So, what is the coupon? Coupon we have considered that is 9% of your per value is equal to 100.

Then here the cash flow will be in the end would be 109 you have to discount it with respect to the interest rate of 12.1% because this value will be 12.1. It starts with 10, 11, 9.5, then further it can go up to 12.1 or it can go down to 10.45 or it can further go down to 9.025. So, we have used these 12.1% one values you can calculate that is 109 by 1.121, that will be 97.2346. You can get your B_{ud} at a rate of 10.45% that is 109 by 1.1045 that will give you 98.6872 or it can be B_{dd} that will be 109 divided by 1.09025 that will give you 99.9771.

B_{uu} = 109/1.121 = 97.2346

B_{ud} = 109/1.1045 = 98.6872

B_{dd} = 109/1.09025 = 99.9771

So, you got the 3 possible values in the 3 different nodes. Then what can you do? Given these values basically, we next roll these trees to the first period and determine the 2 possible values as the present value of the expected cash flow in the period 2. So, we know this 50-50% chance we are considering. The value can become 97.2346 or it can become 98.6872. So, in the first period, the interest rate is 11% or 9.5%.

So, the cash flow what we get that is B_{uu} which is 97.2346 or 98.6872 or 99.9771 then what we have done? First 2 cases, we have considered 0.5 into 97.2346+9+0.5 into 98.6872+9 divided by 1.11 because in this period, this interest rate can prevail at 11%. So, you got the value 96.3612 or it can be it will be B_u we are considering this 11%. So, it can be also B_d, that will be your 0.5 into 98.6872 +9 + 0.5 into 99. 9771 + 9 divided by 1.095 at the rate of 9.5% that you can get 98.9335.

$$B_u = \frac{0.5[97.2346 + 9] + 0.5[98.6872 + 9]}{1.11} = 96.3612$$

$$B_d = \frac{0.5[98.6872 + 9] + 0.5[99.9771 + 9]}{1.095} = 98.9335$$

So finally, you got the 2 values in period 1 this is the period one's value. So, now using the bond values in period 1, we roll the tree to the current period and finally, you can determine the value of the bond. So, then what will be the value of the bond in the current period? That B_0 = 0.5 into 96.3612 + 9 + 0.5 into 98.9335 + 9 divided by 1.1 that will give you 96.9521. Then the value of the bond has become 96 that is what the current period price, current period value.

$$B_0 = \frac{0.5[96.3612 + 9] + 0.5[98.9335 + 9]}{1.10} = 96.9521$$

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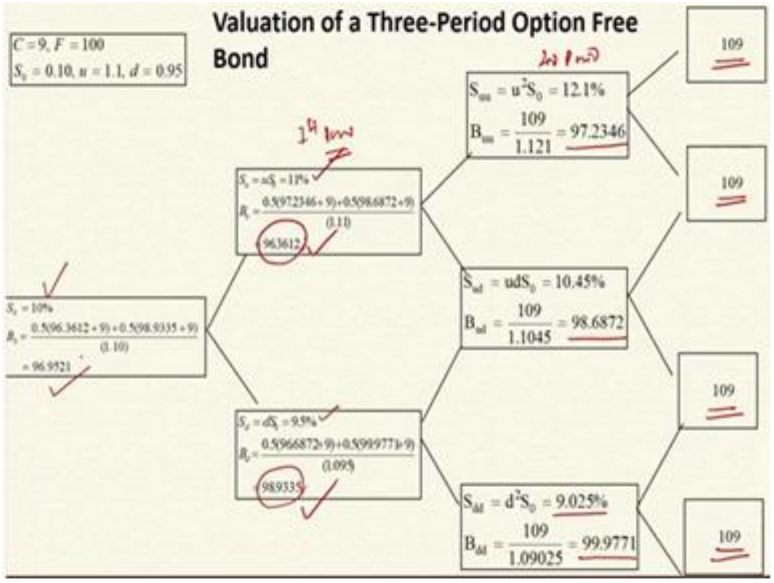

So, if you want to show this then how it will basically, how the calculation really, we have done in this case? You see we are going backward, rolling back your 9% is the coupon and 100 is the par value. So, in the end, the cash flow will be basically 109 every cases it will be 109 and the second-period interest rate already we have seen the 10% can go up to 11%. 11% can go up to 12.1% then we calculated your value of the bond in this particular period which is 109 divided by 1.121 and you got 97.2346.

B_{uu} = 109/1.121 = 97.2346

Here the interest rate either from 11% to it has gone up to 12.1 or can go down to 10.45 or from 9.52 also it can go up to 10.45. So, you can calculate your B_{ud} that is 109 divided by 1.1045, you got 98.6872 in this period of this node. Here, again if the further it is going down you get 109 divided by 1.09025 that will be 99.9771. Now, we have 3 nodes and the cash flow what basically we are calculating in this period. Either it can become 97.2346 or it can become 98.6872 or it will become 99.9771.

B_{ud} = 109/1.1045 = 98.6872

B_{dd} = 109/1.1.09025 = 99.9771

So, these are basically 2 alternatives and they can also be 2 alternatives. So, if you consider these 2 alternatives the probability can be 97 or 98. Then you get 0.5 into 97.2346 + 9 + 0.5 into

98.6872 + 9 and you are discounting it in this period interest rate that is 11%. Then you get 96.3612 or it can also be the 98.6872 and 99.9771. So, then you can discount it with respect to 9.5% and you get 98.9335.

$$B_u = \frac{0.5[97.2346 + 9] + 0.5[98.6872 + 9]}{1.11} = 96.3612$$

$$B_d = \frac{0.5[98.6872 + 9] + 0.5[99.9771 + 9]}{1.095} = 98.9335$$

Now, these 2 values you got in the first period, this is the second period. So, then what has happened in the current period, what do you get? You got the 2 cash flows 96.3612, 98.9335. Then you can discount it with respect to the current period's interest rate that is 10%. Then your probability is 0.5. We are considering 50%, 0.5 into 96.3612 + 9 + 0.5 into 98.9335 + 9. You can discount it with respect to 10% you got 96.9521 that is the current period value. So, that is basically a 3-period option-free bond.

$$B_0 = \frac{0.5[96.3612 + 9] + 0.5[98.9335 + 9]}{1.10} = 96.9521$$

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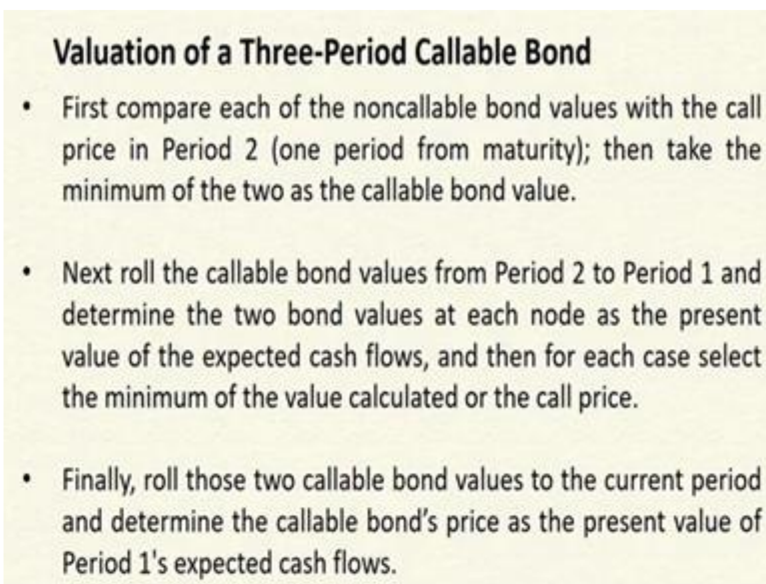

Now, you go to discuss about the valuation of a 3-period callable bond a little bit different. First compare each of the non-callable bond values with the call period, in period 2. Then take the minimum of the 2 as a callable bond value. Next roll the callable bond values from period 2 to period 1 and determine the 2 bond values at each node as the present value of the expected cash flow. And then for each case, select the minimum of the value calculated or the call price.

Finally, roll these 2 callable bond values to the current period and determine the callable bond price as the present value of the period once expected cash flows. Logic whatever we have used for the 2-period model we are just expanding that particular logic to the 3-period callable bond.

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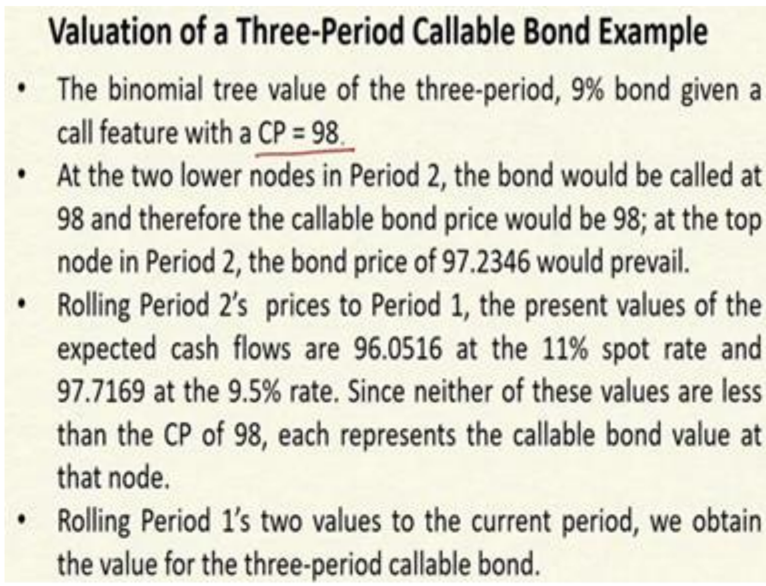

Now, let will continue with this particular example. Let this binomial tree value of the 3-period bond we are trying to do and the coupon is 9%, the bond has a call feature and the call price is 98. So, at the 2 lower nodes in period 2 the bond would be called at 98 and therefore the callable bond price would be 98. But at the top node in period 2 the bond price of 97.2346 will prevail.

So, rolling this period two's price to period one, the present value of the expected cash flows is what 96.0516 are the 11% spot rate and 97.7169 are the 9.5% rate. Since neither of these values is less than the call price of 98 each represents the callable bond value at that node. Rolling period 1's two values to the current period, we can obtain the value of the 3-period callable bond. But how are these values calculated? How we have received or how we got these particular values?

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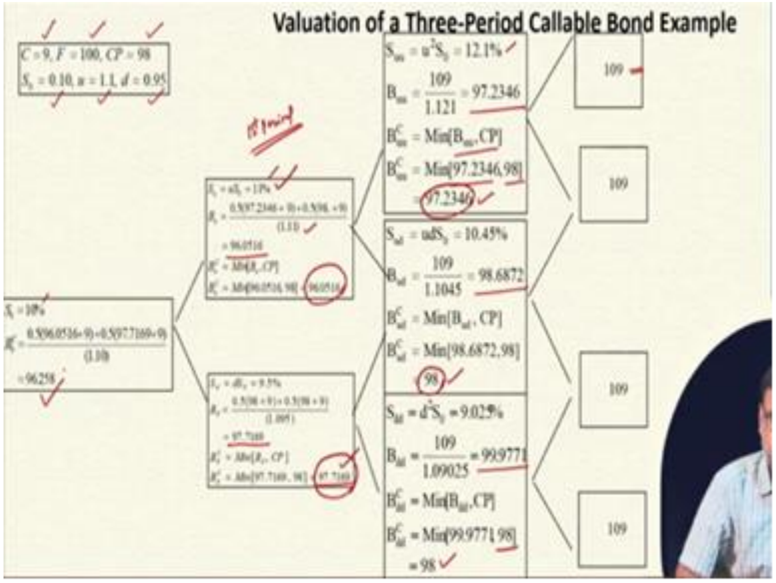

If you see these things, it will be clearer for you here the coupon is 9% or 9 rupees on 100 rupees F = S, CP = 98, S_0 = 10%, u = 1.1, d = 0.95. That; we are considering starting with 10% becoming 11%, 12.1% and now what we have done? This 109 is the cash flow in this period. So, we have discounted it to calculate the B_{uu}, 109 divided by 1.121, we got 97.2346. Now, what is the B_{uu}^C? That is basically the minimum of B_{uu}, CP, that already you know.

$B_{uu} = 109/1.121 = 97.2346$

$B_{uu}^C = \text{Min}\ [B_{uu}, CP] = \text{Min}\ [97.2346, 98] = 97.2346$

Then it will be 97.2346 and 98. So, now 97.2346 less than 98. So, the price remains at 97.2346 only because the price of the bond is less than the call price. In the second case, here if you see what you got 98.6872 and your call price is 98. So, now the call price is less than this. Then this value will prevail. The third node, so is node 1, node 2, and node 3, and the third node becomes 99.9771. The call price is 98 that's why 98 will prevail.

$B_{ud} = 109/1.1045 = 98.6872$

$B_{ud}^C = \text{Min}\ [B_{ud}, CP] = \text{Min}\ [98.6872, 98] = 98$

$B_{dd} = 109/1.09025 = 99.9771$

$B_{dd}^C = \text{Min}\ [B_{dd}, CP] = \text{Min}\ [99.9771, 98] = 98$

Now, what have we considered in this period? 97.2346 and 98, then 98 and 98 there are 2 alternatives. So, the cash flows. So, that's why the B_u in the first period, if you are calculating then it is 0.5 into 97.2346 + 9 + 0.5 into 98 + 9 divided by 1.11; 11% of the discount rate. We got 96.0516. So now in the second case, we got 98 and 98. These are the cash flows and as well as a coupon that is 9 and 9, we got 97.7196 then this has become 97.7169.

$$B_u = \frac{0.5[97.2346 + 9] + 0.5\,[98 + 9]}{1.11} = 96.0516$$

$$B_u^C = \text{Min}\,[B_u, CP] = \text{Min}\,[96.0516, 98] = 96.0516$$

$$B_d = \frac{0.5[98 + 9] + 0.5\,[98 + 9]}{1.095} = 97.7169$$

$$B_d^C = \text{Min}\,[B_d, CP] = \text{Min}\,[97.7169, 98] = 97.7169$$

So, now these 2 cash flows whatever you have considered, these 2 cash flows are less than 98. So, both the prices will prevail. So, this is a minimum of 96.0516 and 98 that will be 96.0516. Here also it will be 97.7169. So, now this cash flow we are using or discounting is rolling back to the current interest rate of that is 10%. So, then your 96.0516+9 into 0.5 + 0.5 into 97.7169 + 9 divided by 1.1 you got 96.258.

$$B_0^C = \frac{0.5[96.0516+ 9] + 0.5\,[97.7169 + 9]}{1.10} = 96.258$$

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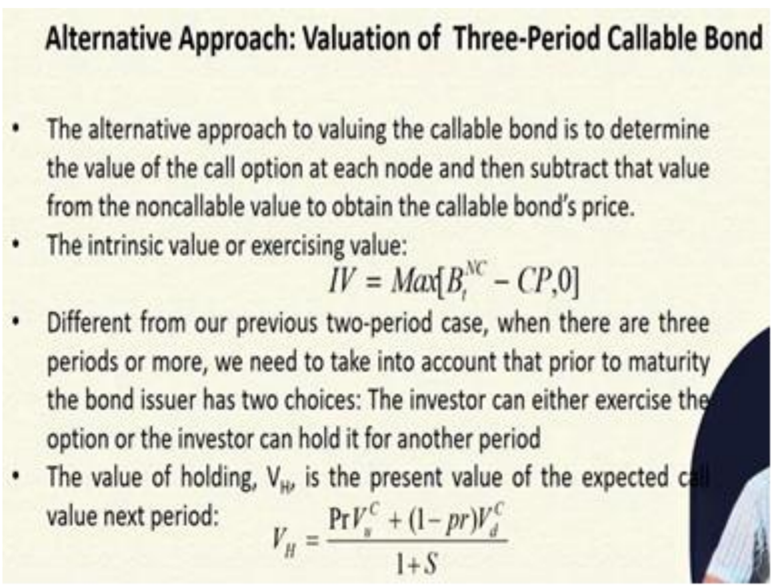

So now, what we have observed here, is that the value of the call option is different because actual cash flow depends upon the market price of the bond or the value of the bond at that node and the call price. So, there is an alternative approach. Whatever approach we have discussed for the 2-period bond to some extent, we can also use it for these 3 periods collaborative bonds also. The alternative approach is to go for a valuation of the callable bond.

First of all, you can find out the call option at each node. Then subtract that call option value from the non-callable bond value. That we have discussed in the previous case. That is called basically the intrinsic value of the exercising value. So, what is the basic difference here if you see? The difference from our previous 2 period case, when there are 3 periods or more, we need to take into account that prior to maturity, the bond issuer to have 2 choices: either the investor can exercise the option or the investor can hold it for another period.

Then let the value of the holding let V_H, if you want to calculate it, be nothing but the present value of the expected cash value in the next period. So, that means it is basically V_H will be your probability of this V_u^C + (1 – pr) of the V_d^C divided by 1 + S. Let us see how this particular concept is working in this case?

$$V_H = \frac{Pr\ V_u^C + (1 - pr)\ V_d^C}{1 + S}$$

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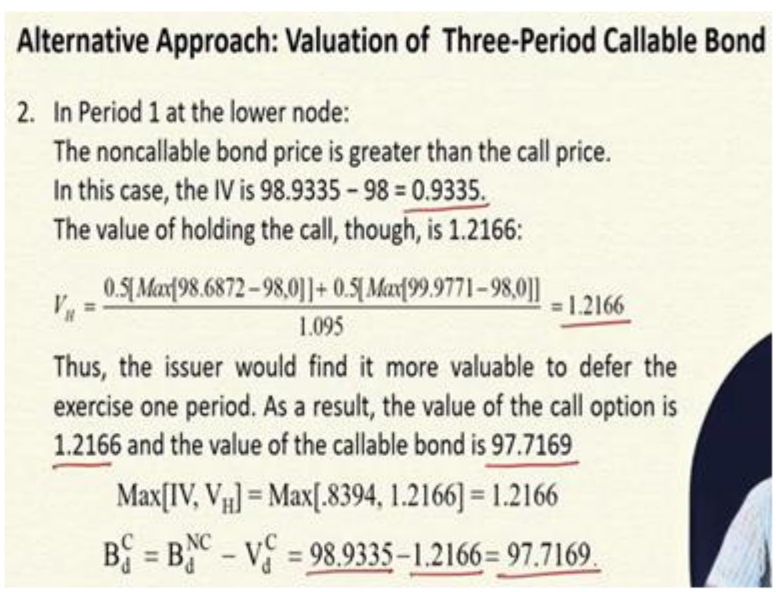

Let in period 2, the value of holding is zero at all 3 nodes. Since, next period is maturity, where it is too late to call. The issuer, though, would find it profitable to exercise in 2 of the 3 cases, where the call price is lower than the bond values. The 3 possible callable bond values in period 2, if you want to calculate that is your 97.2346 that is the value of the non-callable bonds minus the maximum of 97.32346 - 98 and 0.

This will be a minus then minus comma zero, then obviously this 97.2346 - 0 that will be 97.2346. This will be 98.6872 minus the maximum of this and here also maximum will be this will be basically what, how much will be 0.6872, the 98.6872 – 0.6872 that will be 98. Here also, it is more than this 99.9771 minus this that will be giving you 98. In all these cases, you got your 3 possible callable bond values in period 2.

Then what will you do? In period 1 at the lower node, the non-callable bond price is greater than the call price. So, the non-callable bond price is 98.9335 and the callable bond value is 98 then the intrinsic value what you got that is basically 0.9335. So, if you want to calculate the value of holding the call if you want to hold the call option. Then what is the value of holding the call? 0.5 into the maximum of 98.6872 – 98, 0 + 0.5 into the maximum of 98.9771 – 98, 0 divided by 1.095 in that lower node that interest rate is 9.5%.

$$V_H = \frac{0.5[\text{Max}\,[98.6872 - 98,0]] + 0.5\,[\,\text{Max}\,[99.9771 - 98,0]]}{1.095} = 1.2166$$

Then you got that is 1.2166, so the issuer would find it more valuable to defer the exercise 1 period. So, as a result, the value of the call option is 1.2166 and the value of the callable bond will become 97.7169 how? Because the value of the non-colorable bond is 98.9335 and the value of the particular call option becomes 1.2166 then this minus this will give you 97.7169. So, here is basically what we are trying to say. This is a little bit different in the sense that there is a concept of the holding period; it can be whole.

$\text{Max}\,[IV, V_H] = \text{Max}\,[0.8394, 1.2166] = 1{,}2166$

$B_d^C = B_d^C - V_d^C = 98.9335 - 1.2166 = 97.7169$

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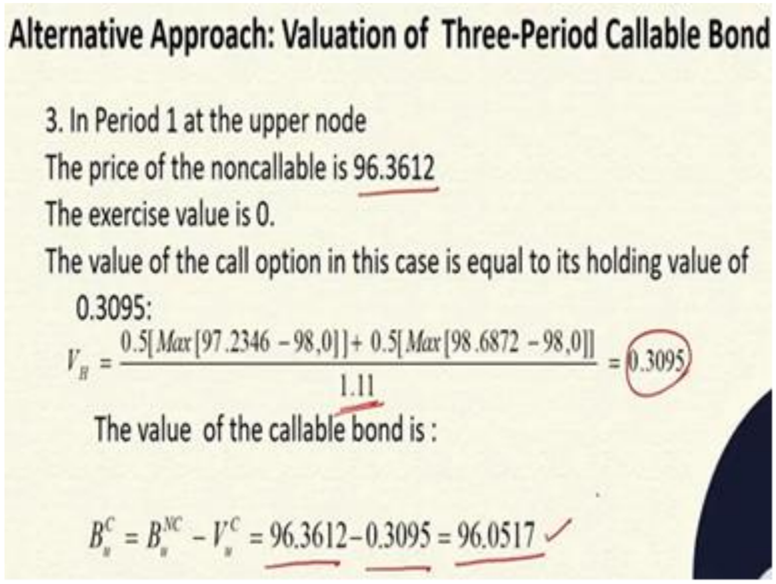

So, then what we have seen in the third step in period 1 at the upper node, the price of the non-callable bond is 96.3612 and the exercise value is 0. Then the value of the call option, in this case, is equal to its holding value if we calculated 0.3095 how we have calculated? That is the 0.5 into a maximum of 97.2346 - 98 and 0 and the 0.5 into a maximum of 98.6872 – 98, 0 divided by 1.11. 11% is the interest rate at the upper node then you got 0.3095.

$$V_H = \frac{0.5[Max [97.2346 – 98,0]] + 0.5 [Max [98.6872 – 98,0]]}{1.11} = 0.3095$$

So, then the value of the callable bond is 96.3612 which is the value of the non-callable bond minus your 0.3095 which is basically your 96.0517.

$B_u^C = B_u^{NC} – V_u^C = 96.3612 – 0.3095 = 96.0517$

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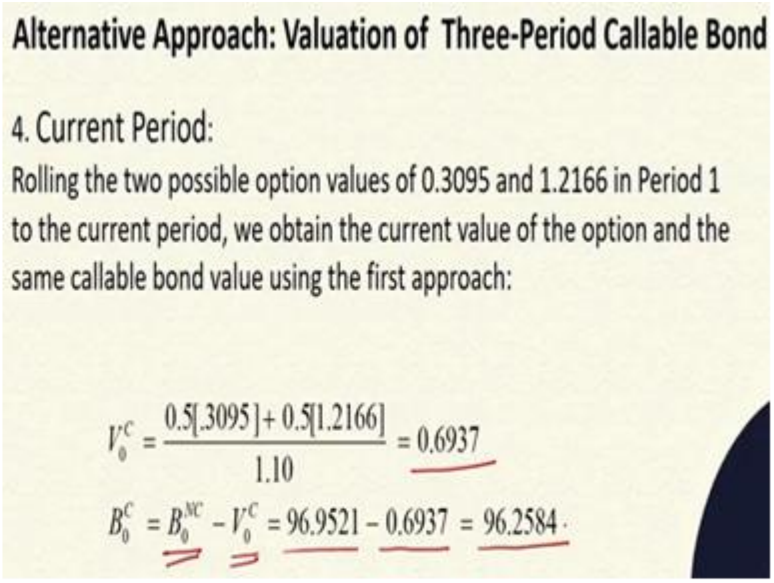

So, if you are rolling these 2 periods of possible options values of 0.3095 and 1.2166 in period 1 to the current period. Then we can find the current value of the option and the same callable bond value using the first approach. So, now you got 0.5 into 0.3095 + 0.5 into 1.2166 divided by 1.1 that is basically giving you 0.6937. So, then your value of the call option is the value of the non-callable bond minus the value of the call option, that is your value of the non-callable bond has become 96.9521, that already we have calculated - 0.6937 that will give you 96.2584.

$$V_0^C = \frac{0.5[0.3095] + 0.5[1.2166]}{1.10} = 0.6937$$

$$B_0^C = B_0^{NC} - V_0^C = 96.9521 - 0.6937 = 96.2584$$

So, this is basically another approach what you can use for the calculation of the 3-period callable bond. This is another alternative approach that also can be used.

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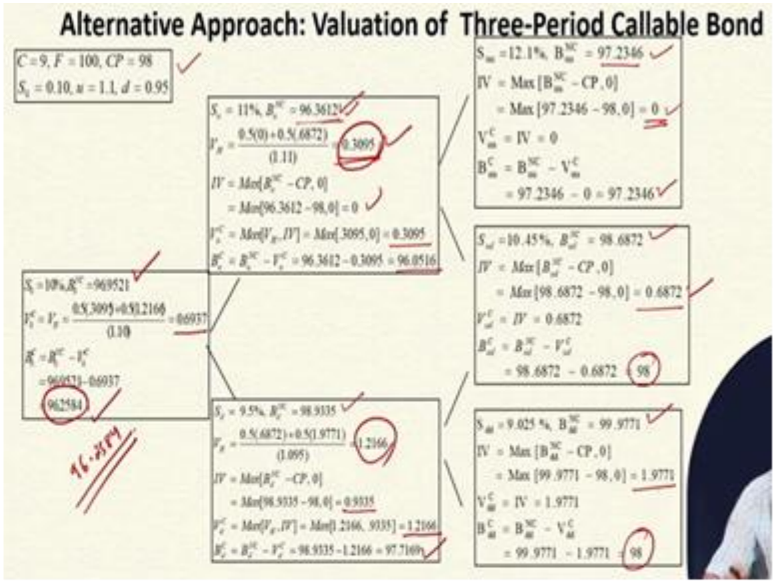

So, what basically we have discussed if you see the summary of this? Then what basically we have done here? Other things we have all these things already it is clear to you and now we have calculated the non-callable bond value is this. Here it is basically this, here it is this, this and this. These are the non-callable bond values then what we have done? The intrinsic value we have calculated the intrinsic value basically in this case is if you see that it is 0 then the B_{uc} that becomes is the same with this.

$S_{uu} = 12.1\%$, $B_{uu}^{NC} = 97.2346$

$IV = Max\ [B_{uu}^{NC} – CP, 0] = Max\ [97.2346 – 98, 0] = 0$

$V_{uu}^{C} = IV = 0$

$B_{uu}^{C} = B_{uu}^{NC} - V_{uu}^{C} = 97.2346 – 0 = 97.2346$

$S_{ud} = 10.45\%$, $B_{ud}^{NC} = 98.6872$

$IV = Max\ [B_{ud}^{NC} – CP, 0] = Max\ [98.6872 – 98, 0] = 0.6872$

$V_{ud}^{C} = IV = 0.6872$

$B_{ud}^{C} = B_{ud}^{NC} - V_{ud}^{C} = 98.6872 – 0.6872 = 98$

$S_{dd} = 9.025\%$, $B_{dd}^{NC} = 99.9771$

$IV = Max\ [B_{dd}^{NC} – CP,0] = Max\ [99.9771 – 98, 0] = 1.9771$

$V_{dd}^{C} = IV = 1.9771$

$B_{dd}^{C} = B_{dd}^{NC} – V_{dd}^{C} = 99.9771 – 1.9771 = 98$

Here it is 0.6872 which is why this value becomes this minus this which will be 98 here also if you see that it is 1.9771. So, this minus this would give you 98. So, now what has happened? We have considered these values here. So, now we got this 96.3612 value. So, here it is either 0 or it is what 0.6872. So, here we have gone 0.5 into 0 + 0.5 into 0.6872 divided by 1.11. We got 0.3095. So, with the maximum of 0 and 0.3095 in the second case, we got 0.

$S_u = 11\%, B_u^{NC} = 96.3612$

$$V_H = \frac{0.5(0) + 0.5(0.6872)}{1.11} = 0.3095$$

$IV = Max\ [B_u^{NC} – CP,0] = Max\ [96.3612 – 98, 0] = 0$

$V_u^{C} = Max\ [V_H, IV] = Max\ [0.3095, 0] = 0.3095$

$B_u^{C} = B_u^{NC} - V_u^{C} = 96.3612 – 0.3095 = 96.0516$

So, either of these 2 basically if you consider 0 and this you got 0.3095. So, the value of this particular call option in this node will be this minus this. That will be 96.0516. In the second case, what have we done? That will be your 1.2166 and here it is 0.9335, then it has become 1.2166 then this 98.9335 – 1.2166 that will give you 97.7169. So, now we again roll back to the current period.

$S_d =9.5\%, B_d^{NC} = 98.9335$

$$V_H = \frac{0.5(0.6872) + 0.5(1.9771)}{1.095} = 1.2166$$

$IV = Max\ [B_d^{NC} – CP,0] = Max\ [98.9335 – 98, 0] = 0.9335$

$V_d^{C} = Max\ [V_H, IV] = Max\ [1.2166, 0.9335] = 1.2166$

$B_d^C = B_d^{NC} – V_d^C = 98.9335 – 1.2166 = 97.7169$

The current period is 0.3095 this is the call option here in this case the call option is 1.2166. So, then it is 0.5 into 0.3095 + 0.5 into 1.2166 divided by 1.1 that will give you 0.6937. So, in this period the value of the non-callable bond is this. So, 96.9521 – 0.6937 will give you 96.2584. So, that is the call option value, in this case, is 96.2584. So, that is the alternative approach of the summary, whatever we have discussed that just we have presented in one case.

$S_0 = 9.5\%$, $B_0^{NC} = 96.9521$

$$V_0^C = V_H = \frac{0.5(0.3095) + 0.5(1.2166)}{1.10} = 0.6937$$

$B_0^C = B_0^{NC} – V_0^C = 96.9521 – 0.6937 = 96.2584$

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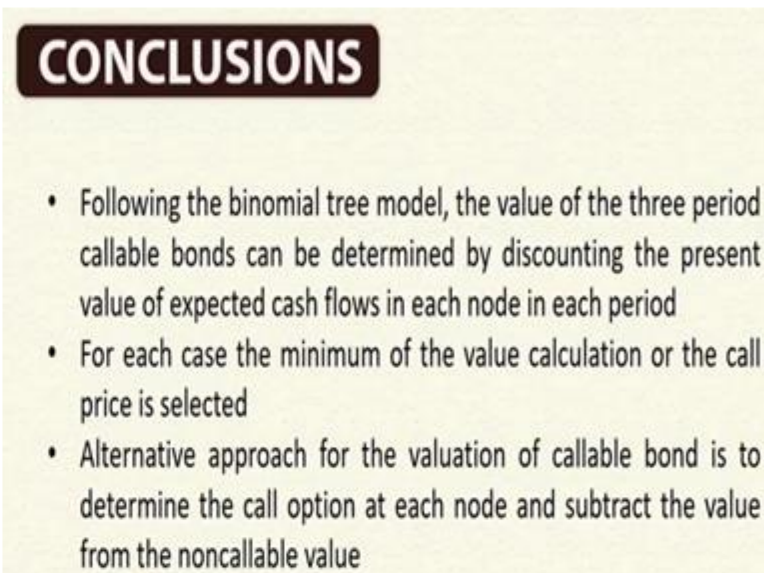
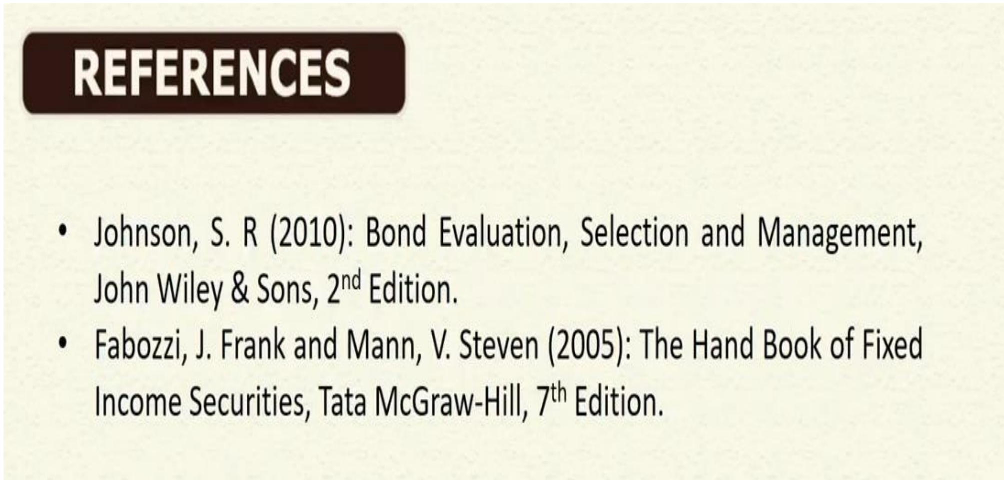


So, what basically we have discussed here is that the value of the 3 period callable bonds can be determined by discounting the present value of the expected cash flows in each node in each period. For each case, the minimum of the value calculation or the call price is selected and the alternative approach for the valuation of callable bonds is to determine the call option at each node and subtract the value from the non-callable value. That is another approach that also can be followed.



So, these are the references. Thank you