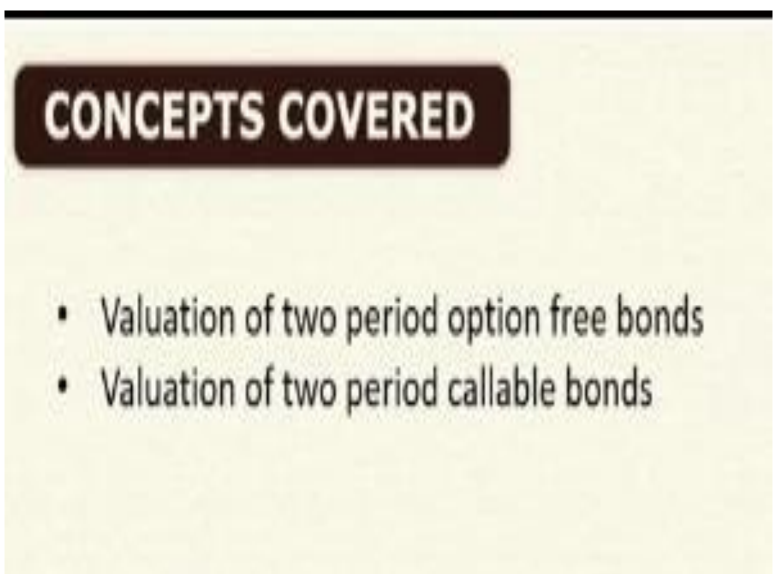
Management of Fixed Income Securities
Prof. Jitendra Mahakud
Department of Humanities and Social Science
Indian Institute of Technology, Kharagpur

Module No # 08
Lecture No # 37
Binomial Valuation of Callable Bond - I

Welcome back. So, in the previous class we have introduced the concept of the binomial tree model. Basically, whenever it is assumed that the interest rate may change in the future or the yield curve is going to be changed. And in that particular context how the valuation of the different type of bonds which have certain kind of embedded features or embedded options can be evaluated. Or how the evaluation of those bonds can be done with some kind of embedded options with this concept of volatility of the interest rate.

In that particular context we just introduced the binomial tree of the interest rate movements in a particular point of time. Then accordingly we can use that particular probable interest rate which is going to prevail in the market, and accordingly the valuation of that particular callable bonds or the bonds having the call features can be estimated.

(Refer Slide Time: 01:27)



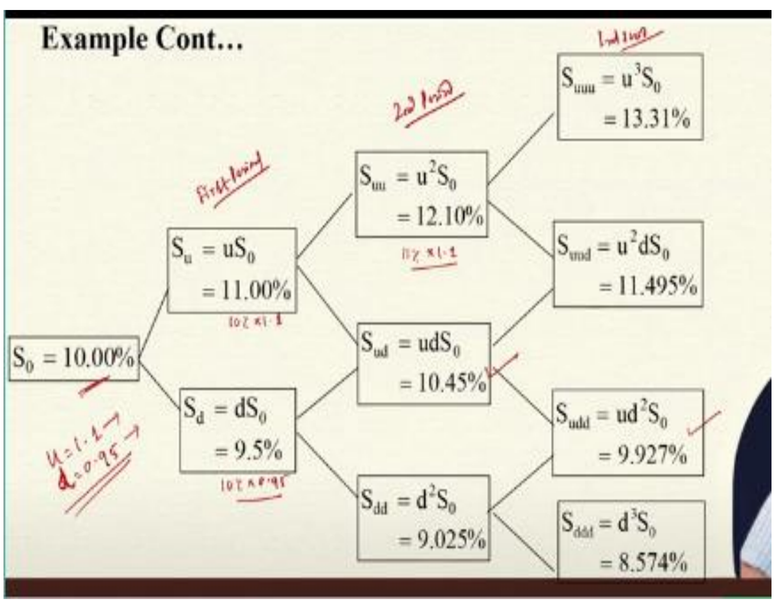
So let us see that what are those concepts, we are going to cover in today's class. We will be covering 2 things one is the valuation of the 2-period option free bonds and the valuation of 2 periods collable bonds. So, what is the basic difference between the valuations of the 2 periods of an option free bond.

That means if the interest rate is going to be changed and that interest rate probably, interest rate in the 2 periods first we have to find out using the binomial interest rate introspective model. And from there we can use that particular interest rate for the valuation of this particular bonds.

(Refer Slide Time: 02:17)

So, the different keywords what will be using for today's class or you will come across. These are basically the call options, call price, this spot rates, the intrinsic value all these things. These are the things basically what you will get to know or basically will be used in today's sessions.

(Refer Slide Time: 02:43)



So let us see that how this thing is basically going to work, you just recall in the previous class that what basically the binomial interest rate tree we have constructed. This is the hypothetical example whatever we have taken we have started with the initial spot rate that is 10% and here we have taken your u = 1.1 and d = 0.95. So here this either the interest rate can go up or the interest rate can go down.

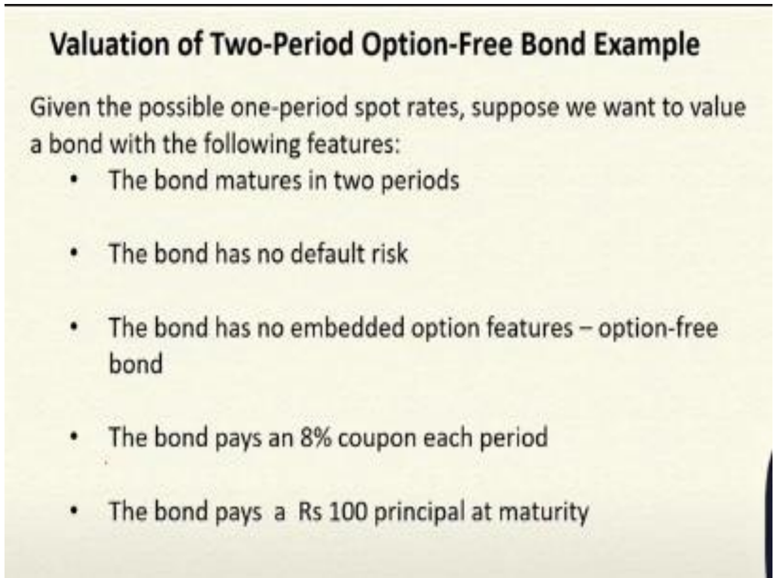
So, if the interest rate will go up it can become 11% so that is nothing but the 10% into 1.1, and it is nothing but this 10% into 0.95; so, they can either become 11% or become 9.5%. Next period it is the one period data the next period from 11% to it can go up to 12.1% it is nothing but this 11% into 1.1. And again, it can also come down to 10.45% the 10.45% is nothing but 11% into 0.95, or it can also go from 9.5% to one point multiplied by 1.1.

So, there is a node here that this node basically gives you the figure that is 10.45%; or again it can further go down that will be 9.5 into 0.95 that will be 9.025%. Further again you can go for this is the first period nodes, and this is your second periods nodes, and this is your third period nodes. So, third period again the 12.1% can become 13.31 that is 12.1 into 1.1, or it can go down also up to 11.495% that basically you get that 12.1 into 0.95; or it may be 10.45 into 1.1 that will become 11.495%.

Or it can go also 10.45 to 9.927% it can go down then 10.45 into 0.95 that you can get here, or it can also 9.025% can also reach here, if you multiply 1.1 you can reach also 9.927%. Or it can again further go down that d cube S 0 that may be 8.574%. So, like that you can also expand this particular interest rate tree for the next period so either, it can from 13.31 to it can go up to 13.31 into 1.1 or it can go down also 13.31 to 0.95.

So, u is equal to 1.1 and d is equal to 0.95 that is what we have assumed, or we have considered. Further we will see that how this u and d can be estimated, but now you can assume that that u = 1.1 and d = 0.95. So, keep this particular figure into your mind then we are going to use this particular numbers for evaluation of a hypothetical callable bonds. And the bonds also which do not have any call features, and what is the difference between the valuation of that callable bond or the valuation of the bonds which have no call options.

(Refer Slide Time: 06:45)

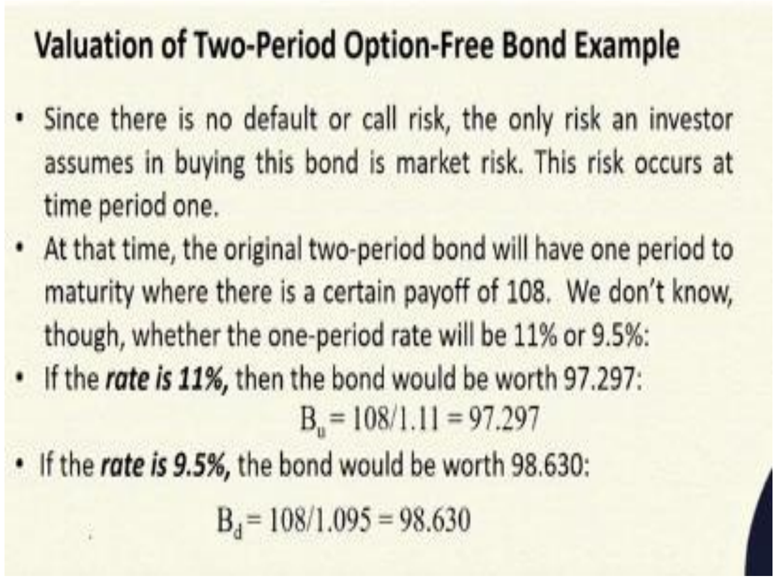


So then here we have taken this example, so given this possible one period spot rates suppose we want to value a bond with the following features. Let you are considering a bond and the bond basically matures in 2 periods that is the first assumption we have taken the bond is going to be matured in two periods. The bond has no default risk this is the second assumption what we have taken.

And the bond let us first case the bond has no embedded option features, let that is option free bond, it has no call option feature, or boot option feature or any sinking fund provisions, no other options are available, the bond has no embedded options features. And you assume that the bond pays a coupon of 8% in each period, and the bond pays rupees 100 the principal at the maturity, the par value of the particular bond is 100 rupees.

Coupon is 8% the par value the bond is 100 rupees and this is option free bond. That is the assumption what basically we have taken and it has no default risk and the bond matures in the 2 periods, that is more important that actually you have to keep in the mind.

(Refer Slide Time: 08:12)



So, in that case what basically we can see first of all; let us see that how the valuation of a 2-period option free bond can be carried out. Since the bond has no default risk or the call risk; the only risk the investor is basically is assuming that is the market risk. And why basically market risk because the interest rate is going to be changed. So, considering that market risk we are trying to see that on the basis of the binomial interest rate tree whatever we have considered this interest rate scenario or the change in the interest rate in the different periods how it is going to play the role for the evaluation of this particular bond.

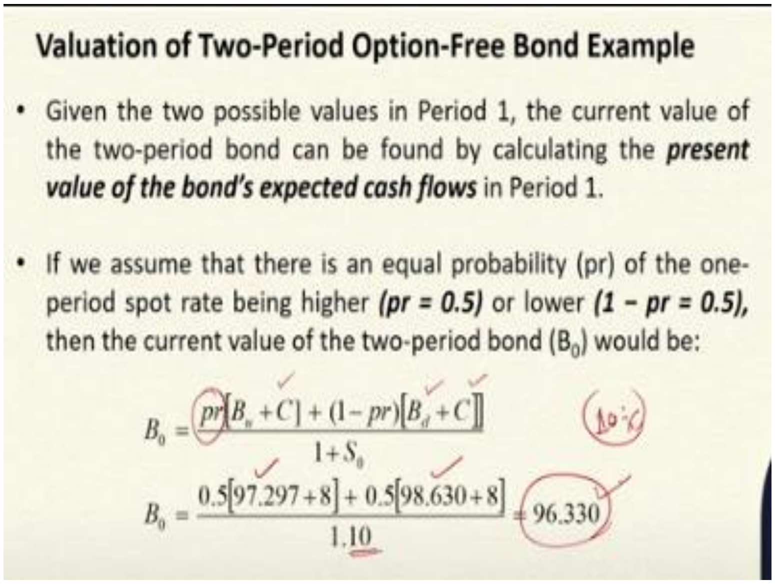
So, at that time the original 2 period bond will have one period to maturity where there is a certain payoff of 1.08. But we do not know whether the one period rate will be 11% or 9.5%. It was started from 10% so either it can go up to 11% or it can go down to 9.5% that is not clear to us. So, if the rate will be 11% then what will be the value of the bond, then it will be the B_u represents if the interest rate is going to be increasing and it will reach 11% from the 10%.

Then the value of the bond will be your 108 the cash flow what you are getting in the end of the period because 8 rupees 8% is the coupon and per value is bond is 100. That we have assumed then 108 divided by 1.11 that is 97.297, so if the interest rate will go down from 10% to 9.5% then the worth of the bond or the value of the bond will be 108 divided by 1.095 that is 98.630 right.

$B_u = 108/1.11 = 97.297$

$B_d = 108/1.095 = 98.630$

(Refer Slide Time: 10:28)



So now if you see that given the 2 possible values in the period one that what we have taken the 2 values one is your with respect to the 11% another one is with respect to the 9.5%. So given this particular 2 possible values in the period one, the current value of the 2-period bond can be found by calculating the present value of the bonds expected cash flow in the period one. Whatever probable cash flows you are getting in the period one that probable cash flow can be used to calculate the value of this particular bond in the present time.

$$B_0 = \frac{pr\,[B_u + C] + (1-pr)\,[B_d + C]}{1+S_0}$$

So, in that case the expected cash flow you have in one case it is 97.297 and other case it is 98.630. If the bond the interest rate is going to be 11% then the value will be 97.297; and if the interest rate is 9.5% then it will be 98.630. So let assume there is an equal probability of the one period spot rate being higher that is 0.5 probability is 50% or it can be lower that is one also 0.5; (1 – pr) that is 0.5.

And probability of cash flows in the 2 different periods will be is 50% and the value of the bond whenever the interest rate is going to be up that is 97.297. And whenever it is going down that will be your 98.630 that we have estimated we have calculated in the previous slide.

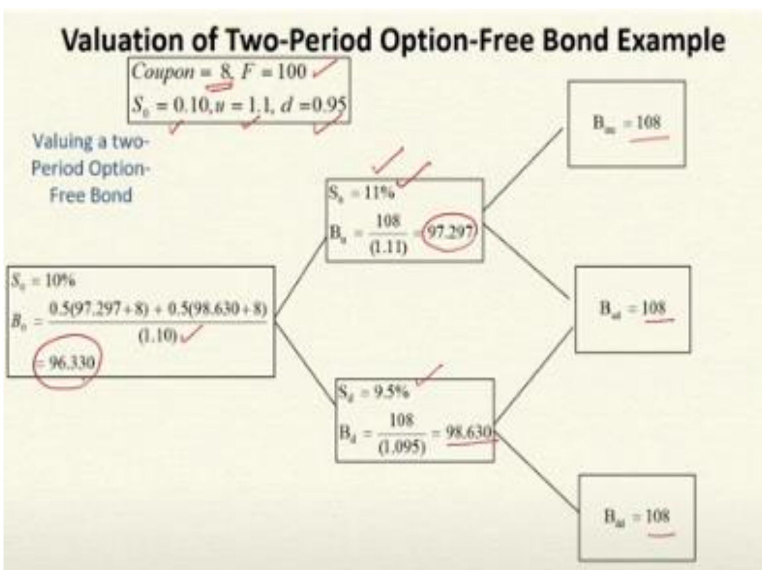
So then how we can calculate the value of for the particular bond in the present time. So, your probability that is the 0.5 whatever we have considered into your B_u which represents whenever the interest rate is going to be up what is the cash flow or the value of the bond in that particular point of time that is 97.297. And you get your coupon that is basically your 8 so 97.297 + 8 +(1 – pr) into your $B_{d;}$ B_d represents the value of the bond of the interest rate is going to be down so that is your 98.630 + your coupon that is 8.

So, this is the expected value what basically you are getting in the current period. You are going rolling down. There is a backward calculation what we are making. So, cash flow what we have calculated on the basis of the change in interest rate in the first period. Then we are basically discounting it with respect to the interest rate which is going to be prevailed or which is already prevailing in the market in the current period.

$$B_o = \frac{0.5\,[97.297 + 8] + 0.5\,[98.630 + 8]}{1.10} = 96.330$$

In the current period the interest rate we have assumed we have started from the interest rate of 10%. So, the cash flow what you get 0.5 into 97.297 + 8 + 0.5 into 98.630 + 8 divided by 1.1 this 1.1 means this is 1 + r this r is the 10%, so then we get it 96.330. So that is the current value of this particular bond if the interest rate either going to be up or down then these are the cash flows what you are getting. Then if you discount it with respect to the current interest rate that is 10% then you get the value of the bond in the current period.

(Refer Slide Time: 14:20)



So now what basically here we have seen we have just summarized these calculations whatever we have made. If you see that we started with this we have assumed this coupon is 8% means 100 rupees is the face value. That's why we have assumed coupon is the amount is 8 rupees face value is 100 rupees your current interest rate is 10%, u = 1.1 d = 0.95 right. So, what has happened, if interest rate was 10% it can go up to 11% or it can go down up to 9.5%.

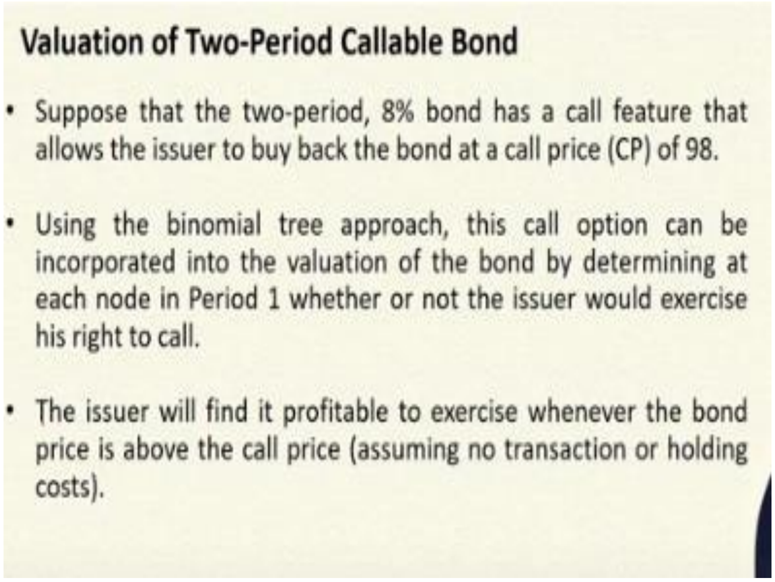
And your cash flow what basically you are getting here it is basically B_{uu} if you consider the 8 rupees is the coupon 100 rupees is the first value that is 108 every cases it will be 108. Now what basically we have done we have discounted that 108 with respect to the expected interest rate which is going to be prevailed in this node that is 11%. So, 108 divided by 1.1 you got 97.297, if assume interest rate has gone down then it will become 9.5% then your value of the bond will be 98.630, now these are the expected cash flows what you are getting.

$B_u = 108/1.1 = 97.297$

$B_d = 108/1.095 = 98.630$

Now what you are doing you discount it with respect to the current interest rate that is basically 10% then you can get the current value of the bond that is 96.330 so that is the current value of that particular bond in the current time. So, this is what basically what we get it for an option free one whenever you are going for a violation of a 2-period option free bond.

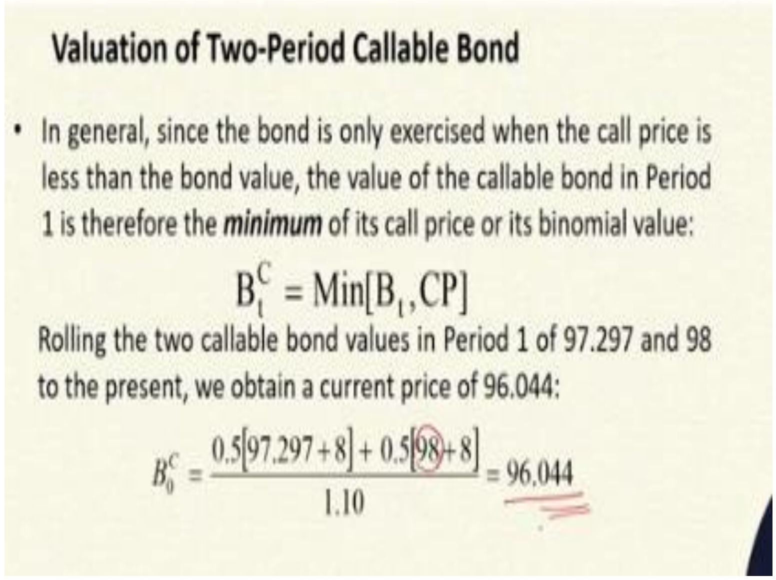
(Refer Slide Time: 16:20)



So let us see that how basically this thing will be prevailed or this thing will work for valuation of a 2-period callable bond. Suppose you assume that there is a 2-period bond 8% is the coupon rate it has a call feature. It has a call feature which allows this particular issuer to buy back that particular bond at a call price of 98. So here what basically you are doing using this binomial tree approach this call option can be incorporated into the valuation of the bond by determining at each node in period one; whether or not the issuer would exercise his right to call. Depending upon the price of the particular bond expected price of the bond in that particular period, you can decide that whether the issuer will go for calling the bond or not if it is profitable, then they will go for utilizing this call option if it is not profitable then they will not exercise. So, the issuer will find it profitable to exercise whenever the bond price is above the call price, whenever the bond price in the market will be more than the call price.

Then only the, it will be profitable for the issuer to call back the bond assuming no transaction or the holding cost you assume that. If your bond price in the market will be more than this call price, whatever has been fixed before then there is a possibility that the issuer can exercise that particular option.

(Refer Slide Time: 18:23)



So now let us see what exactly happens in this case. So, since the bond is only exercised when the call price is less than the bond value. Then the value of the call option in period one is what it is the minimum of its call price or it is a binomial value. So, your B_t^C (t is the time period) will be your minimum of B_t or the call price**.** So, what basically you are doing rolling the 2 callable bond values in period 1 that what basically we are getting that is 97.297 and 98 to the present.

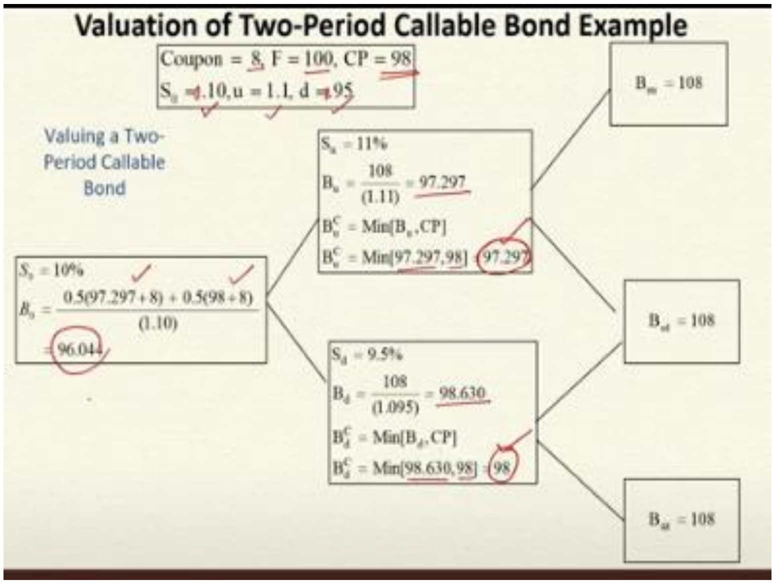
$$B_t^C = \text{Min}\,[B_t, CP]$$

First period if the interest rate has gone up then what was the value of the bond, we have calculated that is 97.297 and the call price we have fixed that is 98, right. Then we can obtain the current price of that particular bond that is your 0.5 into 97.297 + 8 + 0.5 into 98 + 8 and you can discount it with respect to the 10% you get the value of the 96.044. So here what basically we are trying to say that when the bond call option will be exercise. The call option will be exercised whenever the market value will be more than the call price otherwise not.

So, in the lower interested scenario, the value of the bond was 98 point something 98.630, so in that case the call option will be exercised. But in the first case the call option will not be exercised because the 97.297, is less than the 98 rupees. So that’s why instead of using the 98.630 as the cash flow we have used 93 is the cash flow which is the call price. So now the value of the call option has become 96.044.

$$B_0^C = \frac{0.5\,[97.297 + 8] + 0.5\,[98 + 8]}{1.10} = 96.044$$

(Refer Slide Time: 20:59)



So now let us see that what exactly we have done here you see this particular binomial tree then it will be clear for you. This part already you know the coupon is 8, f = 100 call price we have taken 98, then S_0 is 10 % you can write it 0.10, u = 1.1 d is equal to it is also 0.95 right. So here what basically we have done in the previous case we have done that B_u = 97.297 and your B_d = 98.630.

Then your B_u^C is the minimum of B_u or CP, so minimum of B_u and CP means it is 97.297 or 98 whichever is the lower we have considered that that is 97.297. And here in this case you have minimum of the B_d and the CP and that is B_d = 98.630 and CP = 98. So, we have considered 98 so then the cash flow in the first period has become 97.297 and the 98.

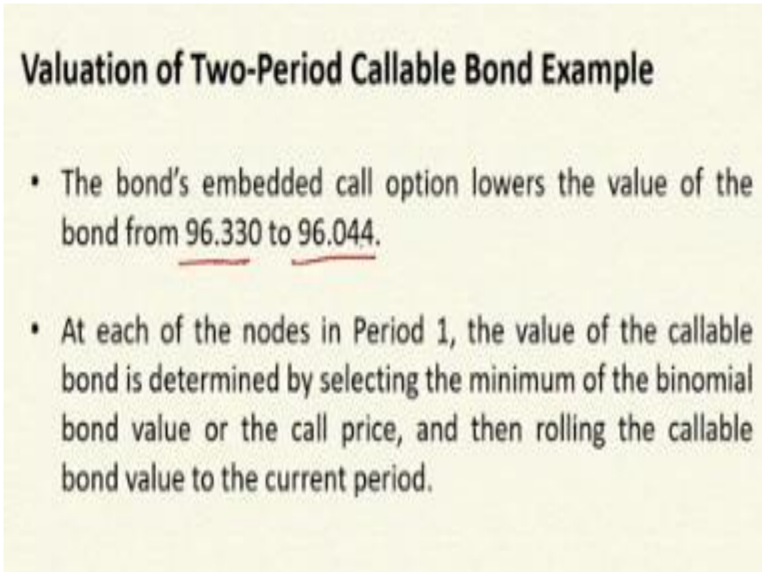
$B_u^C = \text{Min}\ [B_u, CP] = \text{Min}\ [97.297, 98] = 97.297$

$B_d^C = \text{Min}\ [B_d, CP] = \text{Min}\ [98.630, 98] = 98$

So, then the expected cash flow in the end of this first period will be you get this price + obviously the coupon will be there then it will be 97.297 + 8 and it will be 98 + 8 and probability we have considered it has been 50-50 % chance that interest rate can be high interest rate it can be low. So, then it will be 0.5 into 97.297 + 8 + 0.5 into 98 + 8 divided by 1.1 that will give you 96.044.

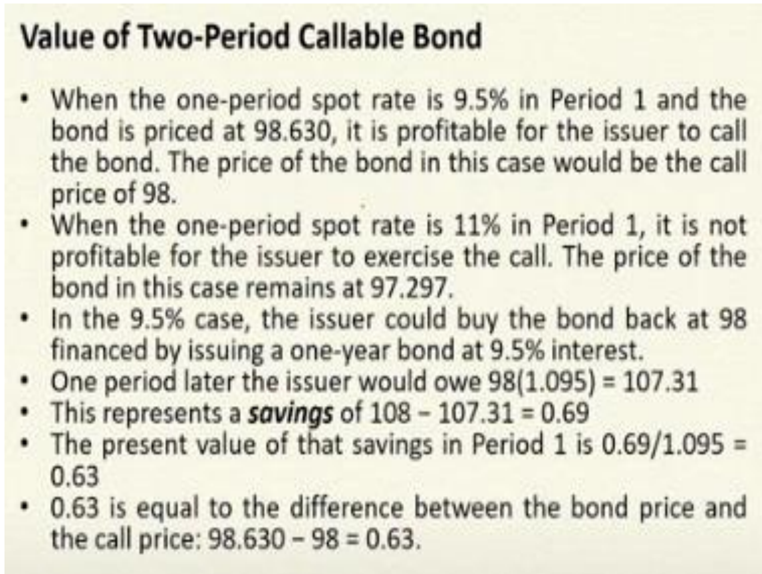
So little bit higher than the value of the option free bond, whatever; we have considered in the previous case. So, this is the way basically the valuation of 2 period callable bond can be calculated.

(Refer Slide Time: 23:14)



There are other methods also so here what we have seen before let us see. The bonds, embedded call option lowers the value of the bond from 96.330 to the 96.044 sorry it has lowered the value of the bond from 96.330 to 96.044. And at each of the nodes in period 1 the value of the callable bond is determined by selecting the minimum of the binomial bond value or the call price. Then rolling the callable bond value to the current period that is the process basically what we have followed for calculation or for valuation of the bond in the current context right.

(Refer Slide Time: 24:08)

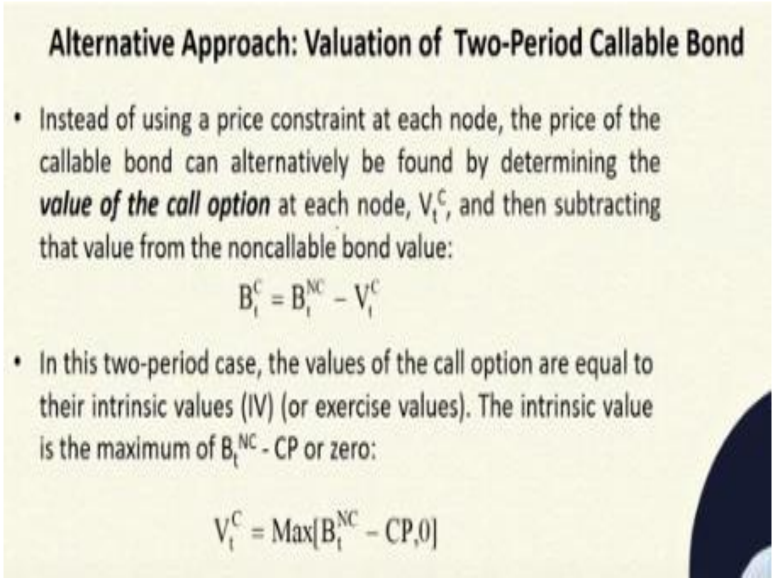


So, then what basically here we have seen when the one period spot rate is 9.5% in period 1, and the bond is priced at 98.360. It is profitable for the issuer to call the bond and the price of the bond in this case would be 98 that is the call price. When the one period spot rate is 11% in period 1, it is not profitable for the issuer to exercise the call and the price of the bond in this case remains as 97.297, anyway that call option is not going to be exercised.

So, in the 9.5% case the issuer could buy the bond back at 98 financed by issuing one period bond at 9.5% interest and one period later the issuer would owe 98 into 1.095 that is 107. 31, so that basically represents the savings of 0.69 that is 108 – 107.31 that is 0.69. And the present value of that particular savings in period 1 will be 0.69 divide by 1.095 that is 0.63. And that 0.63 is nothing but the difference between the bond price and the call price.

The bond price was 98.630 one about the interest rate has come down to 9.5 and your call price is 98 and you get a difference of 0.63. So, this is the process how this particular saving is basically working out in this case that is what basically what we are trying to explain through this.

(Refer Slide Time: 26:10)



So, the other aspect of the valuation of the callable bonds also there within this particular same framework. What actually you can do instead of using a price constraint at each node. The price of the callable bond also can be alternatively found by determining the value of the call option at each node**.** And then subtract that particular value from the non-callable bonds value.

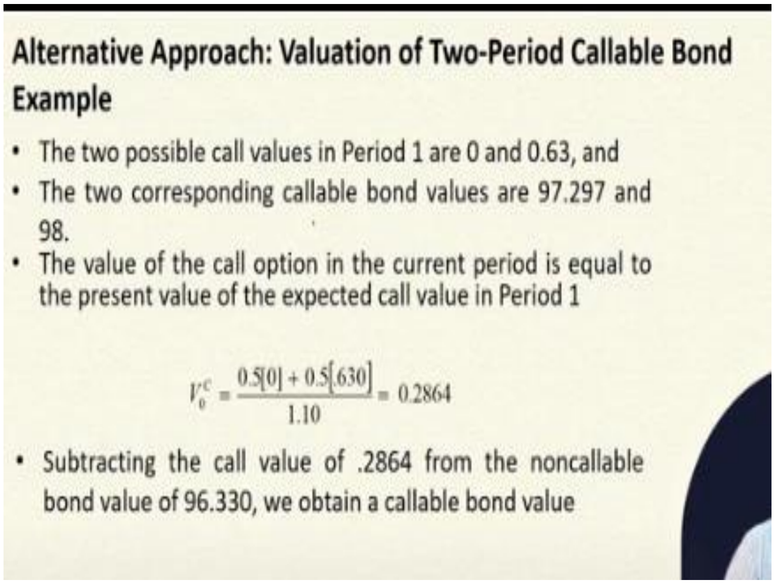
You have the non-callable bonds value without any call feature already you have calculated. If you can calculate the value of the call option and you subtract that value from the non-callable bonds value, then also you can calculate the value of this particular callable bond. In this 2-period case what basically we are considering the values of the call option are equal to their intrinsic value or the exercise value.

$B_t^C = B_t^{NC} - V_t^C$

So, the intrinsic value is nothing but is the maximum of the B_t^{NC}; NC means it is the non-call option; non-callable feature – C P or 0; C P means it is the call price. So, your V_t^C the means it is the value of the call option at each node, that is nothing but the maximum of either B_t^{NC} that value of the bond without any call option - the call price or zero.

$V_t^C = Max\ [B_t^{NC} – CP,0]$

(Refer Slide Time: 28:07)

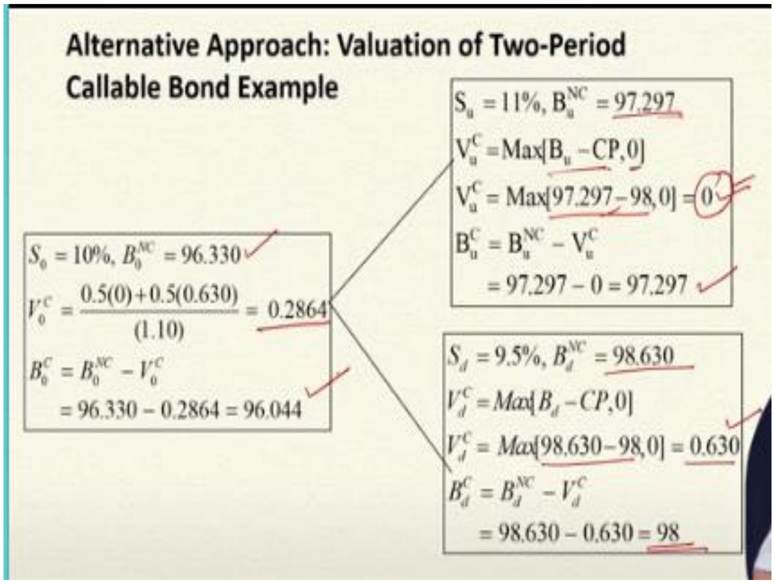


So, in our example if you see the 2 possible call values in the period 1 are either 0 or 0.63 and two corresponding callable bond values are either 97.297 and 98. So then the value of this particular call option in the current period is equal to the present value of the expected call value in the period 1. So how you can calculate this call premium that is 0.5 into 0 if it is not basically exercise then the value will be 0 into 0.5 into 0.63 divided by 1.1 that will give you 0.2864 that is the 0.2864.

$$V_0^C = \frac{0.5[0] + 0.5[0.630]}{1.10} = 0.2864$$

So now what you can do you can subtract the call value of 0.2864 from the non-callable bond value that is 96.330 we have calculated before for the non-callable bond value is 96.330. So, 96.330 - the value of the call option whatever you have calculated that is 0.2864 then we can obtain this value of the callable bond. That is basically another method another process through which our alternative approach through which you can also calculate the value of the callable bond.

(Refer Slide Time: 29:54)



So, this is the example same thing what basically we have done S_0 = 10% non-callable bond value is this then here it can go up to 11 or it could go to 9.5. So non-callable bond value in this period is 97.279 it is 98.630 then your V_u^C is the maximum between these 2 and here it is 97.297 – 98, 0. So obviously this will be minus then the value will be basically 0 the 0 is bigger than this, because the value is minus. So, then your B_u^C will be 97.297 - 0 that is 97.297.

$V_u^C = Max\ [B_u – CP,0] = Max\ [97.297 – 98,0] = 0$

$B_u^C = B_u^{NC} - V_u^C = 97.297 – 0 = 97.297$

Second case it will be 98.630 – 98, 0 zero that is 0.630, then this will be 98.630 – 0.630 that is 98. So, then we have used that particular value here it is 0 here it is 0.630, then 0 into 0.5 into 0.630 divided by 1.1 that is what you got the value of the call of call option. Then you take the difference 96.330 – 0.2864 that will give you 96.044 that is basically the, another approach through which the value of the call option can be calculated.

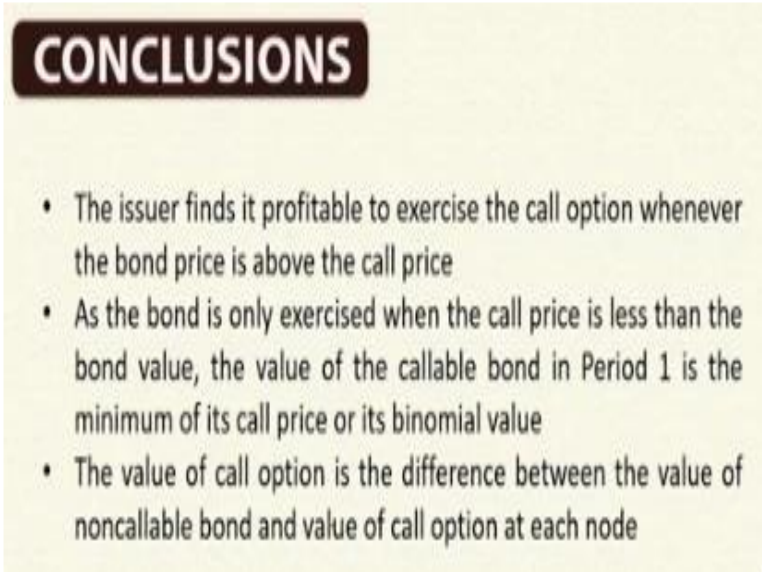
$V_d^C = Max\ [B_d – CP,0] = Max\ [98.630 – 98,0] = 0.630$

$B_d^C = B_d^{NC} – V_d^C = 98.630 – 0.630 = 98$

$$V_0^C = \frac{0.5[0] + 0.5[0.630]}{1.10} = 0.2864$$

$B_0^C = B_0^{NC} – V_0^C = 96.330 – 0.2864 = 96.044$

(Refer Slide Time: 31:24)



So, what basically we discussed here what is the conclusion here. The issuer basically finds it profitable to exercise the call option whenever the bond price is above the call price. As the bond is only exercised when the call price is less than the bond value, the value of the callable bond in period 1 is the minimum of its called price or its binomial value. And the value of the call option is difference between the value of the non-callable bond and the value of the call option at each node. That is the thing what basically we have observed here.

(Refer Slide Time: 32:01)

So, these are the references you can see.

Thank you.