Management of fixed Income Securities
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Module No # 08
Lecture No # 36
Bonds with Embedded Options

Welcome back, in the previous session we discussed about the international bond market so largely we covered of many bond markets that includes the bonds issued by the government, bonds issued by the corporations, bond issued by the municipalities, bonds issued by the banks or the intermediaries that we can say and the bonds also issued by the international agencies. So, then we can discuss about certain things in today's session. There are certain bonds which have some typical embedded options like some bonds have some call features, some put features, some bonds have some sinking fund provisions.

So, there are different types of typical options can be or kind of embedded options can be involved with respect to that. So, in that case how the valuation of those bonds are done number 1.Number 2 is there any typical risk involved with respect to that type of bond whenever they have this kind of extra features are linked to that. So those things basically the prime focus of these particular sessions, what we are going to discuss at least for another 4 or 5 sessions will be discussing about these issues with some typical examples taking some hypothetical examples we can discuss that.

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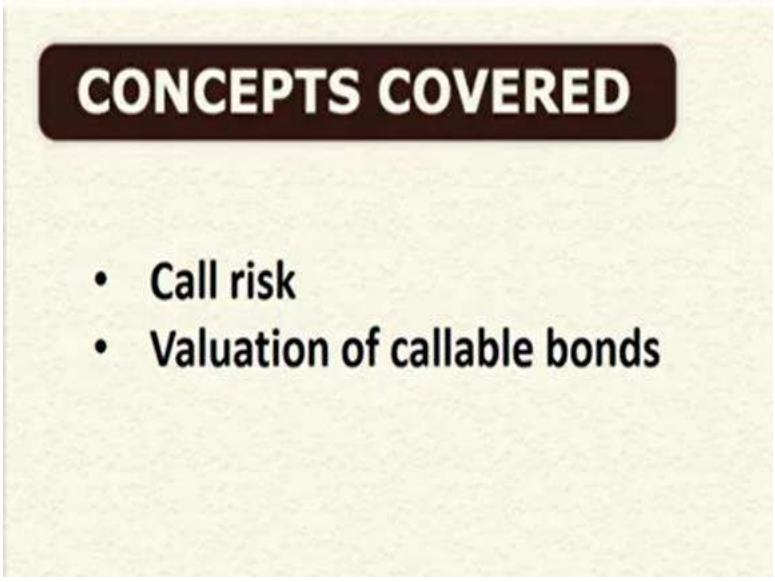

So todaywe will be discussing about certain concepts like the call risk and the valuation of the callable bonds. We can start the valuation of callable bonds but we will carry on this particular discussion. Mostly the session will cover of what we can say that what are those kinds of embedded options are involved in the bonds and how those particular options is basically affecting the valuation and the risk of that particular bond?

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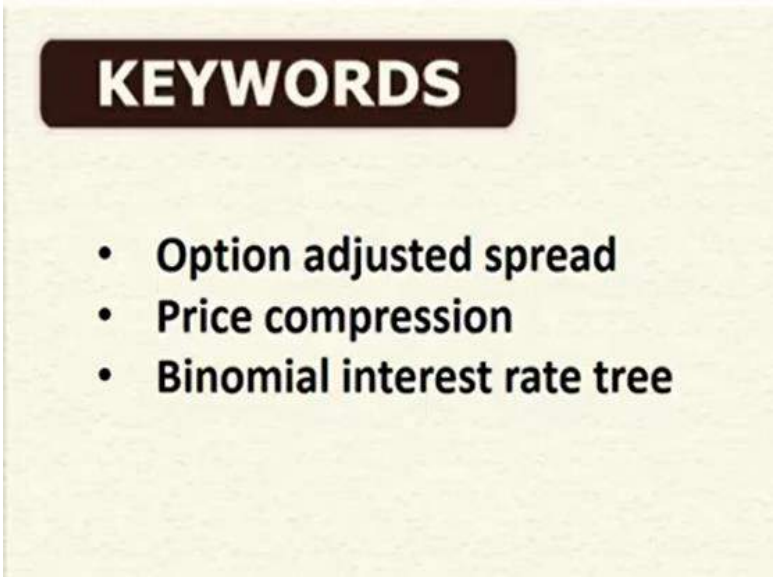

So, we will come across certain concepts like optional adjusted spread, price compression, and binomial interest rate tree. So, these are the different keywords that you will come across in today's session.

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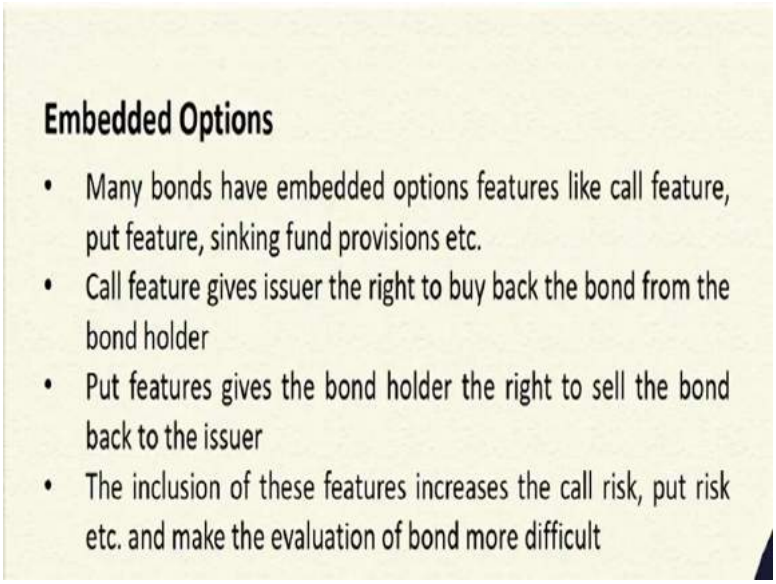

So, let us first see that what are those embedded options generally we have? So many bonds have the embedded options features like call features, they can have a put features, they can

have a sinking fund provision and all these things. So, what does it mean by the call feature? The call feature means it gives the issuer the right to buy back the bond from the bondholder after certain point of time. According to this indenture provision this time period will be mentioned. So, after that particular period the bond issuer can buy back that particular bond from the bond holder.

So, what is the put feature? Put feature is basically here it this particular feature gives the bondholder (the investor) the right to sell the bond back to the issuer. This first one is called the callable bonds; second one is called the puttable bonds. The inclusion of this kind of features increases this call risk, put risk etc. And because of this the evaluation of these bonds is relatively more difficult in comparison to the evaluation of the common bonds whatever we have discussed.

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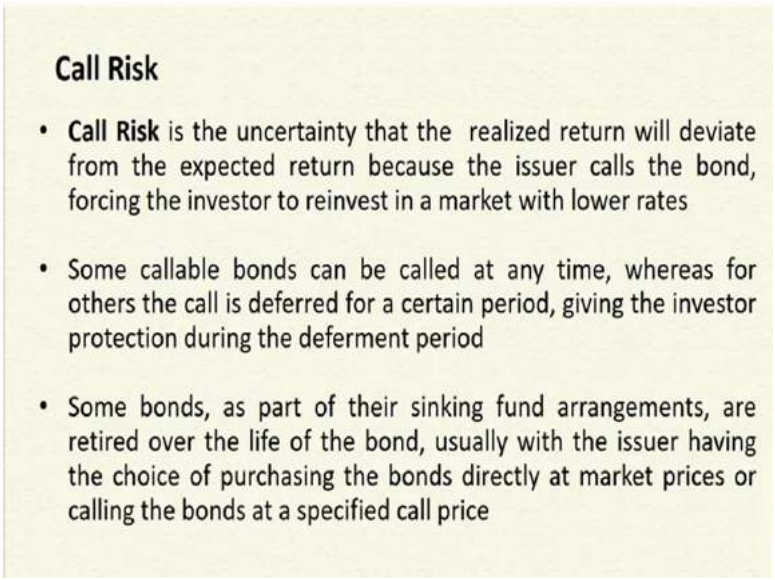

So, then what is the risk involved in that? Let we take the one example that is the call feature. So, if you take the call feature then we call it if any bond has a call feature, we generally face a typical risk that is called the call risk. So, what is the call risk? The call risk is basically a type of uncertainty that the realized return will deviate from the expected return because the issuer calls the bond. Forcing the investor to reinvest in a market with lower rates and when the call feature will work or will be feasible.

The call feature basically will be active whenever the issuer will find the interest rate in the market is low. Whenever the interest rate in the market is low in that particular point of time it will be beneficial for the issuer to buy back that particular bond at a call price which has been fixed before right.So, some callable bonds can be called at any time that depends upon

the agreement or the indentured provision which has been provided which has been mentioned beforehand.

Whereas for some others; the call is deferred for a certain period giving the investor protection during this deferment period. And some bonds as a part of the sinking fund arrangements are retired over the life of the bond usually with the issuer having the choice of purchasing the bond directly at a market price or calling the bonds at a specified call price. That type of provisions also available in some of the bonds not in all the type of bonds but mostly whenever you talk about it is a callable bond.

That means the bond can be called back at any point of time or after a certain point of time with a particular price and that price is generally, we call it the call price. But if there is a call feature, then your realized return may be different from the expected return what you are trying to get. From the investment perspective your realized return will be different from the expected return what basically you are expecting from that particular bond.

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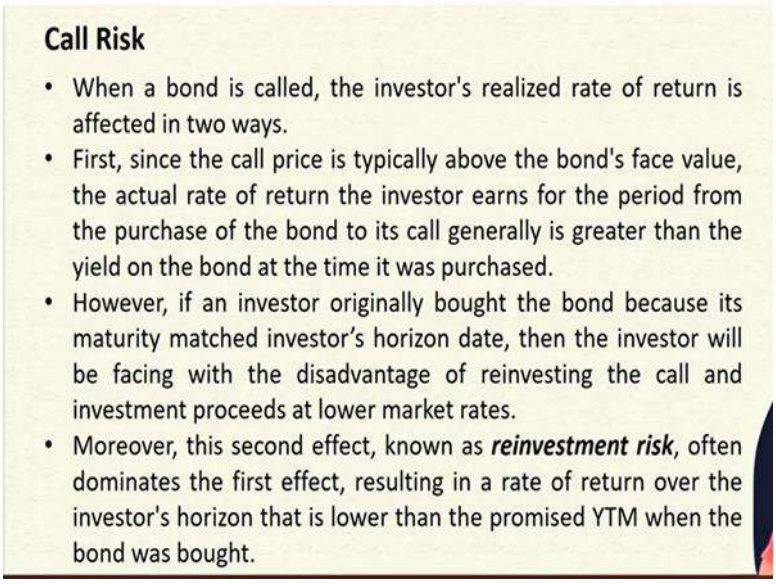

So, let us see that what happens that? When a bond is called the investors realized rate of return is affected in 2 ways. Already for a normal bond we have discussed that first. Since the call price is generally above the bonds face value. Generally, the call price is fixed above this bond space value the actual rate of return the investor earns for the period from the purchase of the bond to its call generally is greater than the yield on the bond at the time it was purchased. There is a price effect. So here we call it the price effect. Because as per the provision the call price is more than the face value of the bond then obviously even if the bond will be called back. Then the return should be more than the return what basically or

yield whatever they have utilized to purchase the bond at the time of purchasing the bond, what was the yield if the call price is more than obviously, they can expect more return from that.

But if the investor originally bought the bond because its maturity basically is matching with the investor's horizon date then the investor will be facing with the disadvantage of reinvesting the call and the investment proceeds are the lower market rates. Now whatever proceeds they will get that process they have to reinvest in the market at a lower rate right. The money they have got so that money has to be reinvested in the market at a lower rate.

So, in that case they will be facing the reinvestment risk. So, let the bonds maturity period was 10 years the call features were possible after 3 years. So, after 3 years if the interest rate has come down even if the face value of the bond was 1000 and the call price was at 1100 whatever money they will get after 3 years that money if the investors horizon period is 10 years or let8 years.

In that case they are basically always exposed to more reinvestment risk. What this money they are going to use? How that particular money is they are going to use? what they will do with that particular process that is basically the risk what always they are facing. So that is called the reinvestment risk and if the reinvestment risk is dominating the first effect that is the price effect, then what will happen?

The rate of return over the investor's horizon will be lower than the promised yield to maturity when the bond was bought. Let the promised yield to mass rate was 10%, if the reinvestment effect or the reinvestment risk is exceeding this price risk or price effect so here the price risk is low because the bond call price is relatively higher. But if the interest rate is quite low and the coupon or the investors horizon period is not matching with their maturity period then what will happen that?

In this kind of case the yield or the return what they are going to get that will be lower than the return what they are expected to get whenever they have bought the bond.

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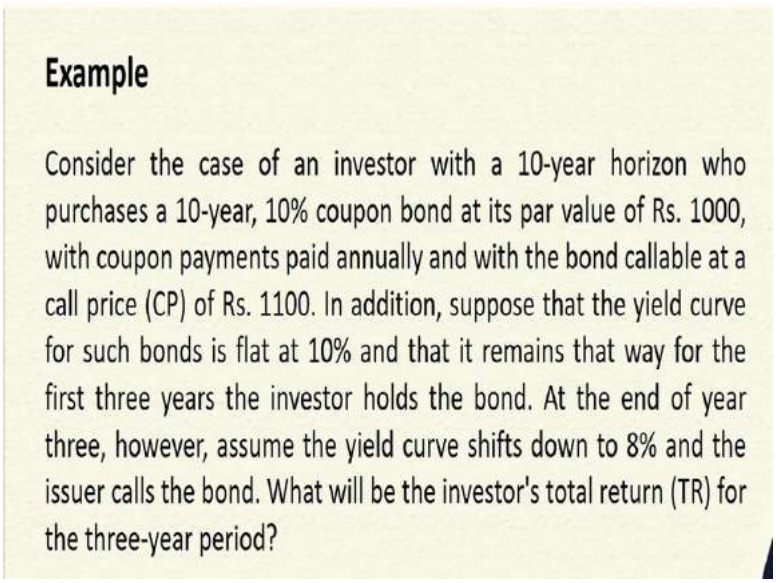

So let us consider one example this if you take this example let an investor with a 10-year horizon that investor's horizon period is 10 years and they purchase a 10-year maturity bond with the 10% coupon at its far value of 1000.It is bond which is issued at let par. And the coupon payments paid annually and the bonds can be callable at a price of 1100 so you assume that the yield curve for such bonds is let fall at 10% and it remains that way for the first 3 years the investor holds the bond and at the end of the 3 years, you assume that the yield curve is shifting down to 8% the interest rate has gone down to 8% it was 10% then it has gone down to 8% and that time let the issuer has gone for calling the bonds. Whenever the interest rate does after 3years the interest rate has gone down from 10% to 8%, you assume that the issuer is now calling the bond. So, in that case what will be the investor's total return for this 3 years period?

You see investors horizon period is 10 years now after 3 years the market interest rate has gone down from 10% to 8% that time the issuer was ready to buy back that particular bond has gone for this utilizing this call option. So, in that case you return what you are expecting that is a 10%.Now whether you are getting the 10% or more than 10% or less than 10% in this particular scenario that is basically our objective to find out. So, let us see what will happen in that case?

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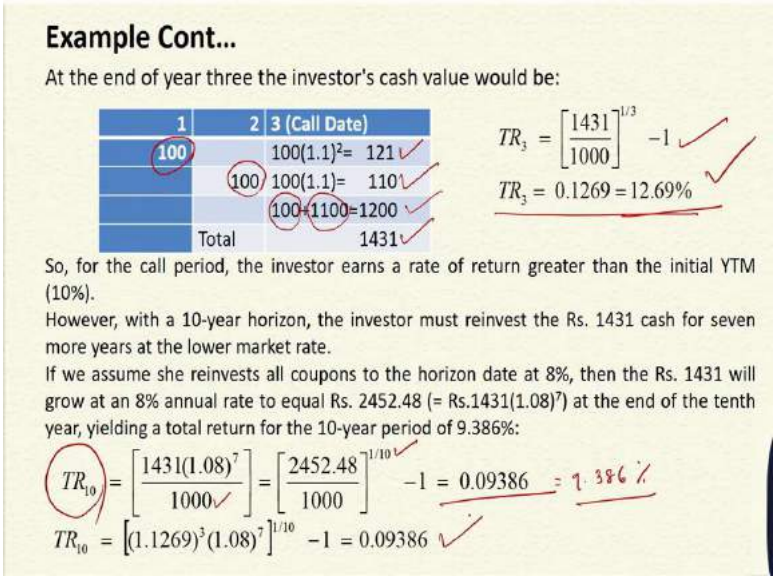

$TR_3 = \left[\frac{1431}{1000}\right]^{1/3} - 1 = 0.1269 = 12.69\%$

$TR_{10} = \left[\frac{1431(1.08)^7}{1000}\right]^{1/3} = TR_3 = \left[\frac{2452.48}{1000}\right]^{1/3} - 1 = 0.09386$

$TR_{10} = [(1.1269)^3(1.08)^7]^{1/10} - 1 = 0.09386$

You see at the end of 3 years the investor's cash flow if you look at or the cash value how much the investor will have. If you see in the first year the 100rupees is the coupon. They got these 100 rupees and whatever 100 rupees they have received they can reinvest it in the market at a rate of 10%.So, in the end of the 3 years that 100 rupees will become 121 rupees and in the end of the second year they will get another 100 rupees.

So that 100 rupees can be reinvested for another one year at a rate of 10%. They will be getting 110 rupees right. So, end of the 3 years how much they will get? Now the issuer has called back the bond utilizes the call option if they will they have exercise the call option then the cash flow will be their coupon 100 rupees then 1100 is the call price then you will get 1200 rupees at the end of the 3 years.

Then the total cash what you will be receiving at the end of the 3 years you are of this 3 here these 2 includes your coupon as well as the reinvestment of the coupons for these 2 years that is 121. And 110 and end of the 3 years you will be getting 1200, so total you will get 1431.In the end of the 3 years that1431 rupees you will be getting and at what price you have bought the bond? You have bought the bond at the price of 1000 rupees the bond was issued at par.

So, at the end of the 3 years if you the particular issuer has gone for the buy back this particular bond or as exercise this call option your total return will be your 1431 divided by

1000 to the power 1 by 3 - 1 that will be 12.69%.You will be getting a return of 12.69% which is more than your promise deal that is the 10%. So, for the call period the investor earns a rate of return greater than the initial yield to maturity that is the 10%.

But the investor's horizon period was 10 years the investors horizon period was not 3 years he was forced to sell that particular bond to the issuer. So now if the investor's horizon period is 10 years, then the investor has to reinvest that 1431 cash whatever they have received for 7 more years yes, or no? They have to invest it for another 7 years. Because the horizon period is 10 years and that time the market interest rate is low that is 8%.

So then if you assume that the investor reinvests all coupons to the horizon date at 8% then your rupees 1431 will grow at an 8% annual rate to get an amount of 2452.48 that you get it 1431 into 1.08 to the power 7 at the end of the tenth year. So, if you calculate the total return for the 10 years period then how will you calculate? So here we have calculated the return at the end of 3 year that is the TR_3.

Now we have to calculate your TR_{10} the total return at the end of the 10 years. Now how much you got that is your 8% interest in the market then your value will become 1431into 1.08 to the power 7 that is your 2452.48. Then your purchase price was 1000 divided by (1000) to the power 1 by n -1 here n =10 years. Then 1 by 10 - 1 effectively you get 9.386%. It is your 9.386%.

But initial promise deal to maturity was 10% in the third year you are getting a return which is more than the initially into maturity. But effectively whenever you calculate it for the 10 years period your return has come down to 9.386% is it clear? So, in that case what basically here also you can calculate here that means what you say that? The reinvestment risk or the reinvestment effect what basically we have observed that basically has the impact on the return expected or the realized return what you are trying to get from that particular kind of features.

So that's why we are always exposed to a call risk. Total return also you can calculate in this way. That is1.1269 to the power 3 into 1.08 to the power 7 whole to the power 1 by n -1 that means 1 by 10-1 that also give you 9.386% that is the way also you can calculate it.So now the return has come down that means there is a risk involved in that.

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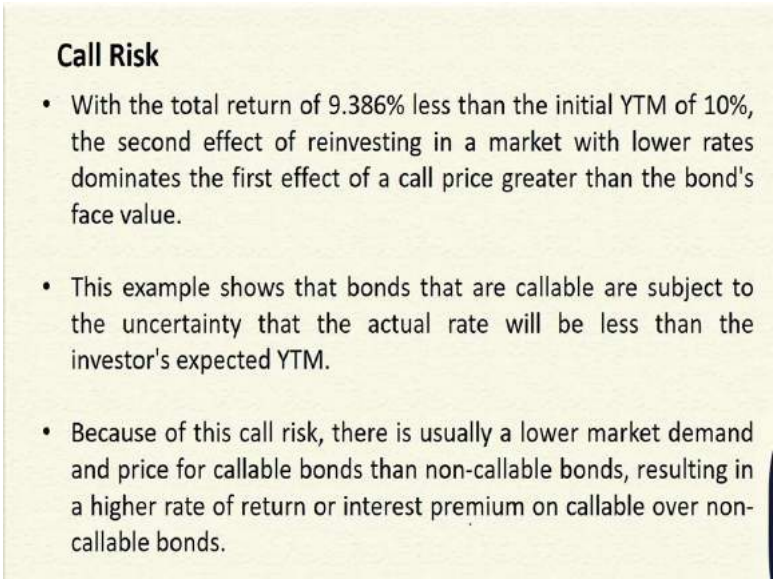

So now with the total return of 9.386% which is less than the initial YTM, the second effect of reinvesting in a market with lower rates dominate this first effect of a call price greater than the bond face value. One face value 1000 call price was 1100 there whatever extra return you are getting that is basically dominated by the reinvestment effect. So, what this example basically shows the bonds are callable are subject to the uncertainty that the actual rate will be less than the investors expected YTM.

Because of this call risk, there is usually a lower market demand and a price for callable bonds than the non-callable bonds which generally result in a higher rate of return or interest premium on callable bonds over the non-callable bonds. They generally give more return or more premium than the non-callable bonds.

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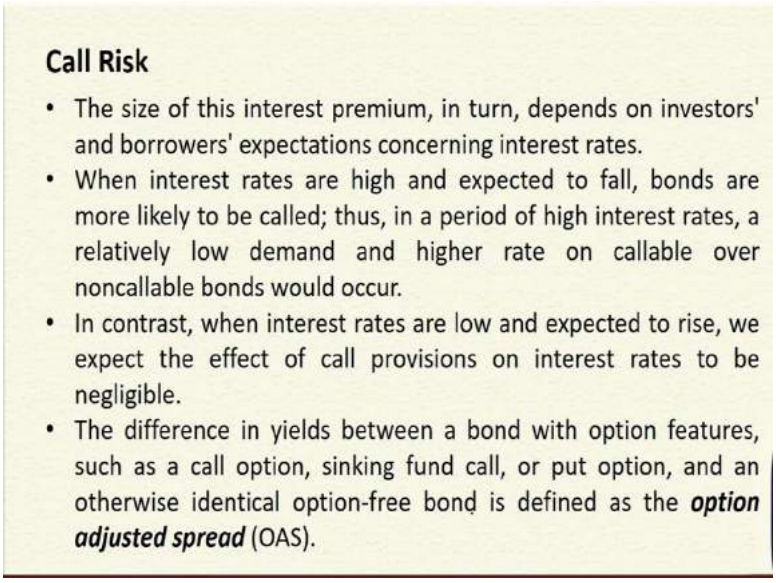

So, this size of the premium generally depends on the investors and the borrowers' expectations about the interest rate. When the interest rates are high and expected to fall then the bonds are more likely to be called. So, in a period of high interest rate, a relatively low demand and higher rate will be on the callable and the non-callable bonds respectively. In contrast if you expecting that interest rates are low and it is expected to rise then we expect effect of call provisions on interest rate to be negligible generally not find much differences between these 2.

The difference in yield between a bond with option features such as call option or sinking fund call or the put option or any kind of features whatever they have. And an identical option free bond generally is called the option adjusted spread (OAS). The bonds having the option features the yield whatever you are getting and the bonds having the no features the option free bond that difference or that spread is called the option adjusted spread.

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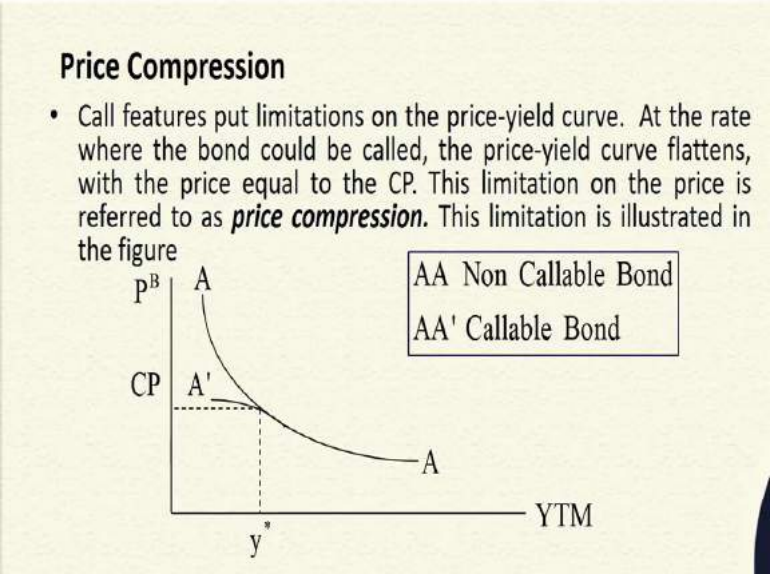

So, the call features generally put the limitations on the price yield curve. So, at the rate where the bond could be called the price yield curve generally becomes flatter with the price equal to the call price. This limitations on the price generally called are the price compression. If you see this figure then AA shows this price yield curve for the non-callable bond but your AA prime shows this yield curve for the callable bond. So, they are basically there is a concept we use that is called the price compression.

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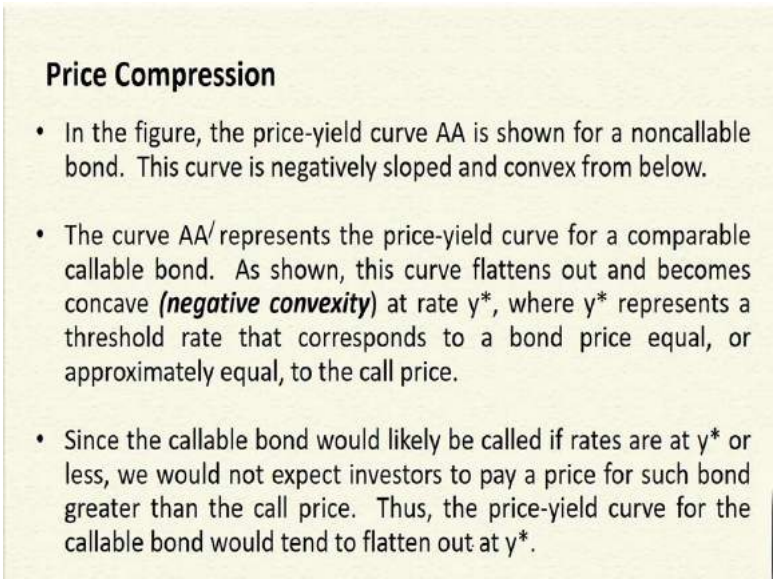

So, if you look at this figure the AA is shown for a non-callable bond and this curve is negatively sloped and convex from below. But if you look at the AA prime that represents the price yield curve for the comparable callable bond and this call basically flattens out and becomes concave at the rate that is Y^*. And here Y*represents a threshold rate that basically corresponds to a bond price equal or approximately equal, to the call price.

Since the callable bonds should likely to be called if rates at Y*or less then you would not expect the investors to pay price for such bond greater than the call price. So, the price yield curve for the callable bonds would tend to flatten out at the Y*. So that then the curve basically becomes concave in that particular point or particular context.

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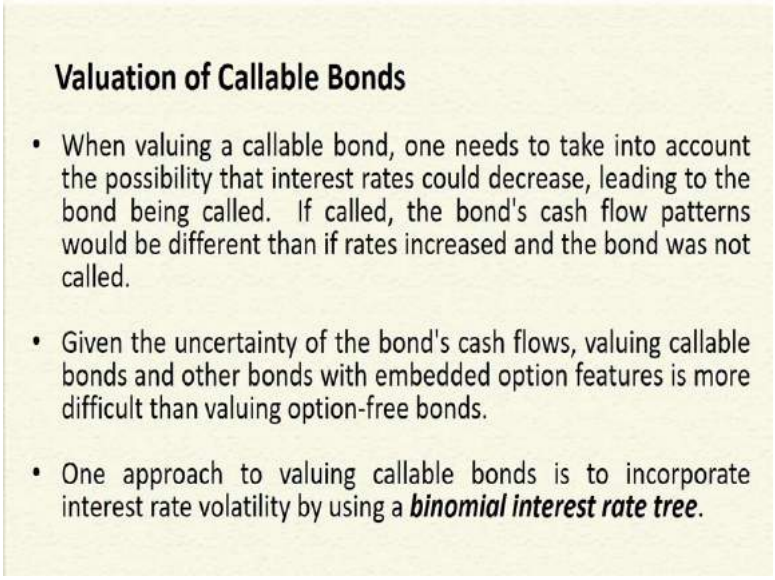

Then we are talking about the valuation of the callable bonds how generally we do the valuation of the callable bonds? So here if you see the valuation of the callable bonds

depends upon the interest rate possible change in the interest rate whether the interest rate will decline or not. If the interest rate will decline then that basically will lead to the calling back the bond unless the issuer is not based very much profitable for the issuer or not lucrative for the issuer to call back the bond.

So, if it is called then the bonds cash flow patterns would be different than if rates increased and the bonds was not called. If it is called then the cash flow pattern will be different and if the bond the particular is not called then the cash flow pattern will be different. So given this kind of uncertainty of the bonds cash flow the valuation of callable bonds and other bonds with some embedded options is more difficult than valuation of option free bond.

One of the approaches what generally, we can use for evaluation of the callable bonds to incorporate the interest rate fluctuations or interested volatility by using the binomial interest rate tree. The binomial interest rate tree model can be used to understand the probable changes of the interest rate and accordingly we have to decide that how this particular value of that particular bond can be determined? So binomial interest rate tree is a one of the approaches which can be used for that.So, what exactly this binomial interest rate model is?

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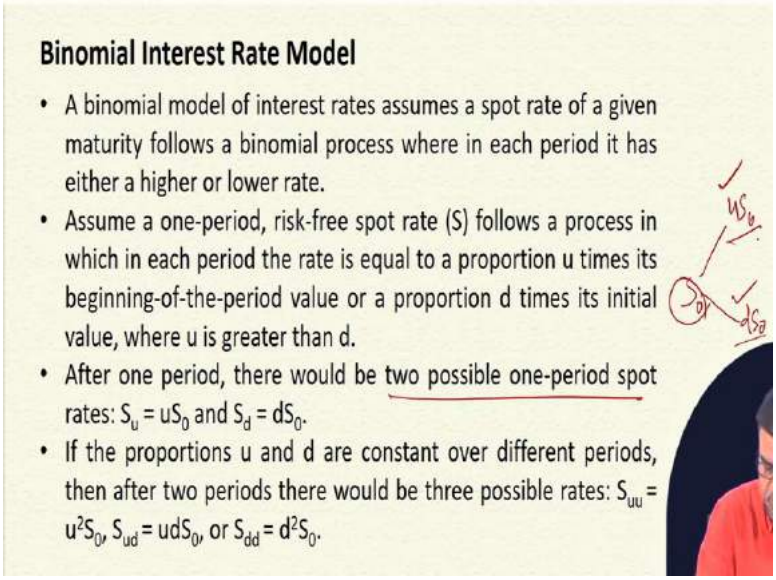

The binomial interest rate model basically assumes the spot rate of a given maturity they follow a binomial process and each period where in each period it has either a high or the lower rate. Let you assume a one period risk free spot rate let that is represented as S follows a process in which in each period the rate is equal to a proportion u times its beginning of the period value or a proportion d times its initial value, where u is greater than d. That means what does it mean in after one period there would be 2 possibilities.

Let this value was S0 in the initial period either it can go up to uS0 or it can go up to dS0. If it is going up then it is uS0 if it is going down it is the dS0. So, if the proportions u and d are constant over the different periods then obviously after 2 periods that will be S_{uu} that means it will be u^2S_0 and it can be S_{ud} that means udS_0or S_{dd} it may be d^2S_0. So, this kind of distribution basically what you can look at.

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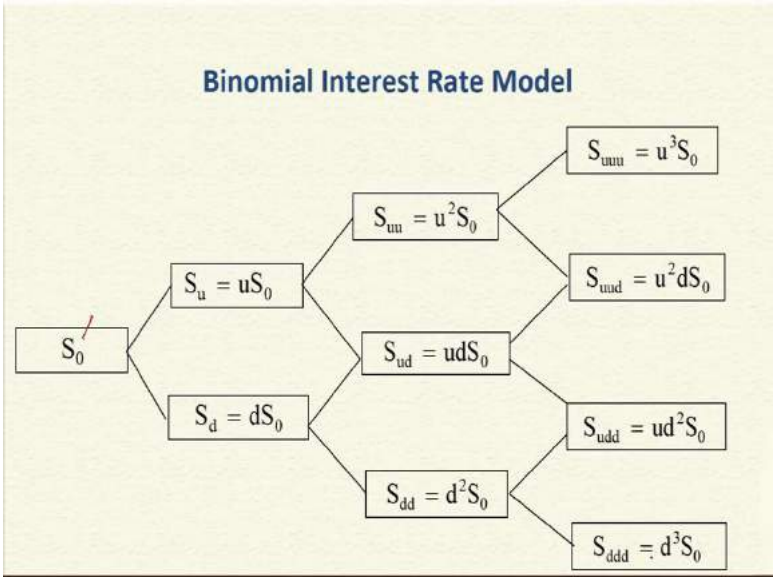

So let me show you that how this distribution basically looks like so this is called the binomial interest rate model. We start with S0 it can go up to Su means this is u into S0 or it can go to Sd which is d S0. So, this again have 2 options after this it can either increase or it can decline. If it is again increasing then Suu that means u^2S_0or it can be uS0 into d or udS0 or it can further decline you can go for d^2S_0.

And again, if you see that another period Suuu then it is u^3S_0or it may be u^2dS0or it may be Sudd that is the ud^2S_0 or it may be Sddd that is the $d^3S_0$is it okay? So that is the way the distribution basically looks like.

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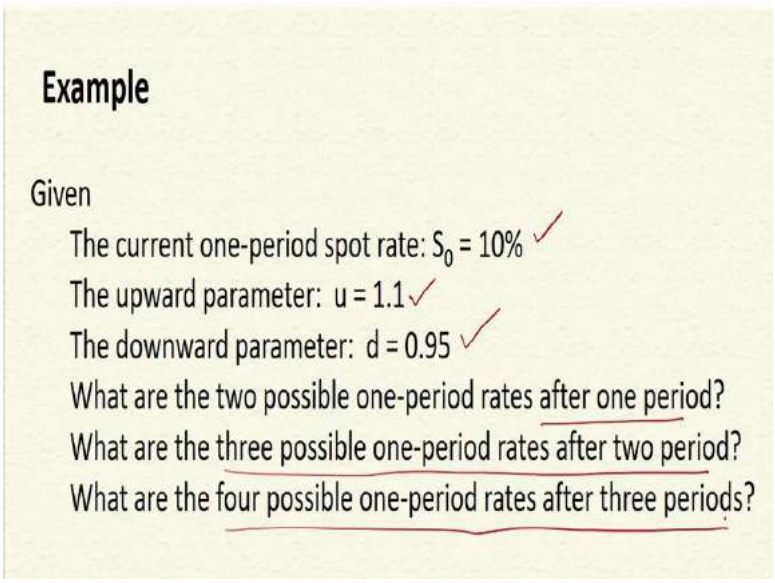

Let us take one example let the current spot rate is 10%u you have taken 1.1 downward parameter you have taken 0.95, then what are the possible one period rates after one period? What are the 3 possible 1 period rates after 2 periods? And what are those 4 possible 1 period rates after the 3 periods? So that is the question.

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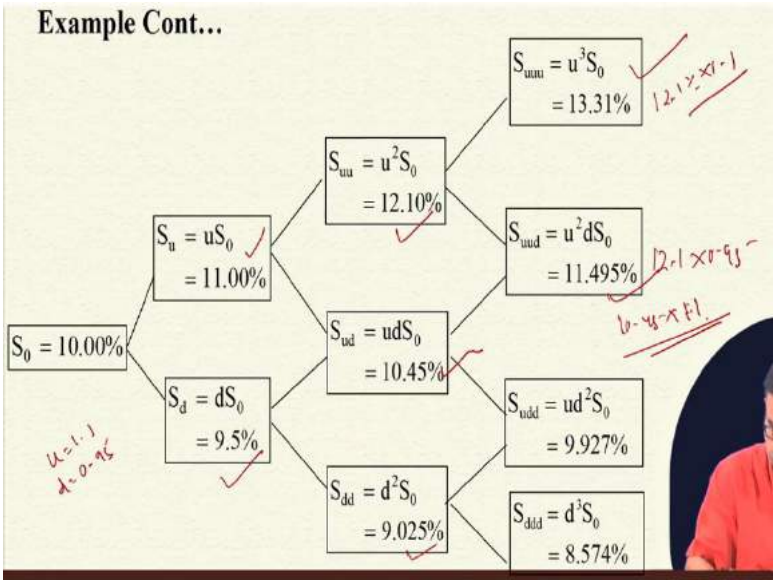

So whatever thing we have discussed if you want to measure this you start with the 10% u =1.1d = 0.95.So then 10% has become if it will increase then it is 1.1 into 10 become 11% or it will be0.95 into 10 it will be 9.5%. Further from 11% to it can go again increase by 1.1 it can become 12.1% that is 11 into 1.1 that will give you 12.1% or it can also come down.So,11 into 0.95 it can become 10.45% again if it is further going down then again 9.5 into 0.95 that will give you 9.025%

So then from 12.1 to it can go up to here it is 12.1% multiplied by 1.1 that will give you this rate or it can decline so this will be 12.1 into 0.95 that will give you this rate and again if it is 10.45.This can again go down that means 10.45 into 0.95 or it can be it can also become this 8.574% if it is further decline. Like that the distribution this 12.1 become 13.1 or 12.1 can become 11.49 or it can also from here; if it is increasing also, it can be also 10.45 into 1.1.

So, there are different ways this particular binomial tree model can work out and, in that case, the different interest rate scenario can be built upon. So once this different interest rate scenario will be built upon accordingly the valuation of the callable bonds can be made. So, we will discuss extensively about the 2 period, 3 period callable bonds, puttable bonds, sinking fund provision bonds and all these things in the subsequent sessions.

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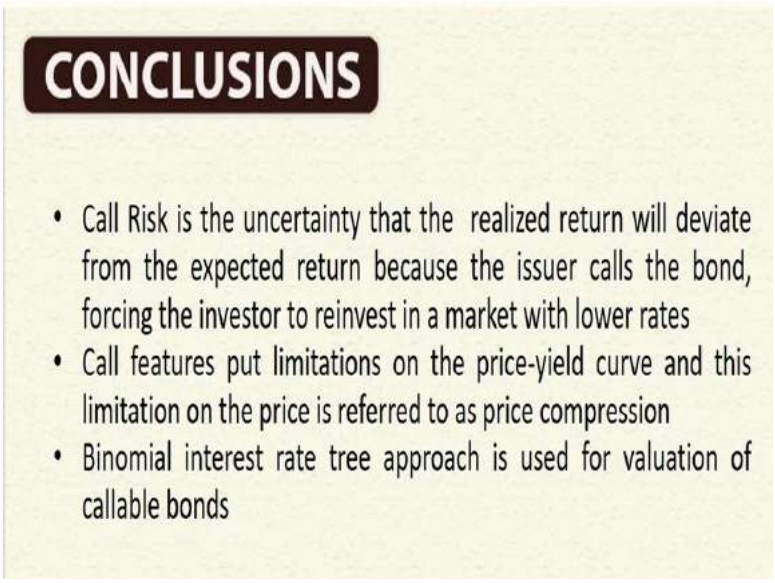

So, what we have discussed today about the call risk which is the risk which makes the uncertainty between the realized return can be deviated from the actual return, what expected return the investor is trying to get. Call features put the limitations on the price yield curve and these limitations on the price generally called as the price compression. And binomial interest rate approach generally used one of the approaches which can be used for the valuation of the callable bonds.

So, future will see with the different examples how exactly the evaluation of the callable bonds, puttable bonds and the sinking fund provision bonds are generally made.

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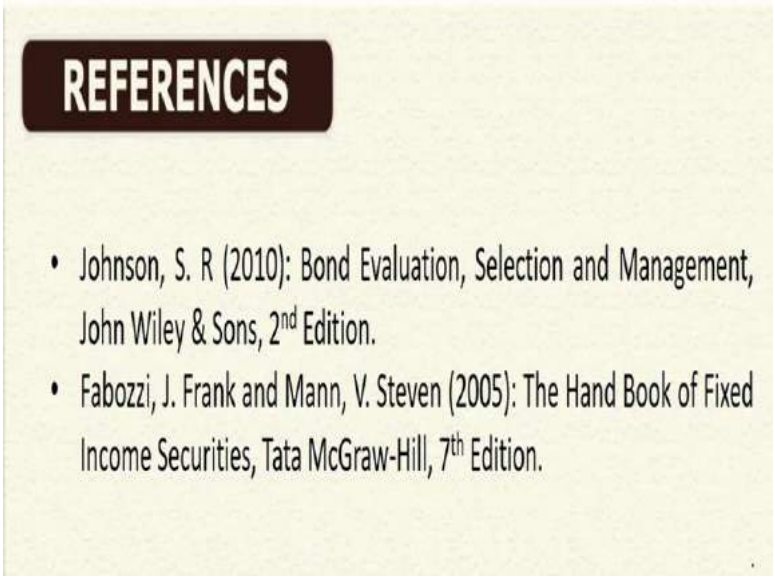

So, these are the references what you can see. Thank you.