Management of Fixed Income Securities
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Module No # 01
Lecture No # 03
Bond Valuation - 1

Good morning, welcome back to the next session of the particular subject on the management of fixed income securities.

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So in the previous sessions, we discussed about the overview of the fixed income securities market and as well as the different type of risk what generally we always face whenever we invest in this particular market. So today, we can start the discussion with respect to the different types of the fixed income securities and how the pricing of those securities are done and how the return of those securities, are basically calculated.

So among all those markets whatever we have discussed, the most important market is the bond market. Then gradually we can move towards the mortgage backed securities and the fixed income derivatives and all. But, today's discussion will mostly focus on the valuation of the bonds.

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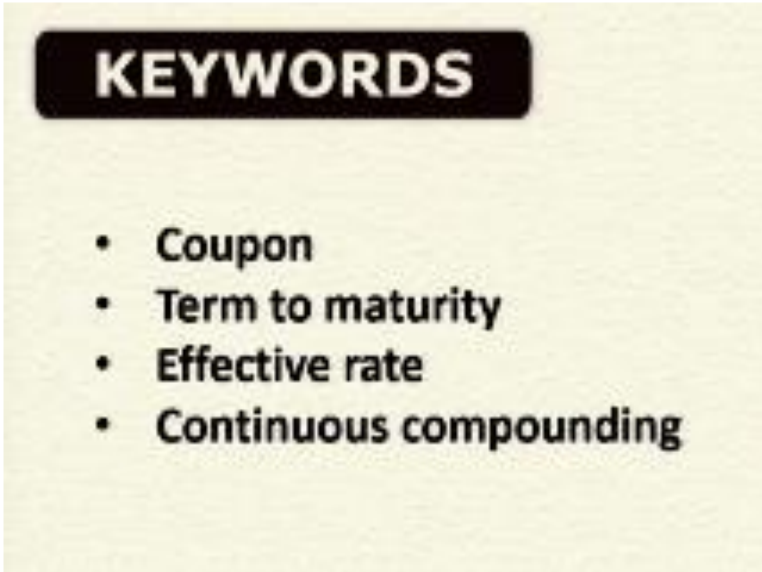

So, here, if you see that whenever we go for the evaluation of the bonds, generally today's discussion, largely cover up the different aspects of the valuation of the bonds. And after the discussion, you will come to understand about the different kind of features of the bond like the coupons, like the term to maturity, how the effective rate is calculated, then, concept of continuous compounding and all these things. Basically, the major of this particular discussion today!

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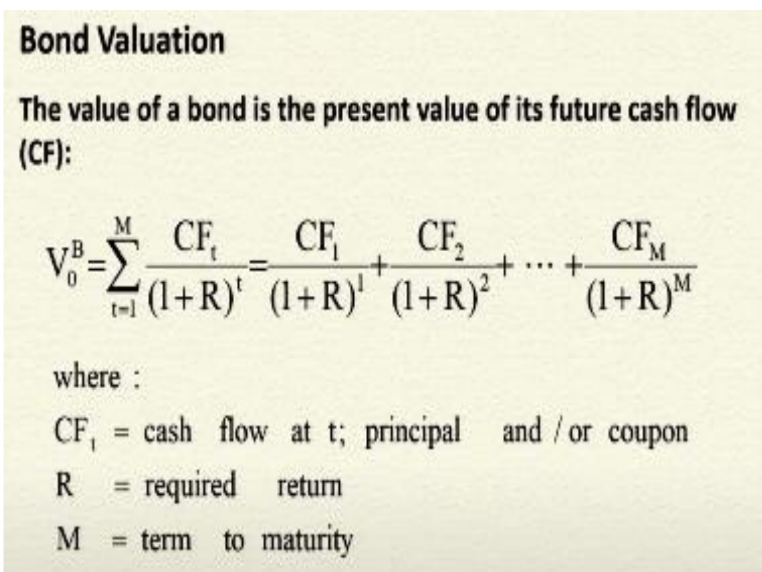

Bond Valuation

The value of a bond is the present value of its future cash flow (CF):

$$V_0^B = \sum_{t=1}^{M} \frac{CF_t}{(1+R)^t} = \frac{CF_1}{(1+R)^1} + \frac{CF_2}{(1+R)^2} + \cdots + \frac{CF_M}{(1+R)^M}$$

where :

CF_1 = cash flow at t; principal and / or coupon

R = required return

M = term to maturity

So let us go to the bond valuation. So all of you know, that whenever we go for the valuation of the bond. Like other assets whether it is a stock or whether it is a bond or any kind of asset, the value of a particular asset is nothing but the present value of its future cash flows. So whenever we talk about the bond part, what is the cash flow involved whenever you talk about the bond.

So now we talk about the bond, largely the cash flow is basically the coupon and as well as the principal or the par value of that particular bond what we are going to expect or we are going to get in the end of the maturity. So, In the generic way, if you want to find out a formula for the evaluation of the bond that is basically nothing but the cash flow what basically we are getting that is the coupon and as well as the principal and that has to be discounted with respect to the required rate of return or the market interest rate, what generally in our term we call it the yield.

So if you discount this particular cash flow with respect to the discount rate which is nothing but the market interest rate. Then what basically we can get we can get the present value of that particular cash flow and this present value of cash flow is nothing but the price of the bond. Then another important feature here is the term to maturity, up to what period the particular cash flow has to be discounted. So that cash flow will be discounted up to the term to maturity of that particular bond.

So therefore if you this look at this particular formula that value of the bond will be:

Value of the Bond = $\frac{CF_t}{(1+R)^t}$

Where, CF_t= cash flows at the time period

R= required rate of return

t= time period

So now already what basically we have discussed, we have two types of cash flow: one is your coupon, other one is the principal or the par value or the price value those words are used interchangeably whenever we go for the valuation of the bonds.

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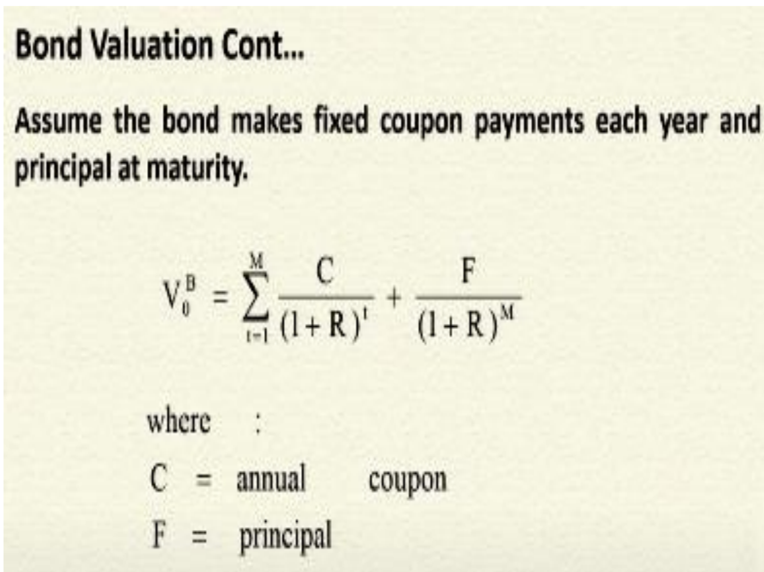

Bond Valuation Cont...

So then if you look at this and you assume that the coupon basically is fixed and it is paid let annually. And the principal what basically you, are expecting that principle you will be getting at the time of the maturity. Then we have two components in terms of the present value calculation of the cash flow: one is your C which is basically the annual coupon what you are receiving. And another one is the F represents the face value of the bond or the par value of the bond.

So now what basically we can do the face value of the bond or par value of the bond you will be realizing once the bond will be matured. So at the end of the maturity, you will be getting the face value of the bond or the par value of the bond. So now first of all, the C has to be discounted periodically over the period of time up to what time you are holding that bond.

And in the end of the maturity you will be getting your face value and that has to be discounted with respect to that particular maturity period and that mature maturity period is represented as m.

$$V_0^B = \sum_{i=1}^{M} \frac{C}{(1+R)^T} + \frac{F}{(1+R)^M}$$

So that is what basically and here you remember this bond basically is making the fixed coupon payments each year and you are realizing or is you are receiving the maturity at the time of maturity you are receiving your principle. So that is the two things basically you have to keep in the mind.

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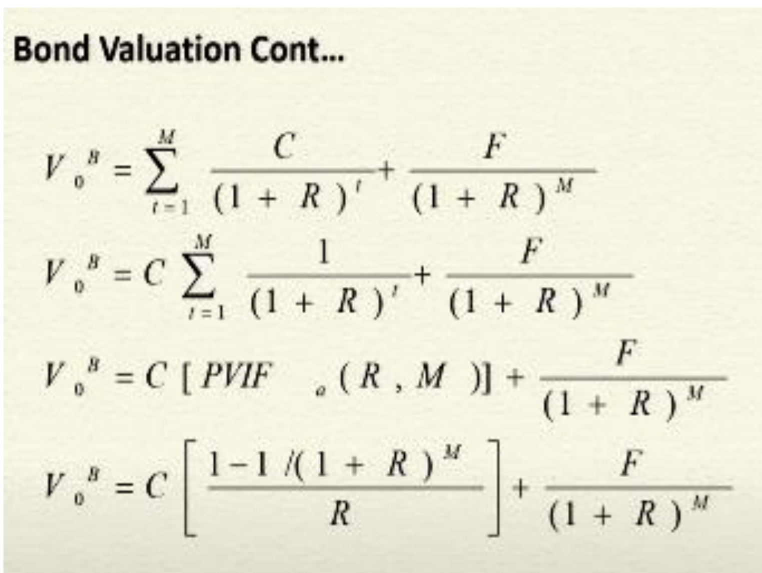

Then if you see if you have to expand it mathematically then how this particular expansion can be possible such as:

$$V_0^B = \sum_{i=1}^{M} \frac{C}{(1+R)^T} + \frac{F}{(1+R)^M}$$

$$V_0^B = C \sum_{i=1}^{M} \frac{1}{(1+R)^t} + \frac{F}{(1+R)^M}$$

$$V_0^B = C[PVIFa_a(R, M)] + \frac{F}{(1+R)^M}$$

$$V_0^B = C\left[\frac{1-1/(1+R)^M}{R}\right] + \frac{F}{(1+R)^M}$$

So now if you get the bonds par value if you get the coupon or coupon rate or if you get the yield to maturity or the discount rate and if you get the term to maturity then you can calculate the value of that particular bond at that particular point of time. That particular period, the price of that particular bond can be calculated by using this four kind of factors or four kinds of components.

What basically we require for the evaluation of the bond: one is your coupon, second one is the par value of the bond, third one is the term to maturity, and fourth one is the discount rate. So these are the four things what basically we need.

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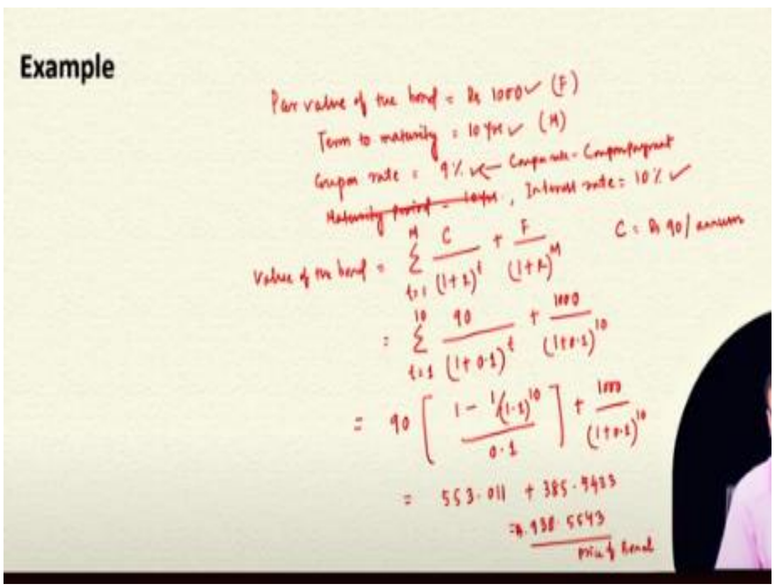

Then let us see one example that how this particular value of the bond can be calculated so if you see that how this particular bond value is calculated. So in this case let us we take one example where,

Par value of the bond= 1000(Face value of the bond)

Term to maturity= 10 years

Coupon rate= 9%, Coupon payment= 90

Interest rate= 10%

So now what else we need we have the par value, we have the term to maturity, we have the coupon rate and we have the interest rate at which the particular bond will be discounted right. So now, what is the formula already we just now we have seen the value of the bond will be

Value of the bond =

$$V_0^B = \sum_{i=1}^{M} \frac{C}{(1+R)^T} + \frac{F}{(1+R)^M}$$

$$\sum_{t=1}^{10} \frac{90}{(1+0.10)^t} + \frac{1000}{(1+0.10)^{10}}$$

$$= 90\left[\frac{1-1/(1\cdot1)^{10}}{0.1}\right] + \frac{1000}{(1+0.10)^{10}}$$

= 553.011 + 385.5433

= 938.5543

Hence, the value of the bond is 938.5543.

So the value of the bond this is basically this is rupees 938.5543 so if the bond will be traded in the market at that particular point of time then the value will be this is the price of the bond. So here one thing you remember that the coupon is basically paid annually and the par value of bond are 1000. So in that case the market price of the bond or the intrinsic value of that particular bond will be 938.5543.

And here we have basically gone for the two components one is discounting the coupons and another one the discounting the par value of that particular bond.

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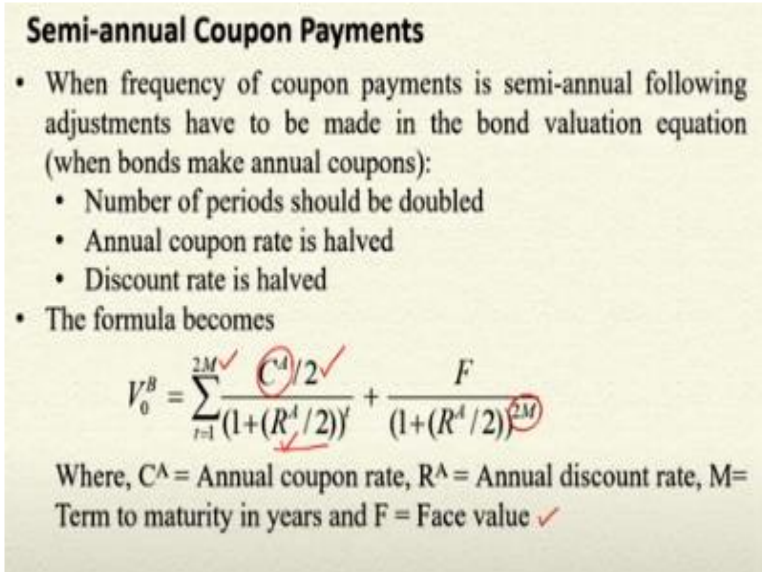

But let us see that if the coupons are let paid semi-annually. Many a times, we have observed that generally coupons are paid semi-annually, every 6 months basis, the bond issuer may pay the coupon. So then whenever the coupons are paid semi-annually, we have to make certain adjustments with respect to that formula. Whatever formula we have derived when the bonds basically make the annual coupons in that case we have made the certain adjustments.

What are those adjustments, we can make? One is your number of period maturity period is basically is mentioned on the basis of the years but now, it is basically 6 months basis the coupons are paid. So that the number of periods should be doubled that is number one. Number two, the coupons basically what you are getting in 6 months that should be half because in the annual you are getting 90 rupees at a 9% coupon rate. But whenever you the coupon is paid semi-annually then the annual coupon rate also should be half then the discount rate also should be half. Because if you annually you are getting 10% then semiannual you should get 5%. So there are 3 things basically what we have to make the adjustment whenever we are talking about the semi-annual coupon payments.

So now if you see your formula will be automatically changed.

$V^B_0 =$

$$\sum_{i=1}^{2M} \frac{C^A/2}{(1 + (R^A\,/2))^t} + \frac{F}{(1 + (R^A/2))^{2M}}$$

- Where, C^A = Annual coupon rate, R^A = Annual discount rate, M = Term to maturity in years and F = Face value

So in the same formula, if you observe here, what basically we have done, your m was the term to maturity in years so automatically your maturity period has become 2 M you observe. Your coupon was annual coupon rate you have taken let C^A, A represents another coupon that would be divided by 2 the annual coupon that whatever we have that is the 2.

So here your C^A represents your annual coupon rate, your R^A represents annual discount rate and M is the term to maturity in years. Then automatically in this case the period will be 2 M then F is the face value or the par value of that particular bond. Now let us with the same example if you see that how that particular thing worked out.

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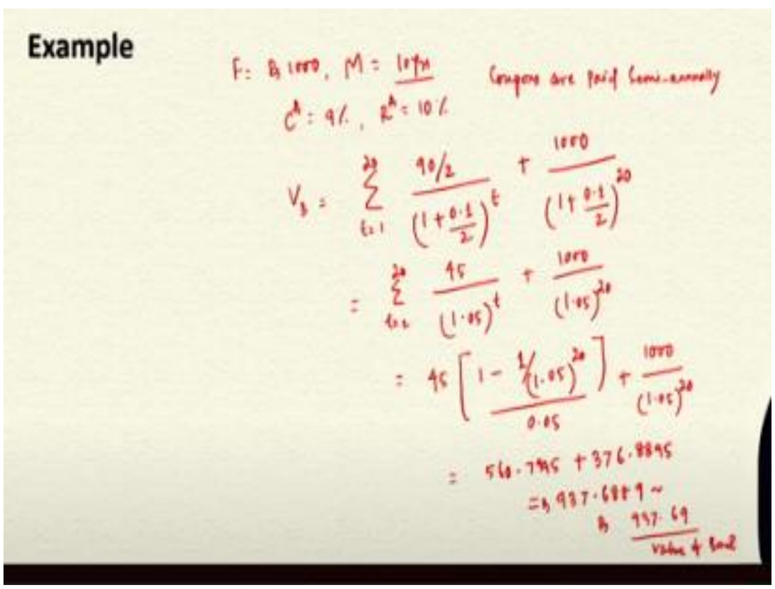

For example, F= 1000, M= 10 years, CA= 9%, Ra= 10% and Coupons are paid semi annually.

$$V_B = \sum_{i=1}^{20} \frac{90/2}{(1+0.1/2)^t} + \frac{1000}{(1+0.1/2)^{20}}$$

=

$$\sum_{t=1}^{20} \frac{45}{(1.05)^t} + \frac{1000}{(1.05)^{20}}$$

$$= 45[\left[\frac{1 - \frac{1}{1.05^{20}}}{.05}\right] + \frac{1000}{(1.05)^{20}}$$

= 560.7995 + 376.8895

=937.6889 or approx.(937.69)

Hence, the value of the bond is 937.69.

This is basically in terms of rupee that is the value of bond if you are paying the coupon semi-annually. If you are paying the coupon semi-annually then that is basically your value of the bond, little bit different from the valuation whatever we have done whenever we have used the annual coupon payments. But we will not find much differences but little bit differences will be there because the cash flow what basically you are getting that frequency of the payment of the cash flow gets changed. So because of that some deviation you can find in terms of the value of that particular bond.

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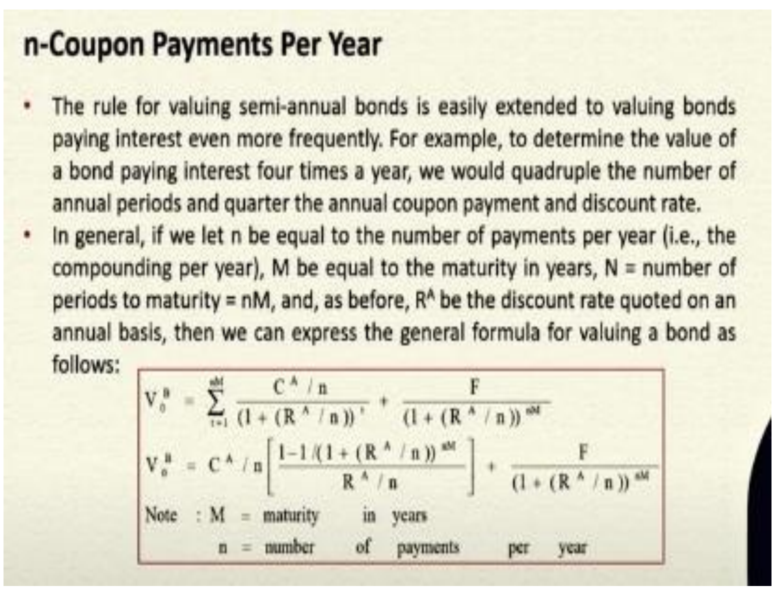

So now we can generalize that formula we have taken the number of frequency of the coupon payment is 2 per year. Now we can extend it even more frequently for example to determine the value of the bond paying interest 4 times a year, we can increase your periods 4 times. And also the coupon payments we can divided by 4, discounted also will be divided by 4 like that. So now if you see that then in general if N is equal to the number of payments per year that means the compounding per year.

Then M is equal to the maturity in years then age N is equal to your number of periods to maturity let that is N M and your R A be the discount rate quoted on an annual basis. So if you express the general formula for valuation of a bond. Then it will:

$$V_0^3 = \sum_{t=1}^{nM} \frac{C^A/n}{(1+(R^A/n))^t} + \frac{F}{(I+(R^A/n))^{nm}} \quad V_0^B = C^A/n\left[\frac{1-1(1+(R^A/n))^{nM}}{R^A/n}\right] + \frac{F}{(1+(R^A/n))^{nM}}$$

Note : M = maturity in years

n = number of payments per year

Then here already I told you that m is the maturity in years and n is equal to the number of payments per year. How many times the coupon is paid accordingly the particular formula can be generalized if the coupons are paid n times in a particular year.

So that is what basically we can generalize it whether it is 4 times or it is 2 times or would be 6 times, depending upon that this kind of adjustments you have to make and accordingly the value of that particular bond can be calculated.

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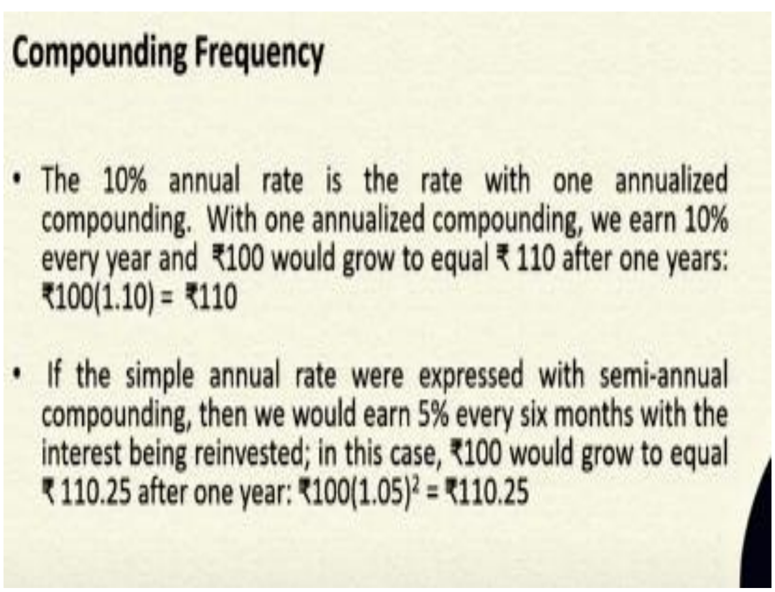

Then we are talking about a concept of the compounding frequency. so what do we mean by this compounding frequency? So whenever you talk about the 10% annual rate, it is basically the rate at which one annualized compounding; it is the rate with one annualized compounding. So if you are using one annualized compounding then what basically you are earning, you are earning 10% every year.

Then if you have invested 100 rupees, then the 100 rupees will grow to 110 rupees, after 1 year this particular value will become 110. So that will be how you get this that is 100 into 1.1 that is 110. So if you are going by the simple annual rate you are expressing with the semi-annual compounding then what will happen? You will get 5% every 6 months and if you assume that the interest is being reinvested again because you get some money after 6 months and that money can be reinvested in the market.

So what will happen in this case, in this case your 100 rupees will grow to 110.25 after one year. In the case of 1 year annualized compounding or one annualized compounding what we got we got 110 but whenever you are using semi-annual compounding, you are basically getting 110.25 rupees

In case of monthly compounding, the future value of the bond will be:

$$\text{Future Value of the bond} = 100(1+R)^2$$
$$= 100\left(1+\frac{0.10}{2}\right)^2$$
$$= 100(1.05)^2$$
$$= 110.25$$

Hence, in case semiannually compounding, the future value of bond will be little bit higher than in case of an annually compounding.

Then the value has become little bit more value you will be getting because you are using that particular money whatever you got after 6 months that has been reinvested in the market then your value has become 110.25 rupees.

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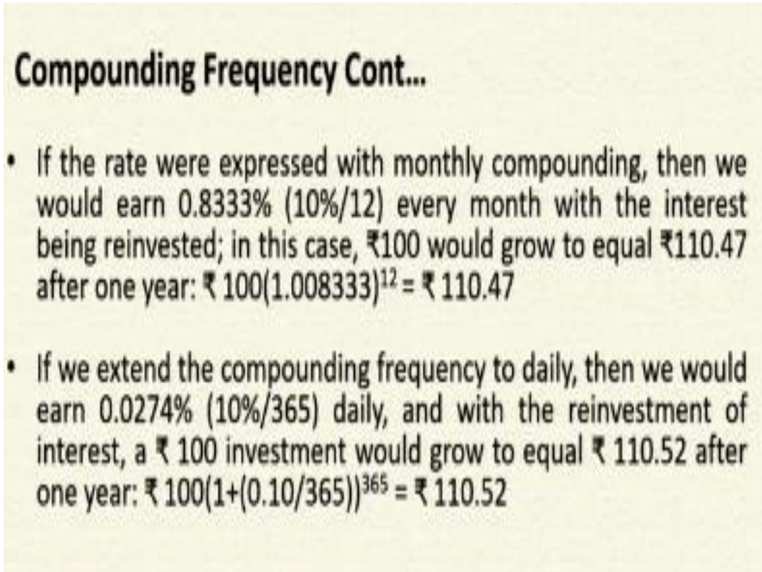

So like that if you increase your compounding let you are going for monthly compounding. In case of monthly compounding, the future value of the bond will be:

$$100(1+R)^n$$
$$= 100\left(1+\frac{0.10}{12}\right)^{12}$$
$$= 100(1.0083)^{12}$$
$$= 110.47$$

In case of daily compounding, the future value of the bond will be

$$100(1+R)^n$$
$$= 100\left(1+\frac{0.10}{365}\right)^{365}$$
$$= 100(1.00027)^{365}$$
$$= 110.52$$

If your compounding frequency will increase then automatically your value this ending value is going to be increasing. Because we are talking about the concept of the reinvestment so that also you have to keep in the mind.

So compounding frequency basically will affect the investment value, the ending investment value of the investor.

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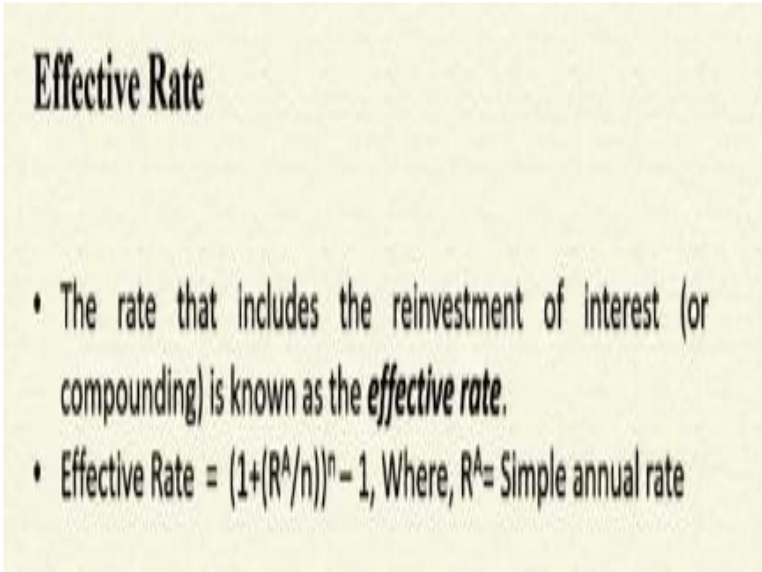

So here another concept you have to keep in the mind that is called the effective rate. So what is effective rate? The effective rate basically is the rate which includes the reinvestment of interest or compounding. So here if you want to calculate this effective rate, it is:

Effective rate= $[1+(R^A/n))^n -1]$.
Your R^A means it is the simple annual rate and from there you can calculate your effective rate.

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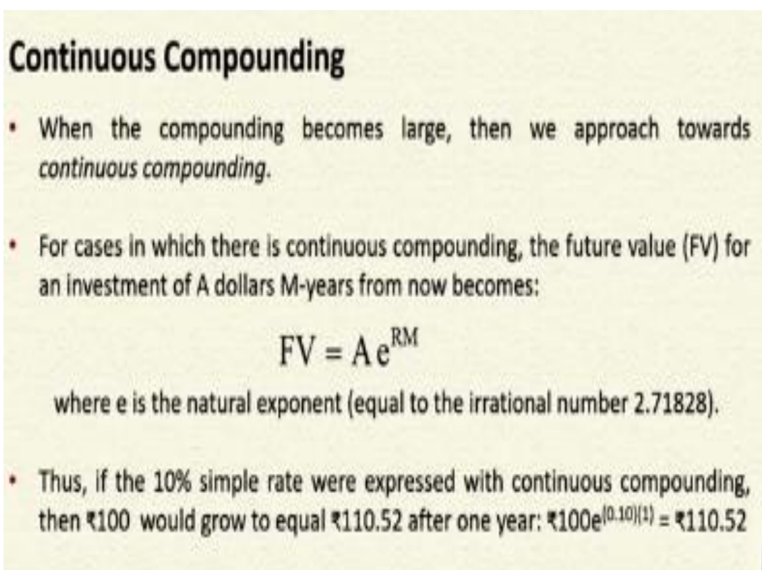

So, let you assume that our cash flow is continuously compounded. When the compounding becomes very large then generally we approach towards the continuous compounding. So wherever there is continuous compounding if you want to calculate the future value for any investment of let 8 dollars for M years from now the future value will become:

$F\ V = A\ e^{RM}$.

$= 100\ e^{RM}$,

$= 100\ e^{0.01*1}$,

$= 100*2.71825^{.01}$,

$= 110.52$

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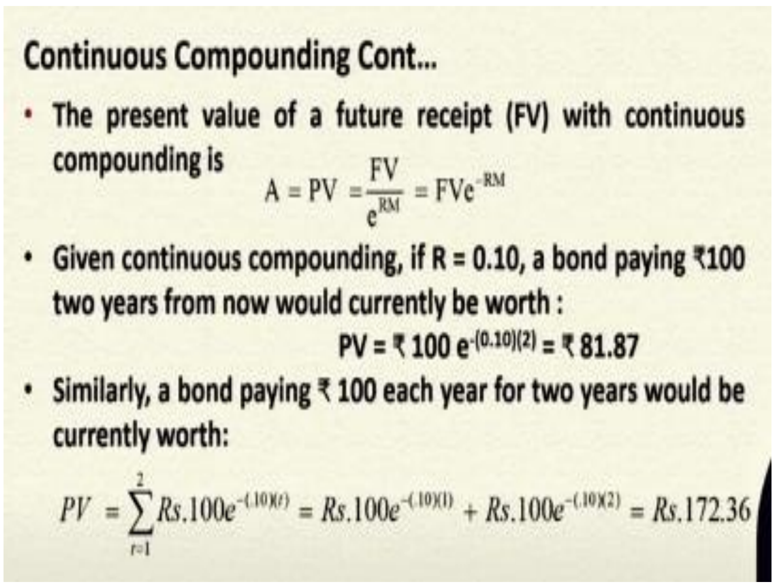

So like that if you go for the calculating the present value from that if you know the future value whatever you are going to get then the present value will be:

The present value of a future receipt (FV) with continuous compounding is

$$A = PV = \frac{FV}{e^{RM}} = FV\ e^{-RM}$$

So given the continuous compounding, let R = 10% and your bond is going to pay 100 rupees 2 years from now then what is the value of the bond?

What is the current worth of 100 particular bonds? The current worth of the particular bond will be:

Given continuous compounding, if $R = 0.10$, a bond paying ₹100 two years from now would currently be worth:

$$PV = ₹100e^{-(0.10)(2)} = ₹81.87$$

So if a bond which is paying 100 rupees each year for 2 years then what is the value of that particular bond now? Then it will be:

Similarly, a bond paying ₹100 each year for two years would be currently worth:

$$PV = \sum_{i=1}^{2} Rs \cdot 100e^{-(10x(1)} = Rs \cdot 100e^{-(10\times1)} + Rs \cdot 100e^{-(10\times2)} = Rs.172.36$$

That means it will give you the current worth of that particular bond will be 172.36 and you will be getting the ending value will be 200 rupees at the end of the maturity. So that is what basically what way we use this continuous compounding.

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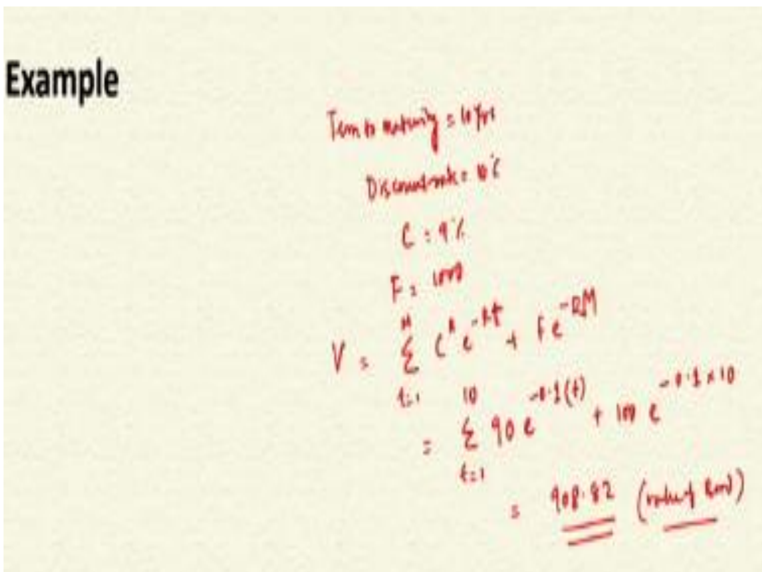

Let us one small example we can we can take with respect to this.

Let, the term to maturity= 10years
Discount rate = 10%
Coupon rate is 9%
Future value = 1000
Then, V=

$$\sum_{t=1}^{m} C^A e^{-rt} + Fe^{-Rm}$$

=

$$\sum_{t=1}^{m} 90e^{-0.1t} + 1000e^{-0.10.10}$$

= 908.82
Hence, the present value of the bond will be 908.82.

So that will basically become your 908.82. It is the value of the bond. So in the same example whenever we have taken this concept of continuous compounding then what basically we got that the value of the bond will become 908.82 with the term to maturity of the 10 years and the face value is 1000 and coupon is 9% and the discount rate is 10%.

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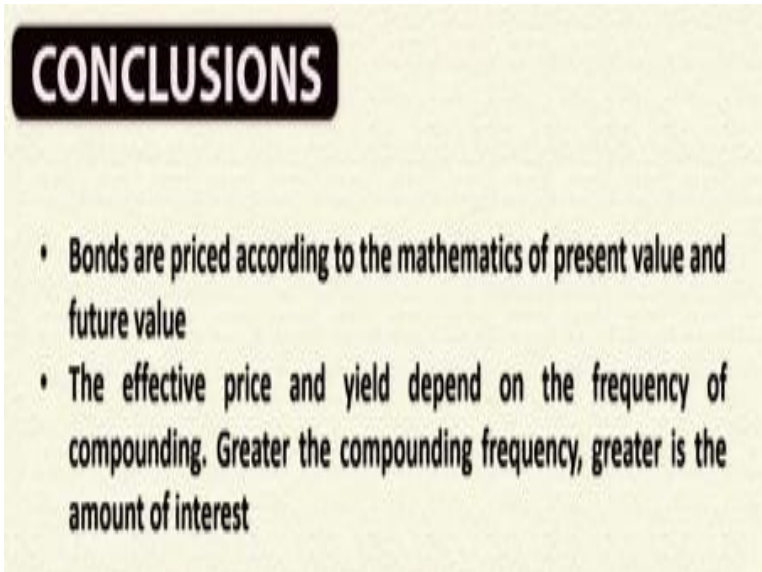

So what basically we have seen here, the bonds are priced according to the mathematics of present value and the future value. And the effective price and yield of the bond generally depends on the frequency of the compounding. Greater the compounding frequency greater would be the amount of interest and accordingly the value of the bond also will to be changed.

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These are the references what you can you can go through then the other discussion with respect to bond valuation, we will discuss in the next class. Thank you.