Management of Fixed Income Securities
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Module No # 05
Lecture No # 24
Bond Convexity

Welcome back! So, in the previous class, we discussed the concept of duration, which is basically a weighted time measure of a particular bond, and as well as also, we have seen that duration can be used as a price sensitivity measure. Due to the change in interest rate, how the price of the bond is going to be changed, that also we can measure, we can judge from the duration measure. So, today we will be discussing another concept related to that that is called bond convexity.

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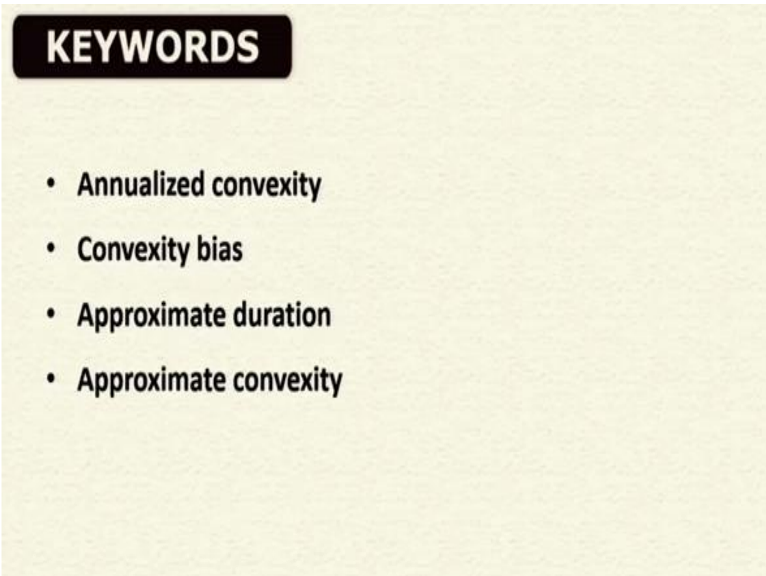

So, whenever you talk about this bond convexity, we will be discussing certain concepts related to that like, what is analyzed convexity, the concept of convexity bias, approximate duration, and approximate convexity. These are the different approximate measures of duration and convexity. Also, we can discuss this because there is a link between duration and convexity. So, what we have seen in the previous class is that you see whenever the interest rate is changing a little bit in a larger percentage or change in interest rate is relatively more.

In that case, your duration was not able to give you an accurate figure in terms of the change in the bond prices. So, that thing can be resolved through the use of the concept of convexity. So, let us see what exactly convexity means, and how convexity is defined?

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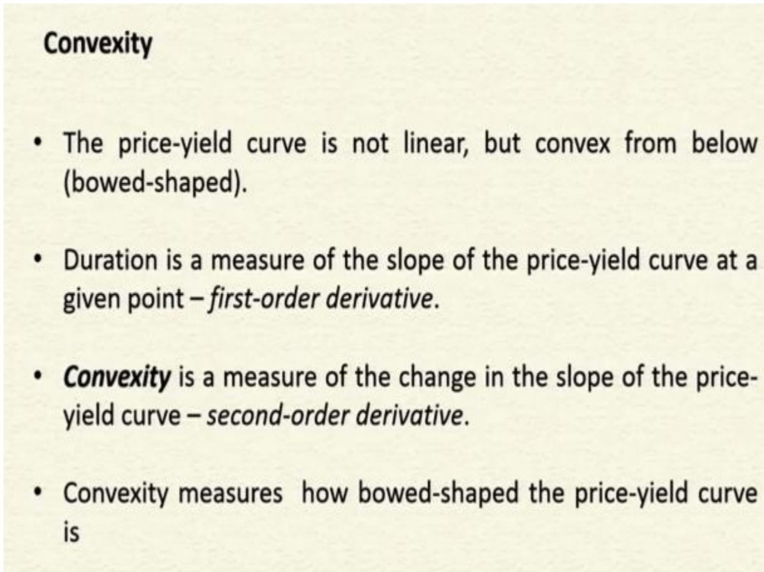

If you see the convexity, you see the convexity is basically a concept, that is very much relevant in terms of the bond risk management strategy. because, the price yield curve is not a linear curve, and it is convex. It is both. just now, we have seen already all of you know that the price yield curve is not linear, but convex from below. So, the duration that we have discussed basically measures the slope of the price yield curve at a given point.

That is the first order derivative of the bond price equation we have estimated and there we have seen the duration is basically, a measure of the slope of the price yield curve. But, whenever we come to the concept of convexity, it basically measures the change in the slope of the price of the yield curve. The duration measures the slope of the pricing curve, but convexity measures the change in the slope of the price yield curve, which means it basically discussed the second-order derivatives.

The convexity is basically the second-order derivative of the price equation, with respect to the change in interest rate. So, in a general sense convexity basically tries to measure, how the bow-shaped price yield curve is? What is the curvature of the price yield curve? that basically is reflected through the concept of convexity? So that is, why convexity has a lot of relevance from the investment point of view. Whenever anybody is investing in the bond market, convexity has its own relevance, and convexity always is used in that particular context.

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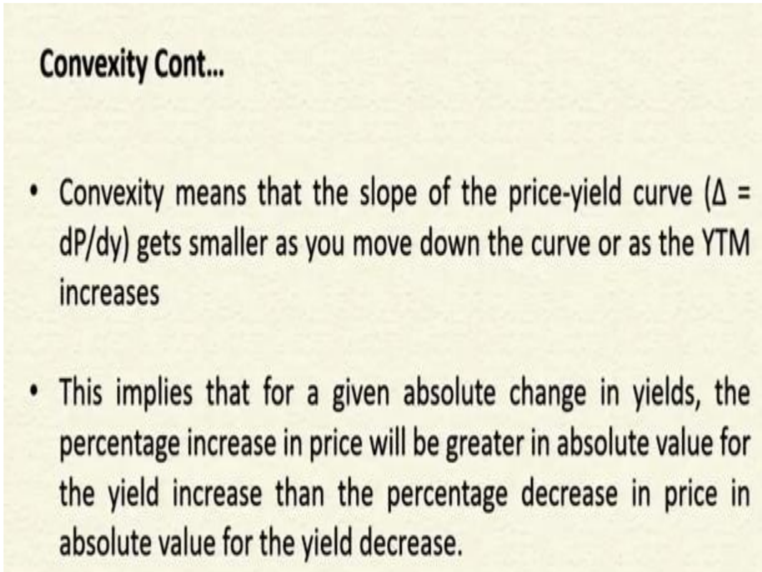

So, let us see in the mathematical sense how convexity is defined. Already, we have discussed that convexity means that the slope of the price yield curve, gets smaller as you move down this particular curve or the YTM is increasing. In the x-axis, we are taking YTM; in the price yield curve. So, whenever convexity, you are trying to measure or you are trying to define, you will find that the slope of the price yield curve, which is basically your delta.

Let we are representing dP by dy which basically gets smaller, as you move down the curve or the YTM is increasing then what does it imply? It implies that for a given absolute change in the yield, the percentage increase in the price will be greater in absolute value for the yield increase than the percentage decrease in the price in absolute value for the yield decrease. You got this point? let the interest rate has changed. So, whenever the yield is increasing, then what will happen?

The price of the bond will change. So, the percentage increase in the price will be greater in absolute value, whenever the yield is changing or increasing, than the percentage decrease in the price in the absolute value for yield is decreasing.

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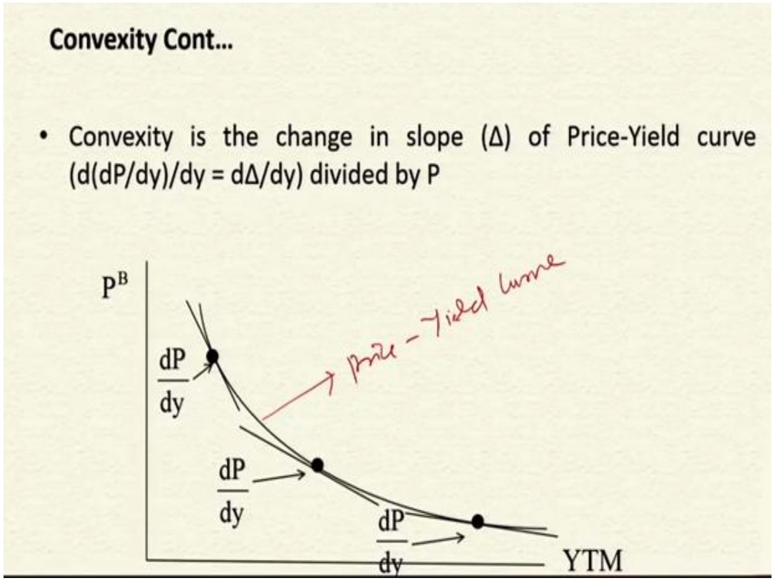

So, what basically here we are trying to see from here? You see already I told you, that convexity is the change in the slope of the price yield curve divided by P. So, we have a price yield curve. So, this is your price yield curve and these are the different slopes the dP by dy, dP by dy, dP by dy. these are the different slopes and the slope is changing whenever you are moving your slope is basically changing.

So, what is the implication you can draw from here? So, whenever the interest rate is increasing or decreasing, if you are talking about whenever your yield is basically increasing you will find that the percentage change in the price, or we can say the slope of the yield curve is not going to be changed or is going to be changed in a different way in the lower zone than in the upper zone. Is it clear?

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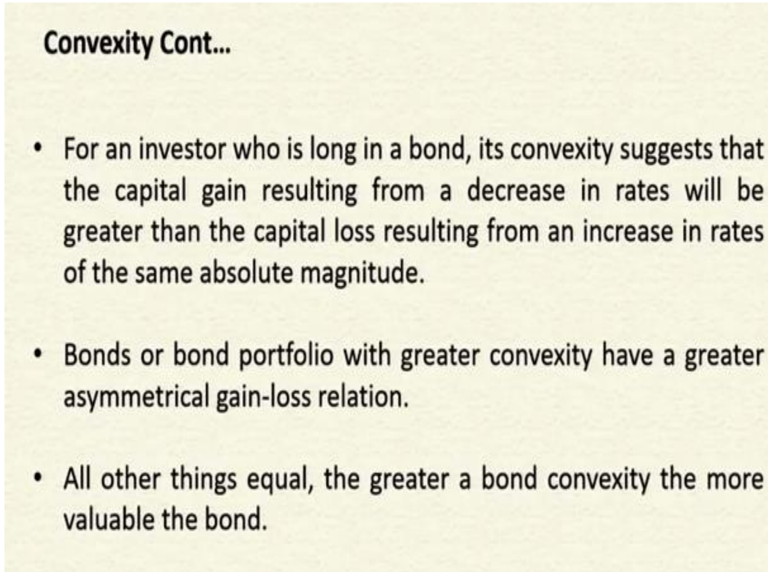

So, let us see what basically the use of that or what is the implication of that? So, for a bond investor who is long in a bond, that means buying the bond. Its convexity basically suggests that capital gain results from a decrease in rates. If the interest rate will decline, then obviously the price will increase. So, the capital gain which is resulting from a decrease in rates will be greater than the capital loss resulting from an increase in rates of the same absolute magnitude.

Let the interest rate will go down by 1% or go up by 1%. Then, whatever capital gain that person will get in terms of the declining interest rate that will be more than, if the interest rate would have increased by 1%, then will have lost in the market. Some capital loss would have occurred. Then, if it will increase by 1%, then the capital gain that he or she will get, will be more than what basically they would have lost.

If the interest rate would have been increased by 1%, that is what basically the convexity is trying to imply to you. So, that is why bonds are the bond portfolio with greater convexity and have a greater asymmetrical gain or loss relationship. The gain and loss relationship is very much asymmetric in nature. So here, from the investment perspective, what you can say is all other things equal greater the bonds convexity more the valuable the bond is from the investor perspective, that actually you can keep in the mind.

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Measurement of Convexity

Mathematically, convexity is the change in the slope of the price-yield curve for a small change in yield; it is the second-order derivative.

It is derived by taking the derivative of equation for dP/dy and dividing the resulting equation by the current price.

Doing this yields:

$$\text{Convexity} = \frac{1}{P_0^b}\left[\sum_{t=1}^{N}\frac{t(t+1)(CF_t)}{(1+y)^{t+2}}\right]$$

The convexity of a bond that pays a fixed coupon each period and the principal at maturity is

$$\text{Convexity} = \frac{\frac{2C}{y^3}\left[1-\frac{1}{(1+y)^N}\right]-\frac{2CN}{y^2(1+y)^{N+1}}+\frac{N(N+1)[F-(C/y)]}{(1+y)^{N+2}}}{P_0^b}$$

So let us see, that how the convexity is measured mathematically? already we have explained it. Convexity is the change in the slope of the pricing curve for a small change in the yield. So, this is basically the second-order derivative of the price equation. So, if you want to derive it, you can take the derivative of the equation of dP by dy and divide the resulting equation by the current price.

So, already we have this first-order derivative equation available to you. Then you take the second derivative with respect to the yield. That is why that means you are I am talking about the dP square by this d square P by dy square. Then, the convexity will be 1 by P0 into your t =1 to N t into t +1 in bracket cash flow at the time t divided by 1+ y to the power t +2. Or if you related the equation whatever we have used for measuring this duration in the previous session.

Convexity = $\frac{1}{{P_0}^b}\left[\sum_{t=1}^{N}\frac{t(t+1)(CF)_1}{(1+y)^{t+2}}\right]$

If you are considering the convexity of a bond that pays a fixed coupon each period and principle at the maturity, then the convexity will be your 2c by y cube into in bracket 1-1 by 1+ y to the power N -2 CN divided by y square 1+ y to the power N +1+ N + N into n +1 F– C by y divided by 1+ y to the power N +2. You can also derive it on your own, whenever you have the price equation available to you.

If you are considering the convexity of a bond that pays a fixed coupon each period and principle at the maturity is

$$\text{Convexity} = \frac{\frac{2C}{Y^3}\left[1-\frac{1}{(1+Y)^N}\right]-\frac{2CN}{y^2(1+y)^{N+1}}+\frac{N(N+1)[F-\left(\frac{C}{y}\right)]}{(1+y)^{N+2}}}{{P_0}^b}$$

Take the first-order derivative with respect to the change in the yield and again, go for a second or derivative with respect to that. Then, you can get this particular equation. Then, you divide it with respect to the current price then you can calculate the convexity. Is it clear? So, this is the second order derivative of the bond price equation with respect to the change in the interest rate.

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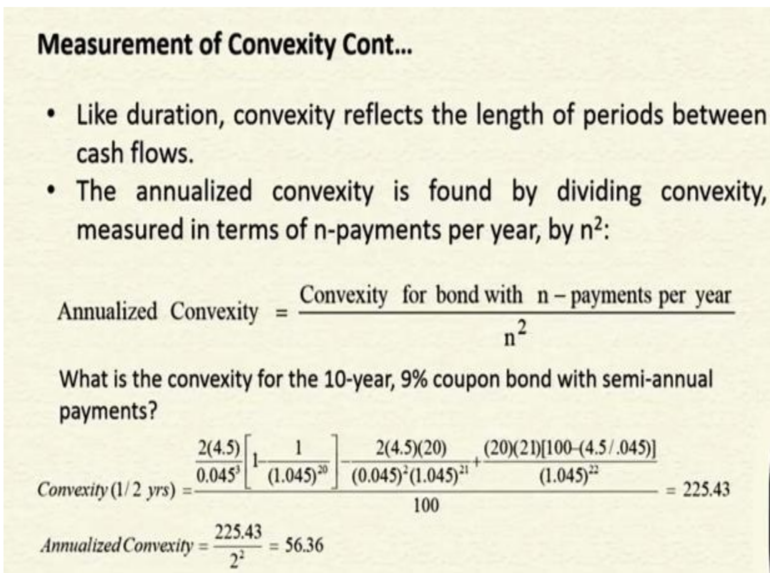

So like duration, convexity also reflects the length of periods between the cash flows. It also reflects the length of the periods between the cash flows and convexities also conventionally always represented annually. So, the annualized convexity is basically found by dividing the convexity measured in terms of n times payments per year by n square. In the duration of what we have done? The convexity for bonds with or the duration of the bonds with n times payments per year divided by n that we have done.

But whenever you are making the convexity analysis, this is convexity for the bond with n payments per year divided by n, square. N payments per year mean, how many times the coupon is paid in a particular year? So, if it is semiannual, then it is 2 to power 2. If it is quarterly, then like that your figure will be changed. So, let what is the convexity for the 10 year 9% coupon bond with semiannual payments and bond is issued at par?

The same example that we are taking, is a semiannual coupon. So that is why 4.5 is the semi-annual coupon payment, what you will be getting? So, then 2 into 4.5 divided by your y to the power or 0.045 to the power 3 into 1-1 by 1.045 to the power 20-2 into 4.5 into 20 divided by 0.045 square into 1.045 to the power n +1 that means 21 plus your 20 n into n +1 that is 20 into 100-4.5 divided by 0.045 divided by 1.045 to the power 22 that is n + 2 that divided by 100. That is the price of the bond.

$$\text{Convexity (1/2 yrs)} = \frac{\frac{2(4.5)}{0.045^3}\left[1-\frac{1}{(1.045)^{20}}\right]-\frac{2(4.5)(20)}{(0.045)^2(1.045)^{21}}+\frac{(20)(21)[100-\left(\frac{4.5}{.045}\right)]}{(1.045)^{22}}}{100} = 225.43$$

We are assuming that the bond is issued at par then your convexity in terms of half yearly, you are getting 225.43. So, if you are calculating the analyzed convexity, then it is 225.43 divided by n square and here n = 2 because it is a semiannual coupon payment bond. Then that will be 225.43 divided by 4 which will be 56.36 approximately. So, 56.36 is the annualized convexity of this particular bond.

$$\text{Annualized convexity} = \frac{225}{2^2} = 56.36$$

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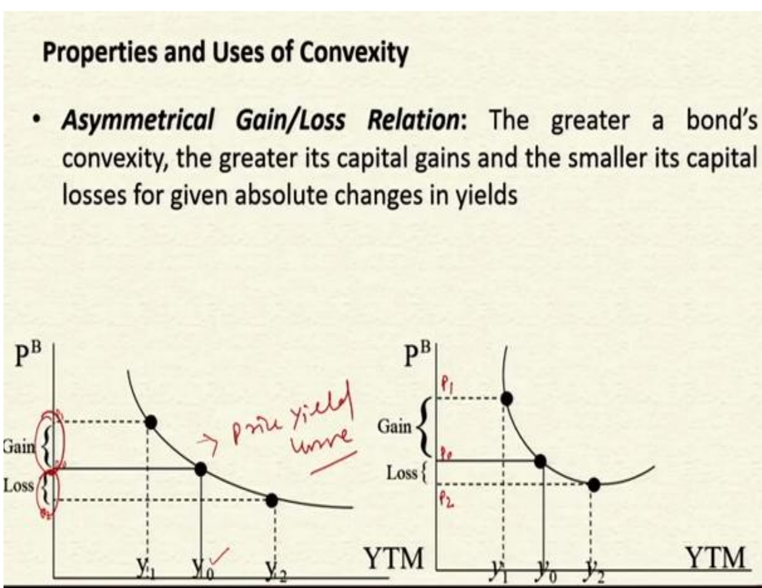

So, then what are the properties of this particular convexity or the use of this particular convexity? First is already we have seen, that there is an asymmetrical gain or loss relation. Let us visualize it, and how basically it will happen? The greater a bond's convexity, the greater its capital gains and smaller its capital losses for a given absolute change in the end. Let the y 0 here, y 0 was the initial YTM, and at that time this was the price.

Now, let the interest rate has gone down, and YTM has gone down from y 0 to y 1. Then what we have observed? the price has gone up let this be P 0, then it becomes P 1. Let this same interest rate has gone up with the same amount that y 0 to y 2. So, then what will happen? There will be the price will go down and this is P 2. So now, what we have seen is that the gain that we are getting is more than the loss, and this is your price yield curve.

Come to this is a little bit more convex in comparison to this. This is a little bit flatter in comparison to this. The curvature of this curve is more than this curve. If you see the same amount of interest rate chains, let this be your original price. This is P 1 and this is your P 2. What we have seen here, the gain is more than the losses, here also gain is more than the losses. But, here it is more in comparison to this, also it is more, that means it shows more the curvature, more the valuable the bond is.

The asymmetrical gain and loss relations if you are trying to measure, then that is basically more visible, more captured on the basis of the curvature of this particular income in this case. So, this one is more than this. This one is also more than this. But here, it is the gain is much higher than the loss in comparison to this. So that is the first property that you can draw, the greater a bond's convexity, greater its capital gains and the smaller the capital losses for absolute changes in the yield and that also depends upon the convexity of that particular price yield curve.

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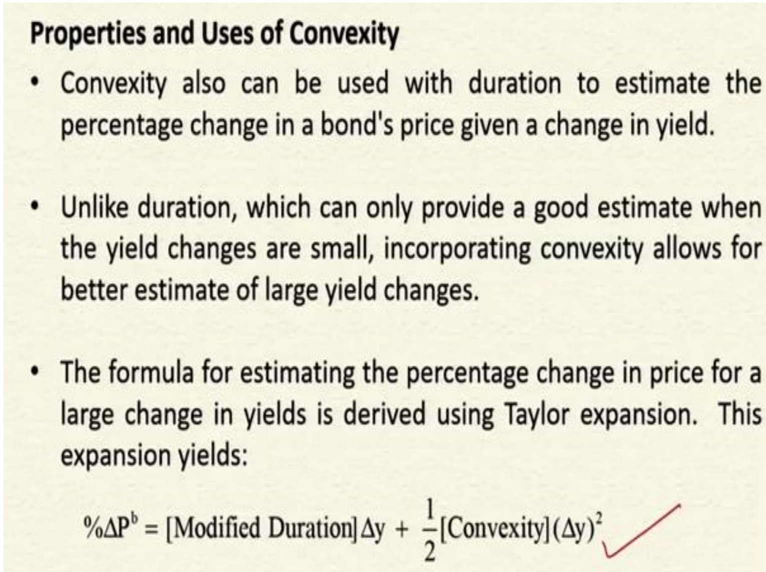

Second thing, convexity also can be used with duration or like duration to estimate the percentage change in the bond price given a change in the yield. We have seen in the duration case, also we can find out the percentage of a change in the price with respect to change in the interest rate by using the concept of duration. But, unlike duration which can only provide a good estimate when the yield change is small, whenever you are using the convexity, it allows for a better estimate for the large head changes.

So, even if the interest rate will increase by a little bit more percentage, the percentage change in the price can be measured through the use of the convexity measure which was not possible in the case of the duration measure. So, if you want to derive the formula which is estimating the percentage change in the price for a large change in the yield, basically you can go for using Taylor's expansion.

Here, I am not giving the complete Taylor's expansion. but this expansion basically will yield the percentage change in the price of the bond is equal to your modified duration multiplied by the change in the yield that is delta y+ 1 by 2 into the convexity into your delta y square.

$\%\Delta P^b$ = [Modified Duration] $\Delta y + \frac{1}{2}$[Convexity] $(\Delta y)^2$

So, that is the formula for calculation of the change in the price, whenever there is a change in a large change in the yield and that thing you are directly calculating using the concept of convexity. Using the concept of convexity basically, you are trying to measure it.

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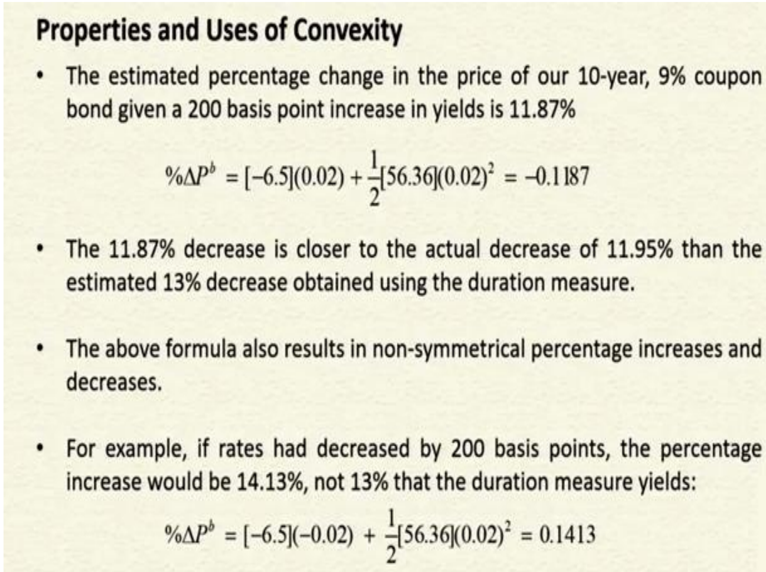

Let us take the example, how basically it happens. We will continue with our previous example also. The estimated percentage change in the price of a 10-year, 9% coupon bond. let the interest rate increase by 200 basis points that means 2%. Now, you use this particular formula. The percentage change in the price of the bond will be your convexity is 6.5. In the previous case, we have measured that is -6.5 into 0.02 then + 0.02 mean 2% increases in the delta y that is 2% plus half of the 56.36 which is the modified duration.Yield curve into your delta y square, that is 0.02 whole square, that you got -0.1187.

$\%\Delta P^b$ = [-6.5] $(0.02) + \frac{1}{2}$[56.36] $(0.02)^2$ = -0.1187

That means -11.87% means the interest rate has gone up. So, because of that, the price will go down. So, the change in the price of the bond percentage the price of the bond will be negative. So, the 11.87% decrease is actually very much close to the actual degrees of or actual decrease per change in the price of the bond, which is 11.95%.

Then the estimated 13% decrease was obtained using the duration measure. In the previous case we have seen, that whenever we are 200 basis points or 2% interest rate we have changed, we got a percentage change in the price will be 13% using the duration. But actually, it is 11.95% but whenever we have used the convexity, we find it is 11.87% and actually it is 11.95%. But whenever you are using the convexity, you are getting 11.87 that mean it is closure.

So, this formula also results in non-symmetrical percentage increases and decreases. How? Let the rate has decreased by 200 basis points. If the rate will decrease by 200 basis points, then the percentage obviously the bond price will increase the percentage of the change in the bond price will be positive. Then we got that is 14.13%, not the 13% that the duration measure basically yields.

$\%\Delta P^b$ = [-6.5] (0.02) + $\frac{1}{2}$[56.36] $(0.02)^2$ = 0.1413

So here, what basically you are we are trying to find out? that convexity is a better measure in terms of measuring the change in the price of the bond or percentage change in the price of the bond, than the duration whenever the change in the interest rate is a little bit higher. Whenever the yield change is a little bit higher, then you can use convexity for calculating the percentage change in the bond price in comparison to the durations. So, that is the advantage that you can get from here.

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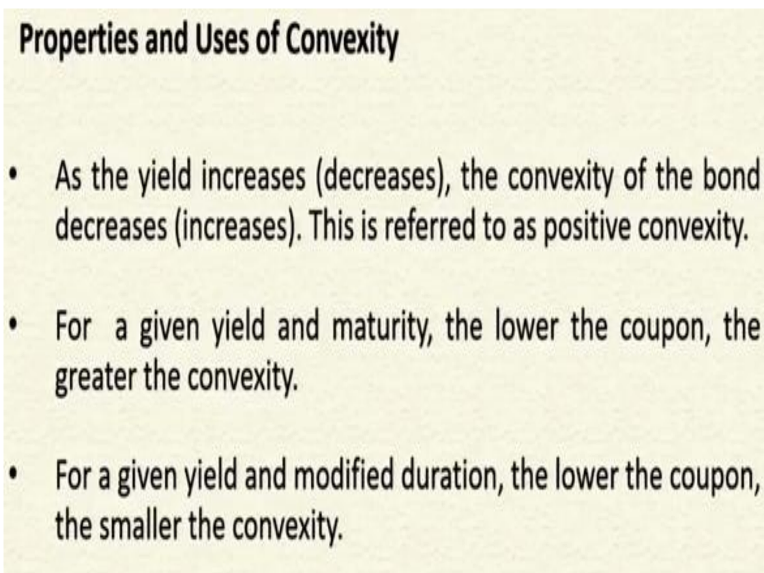

So, as the yield increases or decreases, the convexity of the bond decreases or increases. this is basically referred to as positive convexity. That is, when the yield is increasing, the convexity of the bond is decreasing. Whenever the yield is decreasing, the convexity of the bond is increasing. This is basically called positive convexity. The concept is basically called positive convexity.

For a given yield and maturity lower the coupon, the greater the convexity, and for a given yield and modified duration lower the coupon, the smaller the convexity. So, these are the different properties that basically you can consider you can always try to find out, whenever you are using the concept of convexity.

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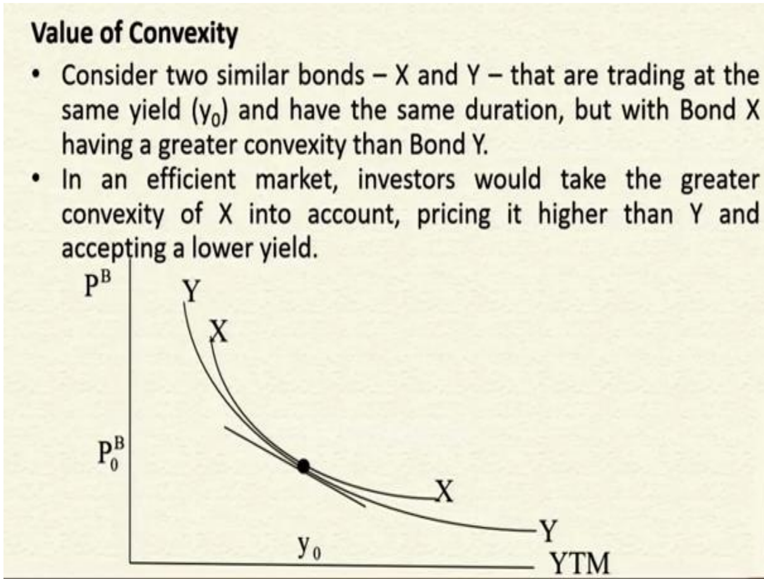

So, the value of the convexity if you are trying to measure, already we say that the bonds having more convexity is more valuable. Let you consider 2 similar bonds X and Y, which are trading at the same yield let y 0 and have the same duration but with bond X having greater convexity than the bond Y. For example, so if you assume that there is an efficient market, then investors always would take the greater convexity of the X into their consideration and price it higher, and accept a lower yield.

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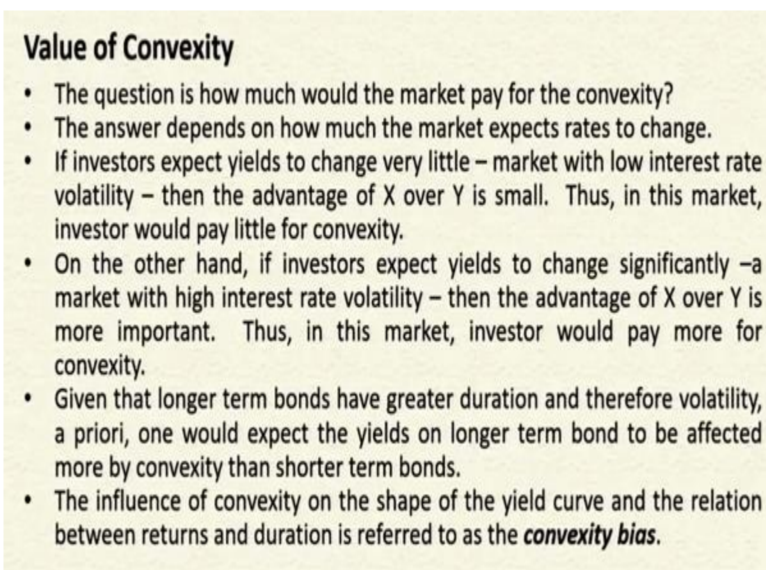

But the question here is how much would the market pay for the convexity? So, the answer basically depends on how much the market expects the rates to change? How this interest rate is going to change? If the investor expects the yield to change very little, that means the market with low-interest rate volatility. Then the advantage of X over Y is very small and in this market, an investor would pay little for the convexity.

On the other hand, if the investor is expecting that change is very much significant which means high-interest rate volatility, then the advantage of X over Y is more important. So, in this market investors will pay more for the convexity and given that longer-term bonds have a greater duration and therefore volatility. One would expect that yields on a long-term bond to be affected more by convexity than the shorter bonds.

Given that the longer-term bonds of greater duration and also the volatility, one can expect the yields on a long-term bond to be affected more by the convexity, than certain bonds. And the influence of the convexity on the shape of the yield curve and the relation between the returns and the duration is called as the convexity bias. That is basically referred to as convexity bias. How the convexity is affecting the shape of the yield curve and the return relationship between the duration and this return and the duration is basically called the convexity bias.

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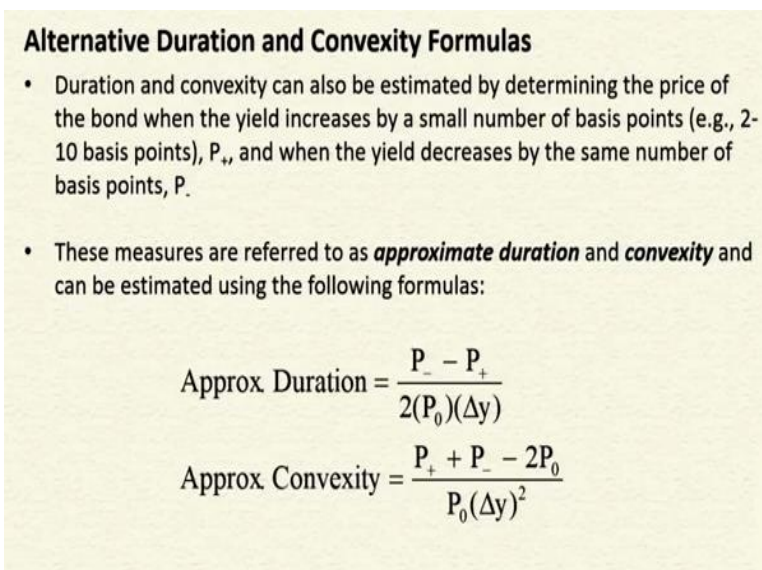

So, there are alternative duration and convexity formulas. Generally, we use that is we call the approximate measures of the duration and complexity. So, duration and convexity can be also estimated by determining the price of the bond. When the yield increases by a small number of basis points like 2 to 10 basis points generally and when the yield decreases by the same number of basis points that is P – or P +.

When it is increased, then obviously the price will be + P plus, and when the yield decreases by the same number of basis points that is P-. So, these measures are basically referred to as approximate duration and approximate convexity. We have seen how the duration and convex it are measured. But, approximately also if you know the price changes due to the change in the yield then you can also calculate the approximate duration and approximate convexity.

So, the approximate duration is your P- change in the price due to whenever the price is declining whenever the interest is increasing. Then it is or vice versa that is, P - v + divided by your 2 into P 0 into delta y. P 0 is the initial price. So, in this case, you can calculate the approximate duration and approximate convexities of your P++ P –2 P0. Whenever the interest rate will increase or decrease you find out your P + and P -, then your initial price is P0 is -2 P0 divided by P0 into delta y square. That will give you the measure of the approximate convexity.

Approx. Duration = $\frac{P_{-} - P_{+}}{2(P_0)\,(\Delta y)}$

Approx. Convexity = $\frac{P_{+} + P_{-} - 2P_0}{(P_0)\,(\Delta y)^2}$

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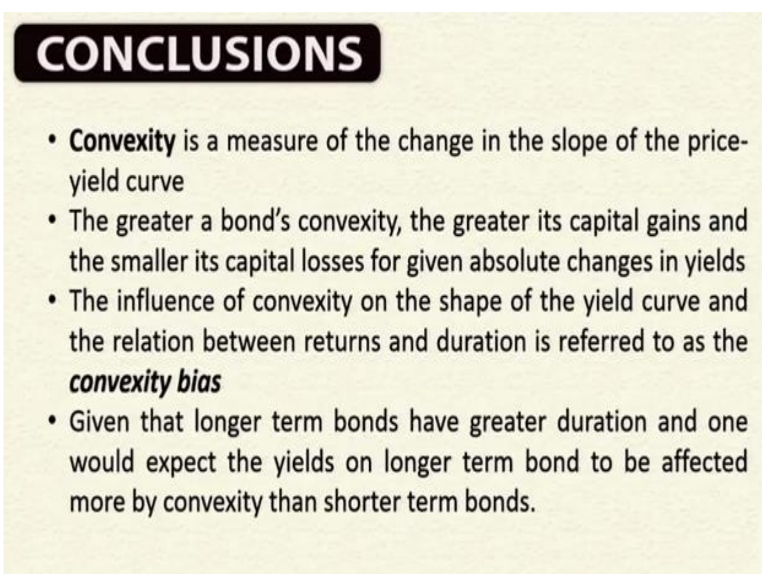

So, what we have seen here is convexity is a measure of change in the slope of the price yield curve. The greater the bond's convexity, greater its capital gain and the smaller its capital losses for absolute change in the yield, and the influence of convexity on the shape of the yield curve generally is referred to as the convexity bias and given that the longer-term bonds of greater duration and one would expect the yields on a long-term bond to be affected more by the convexity than the short-term bonds.

And another thing also you can keep in the mind using convexity, you can also measure the percentage change in the price of the bond. Whenever, the interest rate increases better, whenever the change in the interest rate is a little bit higher. The duration was not able to be approximately measured in a better way that can be always used the concept of convexity. So, whenever we change in the yield is more convexity will give you a more accurate measure in terms of the change in the price of the bond than the use of the duration.

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So, these are the references what you can go through for today's session. thank you.