Managements of Fixed Income Securities
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Module No # 05
Lecture No # 22
Interest Rates Risk

Good morning! So, in the previous class, we discussed about the how to derive this spot rate yield curve? And from there also, we discussed about how to derive forward yield curve? Today, we will start the discussion on the particular important risk, what the fixed income securities market investor always face, that is called the interest rate risk. But, the other name of that thing is also the market risk.

Largely, whenever we are investing in the fixed income securities we are more exposed to interest rate risk because the interest rate fluctuations with the impact of the pricing and other things.

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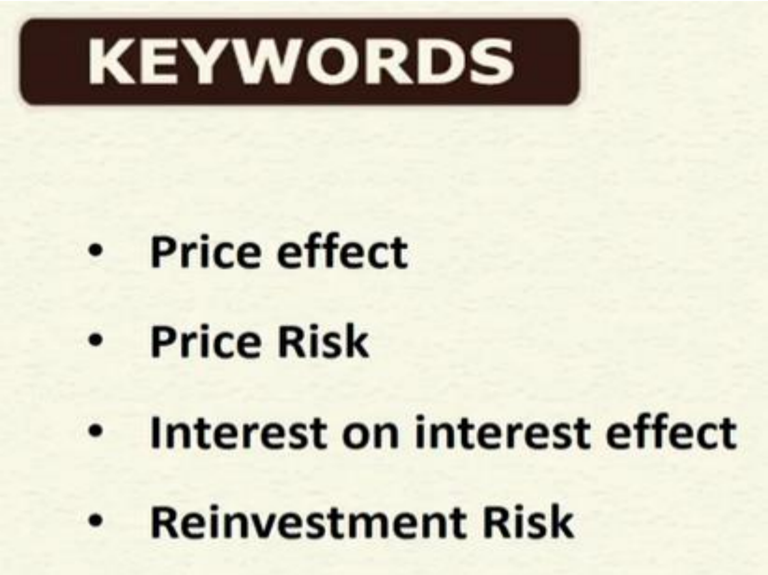

So, in this particular session, we will discuss largely about the concept of the market risk or the interest rate risk. And you will be coming across these particular words like price effect or price risk, interest on interest effect and the reinvestment risk etc. So, these are the different keywords for basically we will come across, whenever this particular session will be going on.

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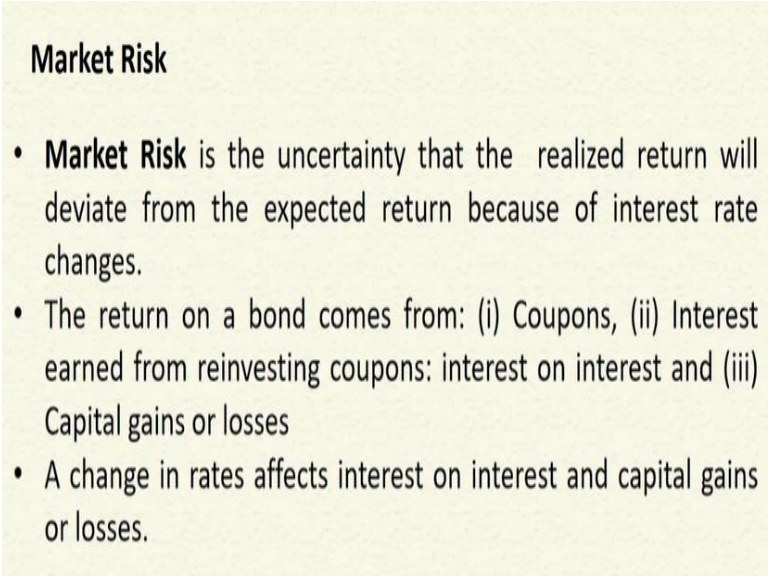

You see? what is the market risk? The market risk is nothing but it is the uncertainty that the realized return will deviate from the expected return because of the interest rate changes. Your total return concept already we have discussed. the total return whatever you are getting, if you are expecting this much return to get and end of the day your total return is not exactly equal to your expected return.

That means, we are exposed to the market risk. So, that is basically formally we defined the market risk. So, the return on a bond basically comes from 3 sources mostly; coupons, interest what basically we earned, whenever you are reinvesting the coupons, that is called the interest on interest and the capital gain or the losses at what price you have bought the bond and what price you have sold the bond.

So, these are the 3 major components of the return always we see. so whenever, there is a interest rate change that will affect basically the interest on interest, the capital gain or the loss part. Because coupons are not going to be affected as coupon is fixed. So, only these 2 components are going to be affected. So, because of this the return can be derivate. So, that is basically what we call it the market risk.

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Market Risk

- A change in interest rates has two effects on a bond's return: **price effect** and **interest-on-interest effect**.
- **Price Effect***:* Interest rate changes affect the price of a bond; this is referred to as **price risk**. If the investor's horizon date, HD, is different from the bond's maturity date, then the investor will be uncertain about the price he will receive from selling the bond (if HD < M), or the price he will have to pay for a new bond (if HD > M).
- **Interest-on-Interest Effect**: Interest rate changes affect the return the investor expects from reinvesting the coupon – **reinvestment risk**. Thus, if an investor buys a coupon bond, he automatically is subject to market risk.

So now, if you see whenever there is a interest rate change, it has 2 effects on the bonds return one is price effect, another one is interest on interest effect. And what is price effect? because the interest rate change affect the price of the bond. if interest rate will go up the price of the bond will go down or the interest rate will go down the price of the bond will go up. So, there is an inverse relationship between the interest rate and the price of the bond.

So if the price of the bond is getting affected we call it the price risk. if you are holding up to the maturity then, your return is not going to be affected. But, if you are going to sell the bond in between; before the maturity, then what basically will happen? you are exposed to some amount of price risk or some amount of interest on interest risk or the investment what we call it.

So that is why, if the investors horizon period is different from the bond maturity date, then the investor generally is uncertain about the price what he or she is going to receive from selling the one. If your horizon date or horizon period is less than the maturity period also the price he will have to pay if somebody is going to buy a bond. If the horizon period is greater than the maturity period, both ways the price effect is basically working there.

Then, the other one is interest on interest effect. So, the interest rate change will affect the return, the investor basically expects from the investing the coupon that is called the reinvestment risk. So, if the interest rate will go up then what will happen? The return from the reinvestment of the coupon will be increasing but the interest rate will go down, then if whenever you are receiving the periodic coupons that coupons will be basically reinvested in the market at the lower rate.

That is why your reinvestment return will go down but your price of the bond will go up. So, the investment effect and the price effect works in the opposite direction.

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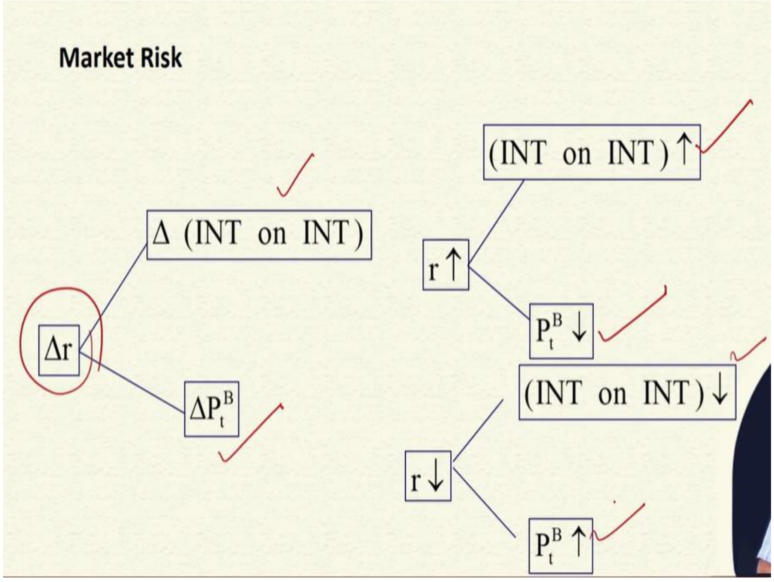

So let us see, how basically this works? I have just summarized here in this particular case. So here, what basically we have seen? let your delta r represent the change in interest rate. So, if your there are 2 components it will affect the interest on interest and also affect the price change in the price. The change in the interest rate will have the impact on the change in price and change in the interest on the interest rate. In this, interest on interest means the reinvestment on the coupons.

So, if the r is increasing, then your interest on interest rate is increasing or price is declining but if r is declining, then your interest on interest rate is declining and price is increasing. So, now the price effect and interest on interest effect, they are moving in the opposite direction whenever there is a change in the interest rate.

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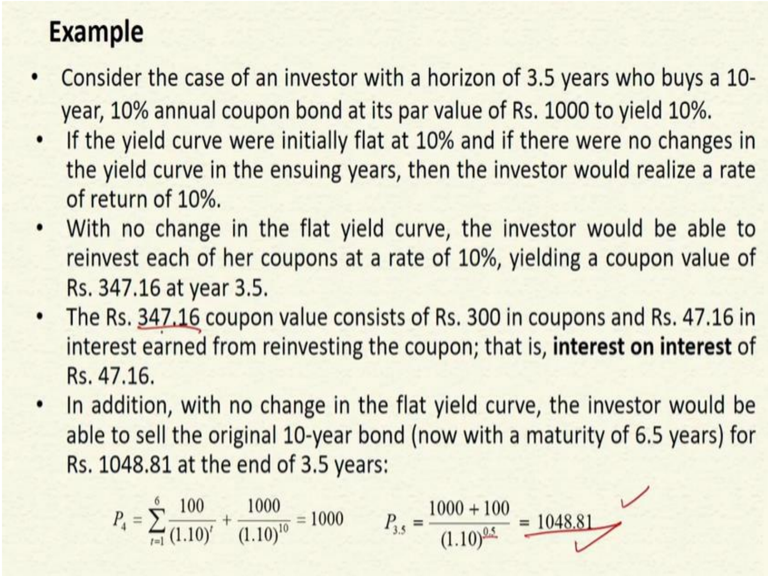

So, let us what happens in that particular context why basically this thing happens in the market? You consider an investor whose horizon period is 3.5 years who buys a 10 years maturity bonds and coupon is 10%. And the par value of the bond is 1000 Rupees which is going to yield 10%. Your expected return from this is the 10%. And you assume that initially the yield curve is flat which is giving 10% return and there are no changes in the yield curve in the forthcoming years.

Then, obviously what will happen? That 10% will be automatically realized because the interest rate is not going to be changed. So, if no change is happening then what will happen? That the investor will be getting the coupons for the 3.5 years at a rate of 10% and the total value of this particular coupon will be 347.16 at the year 3.5 and how we got it? They will got 300 rupees coupon and your 47.16 is the interest which is earned from the reinvesting the coupon.

Annual coupons that 100 rupees each for 3 years he got, and this 300 rupees has been reinvested in the market. Then, he got a interest of 47.16 then the interest on interest basically the reinvestment return amount what they got from the coupon that is 47.16 and the coupon is 300 then total 347.16 they got. Now the investor have to sell the bond because the horizon period is 3.5 years and after 3.5 years he has to sell the bond, then we have to find out the price in which he has sold the bond.

So, the investor will be able to sell the original 10 years bond now the maturity left out that is 6.5 years. Then at what price you will sell the bond? There are 2 ways you can calculate it but you can also calculate it since the bond is sold in a non-coupon rate. Because coupon rate is annually then what this investor can do? You can determine the price by discounting. Then we can say that the value of the bond at the next coupon date is 4 years.

So, therefore the P4 you can calculate then this P4 will be your summation t = 1 to 6 because the 6 years are remaining that is 100 divided by 1.1 to the power t + 1000 divided by 1.1 to the power 10 is it clear. So that will give you the 1000 rupees generally it is the price of the bond. But P 3.5 will be how much? That will be your only the 6 months are left out to reach this particular next coupon date.

That will be 1000 + 100 divided by 1.1 to the power 0.25 so then you can find out the price at the end of the 3.5 years or you can also discount it because it is not in the coupon date then this is the way you can find out the price of the bond. The price of the bond has come 1048.81. so, at this price the investor has sold the bond. he has bought the bond at a price of 1000 and he has sold the bond at the price of 1048.81. And he got the coupon plus reinvestment of the coupon that is 347.16.

Calculation of selling price of 10-year bond at the end of 3.5 years:

$P_4 = \sum_{t=1}^{6} \frac{100}{(1.10)^t} + \frac{1000}{(1.10)^{10}} = 1000$ $\qquad P_{3.5} = \frac{1000+100}{(1.10)^{0.5}} = 1048.81$

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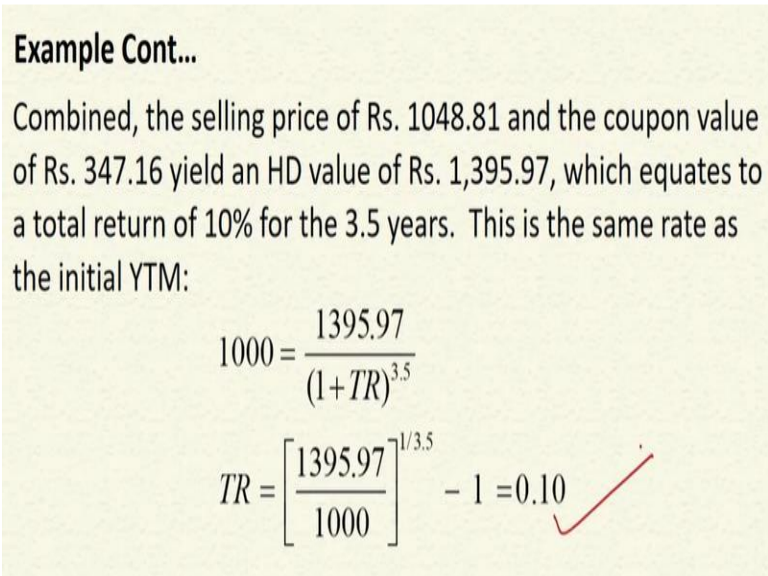

Now what happen you can find out your total return. from this total return is you are this to 1048.81 + 347.16 divided by 1000 to the power 1 by 3.5 -1 that is yours 10%. And why you got 10% total return? because your yield curve has not shifted. the yield curve remains there at 10%. that is why your total return is realized.

Total return = $[\frac{1048.81+347.16}{1000}]^{1/3.5} - 1 = 0.10$

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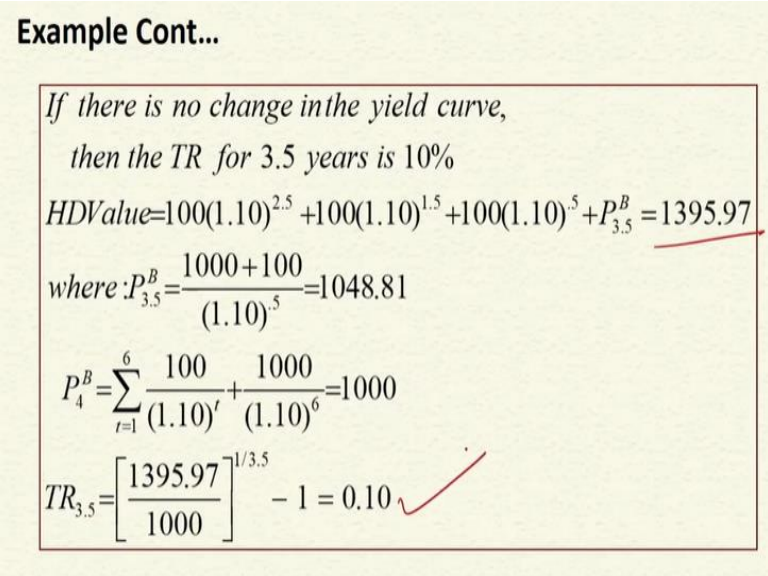

HD value = $100\,(1.10)^{2.5} + 100\,(1.10)^{1.5} + 100\,(1.10)^{0.5} + P_{3.5}^{B} = 1395.97$

$P_{3.5}^{B} = \frac{1000+100}{(1.10)^{0.5}} = 1048.81$

$P_4^B = \sum_{t=1}^{6} \frac{100}{(1.10)^t} + \frac{1000}{(1.10)^6} = 1000$

$TR_{3.5} = [\frac{1395.97}{1000}]^{1/3.5} - 1 = 0.10$

So this is the summary basically has been given so this is the way the horizon value has been calculated and this is your total return basically you are getting.

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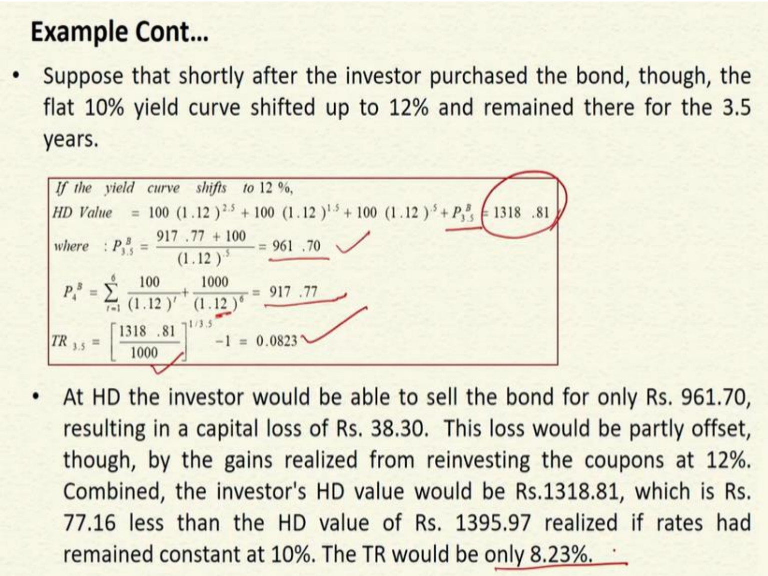

Suppose shortly after investor purchase the bond the flat 10% yield curve shifted to 12% and remained there for the 3.5 years. your horizon period is 3.5 years whenever you have bought the bond the interest rate was 10% but all of sudden it has shifted up to 12%. And remain there are the 3.5 years then what will happen? First of all you find out your horizon value right so 100% coupon you are getting 100 into 1.12 to the power 2.5 because 12% has become the interest rate and you have reinvested it for 2.5 years whatever coupon you have got in the end of the first year.

Then plus 100 into 1.12 to the power 1.5 because that thing you have invested second year coupon you have invested reinvested it 1.4 for 1.5 years that is 100 into 1.12 to the power 1.5 + your 100 into 1.12 to the power 0.5 plus your price of the bond at the period 3.5 years. So what is the price of the bond at the end of the 3.5 years that will be your 961.70 that we have calculated here that is 961.70.

So now your and how about this because price of the bond at the end of the 4 year that is 4 year price that will be 6 years is remaining that is t = 1 to 6 that is 100 divided by 1.12 to the power t + 1000 divided by 1.12 to the power 6 because 12% is the now the discount rate that is got 917.77. So this price will be 917.77 + 100 divided by 1.12 to the power 0.5 that you got 961.7 is it clear?

So now the total return will become so if you add this 961.70 here your P 3.5 B price of the bond at which the investor has sold the bond. Then he got total value has become 1318.81 and you have bought the bond at the price of 1000 which was given in the beginning because the bond basically was that time the yield was 10% and it was issued at par. Then your total return has become 13.181 divided by 1000 to the power 1 by 3.5 that will become 8.23%.

If 10% yield curve shifted upto 12%, then total return at the end of 3.5 years will be:

HD value = $100\,(1.12)^{2.5} + 100\,(1.12)^{1.5} + 100\,(1.12)^{0.5} + P_{3.5}^{B} = 1318.81$

$$P_{3.5}^{B} = \frac{917.77+100}{(1.12)^{0.5}} = 961.70$$

$$P_{4}^{B} = \sum_{t=1}^{6} \frac{100}{(1.12)^{t}} + \frac{1000}{(1.12)^{6}} = 917.77$$

$$TR_{3.5} = \left[\frac{1318.81}{1000}\right]^{1/3.5} - 1 = 0.0823$$

So, in this context what we have seen, there is a capital loss of 38.30 and partially the loss can be offset by the gains what basically you are getting by reinvesting your coupon at the rate of 12%. So combined the investor horizon value has become 1318 which is 77.16 less than the horizon value, what basically we got in the previous case, whenever the yield curve was flat that is 1395.97.

So, whenever the rate was remained constant at 10% your total value was 1395.97. now, the value has become total value has become 1318.81. So, there is a loss in that particular case that is why the total return as come down to 8.23%. Let us take a reverse case here the interest rate has gone off now let the interest rate has gone down to 8%.

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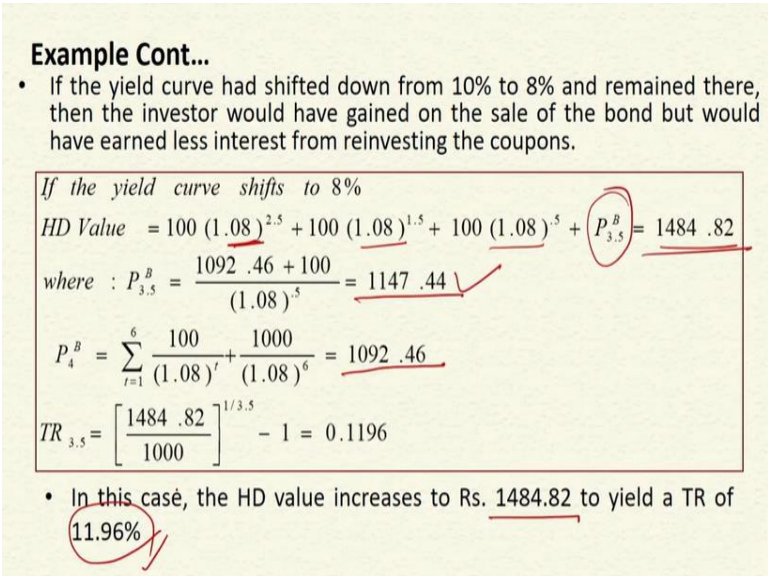

Let the yield curve has shifted down from 10% to 8% and remain there for 3.5 years then what will happen? So, obviously your horizon value will be changed that is 100 into 1.08 to the power 2.5 because 8% has become the interest rate and 100 into 1.08 to the power 1.5 + 100 and 1 positive per 0.5. Then, your price of the bond at 3.5 years that will become 1484.82 because the price of the bond at the end of the 3.5% you got 1147.44.

Same way we have calculated first we calculate the P 4 price of the bond at the end of the 4 that is your t = 1 to 6 100 divided by 1.08 to the power t + 1000 divided by 1.08 to the power 6 that will give you 1092.46. Then here 1092.46 + 100 which is the coupon what you are going to get divide by 1.08 to the power 0.5 that will give 1147.44 the price of the bond bas gone up.

Because the interest rate has gone down so in this case your horizon value has increased from 1484.82 which was 1395 point something in the whenever the yield curve was flat. So that is why the total return has increased from 10% to 11.96%

If 10% yield curve shifted down to 8%, then total return at the end of 3.5 years will be:

HD value = $100\,(1.08)^{2.5} + 100\,(1.08)^{1.5} + 100\,(1.08)^{0.5} + P_{3.5}^{B} = 1484.82$

$P_{3.5}^{B} = \frac{1092.46+100}{(1.08)^{0.5}} = 1147.44$

$P_{4}^{B} = \sum_{t=1}^{6} \frac{100}{(1.08)^{t}} + \frac{1000}{(1.08)^{6}} = 1092.46$

$TR_{3.5} = \left[\frac{1484.82}{1000}\right]^{1/3.5} - 1 = 0.1196$

So, what you have observed here? What is the implication or the findings you got in this particular 2 examples.

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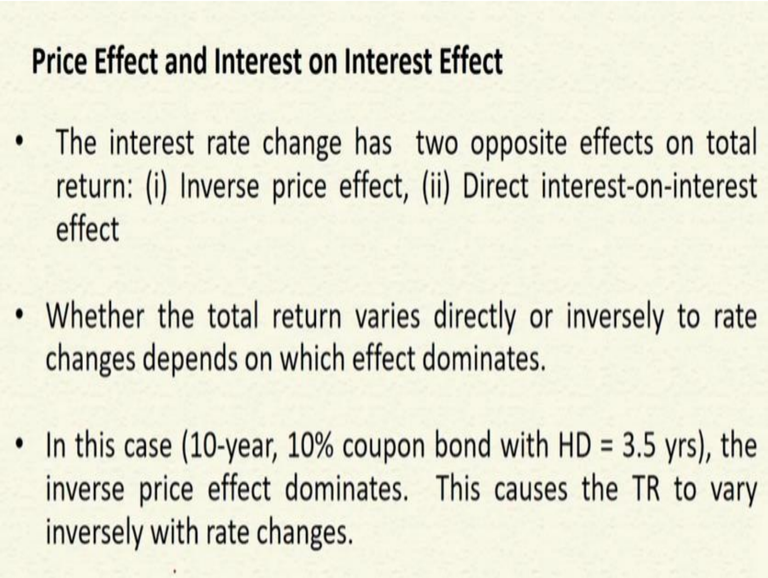

In this example, what basically we have received? that the interest rate change has 2 opposite effects. one is inverse price effect and one is direct interest on interest effect. And the total return varies directly or inversely to the change in the interest rate that will depend on which effect is dominating whether the price effect is dominating or the interest on interest rate effect is dominating. that will decide so, that your total return will be more or total return will be less whatever basically you have expected from the beginning.

So, in this example whatever we have taken the inverse price effect is dominating that is why whenever the interest rate is going up the total return is going down. Whenever the interest rate is going down the total return is going up, so, this basically causes the total return to vary inversely with the change in the interest rate. That is the findings what we got from the example, whatever we have taken. So now, let us see is there any possibility that the interest on interest effect also can dominate the price effect or not. And that is also possible.

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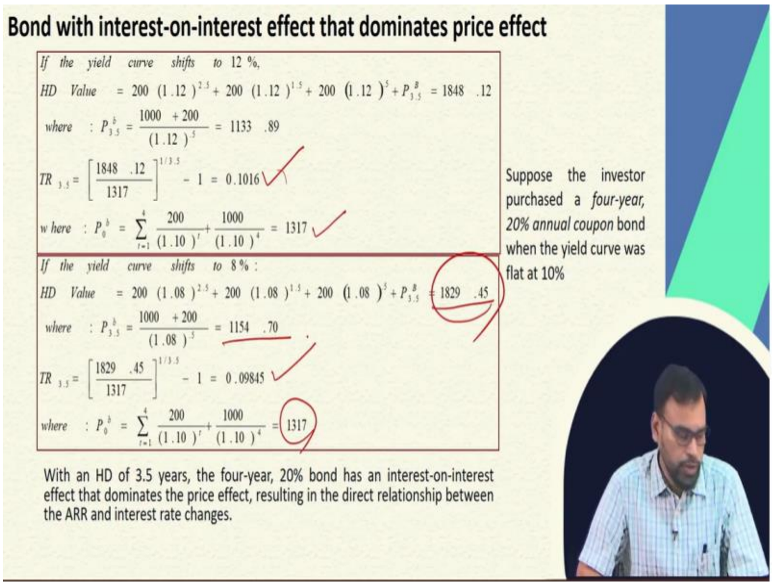

Let us take another example where the interest on interest effect which is dominating the price effect. Let we have taken a bond whose maturity period is 4 years and coupon is 20% and the yield curve was flat at 10%. Let you assume the same thing that the yield curve shift to 12% then you have to find out the price 20% of the coupon the periodic cash flow is 200 per annum. 200 into 1.12 to the power 2.5 + 200 into 1.12 to the power 1.5 + 200 into 1.12 to the power 0.5 plus your price of the bond at the end of the 3.5 years.

That will give you horizon value now what is the price of the bond your price of the bond basically has become this at the end of the 3.5 years that is basically your; what we call it the 1133.89. So your total value of the bond was basically we got it that is your 1848.12 and the price of the bond is basically in the beginning you have bought it that is 1317. So now your total rate has become 1848.12 divided by 1317 to the power 1 by 3.5 -1 that is 10.16%.

If 10% yield curve shifted up to 12%, then total return at the end of 3.5 years will be:

HD value = $200\,(1.12)^{2.5} + 200\,(1.12)^{1.5} + 200\,(1.12)^{0.5} + P_{3.5}^{B} = 1848.12$

$P_{3.5}^{B} = \frac{1000+200}{(1.12)^{0.5}} = 1133.89$

$P_0^{B} = \sum_{t=1}^{4} \frac{200}{(1.10)^t} + \frac{1000}{(1.10)^4} = 1317$

$TR_{3.5} = [\frac{1848.12}{1317}]^{1/3.5} - 1 = 0.1016$

What does it mean? It means the interest rate has gone up but still your return has gone up from 10% to 10.16%. So what it implies? It implies that the interest on interest rate effect is dominating the price effect.

let the interest rate has gone down to 8%. Then the price of the bond at the end of the 3.8 years has become this 1154.70 and the total value at the horizon has become 1829.45 little bit it has gone down because the reinvestments return what you are getting from the coupon investment that is little bit has gone down but the price has increased.

So now the total value got 1828.45 and your price of the bond is 1317 then at this price you have bought the bond then obviously your total return has become 9.845%.
If 10% yield curve shifted down to 8%, then total return at the end of 3.5 years will be:
HD value = $200\,(1.08)^{2.5} + 200\,(1.08)^{1.5} + 200\,(1.08)^{0.5} + P_{3.5}^{B} = 1829.45$

$P_{3.5}^{B} = \frac{1000+200}{(1.08)^{0.5}} = 1154.70$

$P_0^{B} = \sum_{t=1}^{4} \frac{200}{(1.10)^t} + \frac{1000}{(1.10)^4} = 1317$

$TR_{3.5} = [\frac{1829.45}{1317}]^{1/3.5} - 1 = 0.09845$

What you have realized here? If your interest rate will go down in the previous case the return has gone up because the price effect has dominating the reinvestment effect or the interest on interest rate effect.

And in this case what we have observed? Even if the interest rate has come down the return what you got that is lesser than the 10% return. what was there in the beginning? whenever the yield curve was flat at 10%. So clearly in this example we have seen, that the interest on interest rate effect which is dominating the price effect. that is why it will result in the direct relationship between the total return and the interest rate changes.

So that means here you see that is your high coupon rate and term to maturity is also low that is 4 years. So, in that case your interest on interest rate effect generally dominates the price effect. Previous case, we have seen the coupon was 10% term to maturity also 10% so in that case the price effect was dominating the interest on interest effect.

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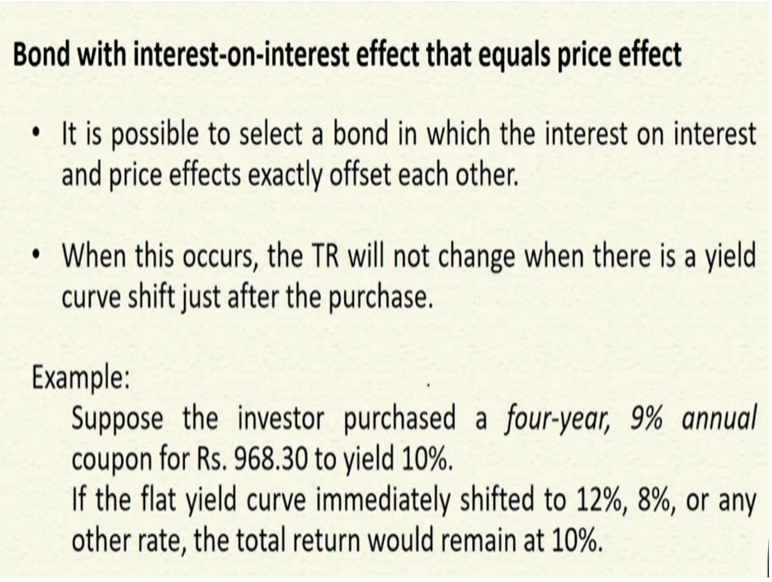

Then let us see, if there is any possibility that interest on interest effect which can equals the price effect. It is also possible to select a bond in which the interest on interest rate price effect exactly offset to each other. We will discuss extensively more about this whenever we go for the bond portfolio strategy that is generally called the immunization strategy. But, I will just give an example here, if the price effect will exactly offset this interest on interest rate effect then the total return will not change.

When there is a yield curve shift just after then purchase even if the interest rate will change the total return will remain same. Suppose, the investor basically purchased a 4 years maturity bond of 9% annual coupon yield is 10% and the price is let 968.30 to assume. So, if the flat yield curve immediately shifted to 12% or 8% or any other rate then the total return will remain at 10%. how it will work?

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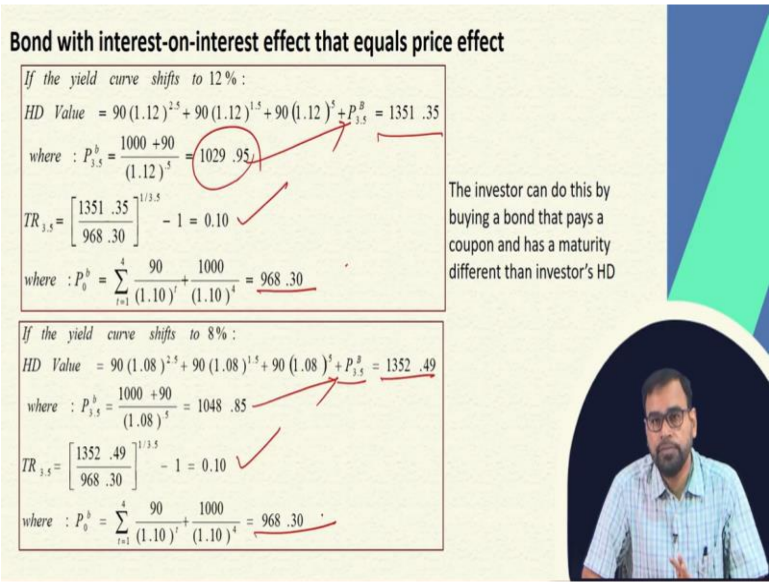

You just see let the interest rate has shifted to 12%. 3.5 years is our horizon period that we are keeping it. coupon is 9% then you got 90 into 1.12 to the power 2.5 + 90 into 1.12 to the power 1.5 + 19 to 1.12 to the power 0.5 plus your price of the bond at the end of the 3.5 years that has become 1351.35. Because the price of the bond at the end of the 3.5 years is 1029.95 that is the price of the bond that basically is here we have added.Price of the bond already we know that is 968.30 is it clear? So now the total return has become this 1351.35 divided 968.30 to the power 1 by 3-5 – 1 that is 10%.

If 10% yield curve shifted up to 12%, then total return at the end of 3.5 years will be:

HD value = $90\,(1.12)^{2.5} + 90\,(1.12)^{1.5} + 90\,(1.12)^{0.5} + P_{3.5}^{B} = 1351.35$

$P_{3.5}^{B} = \frac{1000+90}{(1.12)^{0.5}} = 1029.95$

$P_0^{B} = \sum_{t=1}^{4} \frac{90}{(1.10)^t} + \frac{1000}{(1.10)^4} = 968.30$

$TR_{3.5} = [\frac{1351.35}{968.30}]^{1/3.5} - 1 = 0.10$

Let the yield curve as shifted to 8% then if you go by your understanding or our analysis 90 into 1.80 to the power 2.5 + 90 into 1.08 to the power 1.5 + 90 into 1.08 to the power 0.5 plus the price of the bond which is basically the price that is 1352.49.Then your total return has become 1.352.49 divided by 968.30 to the power 1 by 3.5 that is 10%.

If 10% yield curve shifted down to 8%, then total return at the end of 3.5 years will be:

HD value = $90\,(1.08)^{2.5} + 90\,(1.08)^{1.5} + 90\,(1.08)^{0.5} + P_{3.5}^{B} = 1352.49$

$P_{3.5}^{B} = \frac{1000+90}{(1.08)^{0.5}} = 1048.85$

$P_0^{B} = \sum_{t=1}^{4} \frac{90}{(1.10)^t} + \frac{1000}{(1.10)^4} = 968.30$

$TR_{3.5} = [\frac{1352.49}{968.30}]^{1/3.5} - 1 = 0.10$

Because price of the bond is 968.30. how it is possible? The investor can do this by buying a bond that pays a coupon and as a maturity different than the investors horizon period and up to what particular what should be horizon period the investor should always keep and that should always keep which particular period that also has to be understood.

So, that is basically we call it the bond immunization strategy, where the concept of duration and other things comes into the picture. that we will discuss in the forth coming sessions but that is possible. Price effect can exactly offset this interest on interest effect that possibilities are there.

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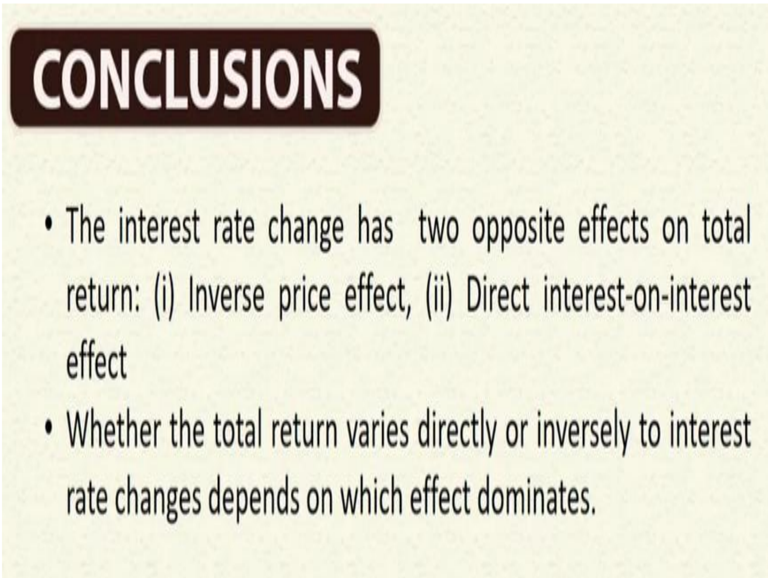

So, what basically we have discussed here? the interest on whenever there is a change in interest rate it has 2 opposite effects on total return, one is inverse price effect, other one is the direct interest on interest effect. And the total return will vary directly or inversely to interest rate changes that depends on that which effect is dominating. The price effect is dominating then the interest rate will increase then the return will go down if the interest on interest rate is dominating then the opposite result will prevail in the market.

So, that is the way basically the concept on interest rate risk can be explained. So, these are the 2 effects always we observe and that is very much important from the investment point of view.

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So, these are the reference what you can go through for the detailed discussion on this and in the future classes, we will be discussing about the other aspects which are related to the bond risk. Thank you.