Management of Fixed Income Securities
Prof. Jitendra Mahakud
Department of Humanities and Social Science
Indian Institute of Technology, Kharagpur

Module No # 04
Lecture No # 20
The Term Structure of Interest Rates – VIII

Welcome back! So, in the previous class we have discussed about the pure expectations theory or we started the discussion on the pure expectations theory. Then, we will continue with that particular discussion and we will learn something about the features of this pure expectations theory. Then, we move forward the other theories of term structure interested like liquidity premium theory and other things.

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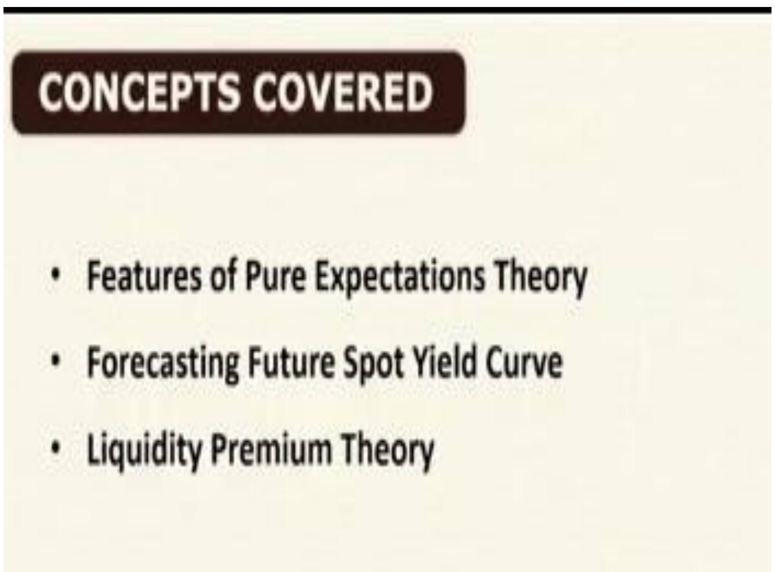

So, in this particular session we will cover up the concepts like the features of the pure expectations theory. Then, how we can forecast the future spot yield curve and as well as the forward yield curve? Then we will discuss about the liquidity premium theory. So, all these concepts basically will be covering up in this particular session.

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So, we will come across certain keywords like your spot yield curve, expected rate of return, the concept called Hedgeable rate, risk premium. All these things basically we will come across. Whenever; we go ahead with this particular discussion with respect to this particular subject or particular session.

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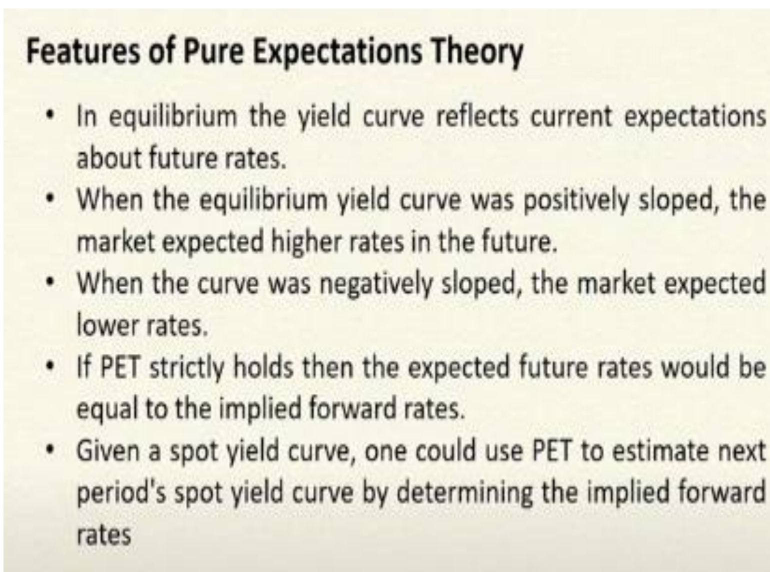

So, let us come to the discussion on the features of the pure expectations theory. So, in the previous class, we discussed about the basic essence of the pure expectations theory. Whenever, the investors and the bond issuers expect something about the movement of the interest rate in the market, then how the yield curve is going to move or how the particular yield curve generally shifts? That part we have discussed.

So, here what basically we have seen from the previous class discussion? That, in equilibrium the yield curve basically reflects the current expectations about the future rates. So, we have introduced the concept called implied forward rate and the expected spot rate. And what we have seen? That, in the equilibrium the implied forward rate should be equal to the expected spot rate. So, what is the implication we have drawn from there when the market is in the equilibrium ,the yield curve is positively sloped?

Or we can say that if in the equilibrium the yield curve is positively sloped, then the market basically expected the higher rates in the future. So, if you observe that, there is a positively yield curve prevailed in the market, then we can say that the expected rates are going to be higher than the current rate. So, when it is negatively sloped, then obviously we can expect that the market basically expected the lower rate.

So, that is the observations what basically we have drawn, whenever we are trying to discuss about the pure expectations theory. So, if the pure expectation theory basically strictly holds, then the expected future rates would be equal to the implied forward rates. That basically just now whatever I told you. So, what basically here we are trying to imply from this particular kind of concept that if you are given a spot yield curve?

Then, investor could use that spot yield curve or they can use this pure expectations theory to estimate the next periods spot yield curve by determining the implied forward rate. So, if they can determine the implied forward rate, then the next period spot yield curve can be determined in the market. So, that is the basic essence of the pure expectations theory or basic feature of the pure expectations theory.

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Forecasting Future Spot Yield Curves (Example)

Maturity	Spot Rates	Expected Spot Rates One year from Present	Expected Spot Rates Two Years from Present
(1)	(2)	(3)	(4)
1	10.0%		
2	10.5%		
3	11.0%		
4	11.5%		
5	12.0%		

This Table shows spot rates on bonds with maturities ranging from one year to five years (Column-2). From these rates, expected spot rates (S_t) can generated for bonds one year from the present (Column-3) and two years from the present (Column-4). The expected spot rates shown are equal to their corresponding implied forward rates.

So, here let us see how this pure expectations theory is helping us to forecast the future spot yield curve? let us take one example. So, here if you observe what basically is given to you one is that there are, let maturity period is given and against this maturity the spot rates are given. And what is our job? our job is to find out the expected spot rate 1year from the present and the expected spot rate 2 years from the present. So, these are the 2 things what basically we have to determine.

So, this table shows the spot, rates on the bonds whose maturity is basically is varying from 1 year to 5 years which is given in the column 2. And from these rates the expected spot rates can be generated for the bonds 1 year from the present which should be we have to fill the table column 3. And the 2 years from the present which has to be reported in the column 4. And the expected spot rates whatever are shown are equal to their current spending implied forward rates.

So, that is the essence of the pure expectations theory whatever we have discussed. Let us see how the expected spot rates one year from the present and the expected spot rates the two year from the present can be estimated by using this particular data on the basis of the pure expectations theory?

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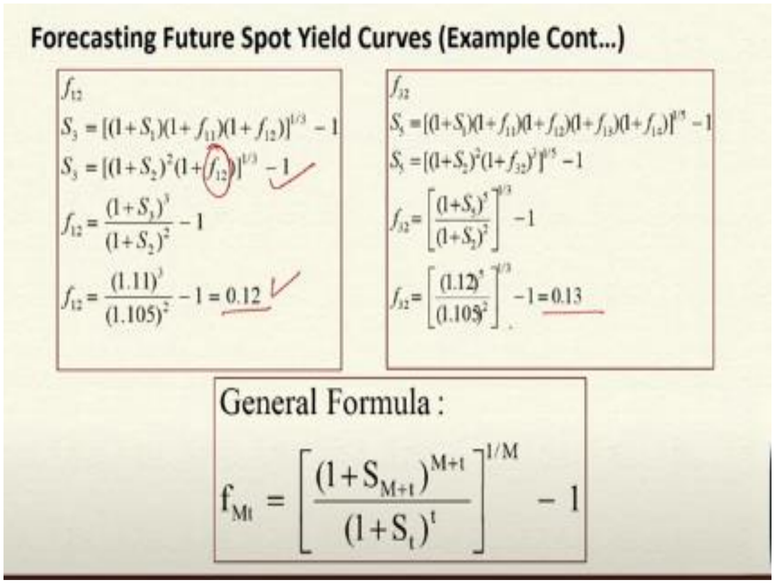

So, if you see the rates are given to you and already, we have discussed this particular formula, whenever we started the discussion on the pure expectations theory. So, here if you see that for example, we are calculating f12. So, here the f12 if you are calculating. So, in the concept wise as per the pure expectations theory whatever we have seen year s3 which is the spot rate for the period 3.

Should be 1 + s1 into 1 + f 11 + into 1 + f 12 to the power 1 by 3 – 1. So then year s 3 will be 1 + S2 whole square into 1 + f 12 to the power 1 by 3 - 1 so this part. So now from here our job is to find out your f 1 2 then your f 12 will become 1 + s 3 to the power 3 divided by 1 + S2 to the power 2 we are deriving this from this particular equation that is – 1. 1 + S3 to the; power q divided by 1 + S2 to the power 2 or the square – 1. Then, in the previous table, it has been given year s3 is equal to 11% right this spot rate or the maturity period 3 that is 11%, then it is 1.11 to the power 3 divided by and year S2 is 10.5% then 1.105 to the power 2 - 1 then we got the 0.12 that means 12%.

Calculation of f_{12}: -

$S_3=[(1+S_1)(1+f_{11})(1+f_{12})]^{1/3}-1$

$S_3=[(1+S_2)^2(1+f_{12})]^{1/3}-1$

$$f_{12} = \frac{(1+s_3)^3}{(1+s_2)^2} - 1$$

$$f_{12} = \frac{(1.11)^3}{(1.105)^2} - 1 = 0.12$$

Like that, if you are trying to calculate the f 32 then it will be 1 + S1 into 1 + f 11 into 1 + f 12 into 1 + f 13 into 1 + f 14 to the power 1 by 5 – 1.So, if you are going to solve this then ultimately you will find your f 32 is equal to your 1 + S5 to the power 5 divided by 1 + S2 to the power 2 whole to the power 1 by 3 - 1. So, your S5 is 12 % which was given in the previous table and year S2 = 10.5%. then using putting those particular values, you got your f 32 = 0.13.

Calculation of f_{32}: -

$S_5=[(1+S_1)(1+f_{11})(1+f_{12})(1+f_{13})(1+f_{14})]^{1/5}-1$

$S_5=[(1+S_2)^2(1+f_{32})]^{1/5}-1$

$F_{32} = \frac{(1+s_5)^5}{(1+s_2)^2} - 1$

$F_{32} = \frac{(1.12)^5}{(1.105)^2} - 1 = 0.13$

So, from this we can derive a general formula then your f M t is equal to your 1 + S M + t to the power M + t divided by 1 + S t to the power t whole to the power 1 by M - 1.

General Formula:

$$F_{Mt} = [\frac{(1+s_{M+t})^{M+t}}{1+s_t{}^t}]^{1/M} - 1$$

So, this formula has been derived by using this concept of the pure expectations theory. So now you can calculate the all those kinds of rates whatever was blank in the previous table. So, here we have given the 2 cases. you can also use the other you can use this formula to calculate the other rates which has to be filled out in that particular table.

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Forecasting Future Spot Yield Curves (Example Cont...)

Maturity	Spot Rates	Expected Spot Rates One year from Present	Expected Spot Rates Two Years from Present
(1)	(2)	(3)	(4)
1	10.0%	f_{11} = 11.0%	f_{12} = 12.0%
2	10.5%	f_{21} = 11.5%	f_{22} = 12.5%
3	11.0%	f_{31} = 12.0%	f_{32} = 13.0%
4	11.5%	f_{41} = 12.5%	
5	12.0%		

So, now your spot rate was given so now whatever we have calculated your f 11 we got 11.1 11% and f 32 we got 13%. So, like that you can calculate f 21, you can calculate f 31, your f 41, f 12,

f 22 like that. So, like that you can use that particular formula and derive all those expected spot rates 1 year from present and the expected spot rate 2 year from the present that actually you can estimate using that particular formula.

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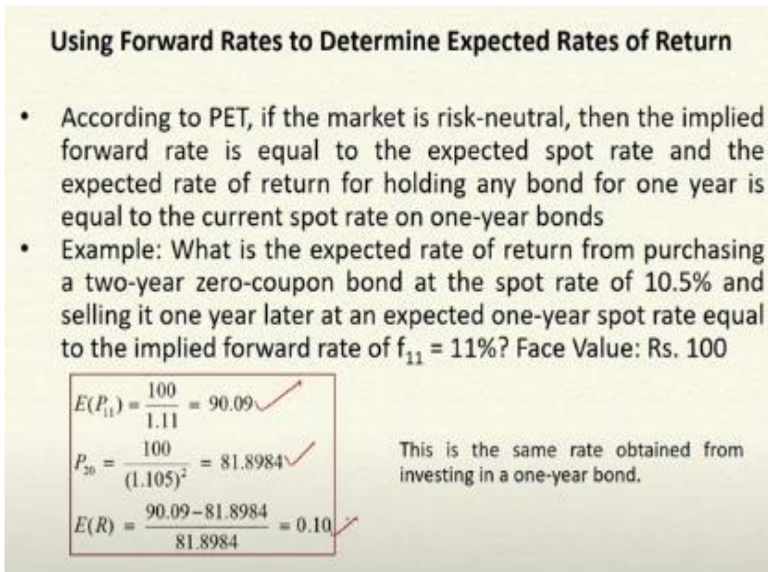

Now, we see that how to determine this expected rates of return using this forward rates? Right? So, according to this pure expectations theory, if you assume that the market is risk neutral we know that, the implied forward rate is equal to the expected spot rate and the expected rate of return for holding any bonds for one year is equal to the current spot rate on one year bonds. So, let us take one example. what is the expected rate of return from purchasing let a 2-year zero coupon bond at the spot rate of 10.5% which was given to you year s 2 is 10.5%?

And selling it one year later at an expected one-year spot rate equal to the implied forward rate that is f 11, that is your 11% and you take the phase value of 100 rupees right. So, your expected price P 11 will be how much? your forward rate is 11 %. so then if you are using that one for discounting then the value will be 90.09 right. Then, what will be the price on that particular day? you have to use the discount rate for the spot rate for the period 2 so that means your price will be 100 divided by 1.105 to the power 2.

$E(P_{11}) = (100/1.11) = 90.09$

P_{20}= 100/(1.105)^2= 81.8984

Because basically it is a 2 year bond zero coupon bond your face value divided by 1 + r to the power 2 two year zero coupon bond then and the beginning the price is p 20 the price is basically 81.8984. Then how much expected return you got your 90.90 – 81.8984 divide by 81.8984 that will be 10%. So this is the same rate which basically is obtained from investing in a 1 year bond right.

Expected rate of return will be= (90.09-81.8984)/81.8984= 0.10

So, that is what basically what we are trying to justify here, that how the expected spot rate basically what we are trying to measure it is equal to the forward rate as per the pure expectations theory. And you will get the same return what was given for holding a one-year bond, if you are using this particular implied forward rate for your calculation.

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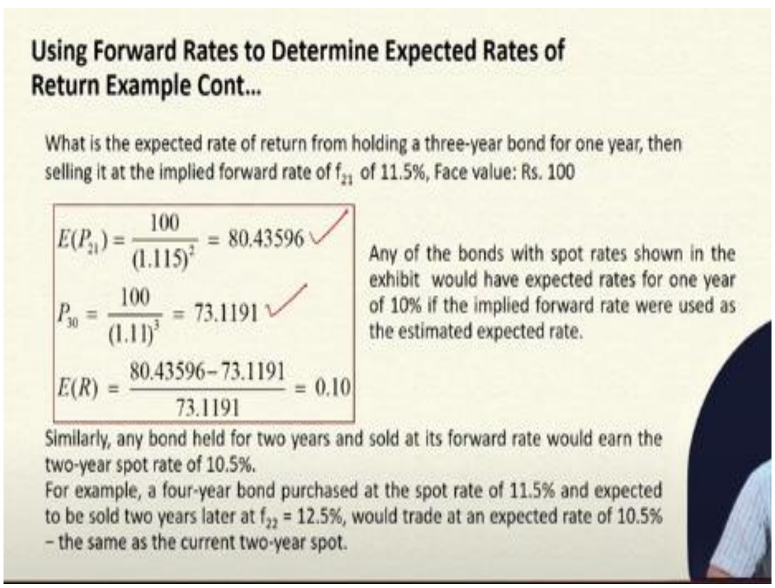

Now, if same way you want to calculate rate, what is the expected return rate of return from holding a 3 year bond from one year? Then selling it at the implied forward rate of f 21 right of 11.5% what already we have calculated. Then, you take the phase value of 100, then same logic you can apply your 100 divided by 1.115 square that will give you the expected price on that particular day. Then, initial price that is p 30 that is 100 divided by 1.11, which is the S3.Right? 3 years zero coupon bond then that will give you 73.1191. then what is the return you got that 80.43596 - 73.1191 divided by 73.1191 = 10%.

$E(P_{21})= (100/1.115)^2=80.43596$

$P_{30}= 100/(1.11)^3= 73.1191$

Expected rate of return will be= (80.43596-73.1191)/73.1191= 0.10

That means, any of the bonds with spot rate shown in this particular exhibit or the table would have expected rates for one year of 10%. If the implied forward rate is used as the estimated expected rate, then you will get the same return expected your one-year return will be same.

Similarly for other things any bond held for 2 years and sold at its forward rate will earn the 2-year spot rate; that is 10.5%. If you see the example, let a 4 year bond persist at the spot rate of 11.5% and expected to be sold 2 years later at f 22, what we have calculated that is 12.5%, then obviously it would trade at an expected rate of 10.5% which is the same as the current two year spot rate. So, the forward rates can be used to determine the expected rates of this particular bonds so that is the way basically it is estimated.

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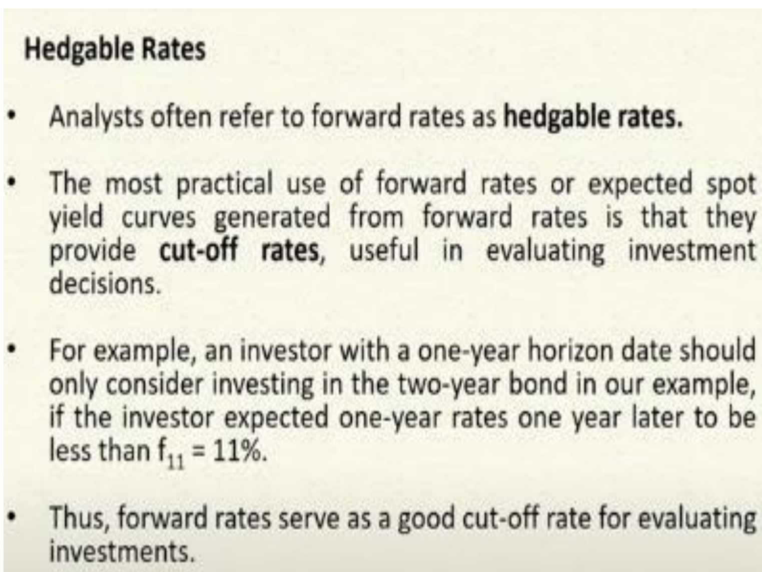

So, this particular rate whatever basically we have just now used, the analyst many a time refer this forward rate, as the hedgable rates. these particular rates are called as the hedgable rates. The most practical use of the forward rates are the expected spot rate yield curves, basically generated from the forward rate is that they provide the cut off rates. And these rates are generally very much important for the investment decisions. How for example, if an investor has a one year horizon, then they should only consider investing in the 2 year bond is horizon period is 1 year.

But, they can consider also a 2 year bond if the investor expects 1 year rates one year later to be less than your forward rate which is 11%. Even if his horizon period is 1 year, they can go for the 2-year bond if this condition will prevail. So that means, this forward rate has been used as a cut off rate so accordingly he or she can take the decisions. So, therefore the forward rates basically always serve as a good cut off rate for evaluation of the investments in the market that actually you have to keep in the mind.

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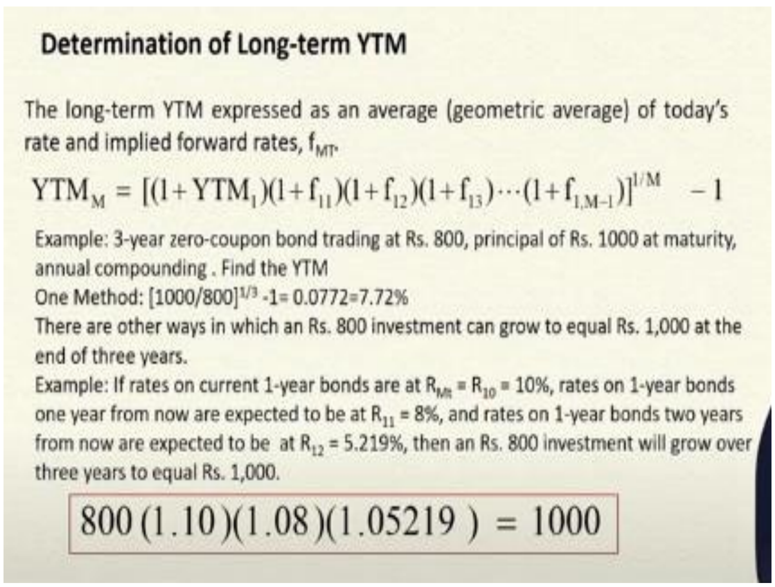

So, in this context the basic essence of the pure expectations theory, if you see we can calculate the long term yield to maturity by using the current spot rate and the implied forward rates. So, the long term YTM generally is nothing but a geometric mean of today's rate and the implied forward rates. So, therefore you can write a formula that YTM = 1 + YTM 1 which is the current spot rate into 1 + f 11 into 1 + f 12 into 1 + f 13 so on up to 1 + f 1 m - 1 to the power whole to the power 1 1 by m – 1.

The long-term YTM expressed as a geometric average of today's rate and implied forward rates, f_{MT}:

$YTM_M = [(1+YTM_1)(1+f_{11})(1+f_{12})(1+f_{13}) \ldots\ldots (1+f_{1,M-1})]^{1/M} - 1$

Take an example let a 3-year zero coupon bond which is trading at 800 rupees, which will give you 1000 rupees which is the principal at the maturity and it is annually compounded. So, I will look at the YTM straight forward, you can say that the particular rate which can give you 1000 rupees if you are investing 800 rupees today. it is simply one 1000 divided by 800 to the power 1 by 3 - 1 that is 7.72 %.

YTM= $[1000/800]^{1/3} – 1$= 7.72%

There are other ways also these 800 rupees can grow to 1000 rupees at the end of the three years.how? How basically it can happen?If the rates on current one year bond rate is 10 % rates on one year bond 1 year from now are expected to be let 8% and rates on one year bond 2 years from now that is r 12 that is expected to be 5.219%. Then your 800 rupees investment also can grow to 1000 in the 3 years time because the 800 into 1.1 into 1.08 into 1.05219 that will give you 1000 rupees so that if this 8 % is f 1 1 this 5.219% is f 1 2 and so on.

800(1.10) (1.08) (1.05219) = 1000

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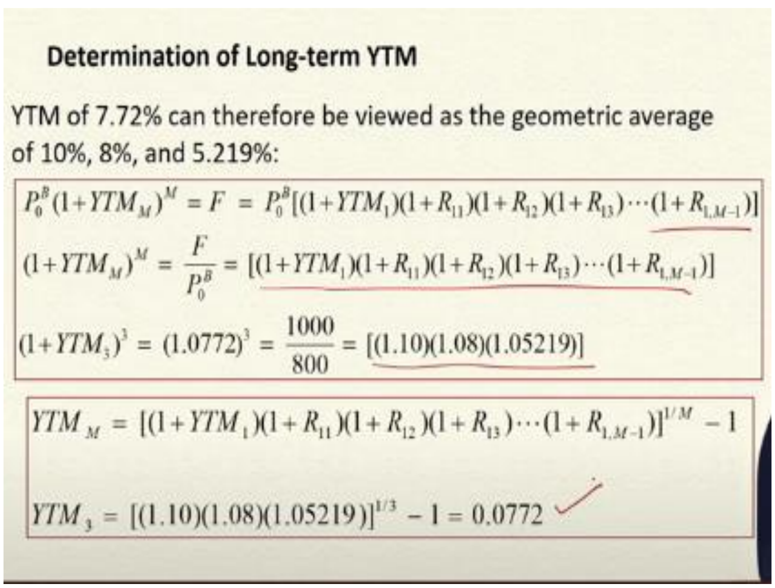

So here this YTM of 7.72% can be considered or can be viewed as a geometric average of the 10%, 5% and 5.219% is it. So you can derive this in this way your price into 1 + YTM m to the power m is equal to your phase future value of this particular bond which is 1000 rupees. That is nothing but your price of the bond into 1 + YTM 1 into 1 + r 1 1 + into 1 + 1 12 + 1 + r 13 up to your 1 + r 1 m – 1.

Then if you take this one into this side then it will be f by p 0 that will be equal to this then now your 1 + YTM 3 here in our example to the power 3 is equal to your 1.0772 your 100 by 800 right is equal to your 1.10 into 1.08 into 1.05219. So finally what; basically we have seen your YTM 3 is equal to your 1.10 into 1.08 into 1.05219 to the power 1 by 3 - 1 that is (()) (22:56). So that means the long term interest rate is a geometric mean of the current spot rate and the implied forward rates so that is the basically thing what basically you have to keep in the mind.

YTM of 7.72% can therefore be viewed as the geometric average of 10%,8% and 5.219%:

$P_0^B(1+YTM_M)^M = F = P_0^B[[(1+YTM_1)(1+R_{11})(1+R_{12})(1+R_{13}) \ldots\ldots (1+R_{1,M-1})]$

$(1+YTM_M)^M = \frac{F}{P_0{}^B} = [(1+YTM_1)(1+R_{11})(1+R_{12})(1+R_{13}) \ldots\ldots (1+R_{1,M-1})]$

$(1+YTM_3)^3 = (1.0772)^3 = \frac{1000}{800} = [(1.10)(1.08)(1.05219)]$

$YTM_M = [(1+YTM_1)(1+R_{11})(1+R_{12})(1+R_{13}) \ldots\ldots (1+R_{1,M-1})]^{1/M} - 1$

$YTM_3 = [(1.10)(1.08)(1.05219)]^{1/3} - 1 = 0.0772$

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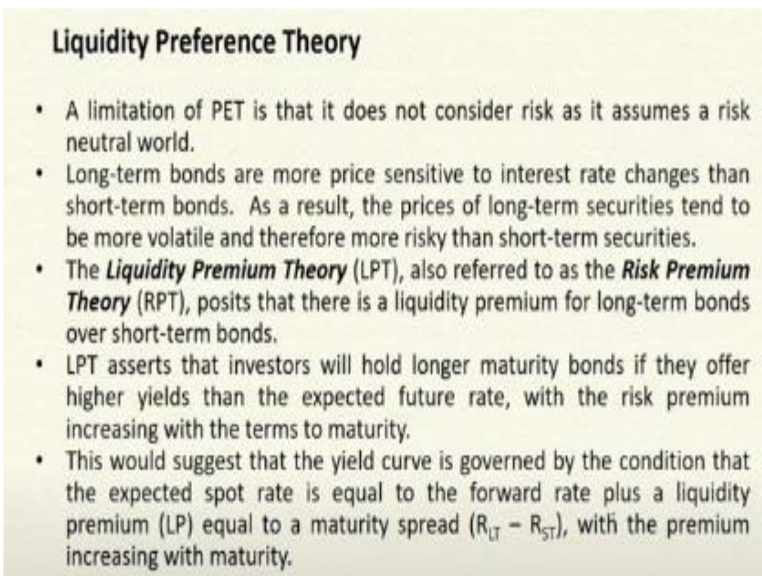

Then, the other theory is basically your liquidity preference theory and why basically the liquidity preference theory was emerged? Because, whenever we have explained this pure expectations theory it always assumes that, there is a risk neutral world**.** So, that is why we can consider that is the limitation of the pure expectations theory because it does not consider the risk.

And where the risk comes? because all of you know that investing in a long-term bond is riskier than the investment in a short-term bond. that all of us know, because the interest rate is very much uncertain. And also, we have explained in the previous sessions that the long-term bonds are more price sensitive to interest rate changes than the certain bonds. So, if anybody is holding a long-term bond he is always exposed to more risk in comparison to the person or investor who is holding a short-term bond.

That is why the price of the long-term securities or long-term bonds always tends to be more volatile and therefore more risky, the price volatility is more for the long-term bonds. So, that is why there is an emergence of the liquidity preference theory which is also popularly called as the risk premium theory. So, according to this liquidity premium theory always that should be premium involved in the long-term bonds.

That means, definitely if anybody is investing in the long-term bond that long term bonds should give more return than the short-term bonds because there is a risk premium, there is a liquidity premium that is basically involved in that particular case. But, the investor also invest in the long term bond, investor also invest in the short term bond. then when? in what condition the investor will go and invest in the long term bond?

The investor will go for a long-term bond obviously if the long-term bonds will give higher rates than the expected future rate. So, what we have seen? that the expected future rate should be implied forward rate as per the pure expectations theory. But as for the liquidity premium theory, they said that if anybody is investing in the long term bonds then, whatever calculations we are doing on the basis of the current spot rate, there we have to add the premium.

So, the risk premium concept if you are bringing into the considerations then this particular premium should be increasing with the term to maturity. So, higher the bonds term to maturity more the return that basically we should expect. So, what basically then this theory suggests? the yield curve is governed by the condition that the expected spot rate is equal to the forward rate plus a liquidity premium.

Previous case what we have seen? that expected spot rate is equal to the forward rate but as per the liquidity premium theory the expected spot rate should be equal to the forward rate plus the liquidity premium which is basically nothing. But the maturity spread what we call it that is your return from the long-term bond, the return from the short-term bond. And if you assume that investment in the longer period is riskier than the investment in the short period or the investment in the short-term bond then the premium should be increasing with the maturity.

So, therefore as per the liquidity preference theory or liquidity premium theory what basically we can call it always the long-term bonds should give more return than the short-term bonds because there is a risk involved. So, the limitations what basically we have seen whenever we have discussed about the pure expectations theory. there we have seen there is a risk neutral world. But you assume that the risk is also playing the role and the investor who is investing in the long-term assets.

This will be compensated against the extra risk what they are taking whenever they are holding the long-term bonds or they are trying to invest in the long-term bond. So, that particular extra return what they are expecting that basically we call it the liquidity premium and that premium is basically increasing with the maturity. So, that is the thing basically what basically you have to keep in the mind.

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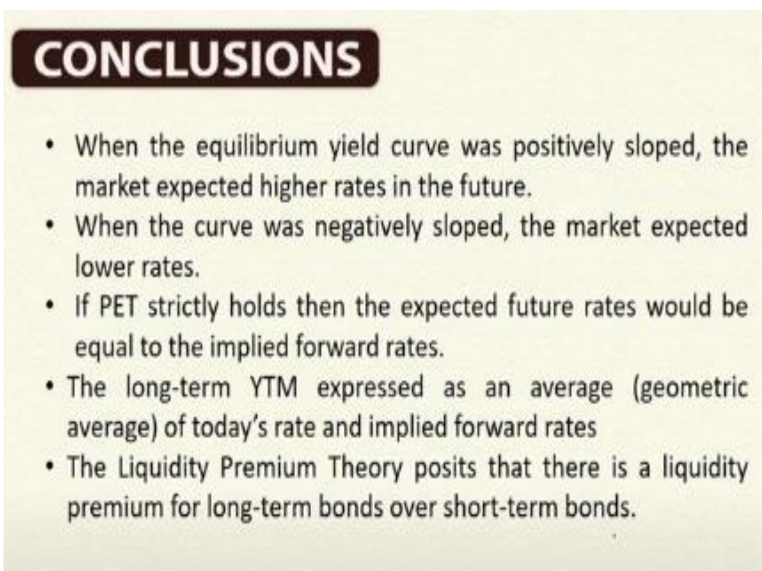

Then what basically we have discussed in today's class? that when the equilibrium yield curve was positively sloped the market always, expect higher rates in the future. Well then, it is negatively sloped then the market, basically expect a lower rates in the future. Then, if the pure expectations theory holds, then the expected future rate should be equal to the implied forward rates.

And the as for the pure expectations theory, the yield to maturity can be expressed as a geometric mean of or the average of the today's rate and the implied forward rate. And the implied forward rate is equal to the expected spot rate, as per the pure expectations theory. But whenever, we have brought the liquidity premium theory that basically tells that there is a liquidity premium for the long-term bonds over the short bonds.

So, in view of this long term bonds always give more return than the short term bonds to compensate that particular extra risk what the investor takes. So, we can also say that the liquidity premium theory is an extension of the pure expectations theory. So, you can assume your expected spot rate you can determine your expected spot rate or you can calculate your implied forward rate using the pure expectations theory.

But, we have to add that risk premium into that to find out the long term interest rate. that is what basically always you have to keep in the mind. So, these are the different theories which, basically tries to explain the term structure of the interest rates. Why there is a difference between the long-term rate and this short term rate and as well as the shift of the yield curve in this particular way can be determined.

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So, these are the references you can go through for the detailed discussion on this. Thank you.