Management of Fixed Income Securities
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Module No # 04
Lecture No # 18
The Term Structure of Interest Rates-VI

Welcome back! So, in the previous class we have discussed about the preferred habitat theory, which is necessarily an extension of the market segmentation theory.

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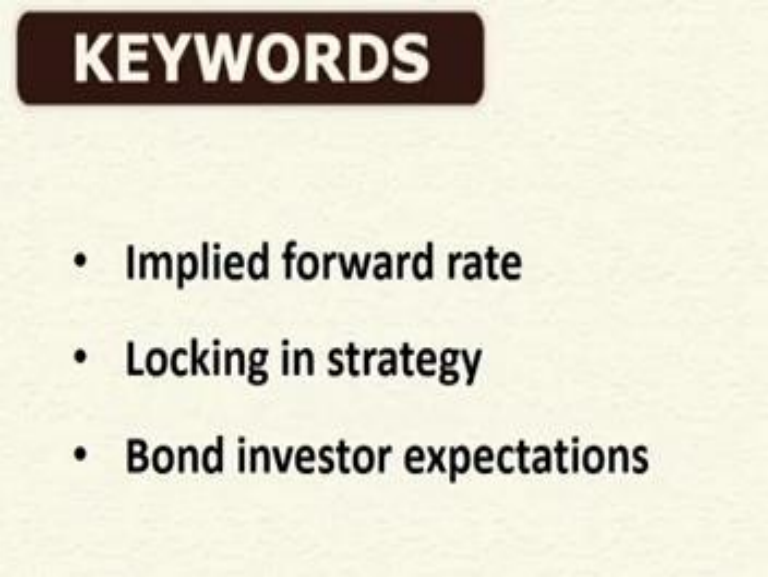

In today's lesson, we will be discussing another popular theory what we call the pure expectations theory. That means, how the expectations of the investors and the issuers about the future interest are going to change the yield curve or decide the shape of the yield curve or the slope of the yield curve. So, that is the basic essence of the pure expectations theory. If you are expecting the interest rate is going to be up, what the shape of the yield curve will look like?

If you are expecting that the interest rate is going to down, then what should be the shape of the yield curve? If you assume that the interest rate is not or you are expecting that the interest rate is not going to change, then what will be the shape of the yield curve? So, basically, the basic objective of the particular session or what in a general sense we call the pure expectations theory. In this case, there are certain things we will discuss; the concept of the implied forward rate, concept of locking strategy and the bond investors expectations.

In the beginning, I am just telling you that in this particular session we will discuss about, how the expectations of the investor are going to affect the yield curve? Or what is the response of the investor expectations about the interest rate changes are going to affect the yield curve? That is the basic essence. we will not consider the expectation of the issuers. That we will discuss in the future sessions. But in today's class, we will be discussing only the expectation of the investors that actually you can keep in the mind.

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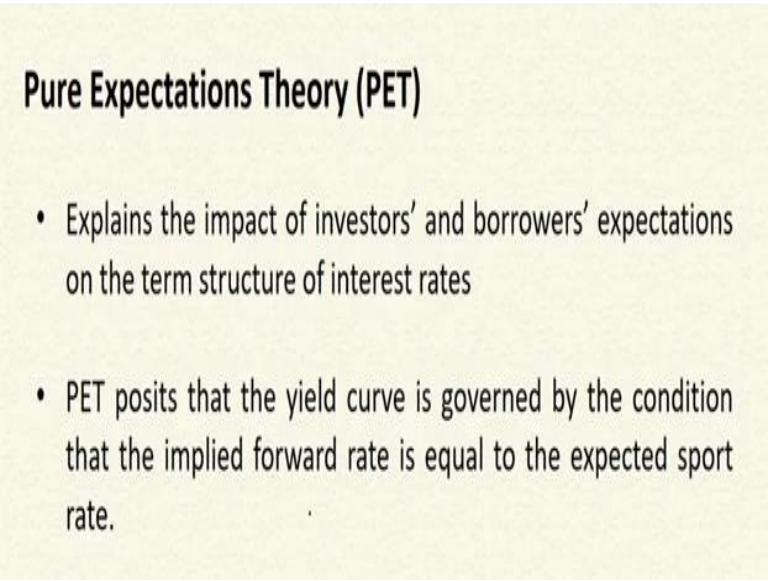

So, what is pure expectations theory basically talking about? The pure expectations theory basically explains the impact of the investor and borrower expectations on the term structure of interest rate or the shape of the yield curve. How the yield curve shape is going to be changed if the expectations of the investors or the issuers are going to be changed? That is the basic essence of the pure expectations theory and as per the definition; the pure expectations theory basically tells that the yield curve is governed by the condition that the implied forward rate is equal to the expected spot rate.

There are 2 things you keep in the mind. one is implied forward rates and another one is the expected spot rate. So, whatever spot rate is going to happen after 2 years that basically, we have to decide today and for today that particular rate is basically called the forward rate, and that is implied upon something, and what is that something? That something is the spot rate of the

today. The forward rate is what basically we are going to calculate on the basis of the spot rate that is called the implied forward rate.

So, on the basis of the pure expectations theory, we assume that the implied forward rate whatever, we are calculating today or we are deciding today should be equal to the expected spot rate.

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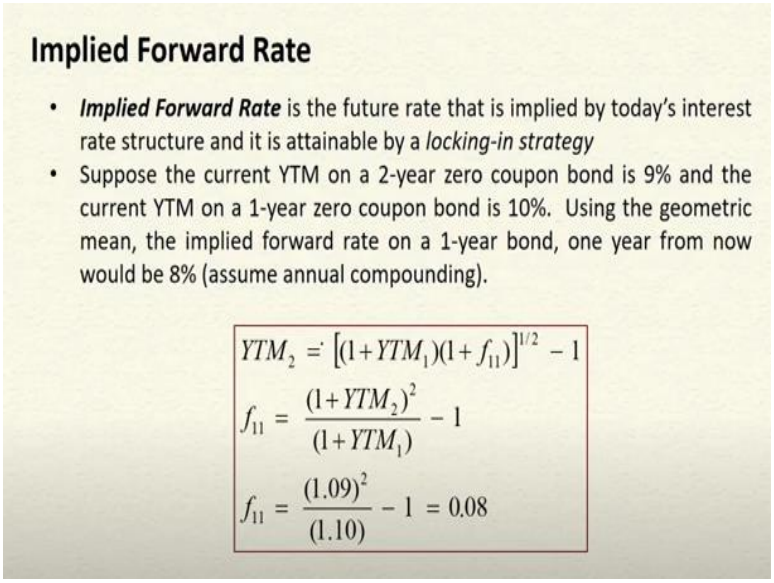

So let us see, what is the implied forward rate? So, whenever you talk about the implied forward rate, already what I told you? Implied forward rate is the future rate which is implied by today's interest rate structure. That is why I told you that its implied upon today's spot rate, today's interest rate. So, that is why we use this word implied. It is conditioned upon today's spot rate. Let us see, how it is basically attained? We can explain it using a concept called the locking-in strategy.

What is this locking-in strategy? You assume the current yield to maturity on a 2 year zero coupon bond is 9% and the current yield to maturity of a 1 year zero coupon bond is 10%. Because already we have discussed that this spot rate is nothing but zero coupon rates. That was already in the previous sessions we have discussed. So, that is why the zero coupon rates are considered as these spot rates. So, here a 2-year zero coupon bond the maturity period is 2 years, that yield is 9% and the 1-year zero coupon rate let you assume that, is 10%.

So, if you go for an annual compounding and use this geometric mean concept, all of you know what is geometric mean and arithmetic mean. So, using the geometric mean; the implied forward rate on a 1 year bond. One year from now will be how much? So, your spot rate for the 1 year zero coupon bond and the spot rate of the 2 year zero coupon bond that data is available. So, you are calculating the one year from now, what will be the implied forward rate?

So, what is the formula we can use that your YTM 2 is equal to your 1 + YTM 1into 1 + the forward rate which is 1 year from now, you are calculating the power 1 by 2-2? So, that means your f1 is equal to your 1 + YTM 2 square divided by 1 + YTM 1 - 1. Then what is f 11 + f 11 = 1.09 square divided by 1.10, because 9% is the 2-year spot rate and 10% is the 1 your spot rate - 1, that will give you 8%, right? But, what does it basically imply? That basically explains this locking-in strategy.

Implied forward rate $= \frac{(1+YTM_2)^2}{(1+YTM_1)} - 1$

$= \frac{(1.09)^2}{(1.10)} - 1 = 0.08$

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Locking in Strategy

- Execute a short sale by borrowing the one-year bond and selling it at its market price of Rs. 909.09 = Rs. 1,000/1.10 (or borrow Rs. 909.09 at 10%).
- With two-year bonds trading at Rs. 841.68 = Rs. $1,000/(1.09)^2$, buy Rs. 909.09/Rs. 841.68 = 1.08 issues of the two year bond.
- One year later, cover the short sale by paying the holder of the one-year bond his principal of Rs. 1,000 (or repay loan).
- Two years later receive the principal on the maturing two-year bond issues of (1.08)(Rs. 1,000) = Rs. 1080.

You use these rates and try to explain this. Let, what the investor can do go for a short sell by borrowing 1-year bond and selling it at a market price of 909.09, how we have arrived at this 909.09? That is your 1000 divided by 1.10 or you can also borrow 909.09 rupees at the rate of 10%, which is basically your one-year rate.

The market price of the one-year bond= 1000/1.10= 909.09

The 2 year bond will be trading at that particular time with a market price of 841.68, how we got this go for the valuation of zero coupon bond that is your let you assume that face value is 1000. 1000 divided by 1.09 square that will give you 841.68.

The market price of the two-year bond= $1000/(1.09)^2$= 841.68

So, what you can do with 1.08 issues of the 2 year bond? That is your 909 whatever you have borrowed 909.09 divided by 841.68, which means 1.08 issues of the 2 year bond. You can get it from now.

No.of issues of the two year bond= 909.09/841.68= 1.08 issues

1 year later, what you can do? Cover the short cell or pay the loan by paying the holder of the 1-year bond, which is your 1000 and 2 years later, because you are holding 1.08 issues. 2 years later receive the principle on the maturing this 2-year bond that is 1.08 into 1000 that is 1080. Basically you can do you can get some profit out of this.

Principle on the maturing two-year bond issues of (1.08) (1000)= Rs 1080.

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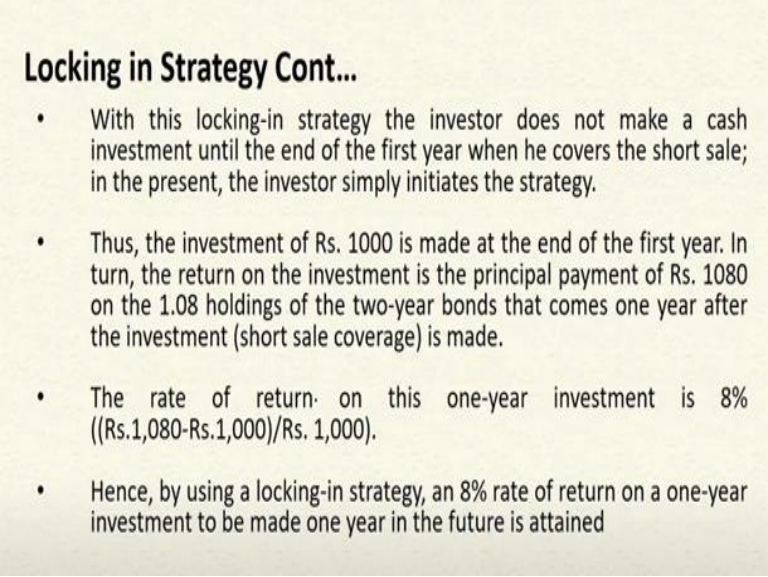

Now, with this locking-in strategy, you see the investors or investor does not make any kind of cash transactions until the end of the first year because he has gone for short selling. When he covers the short cell, then only he has to go for these cash transactions. So, when he covers the

short cell, he needs the money to repay that particular money. So, in the present, the investor simply initiates the strategy in the beginning.

So, the investment of 1000 is made at the end of the first year, and in turn, the return on investment is the principal payment of 1080 which is 1.08 holdings of the 2 year bonds, which comes one year after the investment, that means your short sale coverage is made. So, the return on the investment or the return on rate of return on this 1 year investment is what your 1080 - 1000 divided by 1000 that will give you 8%.

Return on the investment= (1080-1000)/1000=8%

So, if the particular individual is going to use this locking in strategy, then the 8% rate of return on 1-year investment to be made 1 year in the future can be attained. That is the way he got the rate of return of 8%. That concept is called the locking in strategy; through that basically we have observed that your implied forward rate was 8%. Now, he is getting a return of 8% whenever we are going to use this kind of strategy in the market.

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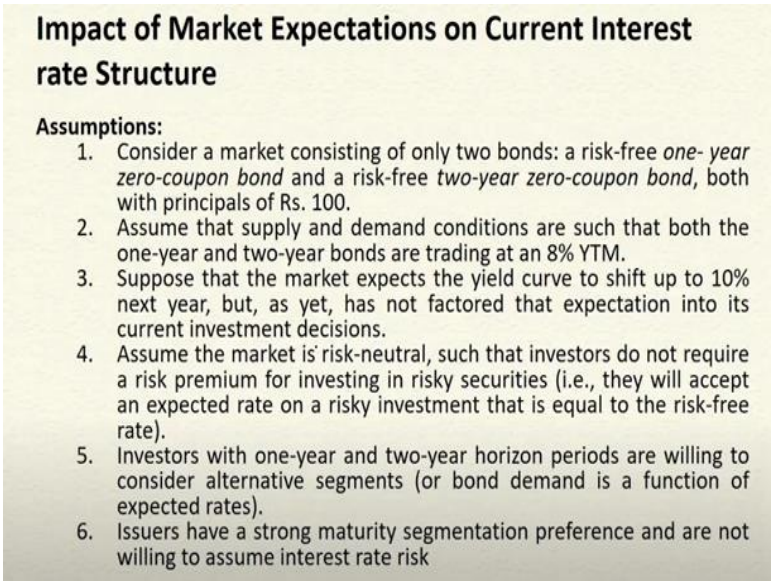

Now, we will come back to our pure expectations theory which basically we have said, that the implied forward rate should be equal to the expected spot rate. So, to explain that there are certain assumptions have been taken. Let you consider a market consisting of only 2 bonds. A risk-free 1-year zero coupon bond and a risk-free 2-year zero coupon bond .let the principal of the 100 rupees.

And supply and demand conditions are such that, both the 1 year and 2-year bonds are trading at 8% yield to maturity. Suppose, the market is expecting the yield curve is going to shift to 10% in the next year. But, this thing is not factored into the current investment decisions now and the market is risk neutral. That means the investors do not require a risk premium in investing in risky securities.

That is the assumption of whatever we are taking? That means, they will accept an expected rate on a risky investment that is equal to the risk rate. That is why you call it that a market is risk neutral and investors with 1 year and 2 year horizon periods are willing to consider alternative segments, that means the bond demand is a function of the expected rates. And another major assumption is issuers have a strong maturity segment segmentation preference and are not willing to assume the interest rate risk.

That means, the issuers are not going to change their segment, the expectations have no role from the issuer perspective. Only from the investor perspective do the expectations basically place the role. Right! So, these are the basic assumptions that basically we are taking.

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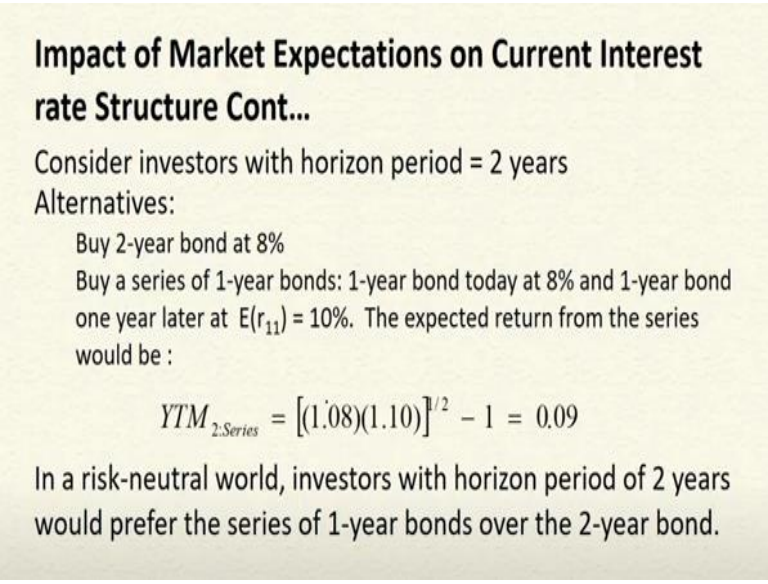

With these assumptions let us see, how the impact of the market expectations will be there on the interest rate structure. There are 2 segments we have seen. we have assumed one is the investor with a horizon period of 2 years. So, he has 2 alternatives one alternative is to buy a 2-year bond

at 8% or buy a series of 1-year bonds today at 8%, and 1 year bond 1 year later, let at the rate of 10%.

Your horizon period is 2 years at a time you can invest in a bond that will give you an 8% return or you can invest in a bond that is giving you 8% for 1 year and after one year again you invest another 1-year bond that will give you 10% return. Then the overall yield to maturity if you see in the series of the bonds in the second alternative, you will get an average return of 9% that is 1.08 into 1.01 to the power one by 2-1, we are using the geometric mean concept, then that is 9%.

$YTM_{2:\text{ series}} = [(1.08)(1.10)^{1/2}] - 1 = 0.09$

So, in a risk-neutral world, investors with a horizon period of 2 years would prefer this series of 1 year over the 2 years bond. Because it is getting more return, instead of investing in a 2 years bond at a rate of 8%, you will get 9%. If, you will go for a series of bonds.

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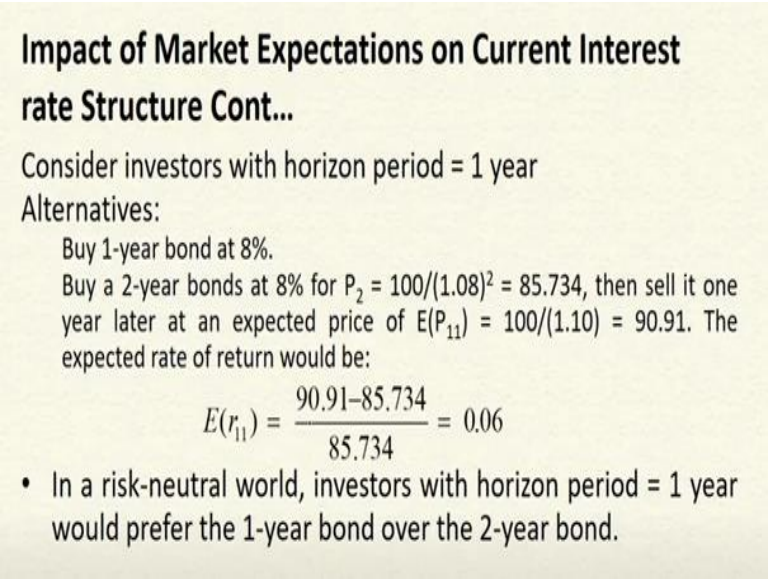

Then another alternative we have, let us have another investor which has a horizon period is 1 year. Then, he has also 2 alternatives. You can buy a 1-year bond at the rate of 8% or buy a 2-year bond at a rate of 8%. Let the price is 100 divided by 1.08 to power 2, that is 85.734. then sell it 1 year later at an expected price of 100 divided by 1.1. The interest rate which is going to prevail for the 2 years bond is 10%. Then you will get 90.91.

Price of two-year bond at 8%= $100/(1.08)^2$= 85.734

Expected price = 100/(1.10)= 90.91

Then, what is the return expected return he will he or she will get that is 90.91- your 85.734 divided by 85.734 which is 6%?

The expected rate of return= $\frac{90.91-85.734}{85.734} = 0.06$

So, then what the investor will do is the investor with a horizon period of 1 year would also prefer a 1-year bond over the 2 year spot. Because he is getting only 6% in the second case first case is getting 8%.

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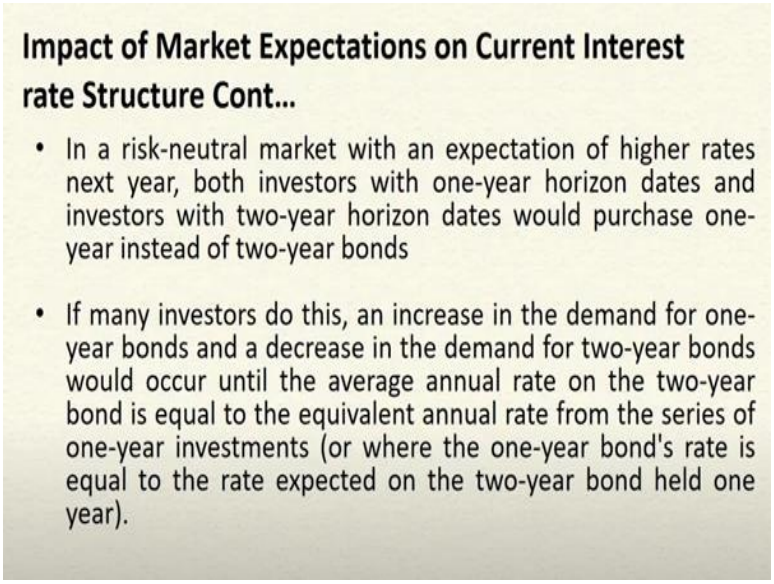

Then, what basically we have seen? in a risk-neutral market with an expectation of higher rates next year, both investors with 1-year horizon dates and investors with 2-year horizon dates would purchase the one-year instead of the 2 years bonds. So, then what will happen? if many investors will do this then, there will be an increase in the demand for the 1-year bonds and a decrease in the demand for the 2-year bond. And that will continue until the average annual rate on the 2 years bond is equal to the equivalent annual rate from the series of the 1-year investments.

To the rate expected on the 2-year bond held for 1 year where generally the 1 year bond rate is equal to the rate expected on the 2 year bond held for one. That particular process will continue unless this particular equality will not arrive.

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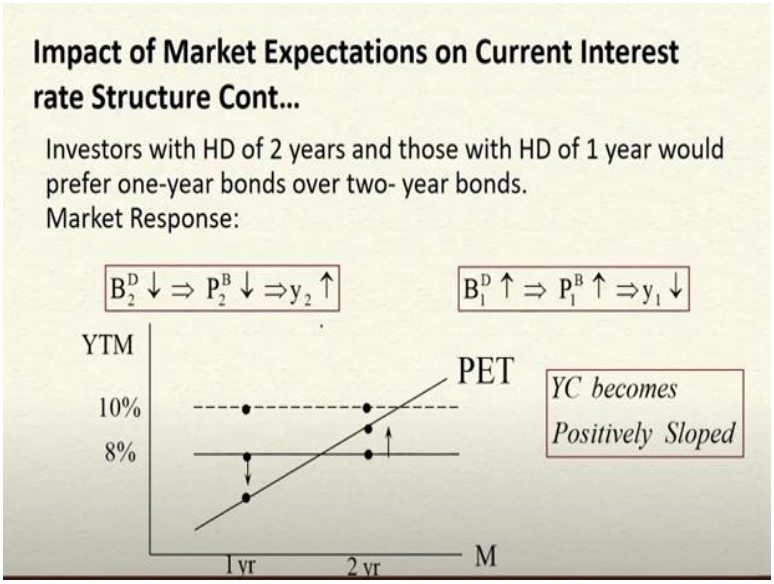

So, if you see then what will happen in this particular case? The investors with a horizon period of 2 years and those with a horizon period of 1 year would prefer 1-year bonds over the 2-year bonds that we have seen. The price of the 2-year bond will go down because the demand for the price of the bond will go down. So, that means your yield of the 2-year bond will go up and the demand for 1-year bond will be up and the price also will be up then the yield will go down. So, in that case, if you see the yield curve that will become positively sloped.

The first-year rate was 8%, the second-year rate was 10%, and if you observe that there will be a shift in the yield curve, the yield curve becomes positively sloped. That is basically what we can observe in this particular case.

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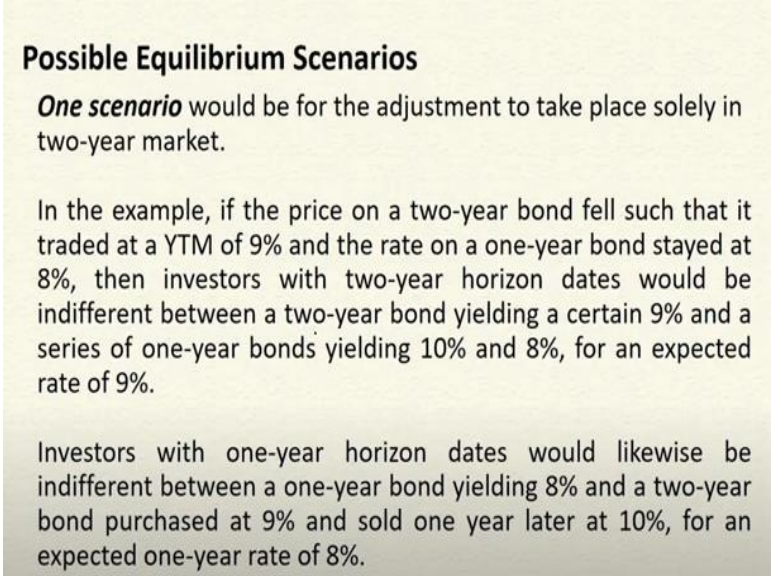

There are possible equilibrium scenarios that can happen in this case. One scenario is the adjustment may take place solely in the 2-year market or adjustment also can take place in the 1-year market. In the example whatever we have considered if the price on a 2-year bond will fall down in such a way that it traded on a yield to maturity of 9% and the rate on a 1-year bond stayed at 8%, then investors with the 2-year horizon dates would be indifferent between the 2 year bond yielding a certain 9% and a series of 1 year bonds yielding 10% and 8% for an expected rate of 9%.

And investors with 1 year horizon dates would also be indifferent between the 1 year bond which is yielding 8% and 2 year bond purchased at 9% and sold 1 year later at 10% for an expected rate of 8%. Is it clear?

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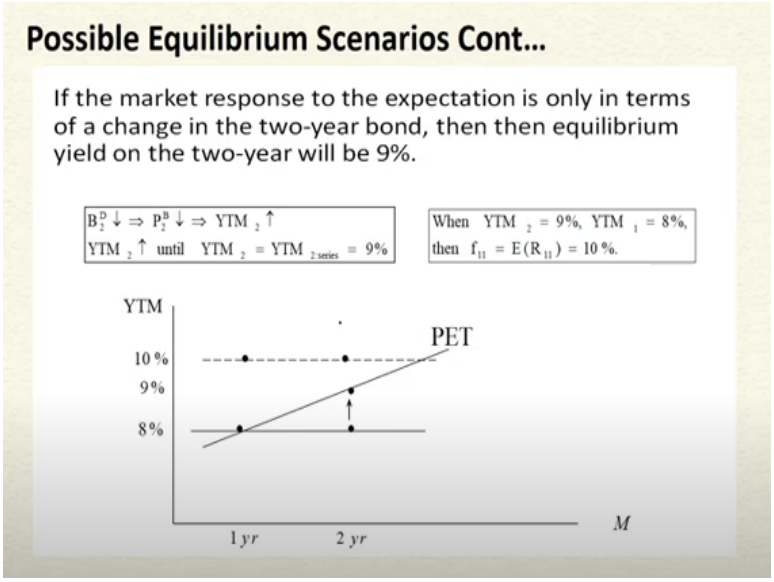

So, in that case, what is going to happen? If the market responds to the expectations only in terms of a change in the 2-year bond, then the equilibrium yield on the 2-year bond will be 9%. Because, the demand for the bond will go down, the price will go down, YTM will go up and the YTM 2 will go up until, the YTM 2 is equal to your YTM of the 2 series of the bonds, which has a maturity period of 1 year. That will be equal to 9%.

Then when YTM 2 = 9% and YTM 1 = 8%, then your f 11 will be 10% so, this is the equilibrium mechanism what you can observe in that particular case which will happen in the 2-year bond case.

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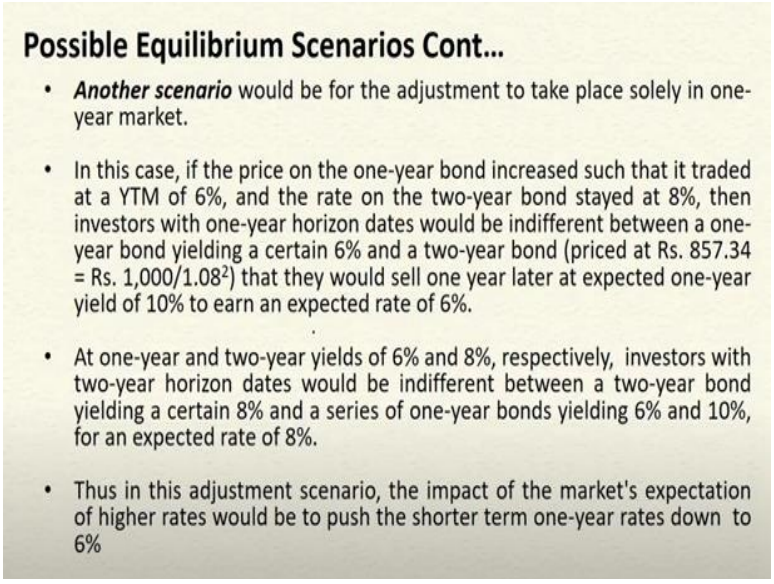

Now, the other scenario is that can happen also in the 1-year market. Let the adjustment will take place in the 1-year market. Then what will happen? If the price on the 1-year bond increased such that it traded at a YTM of 6%, what we have calculated before and the rate on a 2-year bond stayed at 8%, Then investors with 1-year horizon dates would be indifferent between a 1-year bond which is yielding a certain 6% and 2-year bond which is priced at let 1000 divided by 1.08 square that is it is 857.34.

Then, they would sell 1 year later at an expected 1-year yield of 10% to earn an expected return of 6%. So, at 1-year and 2-year yields of 6% and 8% respectively, the investors with 2-year horizon dates would be indifferent between a 2-year bond yielding a certain 8% and a series of 1 year bonds yielding 6% and 10% for an expected rate of 8%. So, in this adjustment scenario, the impact of market expectations of higher rates would be to push the short term 1 year rates down to 6% which was before the 8%.

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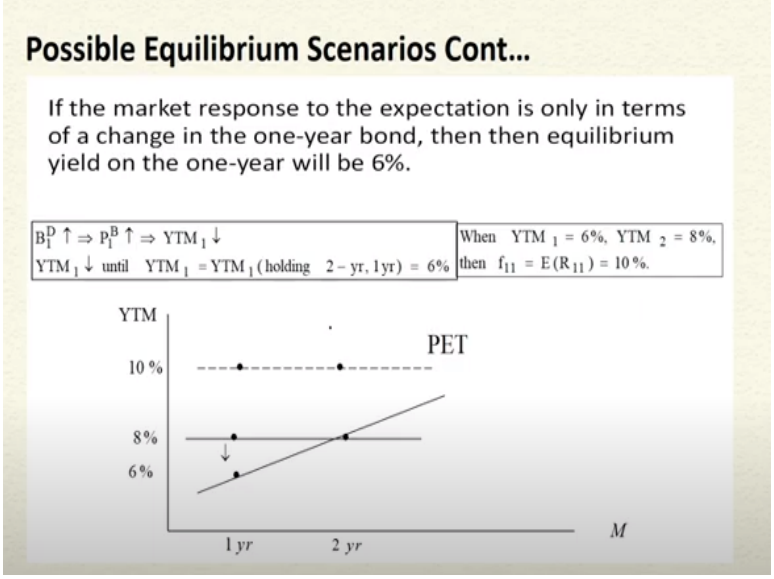

So, you can also explain these things in this way. The demand for the 1-year bond has increased. So, that is why the price has increased, the YTM has gone down and the YTM or the YTM will go down until the YTM 1 is equal to your YTM 1 holding for the 2 years and 1 year that will be equal to 6%, then when YTM 1 = 6% and your yield to maturity of the 2 years bond is 8% then your implied forward rate will be 10%. So, our 2-year spot rate is what we assumed was 10% in the beginning.

Now, we have seen that if you are trying to find the equilibrium in the 2 markets, then that spot rate what we call the expected spot rate should be equal to the forward rate. If that will is equal to the forward rate, then this kind of situation will prevail in the market. That is what basically we are trying to explain. So, in that particular context what basically we have seen? if the expectations in the market are going to be changed and this expectation is coming from the investor side, then this kind of situation will prevail in the market and this is the way basically your equilibrium dynamics will work.

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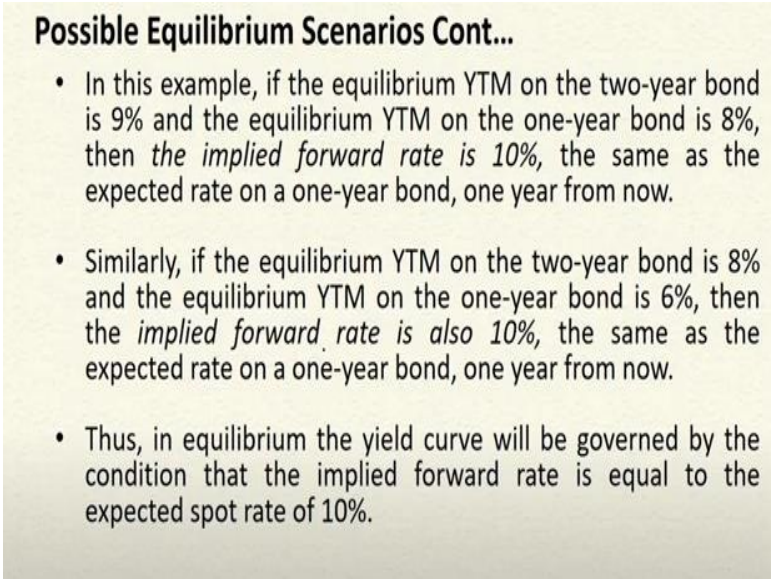

In this example, what we have seen? if the equilibrium yield to maturity on the 2-year bond is 9% and the equilibrium YTM on 1 year bond is 8%, then the implied forward rate is 10%. in both the cases we have seen, Which is the same as the expected rate on a 1-year bond and 2 years and 1 year from now. Similarly, if the equilibrium YTM on the 2-year bond is 8% and 1 year bond is 6%, then the implied forward rate is also 10%.

So, the equilibrium yield curve will be governed by the condition that the implied forward rate is equal to the expected spot rate, which is basically 10%; which is the basic essence of the pure expectations theory. The equilibrium yield curve basically will be attained or will be governed by the condition that the implied forward rate is equal to the expected spot rate which is 10% in this particular case.

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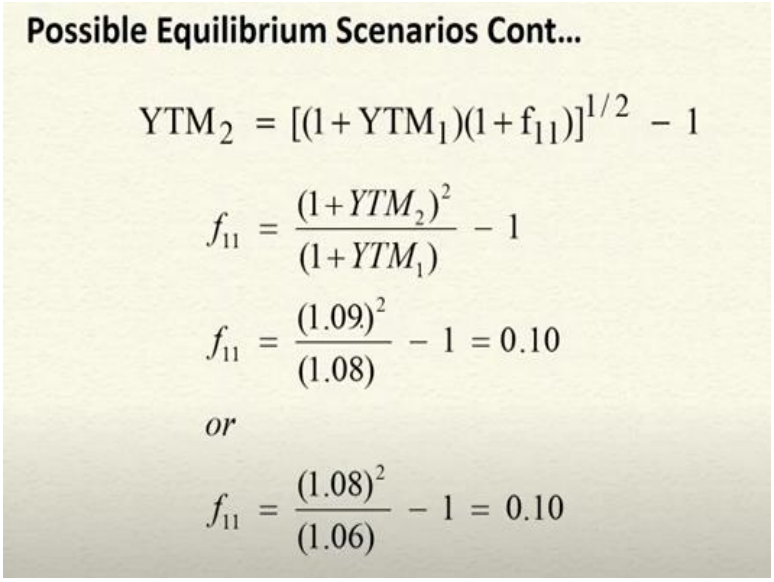

We can derive it with our formula whatever we have seen or we have discussed in the beginning your YTM 2 = 1 + YTM 1 into 1 + f 11 to the power 1 by 2-1 then your f 11 = 1 + YTM whole square divided by t + 1+ y 10 1 - 1. Now, your f 11 in the first case that is 1.09 in the 2 years market, if you are talking about the equilibrium or the adjustment is taking place, then it is 1.09 square divided by 1.08 - 1 that is 10% that means 0.10 second case it is 1.08 square divided by 1.06 – 1 that is the 10%.

$YTM_2 = [(1 + YTM_1)(1 + f_{11})]^{1/2} - 1$

$F_{11} = \frac{(1+YTM_2)^2}{(1+YTM_1)} - 1$

$F_{11} = \frac{(1.09)^2}{(1.08)} - 1 = 0.10$

Or

$F_{11} = \frac{(1.08)^2}{(1.06)} - 1 = 0.10$

So, in both the cases whether the adjustment is taking place in the 1 year bond market or adjustment is taking place in the 2 year bond market that does not make any difference. In both the cases the implied forward rate is equal to the expected spot rate that is the 10%.

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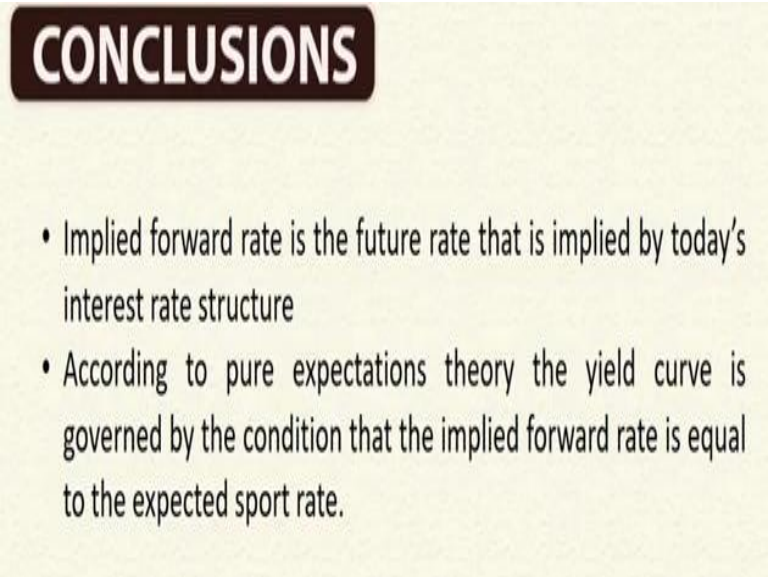

So, what basically conclusions we can derive in this case. The implied forward rate is the future rate that is implied by 2 days interested structure that we have discussed. And the yield curve is basically governed by the condition that implied forward rate is equal to the expected spot rate. Whether the adjustment is taking place in the 1-year bond market or the adjustment is taking place in the 2-year one market, that does not make any difference in both the markets you will find that the implant forward rate is always equal to the expected spot rate.

That is what basically what we can see from the concept of the pure expectations theory. Further, we will continue with this discussion whenever we assume that both the investors and this issuer's expectation are also going to be changed. Here, we have assumed that the resource expectation is not playing any role. Only the investors, expectations we are considering but if you are going to accommodate that thing also.

Then how it is going to change the concept of the yield curve and how the yield curve is going to be shifted? and what is the adjustment mechanism will work in that particular context that basically will be discussing in the future?

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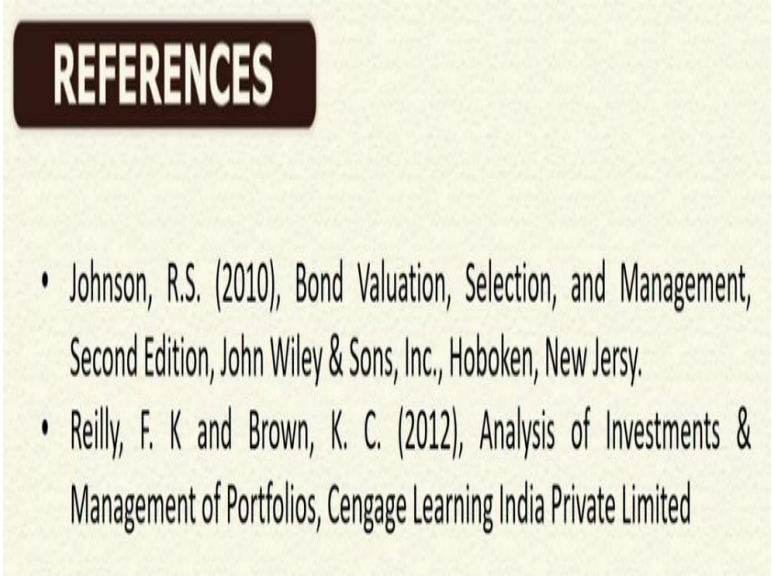

So, these are the references what you can go through for detailed discussion on this, Thank you.