

Symbolic Logic
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Lecture – 09
Truth-Functional Connectives
Propositional Variables
Propositional Constants

Hello. So we are into module 9 of the NOC course in Symbolic Logic. I have already introduced you to the connectives of the PL and we are going to module 9, symbolization with the use of the connectives.

So, we are going to learn this is our first lesson in symbolization. So we are going to first understand what the propositional variables are, what propositional constants are, remember this is syntax. So there is going to be certain guide lines for what to use and what not to use and ultimately our goal is to learn how to symbolize with the PL connectives. So that is what is in the agenda for today, but before I go and start talking about the symbolization, let me just quickly find out whether and how much of the discussion on connectives you remember, so will start with the small and simple problem.

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suppose:
A is true
B is false
C is true
D is unknown

What can you tell about the truth value of

1. $(B \cdot D) \supset [(A \cdot C) \supset (B \vee C)]$

It will be True. For, even if D is unknown, B is False, hence $B \cdot D$ will be false, a conditional with false antecedent will always be true.

Suppose I tell you that you, we do not know anything more than this. There we know that the proposition A is true, proposition B is false, proposition C is true and D is unknown. So these are what is given and then I am going to ask you what can you tell me about the truth value of say this compound proposition. This is B dot D horseshoe, A dot C horseshoe, B with C. This whole thing is the consequent and this is your antecedent. So plugging this value, can you tell me what is going to be the truth value of this? I will give you a minute to think about remember A is true, B is false, C is true and D is unknown.

Some of you I am sure have already figured it out and some in case you are in trouble or in case you are a little bit confused I am going to explain this, but this should tell you how much you remember from the previous discussion of the connectives, because you know unless you know how the dot works, the horseshoe works and the wedge works you cannot really do the computation of the truth value of this compound.

So you recall that this is actually a test of that, that knowledge that we have already discussed. So what is the truth value of this whole compound, can you tell us the answer is yes we can tell. Because these are all truth functional connectives, we can figure out only if we can compute the value of the components. So the answer is Computable. Then the question is what is the truth value of this compound. Now there are more than one way to do this answer, but no matter which method you follow the answer is going to be the truth value of this compound is going to be always true.

Now let us see how, I will explain see though I understand you will say that these are unknown, but even if these are unknown there is a way to find this out for example, if you start at the antecedent see B is false and D is unknown no matter what the value of D is, B is false therefore B dot D is going to be false and once you know the B dot D is false, you can say the whole horseshoe proposition is going to be true. Why? Because a horseshoe with a false antecedent is always true.

Now this is one way to go about it, the antecedent you figured out. Those of you are saying, but these are unknown, what do you know how can you know the value of that question is the first of all even if it is unknown remember the system is such that it believes in only two truth values. So either D can be true or D has to be false. So that is one way, that is A

one answer right there the D is unknown does not mean that it is completely unknown you can still make the possibilities clear.

The other thing is that you really do not need to know what the value of D is, if you following this can of matter to figure out the value or some of you may have to look here B wedge C, both are known values B is false and C is true. If that happens then B wedge C the well the way it works this junction it is going to be true and when you know that you know that any horseshoe statement with a true consequent is going to be true. So this is true and which happens to be other consequent of the whole compound proposition. So if the consequent is true then same logic applies to this horseshoe, the whole compound is going to be true alright.

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C is true
D is unknown

What can you tell about the truth value of.

1. $(B \cdot D) \supset [(A \cdot C) \supset (B \vee C)]$

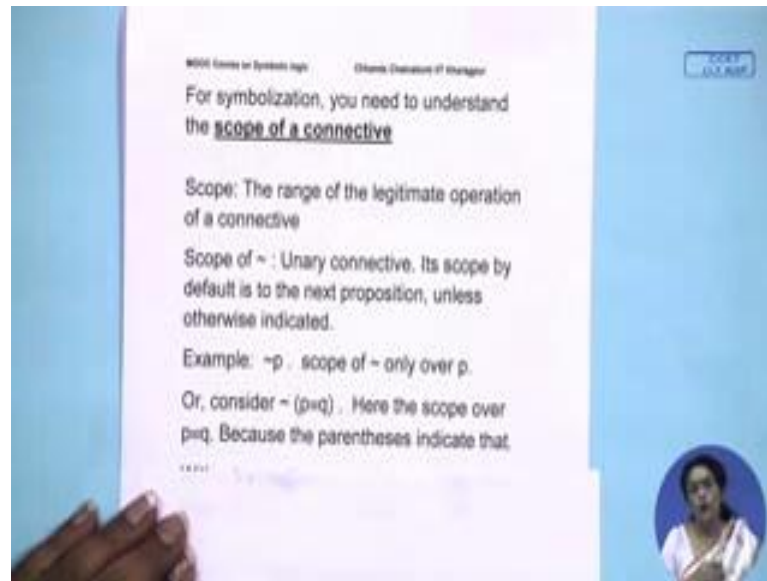
It will be True. For, even if D is unknown, B is False, hence $B \cdot D$ will be false, a conditional with false antecedent will always be true.

Or, you may also argue since C is true, $B \vee C$ will be true, and a conditional with true consequent will always be true.

So this is one way to figure out how you can know that the sentence has to be true from the given values. As I said this is just an exercise sort of A reminder to yourself, whether you know the meaning of the connectives that we explain earlier in terms of the truth tables.

So that is the test that was about if you thing that you are still unsure, I suggest that you look it up that you come to know about that as confidently as possible otherwise the more you progress towards this PL system, you are going to be slightly unsure about your moves. So that was our first round, but as I said our goal is to learn how to do the symbolization today.

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When you are going to do symbolization, my first step would be with the scope of connectives. I have introduced already the connectives. You know there are 5 connectives, but the point is that 5 connectives are actually at par there is no weightage, there is no relative weightage by which you can say this connective is going to rule over the other connectives, it is not exactly like algebra.

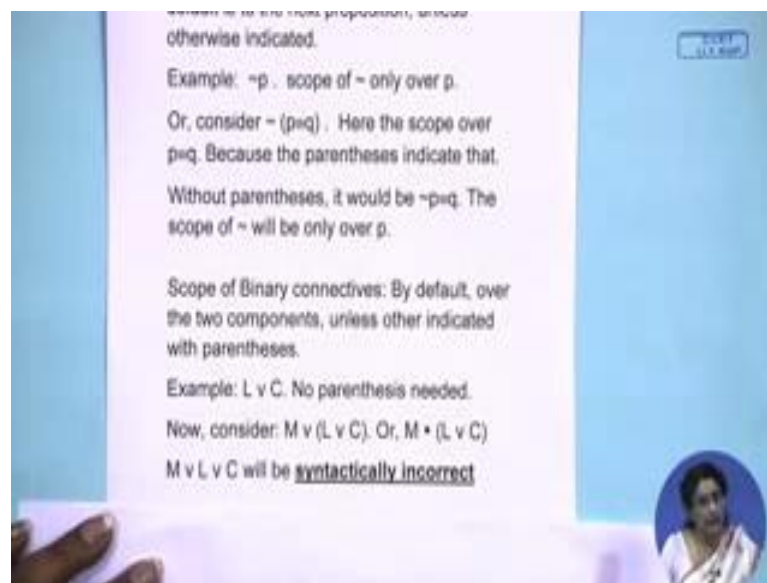
So all connectives are at par, therefore there has to be other mechanisms to know which one to compute first, which one to do it say for the later and so on and so for. And for that the concept of scope of connective is going to be very handy. What is scope? Scope here means the range of legitimate operation of a connective. How far can it legitimately operate on what is its power, what is the limit of its power that is what scope indicates. Now if you are dealing with Unary connectives remember these are those ones that can pick up utmost one proposition when it is connecting.

So there is only one Unary connective that is your tilde and the tilde the way it works it is default scope is only up to the adjacent proposition so up to the next proposition. Unless there are some other indications in the proposition right, let us see what we mean. So suppose you find tilde P that is a unary connective. As I said default is that the scope starts from here and it goes only up to the adjacent proposition which is P. So the scope starts here and it ends here only over P. Now you compare that with say this

statement this says tilde and then there is a bracket a parentheses starting, within the parentheses there is P triple bar Q.

Again the same logic applies that the scope start here it goes up to the adjacent proposition, but here the adjacent proposition is not single P. The adjacent proposition because of the presence of the parentheses this is P triple bar Q. So the range of this tilde will start here and will go till the end of the proposition. That is what this parentheses does, it indicates that the tilde scope goes over the entire thing within the parentheses. So this, now if we had missed out doing the parentheses and we just left it like this. That is an important lesson for you also. Suppose we do not use the parentheses here and we just leave it like this, then what happens. Then the answer is that this tilde the range of it will start here it will go only up to the adjacent P. It will not range over the whole proposition.

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So what you have? We have to completely different kind of propositions. This one is A negation. Its main connective is tilde, remember main connective means though one with the largest scope whereas here you are dealing with a triple bar proposition. Why? Because the triple bar, the direct connective it the range starts from here it goes up to here. Tilde is only starting here it ranges only over the P. All right then, the truth table will differ the meanings will differ, therefore you need to understand the scope very

carefully and wherever possible if you want to mean it in a certain sort of way use the mechanism called parentheses or the brackets.

Now talking about the Binary connectives so far we saw the Unary connective. Binary connective means that they are going to connect two, utmost two. By default the range of the direct connective is going to be over the two components, unless you indicate it in the different sort of way. So let us take an example for example here, $L \wedge C$, because way just dyadic or A binary connective, no need if you want to this to be the main connective, no need to have any parentheses because default is that it will go over the both the components, that it is connecting. Now suppose you have more than two and this is where you will understand why it is importance to have indicated the scope. See here for example, you have $M \wedge L \wedge C$ same connective, but you need to remember that each Well or \vee is a dyadic connectives.

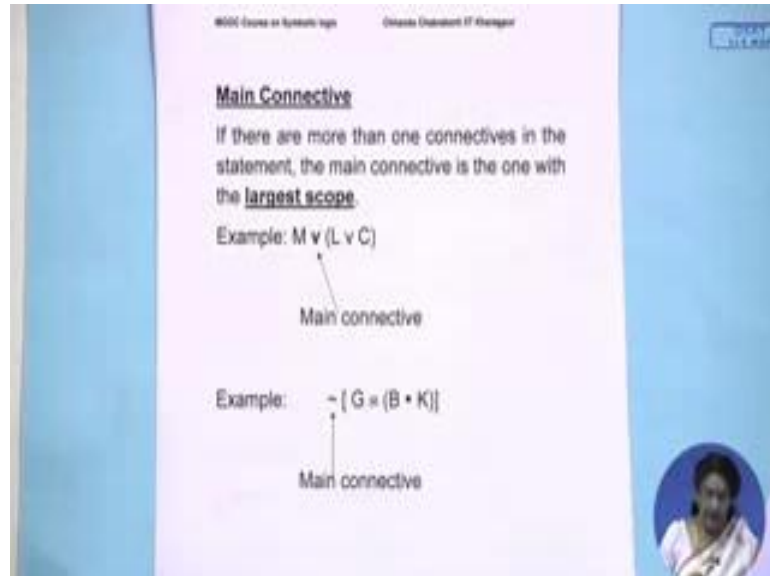
So at a time it is going to only pick two. Therefore there is need to group it in such a way that you have ultimately you have two disjoints. So here is this wedge and this is where the range ends, it goes up to here and goes up to here. This one on the other hand is the main connective, but here is the disjoint and the whole thing here is a disjoint.

Now many of will think that, but the truth value is this same why do I need the parentheses. Please remember that what you talking about the here is the emphasis, how we are disjoining it. We are saying here either M or $L \wedge C$. So the emphasis is that this is my one option and the other option is a neither or. If you regroup it in the different sort of way for example, if you want to put the brackets here between M and L and you say either M or L or C . The emphasis is different and the meaning also changes therefore, remember the grouping is important. This, on the other hand dealing with two kinds of connectives, here is wedge here is a dot and you need to indicate which one is your main connective this one is a conjunction.

So here is a conjunct and then there is other conjunct which happens to be your disjunction by itself. So this is how you are going to read, what we will not allowed syntactically is to leave this kind of sentences stand alone. Though it is the same connective if you do not indicate what grouping you are intending it for or which to are the disjoints. Then just by itself this is going to give you a syntax error. Therefore the

mechanism of using parentheses is going to be quit important for you to learn at this point.

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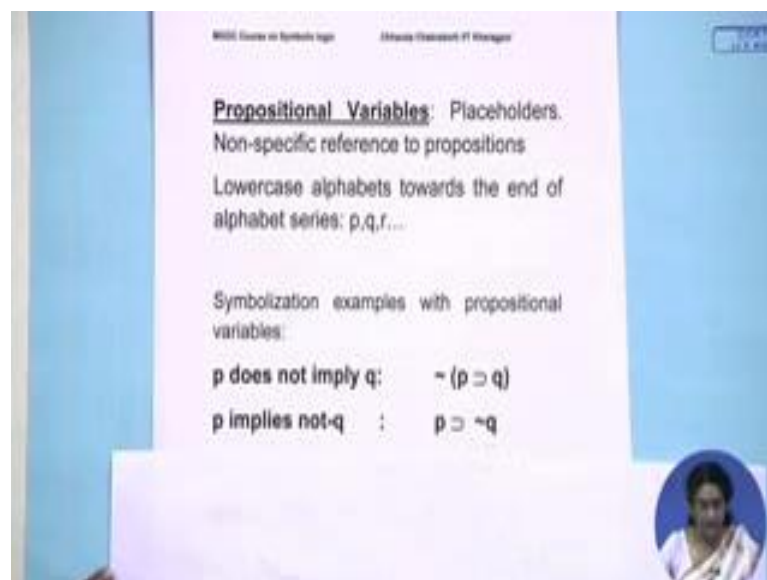
Now main connective I have already mentioned this, but let me take examples so that we are here. Whenever you have more than one connective in the statement, remember the main connective is the one that I said will have the largest scope.

So the subsequent connectives are going to be sub connectives, but the one which has the maximum scope within the proposition is going to be the main connective. For instance here, as I was saying, if you group it like this and the brackets are here parentheses is here then what you indicating this is your main connective. Though the wedge is here, the wedge is here also, please note the range of this wedge as I said starts from l ends with C that is a sub connective. This one on the other hand has A larger scope. It is start from the m it is goes all the way over to the end of the proposition; hence this is your main connective.

Let me take another example to make this point clear say here. There is tilde, there is triple bar, and there is dot. Which one is the main connective? The answer is the tilde. why? Because every other, if you take every other connective, the tilde has the maximum scope, this dot it ranges over B and ends with K. This triple bar it is start with G and ends with this sentence, but this tilde because of the way we have used the brackets it is starts from here beginning of the proposition and goes until the end.

So this is how to understand what is your main connective. Why knowing main connective is important because soon we are going to go and talk about truth table and other things, you will see that the main connective holds a primary position. How we understand, how we approach the proposition, you will see that the main connective tells you what kind of proposition you are dealing with what truth table you are going to reach etcetera. So at this time please pay attention to the main connective, how to understand the main connective. Now we are in A position to talk about the propositional variables, Remember we said we are going to understand the propositional variables. So here we are.

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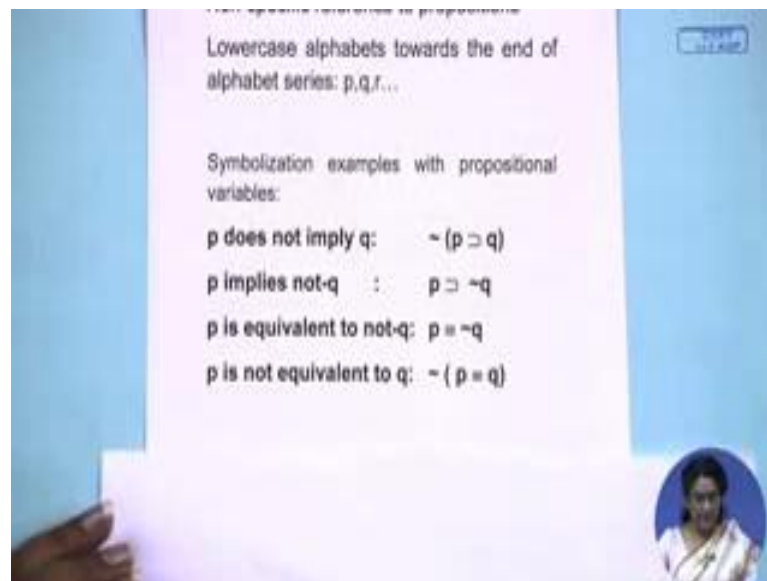


Variables I sure we have been exposed to the idea of variables are earlier, if not there is always a first time what variables are place holders. Therefore, us position and there is no in some other thing can go into that place. So they just occupy space, when you want to use this kind of variables with proposition, reference. What do you doing is making a non specific reference to propositions. You do not mean a particular proposition like today is Tuesday or something like that. You just have any proposition when we want to talk about any proposition.

Then symbolically we say this is K sensitive, choose lower case alphabets towards the end of the alphabets series. In fact we choose p q r etcetera. Please note I am going to remind you again, the lower case. So if you are using propositional variables,

symbolization might work like this, say we want to say for any two proposition p and q. Suppose we want to say p does not imply q. I have already introduced to to the connectives. How to read this we are talking about given any two proposition p and q, it is not the case that p implies q. So the translation is going to be tilde it is not the case that p horseshoe q, all right. Similarly when we are saying p implies not q again we are using propositional variables, not actual propositions. What we mean is that p implies that it is not the case that q. So p implies, if p then it is not the case that q, p horseshoe tilde q.

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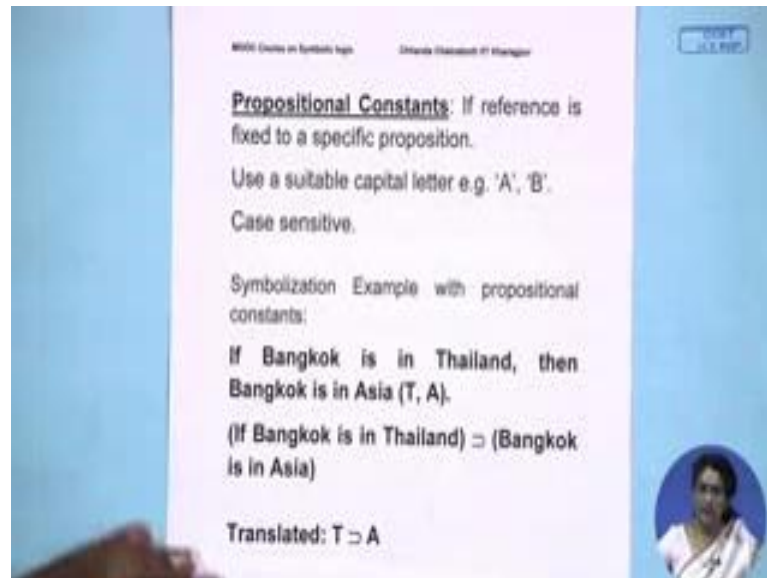


You can try this yourself so that your grasp over symbolization grows a little bit. Here is another kind of translation. See p is equivalent to not q. So the main connective is this, p is equivalent to not q. So here is p triple bar tilde q, which one is the main connective, the equivalent of the triple bar.

Last one here p is not equivalent to q. Very different from what I am saying here. What I am saying here? It is not the case p is equivalent to q, try to read it and you will see why the translation has to follow in this way. It is not the case that p triple bar q. The tilde attaches and rules over the entire sentence. So take a look, the main connective here is the tilde. In order to indicate it is large scope, we need to use these parentheses.

Let us now move to propositional constants. So propositional variables we saw what they are and how they work with them. Remember the lower case alphabets towards the end of the series.

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Propositional constants on the other hand are like numerical constants, within the context they refer to once specific proposition. So they remain constantly referred to two some actual proposition. Symbolically saying we need to use capital letter and with respect the proposition you will see soon, when we going to actual problems that we need to find the suitable capital letters something that reminds you of that proposition, but the point here to remember that you need to use capital letters.

So something like A, B the inverted commas are not required, but this is capital letter that you need to. So if you are using capital letter we understand that you are referring to actual propositions, if you are using lower case we know that you are talking in terms of variables, so case sensitivity is something to remember by. Here is example of symbolization with propositional constants. Suppose we have if Bangkok is in Thailand then Bangkok is an Asia. Here is a proposition here is a proposition and because these are actual propositions we will pick up capital letters and suitable capital letters in the sense that in order to remind us, we are talking about this proposition which has Thailand in it, we choose the T capital T. This one has the reference to Asia. So we have chosen capital letter A.

This is the 'if then' sentence. So very easy to understand the main connective is going to be horseshoe. So this is how to read and then once your practice this is the translation

that should be of that proposition, but remember these are all constants and you are talking about the horseshoe \wedge as the symbolization for an actual proposition such as this.

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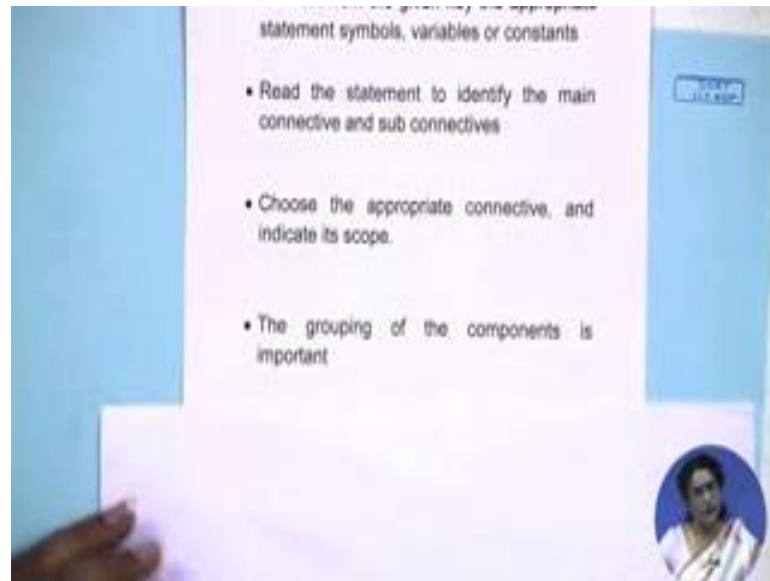
Symbolization in PL:

- Find out from the given key the appropriate statement symbols, variables or constants
- Read the statement to identify the main connective and sub connectives
- Choose the appropriate connective, and indicate its scope.
- The grouping of the components is important

So in general therefore, how to do symbolization in PL and we will see more symbolization in the next module, but remember that first of all you need to choose the statement symbol. If the P is already given in which symbols are mentioned then your job is done, you just follow those symbols, but the main point is whether you are using the variables or constants. By now you should know what we are talking about and there is a difference, but most of the time along with the problem you will be given the statements symbols to use for with clear indication what the symbol stand for.

Once you have the symbols then read the statement carefully. Why? So that you can identify the connectives especially what is going to be the main connective. There may be sub connectives also, but this symbolization means you are translating from one language to another. So there will be English words and then you need to figure out what with the corresponding PL connective to use in place of that English word that is going to be how the symbolization is going to be done. And the thing is finally, is that once you have chosen the appropriate connective in each case; remember to check it is scope because if there is an error to approach the scope you are changing the meaning of the proposition. Your job is to translate not distort. not completely change the meaning of the proposition itself.

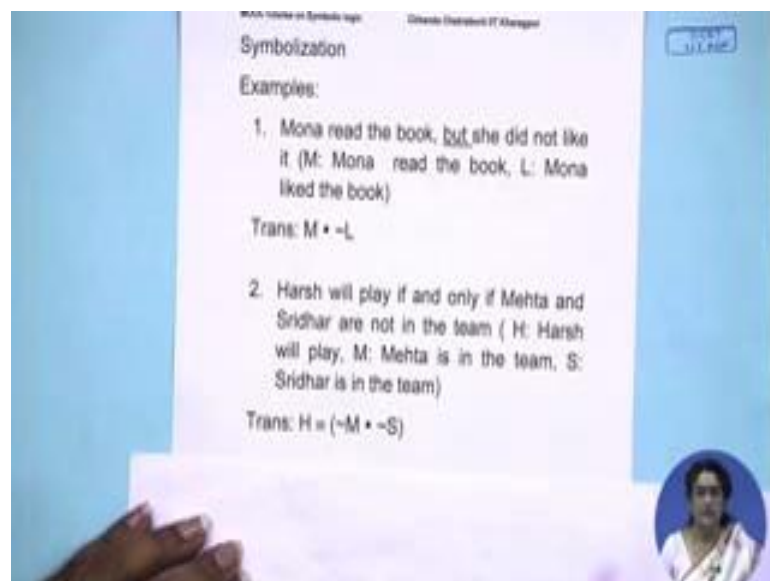
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And the grouping of the competence is going to be important also and this is what do you learn in terms of use of the mechanism of parentheses.

So how the intended proposition is meant, that, grouping you have sort of keep as is. Once this is done then we are in a position to talk about the actual symbolizations. So I am going to try out certain things with you.

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Here is an English sentence. Mona read the book, but she did not like it. Two sentences or two propositions and here is the given key m stands for Mona read the book L for

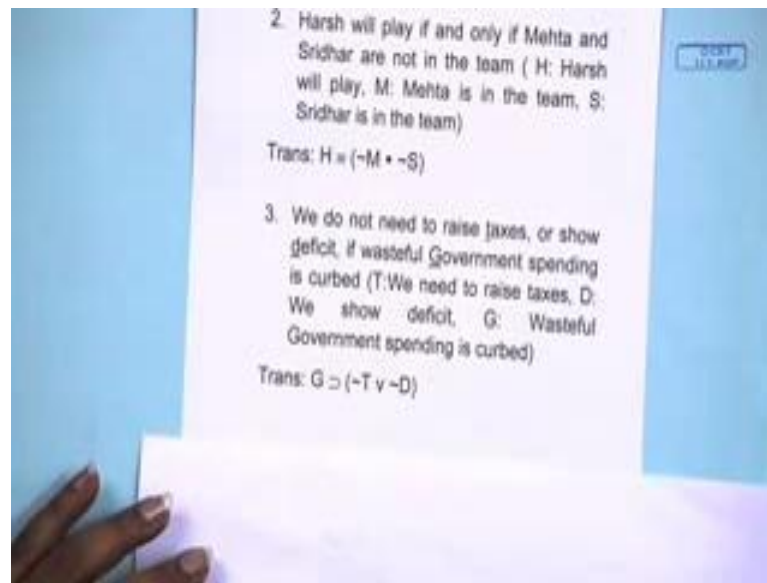
Mona like the book and here is but, this but is to be translated as what in PL, it is corresponding is the or the conjunction or the dot. So if you have done that correctly, then the translation is going to be very easy M dot not L. Where is this not coming from the tilde is coming from not did not like it. Remember the key is Mona like the book. So if you want to say Mona did not like the book. This is how it is going to appear.

Here is another sentence. Harsh will play if and only if Mehta and Sridhar are not in the team all right. Now as you can see start reading it with me. So that you can see the there are 3 components in it. One is harsh will play and that is your h, the other one is Mehta is not in the team there is no sentence call Mehta and Sridhar; obviously Mehta is in the team, that is your simple proposition, Sridhar is in the team also a component. SM M is for Mehta is in the team; S is for Sridhar is in the team. Now we are in a position to find out how to translate, which one is the main connective you think and you are reading a properly you will say the even only if is the main connective. Once you know that the rest is easy slowly will go, so this is our one of the component H and then this is going to be triple bar and here comes the next one which is, Mehta and Sridhar are not in the team. So Mehta is not in the team Sridhar is not in the team.

Now how to express that and you see that you need to translate it like this. One is harsh will play if an only if and then you are saying Mehta and Sridhar are not in the team. So neither Mehta is in the team nor Sridhar, is in the team. It is not the case that Mehta is in the team and it is not the case that Sridhar is not in the team right.

In it to work A little bit with problems such as this to see how the translation is happening here.

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Let me go through another example here. This is we do not need to raise taxes or show deficit, if wasteful government spending is curbed clear indication, that we have one, two, three. Three components here, but T stands for we need to raise taxes. Remember usually the key is given in terms of simple proposition. Wherever you have not you have A compound sentence in your hand.

So T stands for we need to raise taxes, D is for we show deficit, G is for wasteful government spending is to be curbed and the translation happens like this. that G if G, then not T or not D. How do we would get this if then, well here take a look we do not need to raise taxes or show deficit if whatever. Now if that is your antecedent. How do we know this is our main connective? Read again there is a comma. So it is separates out the component clearly. This is your antecedent and this whole thing is your consequent. So this translation is to be done carefully with this kind of horseshoe, with that we are going to end this, but not we doubt giving you a little bit of help here.

Is that you know do not get discouraged in the very beginning that if you are missing out translation or I do not know how to do this etcetera, give yourself A little practice, for example do not look at the cues I have given try to do this on your own and see whether you come up with the same result or not. Sometimes we are impatient in reading the proposition, we just did not get it we just thing that in one reading will be able to answer understand it not necessarily. So give yourself A little bit of a practice here for example,

if you missed out this if, then you are missing out the whole structure of the proposition. So a little bit of careful reading, careful understanding, and a little patience in the beginning is going to help you a little with that I am going to end this module and but will come back with more symbolization in the next module.

Thank you very much.