Symbolic Logic

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Lecture – 08
Connectives
Scope of Connectives

Hello. This is module 8 and we are going to talk about the connectives in propositional logic syntaxes part of propositional logic syntax.

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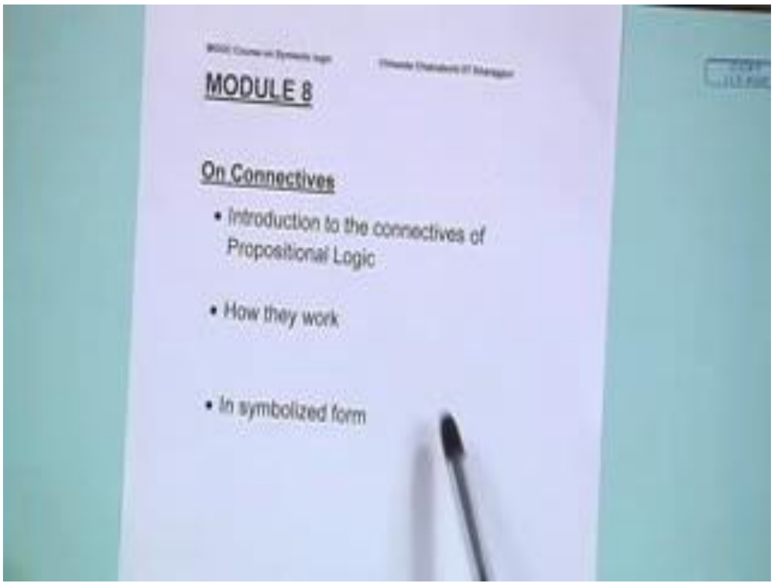

So, we will be introduced to the connectives that they use and specifically how they work and we will start symbolization also in the symbolized form how the connectives are to be represented in the system.

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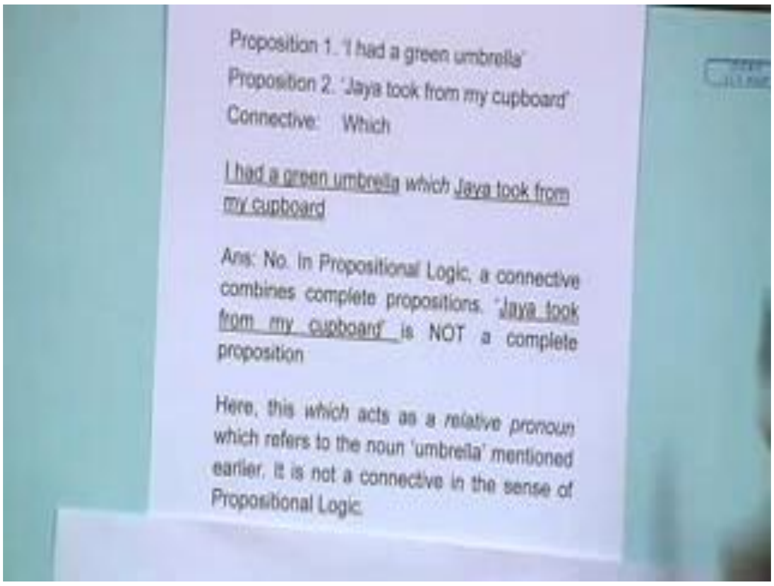

First the idea of connective, see I have shown you some examples, but may be the concept is to be clarified further, what qualifies as a connective in propositional logic system? So let us consider this statement. I had a green umbrella which Jaya took form my cupboard. This looks like a compound statement, where you might think that this is a connective. I had a green umbrella which Jaya took from my cupboard and you will say there are two components. So which connective here, that is what we are trying to find out. Can we read the whole sentence? Say as a combination of two propositions, where which is acting as the connective. The answer is no. If you are thinking in this kind of words that we can proudly analyze the whole sentence like so and which is a connective the answer is clearly no why not.

First of all, understand that in propositional logic a connectively suppose to combine complete propositions. So standle on propositions and if you now look into the sentences itself, then you will find there one of the component is not even the complete proposition. Jaya took from my cupboard, What Jaya took? Jaya took from my cupboard is not even a complete proposition. So the way the connective works in propositional logic is that a complete proposition and then the combination of this. So that is one of the reasons why which here is not a connective.

The further thing that you then you will say then what is its function. Its function is as a relative pronoun. It is a relative pronoun which refers to a noun that has been previously used in the sentence which in this cases the umbrella. So I had a green umbrella which Jaya took from my cupboard. So the noun umbrella is mentioned by referred to by this which. Whatever it is my point is that is not a connective in the sense in which propositional connect logic connective work right. So with that introduction let us go into the proper discussion of the propositional logic connectives.

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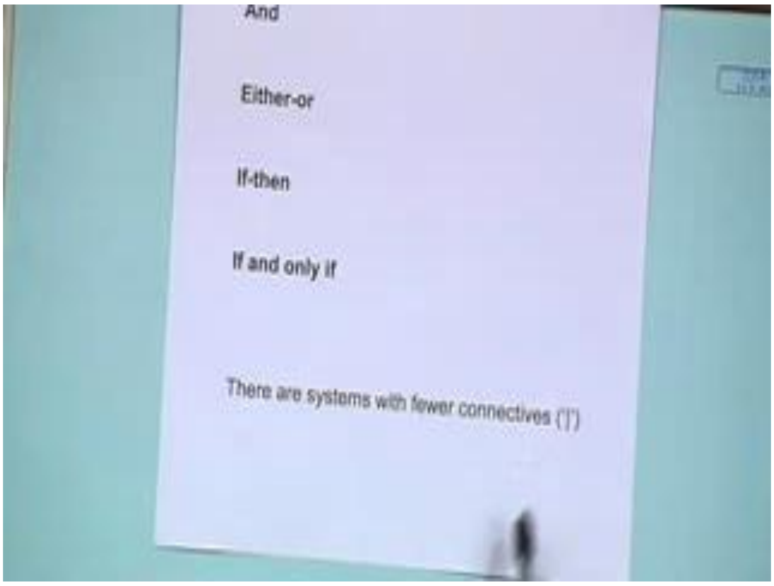

As already discussed propositional logic has five truth functional connectives. What are they? It is important to also know that every logical system does not have to have five connectives, but this system does this system that we are calling PL; it has 5 truth functional connectives. What are those five truth functional connectives, well the first one we have already probably seen is not and it is the only unary connective amongst this not that word. The other four are all binary or dyadic connectives. The other four are and, then, either or and then, if then and also if and only if. So that is the complete set of five connectives that PL uses and each of them is a truth functional connective.

There can be as I was trying to say that there can be system which have fewer connectives, in every system does not have to have five connectives for example, some systems have only one that is stroke function. This is just to give you an idea about the diversity that exists among logical systems, but PL has this five, the not, the either, or the if then and if and only and we are going to now look closely into each of this and try to learn how they are going to be used in the system.

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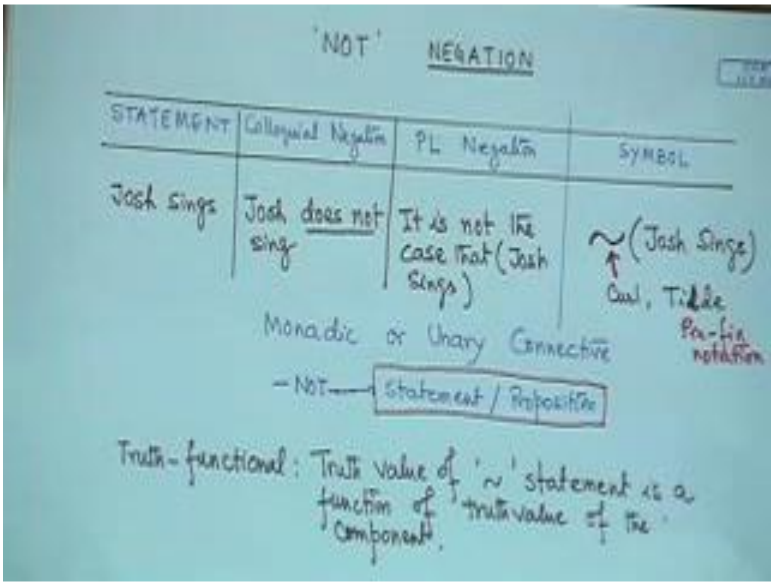

The first one is the NOT, the NOT that we are going to call negation also. So this NOT how does it work. Let me give you one idea about that. See this is a proposition Josh Sings. Colloquially if we attach the not we attach it with the word Josh does not sing. If this is the proposition or statement Josh Sings its negation is Josh does not sing watch, where the NOT is been attached colloquially in ordinary statements it attaches with the word that is not how the PL negation works. So first thing to note is that, that PL negation works rather like this, it attaches itself in the beginning of that sentence or statement. So here is Josh Sings and negation is it is not the cases that josh sings. This is what we call the prefix notation and that is how it is going to be understood any negation has to be understood of the proposition itself.

Also appreciate the fact that this is a compound statement and it does not look like a compound, but if you remember the definition of a compound proposition you will remember that here is a component. Compound propositions of those which have other proposition as its components, here are the components and attachment of the negation mix it into a compound. In symbol form, the symbol of the negation is this sign. What we call the Carl or tilde. Carl or you can call it the tilde. So here is Josh Sings and to that the negation attaches in the beginning that is how the syntax of PL works with negation. We call it prefix notation and because it is attaching in the beginning of the proposition.

Let me also remind you that it is a monadic or unary connective; please see that at a time it is picking up only one component. So it is attaches itself to one component only. This is the kind of structure that you can remember. This is the schema how it works the not attaches in the beginning and here comes a propositions. Here we have taken an example where the proposition in itself is simple, to that the negation is attached, but it is a necessary that it the component has to be always simple it could be a complex proposition or compound proposition also.

To that also when the negation attaches it will attach itself at the beginning that is how negation works in PL and then to remind our self that this negation is truth functional in character. So the truth value of a negation statement or a tilde of statement is going to be a function of truth value of the component itself. If you ask what is the truth value of this you need to know what is the value of this and then compute the value of the negation this is how the truth functional NOT is going to work. So that is our first interaction to one of the very first connective and its one of a kind because it is the only unary connective among the lot.

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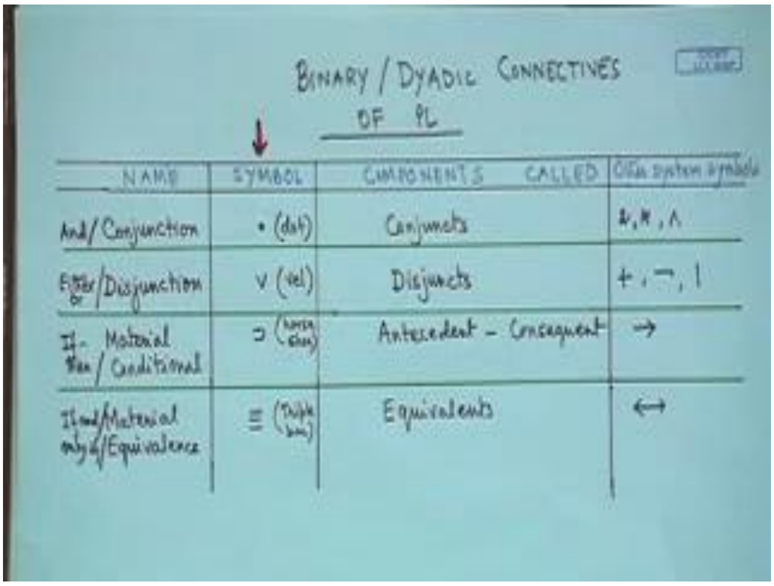

Next we are going to bring in before you the other connectives that we have. And these are as I told you they are all binary or dyadic connectives. We will go over them one by one. Binary or dyadic means this going to be at most two components that we are going to see. The first one is and its formal name is conjunction, as you can see conjunction and the symbol by which it is represented that is called dot. It is a simple dot. The components because remember it is a connective. So there is going to be components and the components are called conjuncts. You are going to have two conjuncts conjoint by this dot.

Now you may have seen another conjunction working in other logical system. So in other programming language and people may use various kinds of other symbols, but syntactically if you use any of this symbols in PL it is going to be syntax error which means that you have to learn you have to orient yourself to learn to use this dot when you want to express conjunction.

Similarly we have either or the formal name for that is disjunction and the symbol that we are going to use is called the vel or the v, it is a small v. See this is the small v and the components are going to be called Disjuncts. You are going to have two disjuncts and the connective is going to be called the vel the v. Again whatever other symbols you may have learnt in other systems, you are required to be using this v when you want to express disjunction or either or.

Next is the If then, the formal name for if then in PL is material conditional and the symbol by which it is represented looks like this. Please do not confuse it with the set symbol, it is the other way. So it is an in worded c and its called horseshoe. You know the horses are earlier days they used to wear iron shoe. So it is called horseshoe. The components are going to be called antecedent and consequent. Which will tell you later when we go into details about then if then, but the components are called one of them is called antecedent, the other one is going to be called consequent. If you have been using so far the arrows symbols for if then I suggest that you orient yourself to this using this symbol the horseshoe symbol.

Finally we come to the fifth connective; it is if and only if which the formal name for in the system is material equivalence and the symbol looks like this. it is called the triple bar. The components are going to be called equivalents and if you have been using double sided arrow in this fashion, again I will remind you that please get acquainted with this triple bar. So these are the symbols for the connectives that we are going to use and even it to learn fast to actually start using them when we see the opportunity to express this kind of connectives. Each of them as I do not have to remind you, but each of these are actually truth function in character and we are going to explain that in our next slide.

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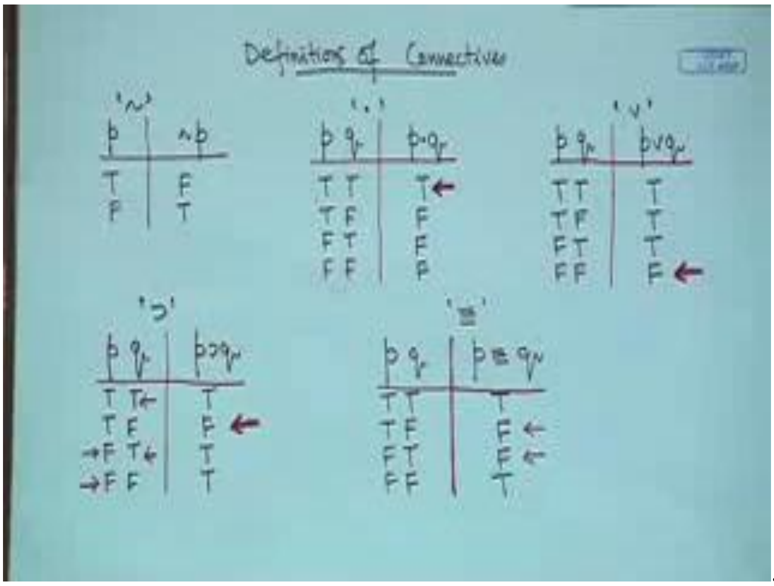
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We are going to now define the connectives and here you will see how the truth functional character of these connectives is going to come through. So we will start with very first one namely the negation. This is the meaning of the negation. Where p is any given proposition, and remember, they are only two truth values possible. So p can be either true or false and when we attach the tilde to it then it returns the other value in your truth values. So, if it p is true not p is false, if p is false not p is true. So if you look into this, how do we know what is the value of not p the answer is that first you have to know the truth value of the component itself. Which makes the tilde a truth functional connective this is self explanatory, but I suggest you remember how the negation works.

Let us now go into the next one this is our dot which you are going to call conjunction. Conjunction or dot is a dyadic connective. So it is going to have two components, two conjuncts and these are the truth values possible. So p has this kind of distribution, q has this kind of distribution and note that p dot q is true, only when both the conjuncts are true. Otherwise in every other case it is false and that is how the dot works. Once more it is a truth functional connective. As you can see entirely determined by the truth value of the components, so this is the definition of dot; I strongly suggest that you get acquainted with this kind of meaning, the tubular sort of meaning of the connectives because we are soon going to use them as we go along and there is if you do not remember this, it will be very difficult for you to apply them there. So remember the conjunction as I have shown true only when both conjuncts are true.

Next one is disjunction. So here you have again the two disjuncts and this is the complete table for the vel or the v of the disjunction. What is happening here is that we find that the v or the disjunction is false only when both disjuncts are false. Otherwise it is true in every other possibility. So normally we would say that either work either or works in this way that. If I ask you will you have tea or coffee and you are suppose to say either tea or coffee, but here in this table it seems like we are getting some other kind of picture.

So here is t or here its coffee and in both cases the value returned is true, but remember the vel is also true when you say yes to both tea and coffee. So in other words that was just a straight example, but the I guess the point goes through, is that the disjunction works in this way that is true when either of the disjuncts is true or when both that is junctions also true and it is false, only when both of them are false.

Let us now come to if then this is our, if then and the symbol that you we were using is the horseshoe. So p q and this is if p then q and notice again when is it false, the only time p horseshoe is false when p is true and q is false. In every other case it is true. So if p then q is false when p is true and q is false. Otherwise in every other case it turns out to be true. That is something to remember that is something very important to remember.

Specifically I want to draw your attention to the last two rows. The third and the fourth row, see what is happening p is false in both cases and q is true in one case false in one case. Look at the result p horseshoe q is true never the less. So when from this we can gather that when p is false, it does not matter what value q has, p horseshoe q will return the value true got it, so the last two rows so to establish that. When you have p has false then it does not matter what value q has, horseshoe will gain the value true. Similarly I would like to draw your attention to the first and the third row. See what is happening q is true in both cases and p is true in one case p is false in another. What do we learn from this that when q is true it does not matter? What value p is p has in each case p horseshoe q will return the value true, from this please read this table closely as you go along, but this is an important lesson to learn about the horseshoe how it behaves.

This is our last one and this is triple bar. What is happening triple bar means equivalent. So, whenever the values match or equal then p triple bar q assumes the value true. So when they are true p triple bar q is true, when they are false p triple bar q is true. Only when the value is mismatch that is one of them is true one of them is false, that is when the p triple bar q become false all right. So this is our quick introduction to all the connectives as we see them.

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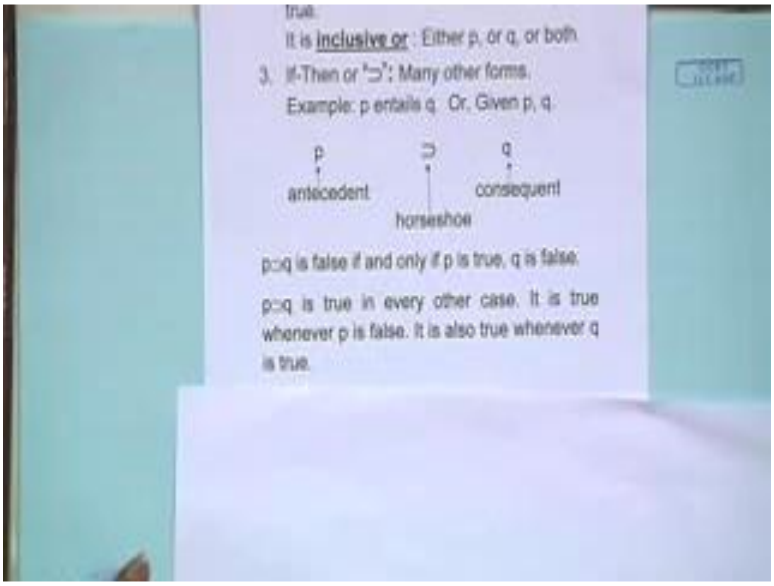

And as I told you and I suggest that you please remember to use this as often as possible, as a reminder because we are going to soon use this as we go along. So, from this what we have learned we will quickly summarize is that first of all that this is our conjunction and something to remember that we have understood how the dot work, but this is also to remind you that many words may express the idea of a conjunction without using the and. So I remind you to more or less tell, call this connective conjunction and fixation is not necessary for example, you may have p, but q that is a conjunction, but the word is not and it expresses the sense of conjunction. Say in addition to p q that is a conjunction, but it does not use the word and.

Regarding the disjunction or the vel, what we have learnt is that it is not the x or, or the exclusive or why because it is true even when both the disjuncts are true. So that something to also put a flag on. Regarding the if then as I have tried to tell you there can be many other forms, you may not use the only if then, but you may use for example, p entails q, that is a clear horseshoe p entails q, but if then is not used, the word if then is not used given p q again if then is not used, but it is a conditional statement.

Now, as I was telling one of them is called antecedent the other one is called consequent. This is if p then q, p horseshoe q. What comes after 'if' and precedes the then that is what we call the antecedent. In this case p is the antecedent and q is called consequent. Something that follows the then that is called consequent and the sign as you know is called horseshoe.

The position is very important. I will from now on I am going to refer to them as antecedent consequent. Let us take a look into various forms, but before that just reminder, that p horseshoe is false only when p is true and q is false. otherwise in every other case it is going to be true. As if we have already discussed.

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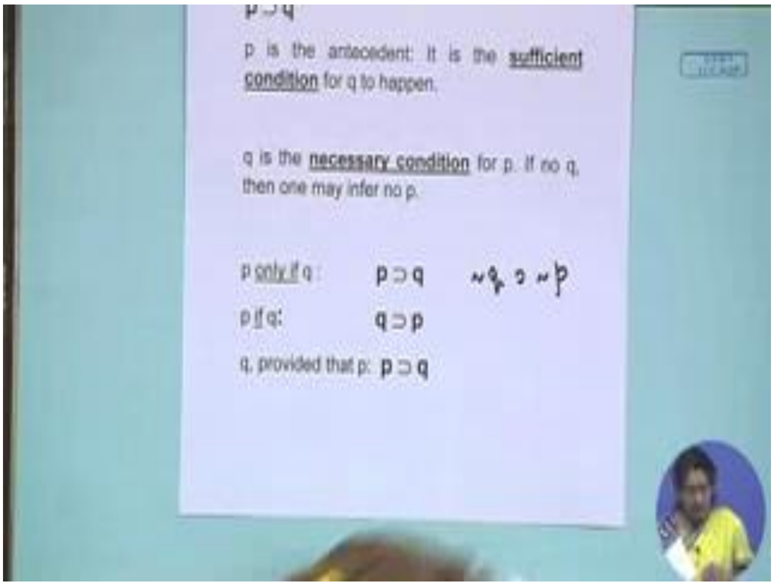

The symbolization slowly we are getting into the symbolization. So here is a simple sort of a quick way to understand this. That this signed has to be read as if p then q symbolized as like this. So, if I push the switch then the light will be on. So p horseshoe q. When we say the p is a antecedent, the other name for the antecedent is sufficient condition, which means that by itself is sufficient to bring q above. It may not be the only condition under which q happens, but it is sufficient, if you say p that q is going to be there the other name of antecedent is sufficient condition and the consequent is known as necessary condition, necessary condition for p.

So the way to understand necessary condition is like this, if there is no q then one may infer no p either. So remember these two terminologies, that yes this is antecedent this is consequent. The other name to know them by that, this antecedent works as the sufficient condition for consequent and the consequent works as the necessary condition for the antecedent.

Now, let us take the various forms, p only if q. When we say that what you were saying is that the q is necessary condition p only if q. So q does not happen, p does not happen and the way to represent that would be this p horseshoe q. You might say that I can also probably write like this not q horseshoe not p and you are right, but these two are equivalent. We will learn that when we go along, but right now as I say you all are beginners. So p only if q, remember. This is the way to represent that p if q which means that q is sufficient condition, if this is the antecedent. Therefore, the symbolization will be q horseshoe p. Q provided that p provided that p means this is your antecedent; this is sufficient, p horseshoe q.

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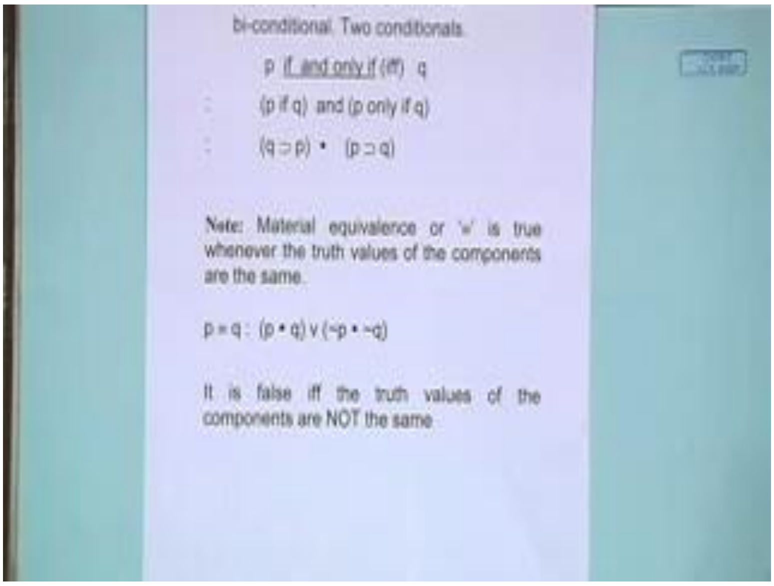

Now these are some things it is not important to memorize but more important that you understand this and then we come towards the triple bar. The triple bar is known or also as bi conditional, so two conditionals are actually packed inside the triple bar. How? Let us take it look see p if and only if q right. So if we break that up, then we find what we are seeing is p if q and p only if q. Once more, material equivalence we express it as p if and only if q right and the short form of 'if' and only if is this I double ff. Now if we break it up then it is p if q and p only if q. What is the translation of that p if q, we just learnt is going to be q horseshoe p, and p only if q we just learnt is p horseshoe q do you see the two conditionals.

So in a way material equivalence is a conjunction of two horseshoe statements, which is why we call it Bi conditional. We also note it the material equivalence is true whenever the truth values are same. So from that see whether you can read this, ,the meaning of triple bar we gain this that p triple bar is q either when p and q both are true or when p and q both are false that is not p and not q are true.

So these are the expressive ways to understand p triple bar q the only time p triple bar q is false when the truth value is do not match. This is how far I will go with this module. We are finished introducing you to the connectives. Please follow these meanings of the connectives because from now on that this is what we are going to use.

Thank you very much.