Symbolic Logic
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Lecture – 07
Propositional Logic: Syntax

Hello. So, today this is our module 7 and we are starting to learn the Syntax of propositional calculus, so propositional logic or propositional calculus is going to have a specific Syntax.

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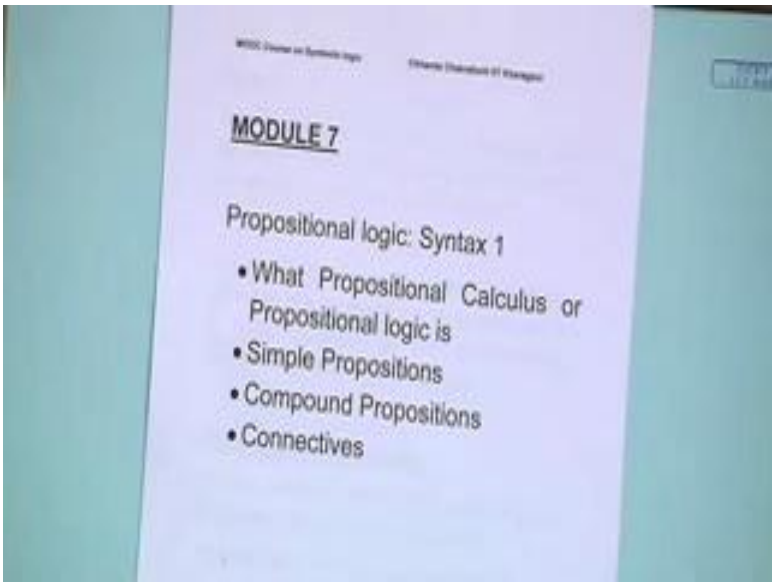

In this module, today's module what we are going to learn is what this logic is about, what are the units that it work from and then what are some of the important classifications of propositions that it deals with and then the idea of connectives.

So let us start slowly by first understanding what Syntax is. See I told you, when a I introduced what logic is formally, I said formal logic has to have three components one is Syntax, the other one is semantics and then there has to be proof theory.

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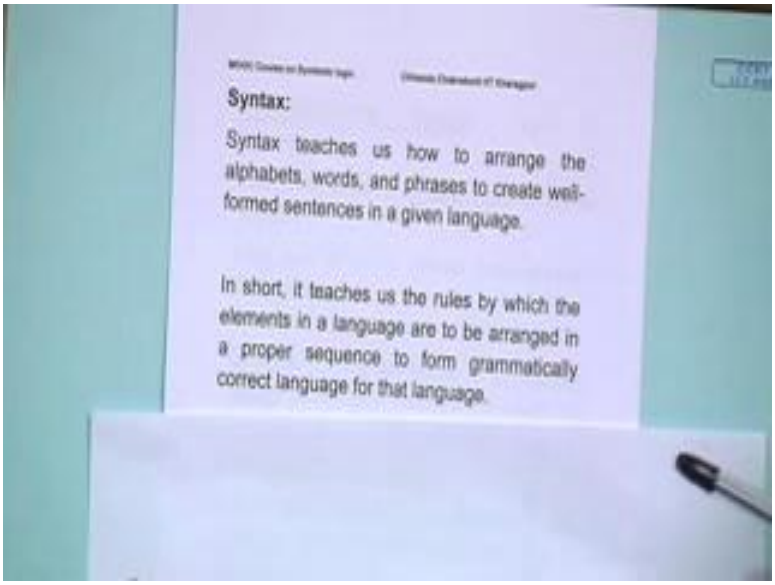

So this is Syntax. What the Syntax do? Syntax is like grammar it teaches us certain rules. What it teaches us is how certain alphabets or how the elements in the language, take any language how the alphabets, words or phrases in that language have to be arranged. There might be certain sequence how to use? You take example of English for example; we have told that whenever there is a q it has to be followed by u. So this is the grammar rule this is a syntactical rule how to have it in the right order. So Syntax teaches us how to create well form sentences in a given language, how to arrange the elements of the language in such a way. So there are well formed sentences the when we say well formed what we mean there are badly formed sentences also. What keeps the badly formed sentences out of the well formed is the syntactical structure. Following the rules and then the arrangement and then comes out the grammatically correct language for that system.

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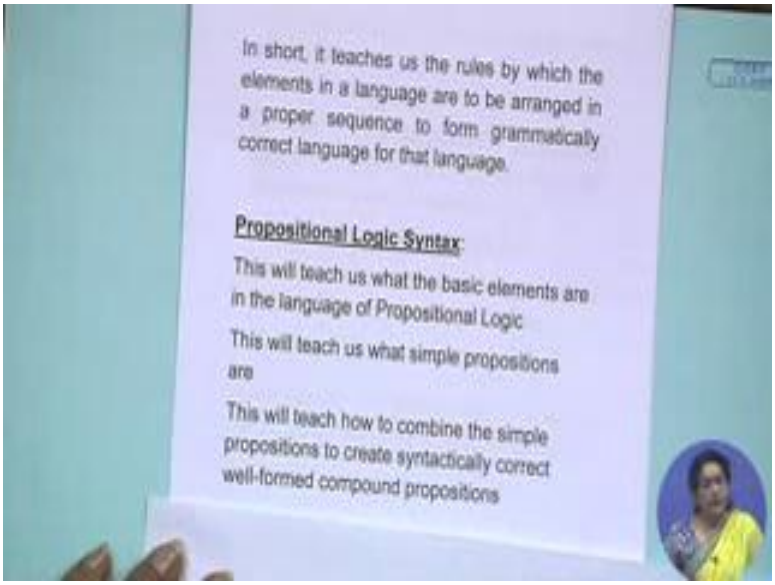

So with that introduction Syntax let us go on to talk about what propositional logic Syntax is all about. So first of all we are going to introduce in this Syntax what is the basic element in propositional logic. What are the units of operations for us? In that we will be introduce to the ideas simple propositions, earlier have in formally introduced to the idea of claims and you will find that idea the concept of claims is going to be closely almost the same as simple propositions. And then it will also teach rules, how to combine the simple propositions to generate, compound or complex propositions. So there is going to be an idea of connective also. So this is what you are going to learn in our introductory lecture of Syntax of propositional logic.

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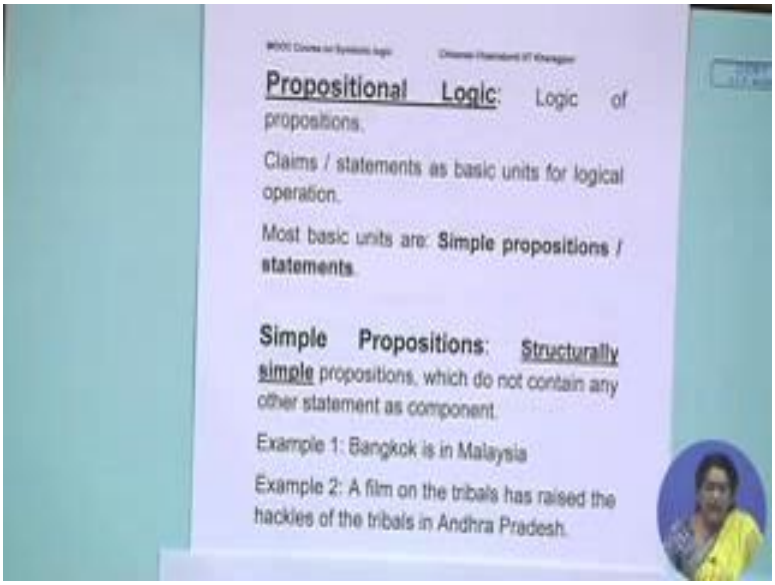

Propositional logic or Propositional calculus has you been told is logical proposition. I have already introduced to what propositions are. If you are forgotten that or cannot recall often let me just remind you propositions are actually strictly speaking of thoughts, when they are articulated, they are sentences statement. We will not make that certain distinction, will synonymously almost use propositions with statement. So I will intermittently use the term proposition, sometimes I am meant to call them statement also.

So propositions calculus is about logic, doing logic with propositions; what are the basic units and this is where we will get to see, propositions are the basic units, so statements are or claims are. This is what we deal with here. The most fundamental if you look at the syntactical structure it is in propositional logic, then the at the very bottom layer the fundamental layer you will find simple propositions or simple statements. What are simple statements? The definition of simple statement is like this, that it is Structurally Simple, Structurally Simple meaning what that if you look at the proposition, you will not find any other statement as its components. So it is a complete thought, but it does not contain any other proposition or statement in it.

For example say Bangkok is in Malaysia alright. Now this is complete thought, but look complete by itself, there is no other statement to be found in it and this structural simplicity is what is to be understood at the intended simple propositions in propositional calculus.

You may now recall what we talked about claim. So claims are at the simple propositions layer. Let us take another example a film on the tribal's has raised the hackles of the tribal's in Andhra Pradesh. See again structurally speaking, it is one thought there is no other statement as component of it conceptually. It may be complex, but structurally simple there is only one thought only one statement in this and that is what it qualifies it as a simple proposition.

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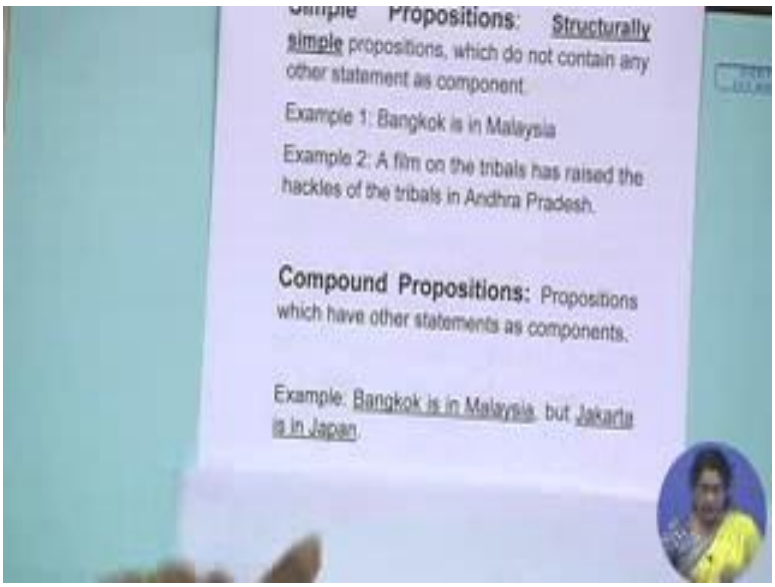

Probably if you compare it with compound proposition will understand the simple propositions of structure simplicity somewhat better. Compound propositions or complex propositions are those propositions which have other statements as components.

So if you look at this proposition file at the presence, distinct presence other propositions statements in it. For example, take a look Bangkok is in Malaysia, but Jakarta is in Japan. Here is a statement, here is another statement. So, this whole statement contains two

statements, which is what makes it compound, compound propositions. So there are simple propositions here, simple proposition number two and they are the components of this compound proposition. This is what I was saying. So, whenever you are looking into simple proposition remember we are talking about structural simplicity. When you are looking into compound compositions what we are looking at a structural compoundness, so that there are the presence of other components.

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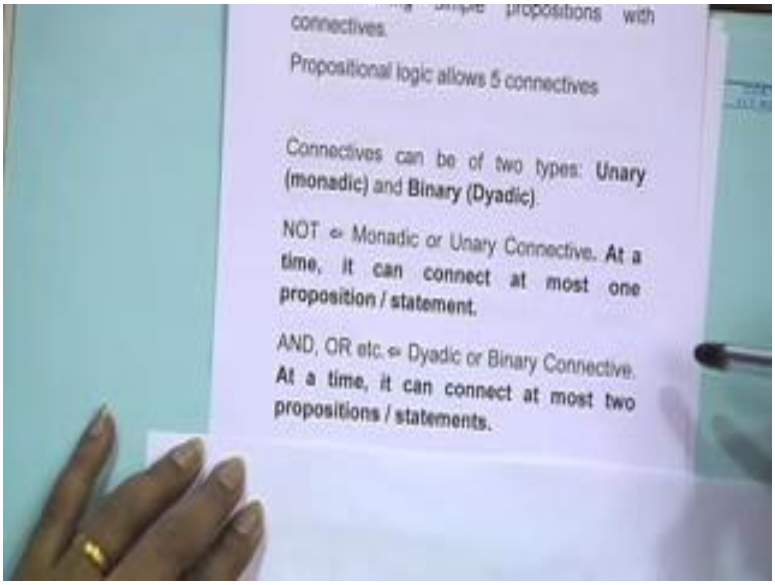

When we are learning the Syntax in proposition logic, we cannot over look the concept of connectives. We have learnt Simple propositions, we have learnt Compound propositions, let us now learn the idea of connectives. What are they? They are basically certain words. They are designated words. So not any word can qualify as a connective, but certain designated words, whose function is to connect statements or propositions, let us take example say Bangkok is in Malaysia, but Jakarta is in Japan. It is a compound proposition, but look how are they connected. They are connected by the presence of this word called but. So here, but is working as a connective. This is the entire statement is a compound proposition, but this is a connective and we required them. In fact, you will see, they are one which helps us to combine simple propositions and to create compound propositions out of the simple propositions.

So syntactical rules will show you how and we talk about connectives in details in our next module, but right now what we are learning is that the way in which in this propositions calculus, you can create compound propositions is with help of connectives. you have to take simple propositions, use connectives and then only compound proposition can be formed. The propositional logic will soon see allows 5 connectives. What they are and what their functions are, what they mean etcetera all those we are going to discuss in next module, but right now let us talk about the broad variety or classification of this connectives.

So we understand that the connectives are words, but not all the words that belong to this category are at far, there is of different kinds. One type is called the Unary or Monadic. The other kind is called Binary or Dyadic. So there are two kinds of connectives, the Unary and the Binary or if you want to use this terms Monadic and the dyadic.

The Unary connectives as they main suggest at a time it connect at most one propositional statement. So remember the job of the connective is to connect, with Unary connective at most it can pick up only one proposition statement. So one example of that would be not, you have the proposition and the not certain attaches itself to it and you get a compound called not says this. So this is one kind of understanding of connective, which connects to only one proposition at a time.

As appose to dyadic or binary connective, which connects at most two proposition statements at a time. So this requires two. This kind of connectives can connect only one. So this is the broad classification that we need to remember as we go along and we will see their functions when we go into details, but this is the classification that you need to remember. Remember that both of these are generating compound statements. Even when you are attaching the Unary connective you are actually generating a compound statements and that is that is something important to remember. This is also generating compound statements and these are the only ways in which out of simple propositions you can get compound propositions.

Connectives we are going to re visit as I said in our next module, but in the mean time we are going to pick up one other rather important concepts Syntactical concepts. One of

them is called truth value all of you have been exposed to numerical values. You know what numbers are, but we are talking about truth values. Why? Because as you know propositions that we are dealing in propositional logic, propositions are the truth value bearers. They are the actual truth values bearers.

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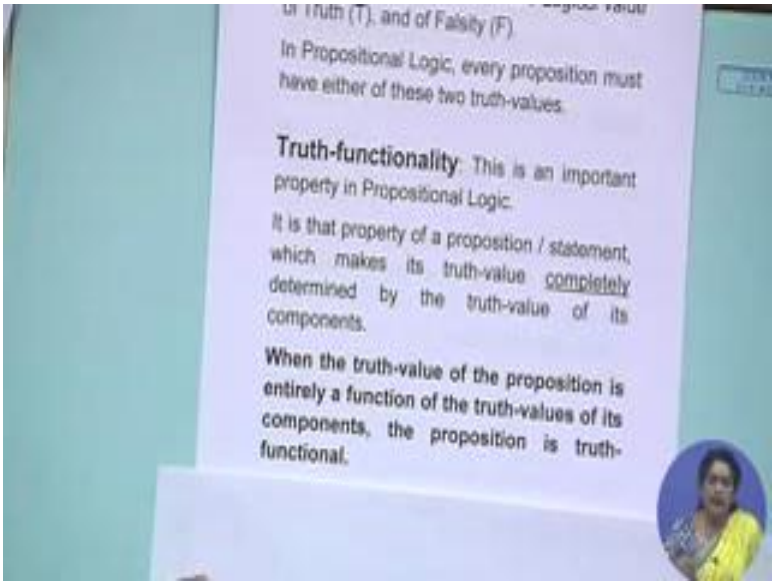

So what are truth values? Please understand these are not numerical values, but logical values that we call truth values. What they are is non numerical values and in this logic proposition logic allows only two truth values. They are the logical values of truth, which abbreviated form is T and of falsity the abbreviated form is F. So these are our truth values and remember propositions bear these values. So propositions are true or false. It is important to remember that in proposition logic every proposition must have a truth value one and must have either of these truth values, none of them kind have both of these truth values, none of them can have neither of these truth values. So if it is a bonafide syntactically correct proposition, in propositional logic it has to have truth value, out of these two choices either it is true or it is false. So truth value, remember. So, We are going to use this to our understanding of connectives. So remember how to use the truth values.

Here is the other very important concept that we are going to learn today. That is called Truth functionality. What it is? It is a property; it is a property of the propositions of the statements. When can we call a proposition or a statements truth functional? The answer is like this, that when it is such that the truth value of the proposition of the statement is completely determined by the truth values of its components. If you know the truth value of the components, then you can figure out the truth value of the whole of the compound proposition itself.

In another way we can say, that when the truth value of the proposition is entirely a function of the truth values of its components, we can call the propositions Truth functional. So remember the property of Truth functionality is like this, that the truth value of the truth functional proposition can be known only by knowing the truth value of its components. you do not need to know anything more, if you know the truth value of the components it can be compute the truth value of the entire whole and that is what truth functionality properties and that is the very important property for propositional logic, you will soon find that the truth tables or anything that you do actually depends heavily on this property. If you do not have propositions which have truth function you cannot compute the truth value in the way we usually do. So remember this property I am going to explain it further.

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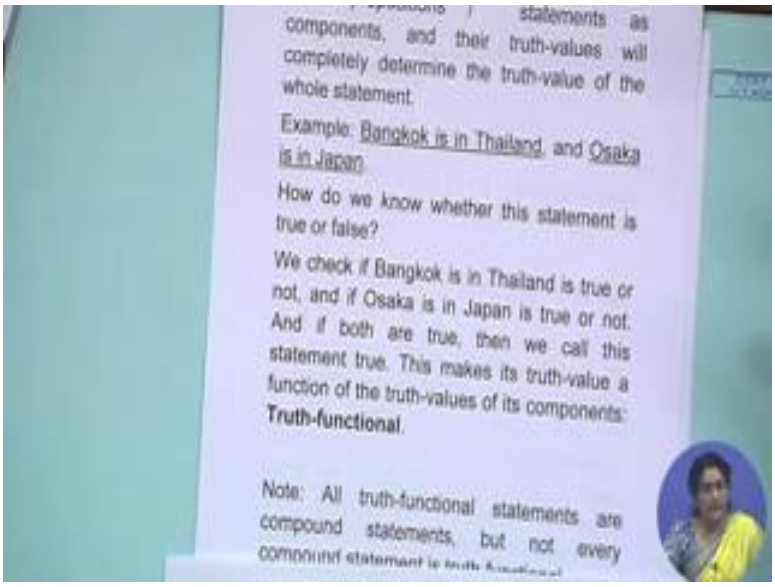

Point number one that if you have understood what Truth functionality is, then you will also understand that only compound statements can be truth functional. Why so? Because the way we have defined it and you heard me that Truth functional statements have those statements whose truth value is determined entirely by the truth value of its components. So unless you have components, how can you be truth functional and which propositions statements have components, those are the compound statements. So only compound statements can be properly be Truth functional. So that is the first thing to understand. We have understood how compound statements can be generated and the answer was by taking simple propositions and combining them with the use of connectives alright.

So for example say Bangkok is in Thailand and Osaka is in Japan that is the compound statement. Now if you ask how do we know whether the statement is true or false? Is this true, the whole compound statements is true or false how do we know that, what is the process and the answer is that we tried to first find out whether Bangkok is in Thailand or not, whether it is true or false, then we try to find out whether Osaka is in Japan or not and depending on what we find out about their truth values. We decide what the truth value of the whole thing is.

So that makes the sentence completely Truth functional because you do not need to know anything more besides the truth value of the components. Note, as we said that all truth functional statements are compound statements, this we have already (Refer Time: 18:57), but with that I am going to now add this further that part it is not the case, that every compound statement is truth functional. it is not necessary just because you have a compound statement in hand, that it is going to be truth functional. So this is the special property and I will give you one example of this to make the point clearer, but this is the property that we need to really look into. In here what happen Bangkok is in Thailand, Osaka is in Japan, is that all I needed to know is whether this is true and this is true in order to figure out about the truth value of the whole thing.

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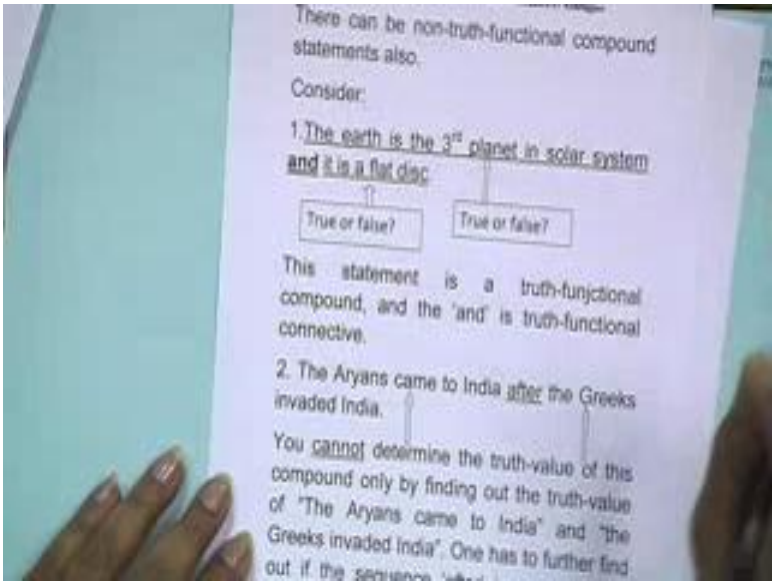

Let us take other example to see the difference so that we can put contrast here. So what we are trying to understand is that there can be compound statements, which can be non Truth functional in nature. Let us move slowly first let us consider this example. Suppose I tell you the earth is a third planet in solar system and it is flat disc. The question is how do you know this is true or false, what will you do and you will probably say well what I will do is find out whether this true or false this component, then I take this one whether true or false and we find that this is true, this is false in somehow we figure out that the whole statement truth value is so that is our process, which makes this proposition truth functional this makes also the connective truth functional alright.

Now, this compound and its truth function. Now compare this scenario with say this one, We give you the second example like this the Aryans came to India after the Greeks invaded India. As you see it is a compound statement, here is the aliens came to India and the Greeks invaded India those are two components. But the question is how we know whether this whole sentence the compound is true or false. Are we going to say that first we try to figure out that one Aryans came to India whether this is true or false and then we tried to figure out the Greeks invaded India or not, if you do that you are in the wrong track, why? Because you cannot determine the truth value of the whole thing just by looking at the truth value of the component; even if it is true, but the Aryans came

to India, even if it is true that the Greeks invaded India, that does not tell you about the truth value of the whole sentence, why not? Because there is an after, so you need to know the historical sequence in which these events have taken place.

If you need to know the historical sequence even after you know the truth value of the components, you are dealing with a compound statement that is not truth functional. Had it been truth functional all you need to know is the truth value of the components that is it. If you need any extra information in order to compute the truth value of the whole thing, then you are dealing with a non truth functional compound statements. So here there is a connective, but note is that it is not the truth functional connective. There is a compound, but the compound is also not Truth functional.

So I leave it in front of you to understand the property of truth functionality better, that this is an example of what we would call truth functional compound. Why? Because all you need is the, is to know what truth value of the components have and that is how you compute the value of the compound. Not so in this case, where you have something else going on, you need something more. Therefore this is non truth functional compound.

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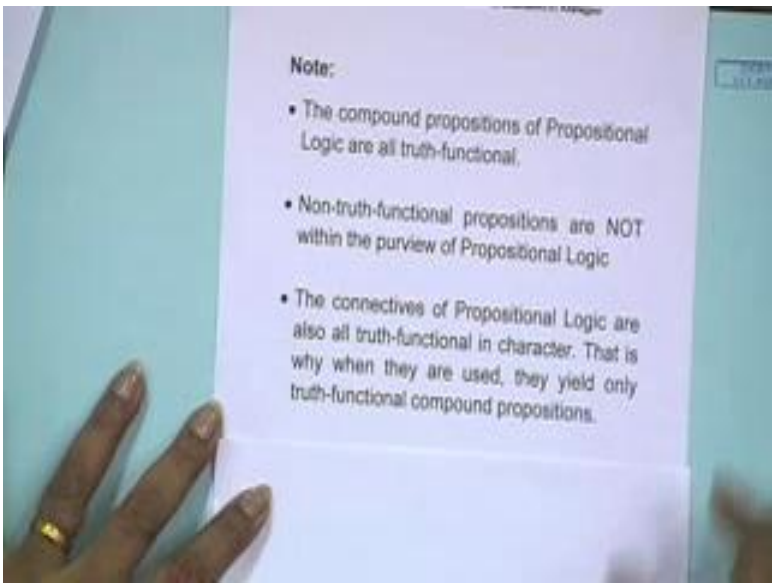

So it is an important property for Syntax of propositional logic and remember the reason why we are looking into it as part of the Syntax, is because the compound propositions of propositional logic are all truth functional. I told you that important logical operations will depend on this property. The procedures that we are going to learn truth tables, truth trees etcetera they are all dependent on this property that the proposition compound propositions are all truth functional in character.

If you find not truth functional propositions, please remember their not within the purview of propositional logic. So if you have non-truth functional proposition syntactically they are not acceptable. You are going to have Syntax error, if you are dealing with non truth functional propositions in this logic.

And finally, we need to remember we are able to soon talk about the connectives, that the connectives of propositional logic. I told you there are 5 of them and we are going to introduce you to them in the next module, but they are all truth functional in character and that is why when they are used to combine other propositions, they always yield, always generate truth functional compound propositions.

With that I am going to end this module and the next module we are going to talk in details about Connectives and other things.

Thank you.