Symbolic Logic
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Lecture – 40
Understanding Overlapping Quantifiers Sharing Predicates and Scope
More on Statements with Multiple Quantifiers
Learning to Use Multiple Quantifiers

Hello. We are at the Module 40 of NOC course in Symbolic Logic. This is the end in the last module of the series in which you are learning about this subject symbolic logic. I have explained what multiple quantifier situations are in our last module. And what I am going to do now is sort of put some more refinement on that, the multiple quantifier situations specifically the one where there are more than one quantifier sharing a predicate, and therefore sometimes sharing the scope also, so embedded scope sharing of a predicate and more than one quantifier; that is what we going to look at.

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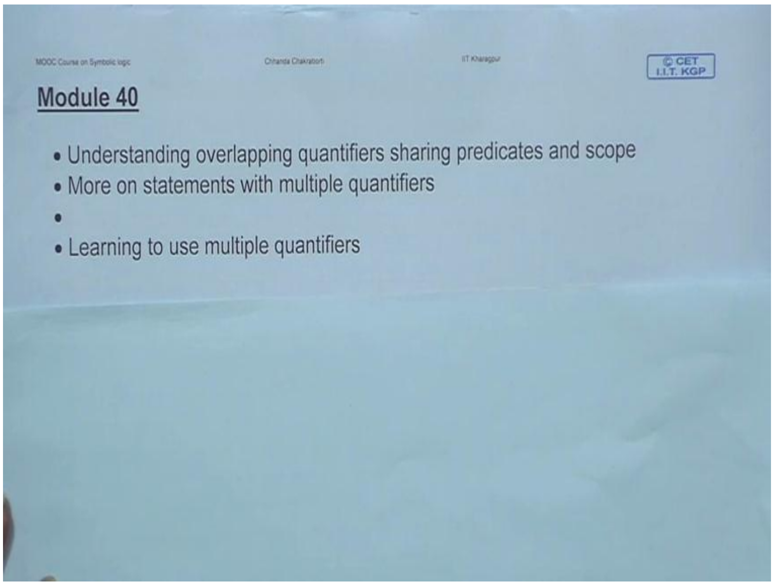

So, that overlapping quantifier situation is going to be our last concern and will try to read it together. And that should give us some idea about how to handle the syntax of statements where there is more than one quantifier present. And this is where we are going to draw the line on first order predicate logic.

Remember we have just talked about the syntax of first order predicate logic; we could not give time to the semantics or the proof systems of first order predicate logic, maybe in some future maybe we can expand the course to cover those also. But right now we are looking into advance label syntax of first order predicate logic, so overlapping quantifiers what to do with them and so on.

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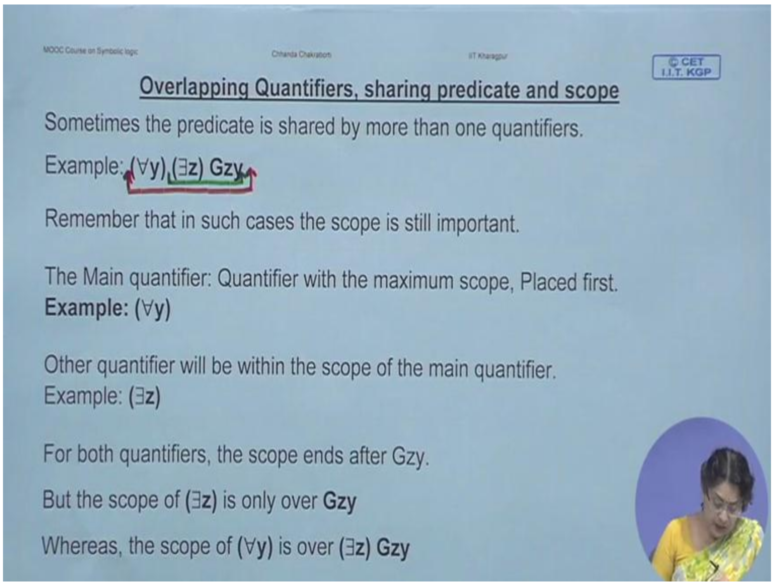

As I said the sometimes the quantifiers may be overlapping. Sharing predicate as well as the scope, we are not just talking about the presence of quantifier in a given statement. So, we are not just talking about same sentence with many quantifiers, but you are actually saying the same predicate is being shared by more than one quantifier. And you can imagine that is very likely to happen in case of the two place predicates. We have seen some samples two place or n place predicates, we seen some samples in module 39 also.

But this is to bring the matter to the sharpest of the focus by say examples like this you have (Refer Time: 03:06) y their exist at least one z such that z is g to y Gzy means z stands in g relationship with y. How to read that? And what does it mean and in this case as you can see both the quantifiers' role over the same predicate and notice that the scope is also kind of within one another. And here also keeping this scopes separate or giving the scope enough important to understand the predication is going to be quite important.

In this case there are two quantifiers, but as you can see there is one main quantifier which is the first one; for all y. Why, because it as the maximum scope. You know the quantifier with the maximum scope will become main quantifier. Just as we discussed in propositional logic the main connective is the one which has the maximum scope. In here, what is the scope of this for all y, as you can see its start from here, and it goes all the way up to here, how? Well, as you can seen the y there are two occurrences of y here one is with the quantifier expressions itself and the other one is adjacent to the quantifier.

Now, this happens to be adjacent to this quantifier also, but this y is within the coverage of main quantifier. The other quantifier that you see here is what you will say the sub quantifier. This one is the main and this one is sub because it does not have the maximum scope, and its scope lies within the scope of the main quantifier. Now what is this scope of there exist at least one z. Take a look it starts from here and it ends here. Now see in the same predicate is within this scope of two quantifiers, get me and this is the kind of situation that you going to see more and more. This is where we need really work with the quantifier themselves the scope and so on so forth.

So, in both cases the scope ends with Gzy, but this one as a smaller scope as you can see and this one as a larger scope. This is what we would call not just multiple quantifier, but this is a overlapping quantifier.

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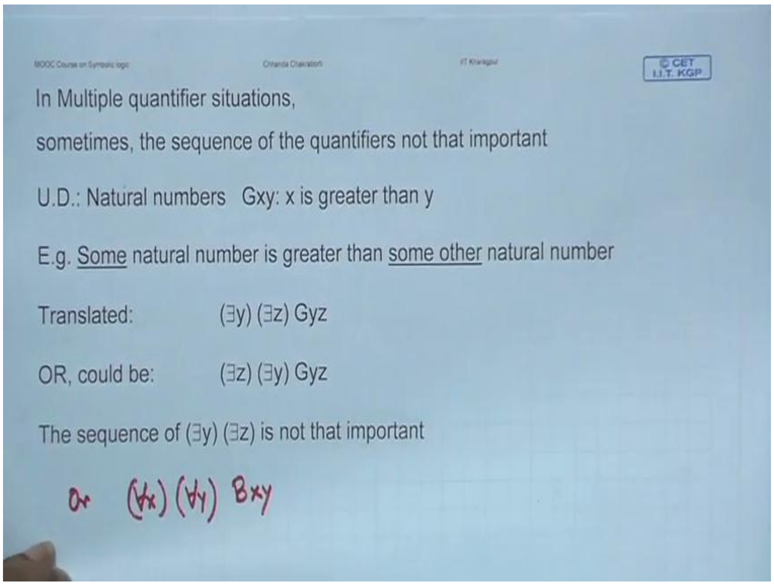

As you saw that in the overlapping quantifier situation we had two quantifiers lined up one next to the other, the sequence. Now sometimes this sequence is not important. The weight edge of one quantifier is not over the other. Let us take a look, sometimes this sequence is not going to matter that much supposed we have this sentence in front of you. The UD is restricted and it is restricted to natural numbers. So, there is no property being a natural number, because the domain is full of natural numbers only. And the only property that we are allowing here is Gxy which stands for x is greater than y, it is a two place predicate.

There we have to express some natural number is greater than some other natural number. Take an arbitrary; the sum is there is at least one if there exist at least one natural number such that it is greater than some other natural number. When you say some other we are referring to two different individuals. Two different individuals without naming them so unspecified therefore we are going to need quantifiers. How many quantifiers? Two; one is greater than the other. Now this is where as you can see there will be a overlapping quantifier situation sharing the predicate namely Gxy.

Take a look, the translation should be easy you probably already will do this. Does not matter with you using y z you can use x y you can use u v, but the matter is stands. So, there exist at least one y and there exist at least one z such that y is greater than z. Now in here, will it matter if I put this z first and y next the sequence. The answer is no, it does not matter. Because all we are planning to say is that some natural number is greater than some other natural number, both are unspecified. Had it been a specified individual, if you wanted to say 12 is greater than 9 that is a different story, 12 is greater than 9 is not equivalent to saying 9 is greater than 12; but there we would not be using quantifiers 12 and 9 these are completely specified individuals unique individuals in your natural number domain. So, you should have used individual constant instead.

But here the reference is unspecified and we are referring to unspecified individual we want to keep the reference unspecified, so which is why we are using the quantifiers. All I am saying is when you have some connected to some other in your domain the sequence of the quantifiers is not really that important. Either way you can make sense of that. So, the sequence in this case will not be important as suppose to other cases which I am go to show now that there the sequence is going to be quite important.

Similarly, when you have two universal qualifiers lined up rolling over the same predicate Bxy. If you interchange these quantifiers it is not really going to matter much, the meaning is not going to be hampered much. If you want to keep it that everything in your domain is connect into every other y in this way (Refer Time: 10:20) that soviet. So, when you have this kind of same kind of quantifier lined up the sequence is not that important could be important, but it is not that important as in the case when you have a mix sort of a quantifier. Mix sort of quantifier why because the first one which comes determines the meaning that is the main quantifier and it changes the meaning, because the whole emphasis on the main quantifier. So, if you met the sub quantifier domain the meaning changes.

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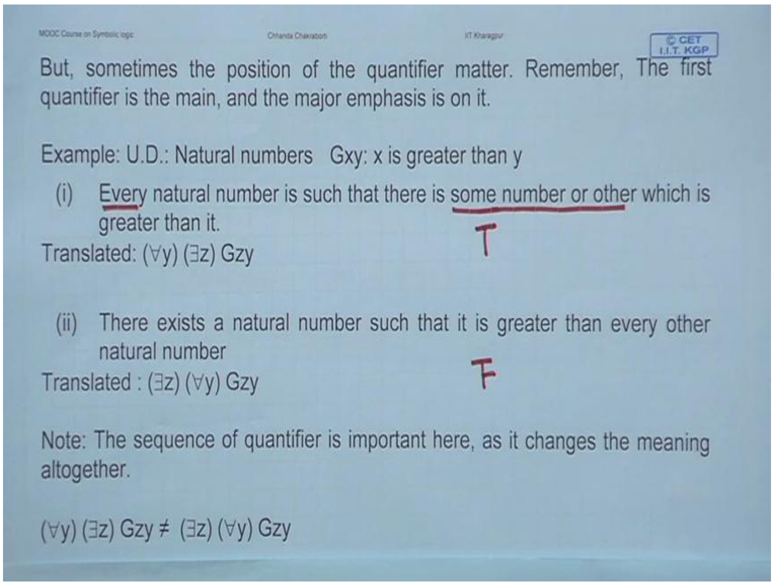

Take a look, again we have the same UD natural numbers and we have Gxy which is x is greater than y. There if your sentence is every natural number is such that there is some number or other which is greater than it. In other words what you are trying to say is true about the natural number series that for every natural number there is at least another number which is greater than it, which is why we say there is no greatest natural number right there is no such thing.

So, that is what you here trying to explain that take any x if x is a natural number then there is some other natural number which is greater than it. First of all notice that we unspecified references to natural numbers and there are two quantity term is every some number or other, every stands for the universal quantifier. Some number or other would refer to existential quantifier. And here we are not or we cannot use the same variable. If you use the same variable what will happen, the meaning will be destroyed the scope compact will be there and you cannot make out what is being said in the sentence.

But what is the translation of this? Translation might be like this. Now, again it does not matter with you using x y instead of y z I have used y z. This is for all y there exist at least one z. Remember the UD is natural numbers, so we are referring to natural numbers only. For all natural numbers there is at least one natural number such that that z is greater than the y. This is things; this is one kind of a thing where the point that I am coming to that this sequence here is important. Just see that, that if we change the sequence you are going to give raise to another sentences all together which says there exist a natural number such that it is greater than every other natural number. What have you done? Change the position of the quantifier. Take a look now, we have there exist at least one z such that for all y Gzy.

And this sentence reads as there is such a number such that it is greater than every other natural number. As you can see the two sentences are different, so the sequence change that and this is not equivalent to saying this. This one is true, if you know anything about the natural number then this is true but this is false and this is true. There is nothing called the greatest number and this is perfectly with natural number series.

Now, what we need to learn from there is a take away from all of this is that the sequence of quantifier can be very important it changes the meaning emphasis the way to read the sentence you change that if you changes the sequence. So, there are cases when tempering with it is going to really matter and those cases specifically are where you have a mix sort of quantifiers, there is a mix sort of universal and the existential.

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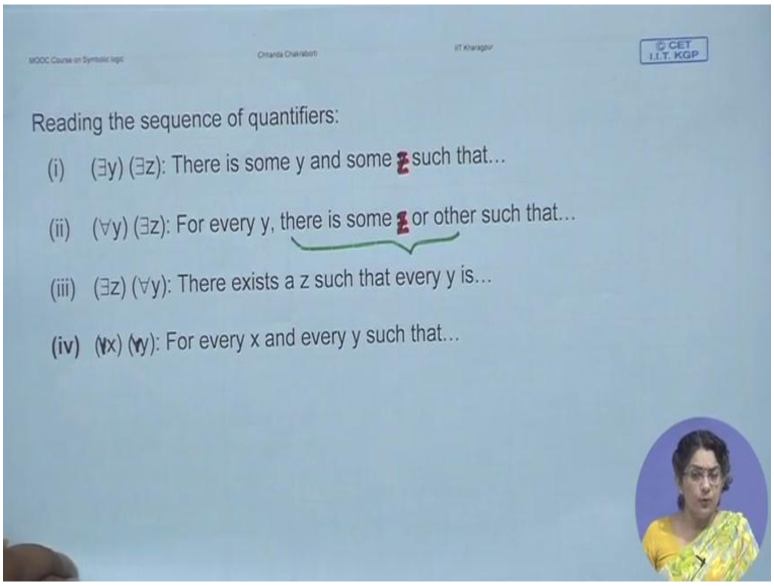

Now how do we read the sequence of quantifiers is like this, that when you have only the existential the way to read it is there exist some y and some z such that etcetera. This is for all y or every y there is some z such that etcetera, some z or other. Notice that this is somewhat lose, some z or other there is nothing definite, because the way you put it in the second position it sure sort of relegate said to rather loose interpretation. The main thing is this for every y; some z there some z or other there exist some z or other. This on the other hand is empathically saying there exist z such that for every y etcetera, etcetera. So, this is where the emphasis place and this is where for all x and for every y such that etcetera. So, we have covered what is known as the overlapping quantifiers and where they are sharing the predicate as well as the scope and so on.

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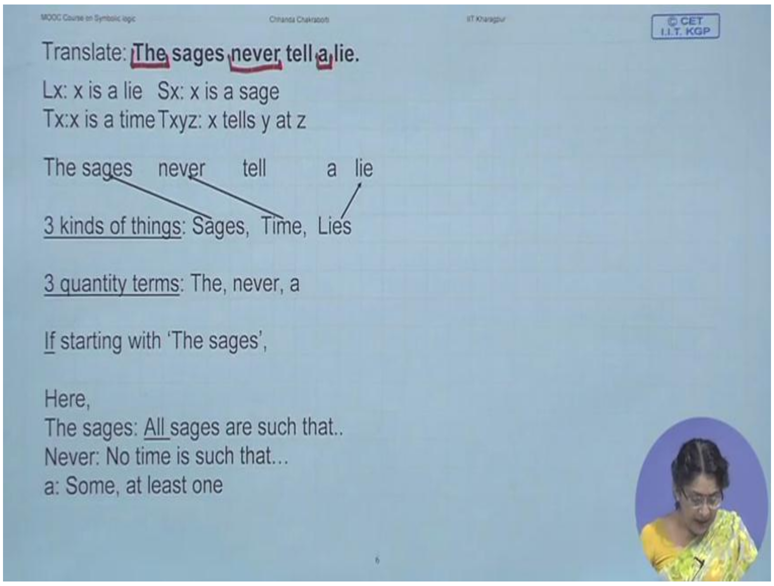

Now, the last thing that our agenda is to sort of put it all together all this knowledge that we have about multiple quantifiers, about reading the predication correctly, about the scope and so on. Let us try it out with the complex sentence like this. The sages never tell a lie. Is that a quantifier sentence? Can we use quantifiers here or their quantity terms here, because we do not find any 'all' we do not find any 'no' we do not find any 'some'. But, we took (Refer Time: 16:59) I am sure you have notice that there is remain this possibility.

Let us take a look first of all what we have here, the properties. Lx is x is a lie, Tx x is a time Sx x is a sage, Txyz that is a three place predicate; x tells y at z. How many kinds of things we are talking about? We have the sages, we have the lies and in this never we are referring to time you see. So, that should tell you that we have three kinds of things. Three kinds of things mean we have going to need at least three kinds of quantifiers. How many of these things. So, sages are one, never as I said never as a not ever no time, so time is one kind of thing and then there the lies, so those things. Three quantity terms are there in case you have noticed, if you have not let me help you. There is the sages, that article mean something, it is to be interpreted but it is a quantity term. Then there is obviously the never that is a quantifier and then there is this article again called a or a. How to interpret those things?

At this point let me tell you have three different things, you have three different choices how you are going to read this sentence, and how you are going to see the predication, right. You can start with the sages as the main group and say that we are talking about the sages about whom the rest of the sentences predication. So, sages are such that they never tell a lie, get the predication sages is the subject and they never tell a lie; are such that they never tell a lie the whole thing is predicated to the sages.

But you have a choice you can also go with the time, instead of going with the sages you can target time as your main subject. All kinds of such that sages never tell a lie or better yet no times as such that the sages tell a lie at a time. This is one kind of predication or if you want you can also start with the lies, so all lies as such that the sages never tell them. Now, this is the kind of libration that you have achieved. This is the kind of freedom that you have achieved after learning all that predication and first order syntax this that; where do I start? You can start I am telling you. You can you all unit do is to fix one subject group and then there remaining predication can be done very easily.

Now, suppose you are starting with the sages then you are saying that the sages or such that etcetera, here what you mean by that the. One particular sage, at least one sage is such or all sages are such. You are right it is a all sages are such that etcetera. So, you understand if you are going with sages you are going to deed a universal quantifier to start with. When you said never and I had already tell you never means no time is such that. Even here you have a reference to a universal quantifier. And then a, a lie, how many lies you what the sages to be telling not even 1. So, a here is at least one or some.

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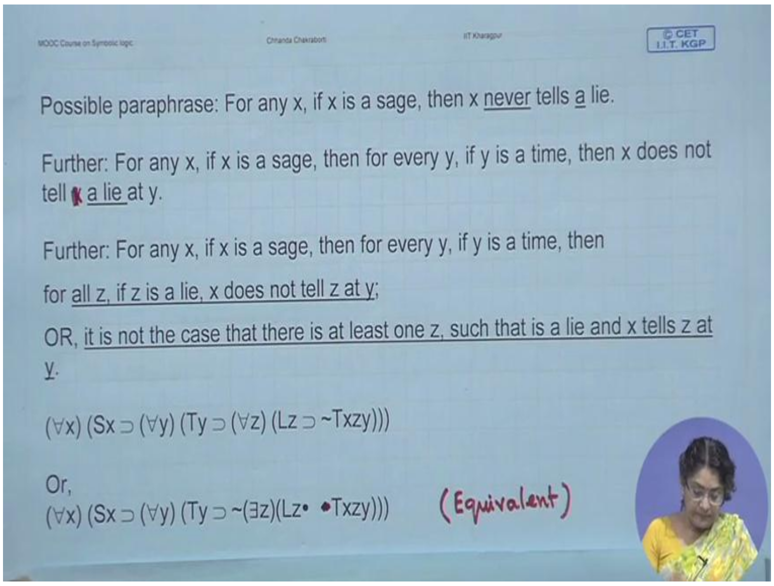

Let us work with the paraphrase because that is where we are going to spend some time. So, we are starting with for any x or for every x if x is such sage then x never tells a lie, that is the first thing to do. So, we have fixed the subject term and then the remaining predication we have put together.

Now, let us break this part keeping this part constant. For any x if x is a sage then x never; never means what no time. Then we go like this for any x if x is a sage then for every y if y is a time then x does not tell a lie at y. Once more, for any x if x is a sage then for every y if y is a time then x does not tell a lie at y. We have kept the a lie untouched, we have only un fact they never and this sentence keeping the first part and the n part intact; did you understand?

Next is then a lie. So, we keep it you all together for any x if x is sage then for every y if y is a time then for all z if z is a lie x does not tell z at y. So, all lies are such that x does not tell them at y, let us see. This part notice that we can rewrite if you recall the contradiction relation then this is e we can also replace it with negation of I. So, this part we can rewrite it also, it is your choice whichever way you want go you can rewrite it as it is not the case that there is at least one z such that z is a lie and x tell z at y. Once more you can go with the e, so for all z if x is a lie x does not tell z at y this is the not. Or its equivalent is I proposition but its negation of I it is not the case that is a negation that there is at least one z such that z is a lie and x tells z at y.

Let us take a look into the translation the translation will may look like this for all x if x is a sage then for all y if y is a time then for all z if z is a lie then x does not tell z at y. All we said it is equivalent fine the equivalent would be something like this, for all x if x is a sage then for all y if x is a time then it is not the case that there exist at least one z such that z is a lie and its tell y, sorry for that. So, x tells z at y.

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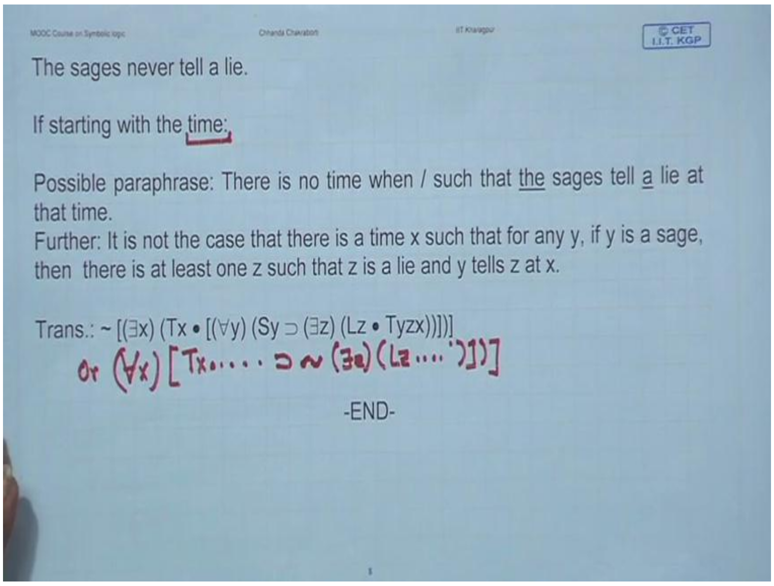

So, this was our never telling a lie with the sages, but the grouping if you starting with time then how do we do that. Remember then time becomes the main subject. So, there is no time such that sages tell a lie at that time, this is what you will say. So, the paraphrase will look like this it is not the case, that there is at least one time x such that for any y if y is a sage then there is at least one z such that z is a lie and y tell z at x.

And the translation would look either like this or like this because they are equivalent. This is it is not the case that there is exist at least one x such that x is a time and for all y if y is a sage then there is at least one z which is a lie and y tells z at x. Now whole thing they changed y because we started with time, no time is such that or it is not the case that there is even one movement of time when x the sage is would do this. This is the equivalent except that it is a e proposition. You are starting with universal quantifier and tilde would appear at the consequent because that is what the propositions do.

This is just give you an idea how to handle this complex and overlapping quantifier situation. Did you see that that there are embedded quantifiers in it and we are rolling over the same predicate. So, each one sort of his connected, it is not a very clearly separate scope no conflict and so but even there we have to ensure that the variable that we have chosen for one group and we keep the reference to that group alive until it is needed. So, in this case the whole three place predicate appears at the end of the proposition, so the quantifiers also remain the scope remains open until the very end.

That brings us to the end of our lessons. I hope you have learnt something out of this, I really hope that you will be able to use it somewhere in your studies or any of other research maybe. And its always hope that will be able to expand this course more and that there will be a greater utility coming out of it to every each of you. So, all the best to all of you and thank you very much for your time and patience, enjoyed teaching you.

Thank you very much bye.