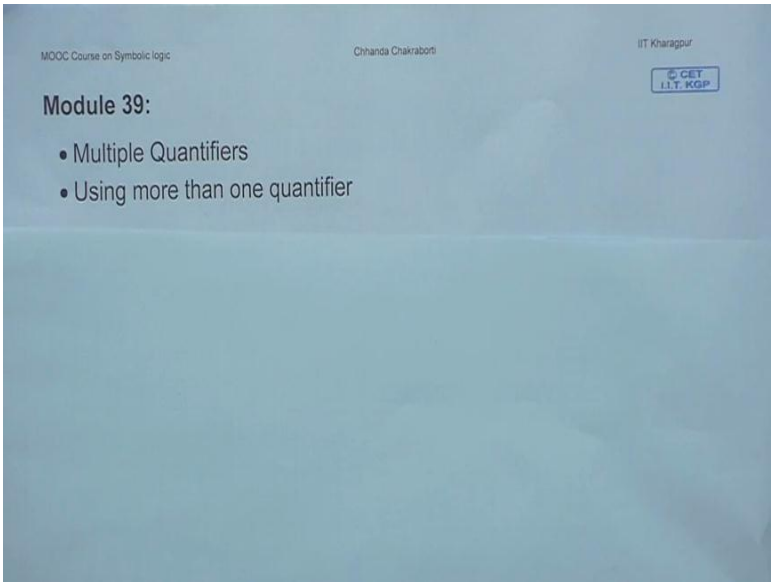
Symbolic Logic
Prof. Chhanda Chakraborti
Department of Humanities and Social Sciences
Indian Institute of Technology, Kharagpur

Lecture – 39
Multiple Quantifiers
Using More than One Quantifier

Hello, we are in module 39 of the Symbolic Logic Course and we are going to start talking about what is known as the Multiple Quantifier Situation. So far when I have shown you what quantifiers are how to use them in a sentence I have shown you with one quantifier, but there can be situations where you need to use more than one quantifier that is called multiple quantifiers proposition.

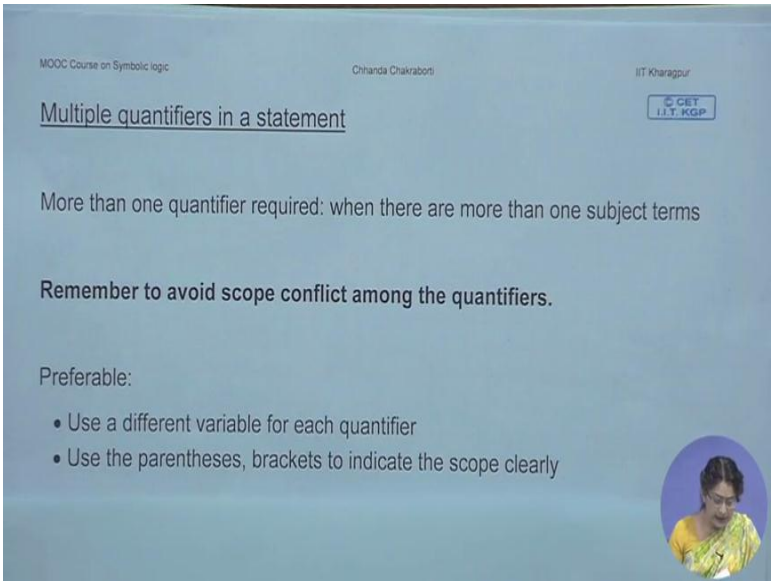
(Refer Slide Time: 00:49)



And when you are using more than one quantifier what is it that you need to be careful about how to use these lessons to understand the multiple quantified proposition of first order predicate logic this is what we are going to talk about today.

So, with that I shall begin the module 39, say multiple quantifiers in a statement.

(Refer Slide Time: 01:23)



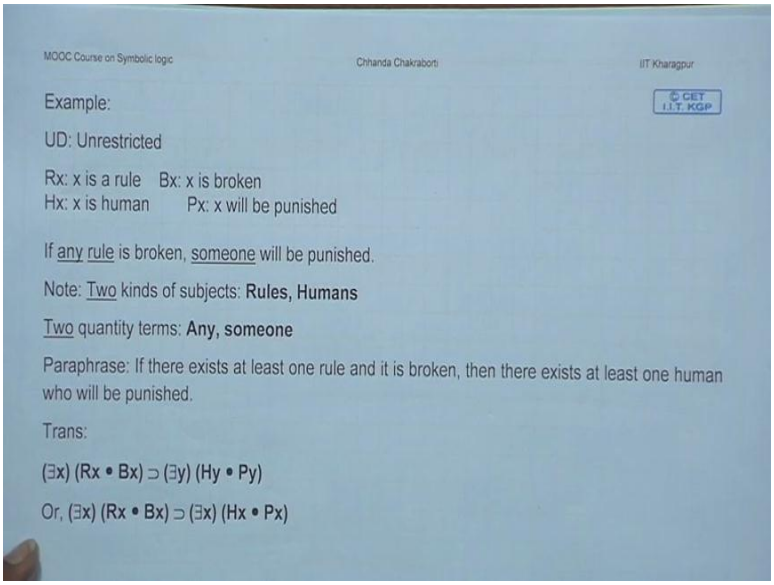
Now, you might think that why would I even think about using more than one quantifier because it seems like a given sentence we are talking about a certain group and we are talking about the quantity, my answer to that is you know you need to look at proposition slightly carefully from now on. There will be some times references is to more than one subject terms see we are doing predication right, I mean we are attributing properties to individuals to some non specified groups and so on. It not necessary that we shall always talk about only one group or one individual, so predication can happen simultaneously in a proposition using the reference to separate groups whenever that happens when you have more than one subject term reference you need more than one quantifier I will explain this and I will exemplify this with; obviously, with actual proposition.

But this is the first lesson of today that if you are asking yourself when do I need to use more than one quantifier or how do I know that I am going to need more than one quantifier the answer is that it depends on the proposition that you have if the references are to more than one subject terms then you know that the proposition or the statement calls for the use of more than one quantifier. We will show you as I said with example also.

But when you using multiple quantifier more than one quantifier that is, the first thing to remember is that you need to avoid scope conflict. Remember every quantifier has scope all right and that is specified, what you need to avoid at all cost when you are using more than one quantifier is the conflict of scope between or among these quantifiers because otherwise you are going to have a terrible problem interpreting the variable that you using. The sentence will not make any sense or the sentence might make more than one sense neither is desirable in this logic. So, to avoid scope conflict is our primary concern when you have a multiple quantifiers in a statement or more than one quantifier actually occurring in a sentence. We will show you how do you do that, how do you avoid scope conflict - well one of the time tested method is that that you choose a different variable for each quantifier each time you are using a quantifier pick a fresh variable. So, this is one way to do that.

So, that because it is a new variable and it is a different variable there is no question about that variable to come under the scope of the any another quantifier. So, this is one way to do that. The other one is that you de market the scope so clearly that even if you are using the same variable it is completely clear where one quantifier scope begins and ends and where the scope the other quantifiers starts and ends we will show this each of these things separately. So, what did I say? I said that first of all you can use more than one quantifier in a proposition if the situation demands that, that is not a problem, but when you are doing that and that situation is called multiple quantifier situation what you need to do is to ensure that there is no scope conflict. In order to avoid scope conflict my suggestion is choose a fresh variable each time you are using a new quantifier because quantifier is going to require the use a variable just choose a fresh variable to go with your quantifier this is one. Or you use the grouping procedures that you have namely the brackets, the parenthesis, the curly brackets, whichever one you are going to use, but use them so that the scope of one quantifier is completely clear.

(Refer Slide Time: 06:26)



So, let us take a look at this kind of situation with some examples. Suppose we have see we are beginners so I am have mentioning that UD is unrestricted, but I told you that if nothing is mentioned then we are going to assume that the UD is unrestricted. So, if I do not mention anything the default is the UD is unrestricted, but since we are just learning it all for the first time, so I thought about making it absolutely explicit. In that you have this kind of predication possibility, so Rx stands for x is rule Bx x is broken Hx x is human and Px x will be punished and we are going to use this kind of translation key to capture this English sentence which says if any rule is broken someone will be punished, if any rule is broken someone will be punished.

As you see that there is this property being broken there is this property being punished, but note that what we also have is two kinds of subjects, two kinds of things you are talking about; not just two kinds of predicates, but two kinds of things which have those properties, what are they? The two kinds of subjects are rules this is one cluster and persons or humans. So, this is rule and you are predicating something to it, namely being broken and there is the humans to which you are predicating the property of being punished and it does not mix, we do not call about rules being punished and humans being broken. So, that is not done. So, that is clear that we have references to different

sets or groups of things and as I told you when the reference is two more than one groups for subject that is when you need more than one quantifier.

So, let us take a look at that. So, we need we are going to need it seems like two quantifier in this sentence, but what kind of quantifiers, what are the quantity terms here well as you can see there are two quantity terms one is any, the other one is someone. So, we need to and none of this are clearly I mean it is not clear that we want we use this quantifier that quantifier now let us see, what does any rule mean here. If any rule is broken the best way to understand that is to ask yourself how many rules need to broken before someone is punished, how many rule - that is what quantifier does, all rules will have to be broken or if even one rule is broken someone will be punished. If you have a answer in your mind (Refer Time: 09:51) at least one rule is broken you are right.

Here this any means if at least one rule is broken then how many people will be punished, someone will be punished means what - at least one human will be punished now then this paraphrases as I say always say this paraphrases is the most important step before you go into capturing this English sentence in your in the syntax of your first order predicate logic. So, the paraphrase of this what we have just discussed we can put it like this, that if there exists at least one rule and it is broken then there exists at least one human who will be punished. Now when we said this this still not in the first order language, so what we will have to say is that if there exists at least one x such that x is rule and this property also you have plug in and it is broken then there exists at least one y such that y is human and y will be punished

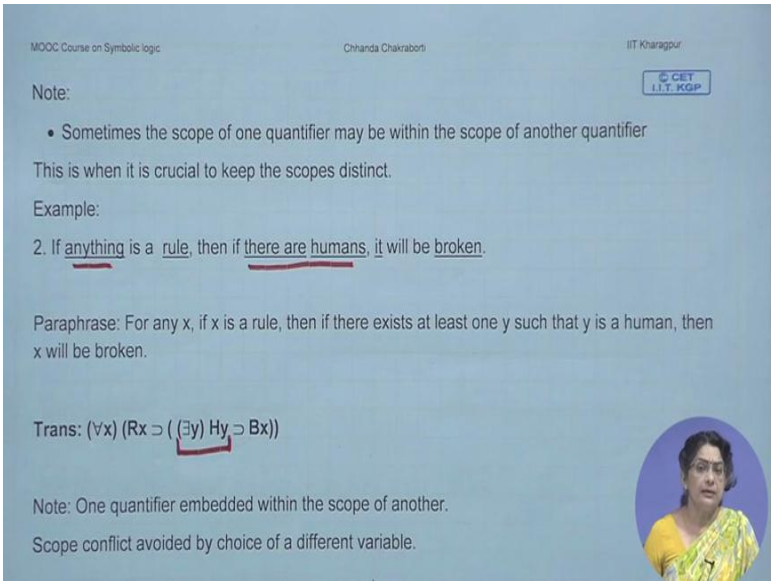
So, there was several steps that in which you can you can divide it up, one is first of all noticing the references and deciding in your mind how many quantifier you are going to need. Once you have decided that, the next thing is what kind of quantity terms or quantifiers to we need here - universal or particular universal or existential. So, that depends on the kind of how good you are in the paraphrase and the paraphrase shows you the entire logical structure which you follow and instead of translate, here we are. We are going to do this then there is this, if any rule is broken, comma then someone will be punished and the translation which going to look like this, two quantifiers.

Multiple quantifiers situation take a look the first one say if there is this is if then, so his whole thing is antecedent if there is at least one x such that x is a rule and x is broken then there exist at least y, one y such that y is human and y will be punished, got it. Now watch what we said you have two quantifiers you also know the scope, the first one scope starts from here and no (Refer Time: 12:58) ends here; it starts from here it ends here because it is a horseshoe when there is no other indication that the entire quantifier rules of the whole thing, no need. If any rule is broken someone will be punished period.

So, this quantifier which rule rules over the rule does not have to rule over the whole sentence correct, it simply starts here and ends here. Then comes the horseshoe sign if then and then comes the new quantifier whose scope starts from here ends here, no scope conflict, but we choose a different variable also. Though in this case there is no question about scope conflict and I will show you how else you can do that, but still we choose a different variable to keep this completely-completely specified and distinct from each other. You could have done this also in this case, because the scope is completely different as you can see starting from here, here, starting from here, ends here.

So, here if you had used even the same variable you have the brackets in place, parenthesis in place to indicate that the scopes are not going to ever come in conflict, get it. So, is this allowed yes this is allowed you can use the same variable provided you are indicating this scope of two quantifier completely distinctly if you have doubt regarding that I suggest you use a fresh variable. So, that is where translation is. So, did you understand that this is the kind of situation where you are going to need more than one quantifier this what we are calling the multiple quantifier situation and handling them is as you have done in the case of single quantifier, but the point is to keep this scope separate to know which group is being governed by which quantifier and so on and so forth and you paraphrase must be done, your paraphrase is going to actually show you the way to the proper translation.

(Refer Slide Time: 15:21)



Now, this example that you saw just now we had two quantifiers, but the scopes where quite different, so no none of the quantifier interfere with this scope or came within the scope of other quantifier that may not always be the case. So, sometimes you may have situation where the scope of a quantifier may be within the scope another quantifier. So, scope of one quantifier embedded within the scope of another quantifier that also is impossibility that we need to sort of look into and that is the situation where I have to again emphasize that the scope conflict must be avoided, must be avoided because here it is not automatically separate you need to take some steps (Refer Time: 16:16) steps to keep this scope separate. Let me show you what we mean when a quantifier scope comes within the scope another quantifier. Earlier we have seen that sometimes quantifiers share a predicate, right and then there you have seen how the meaning changes with the sequencing of the quantifiers and so on.

But here is another kind of an example, take a look. If anything is a rule then if there are humans it will be broken the sentence is if anything is a rule same very properties. So, being a rule then if there are humans it will be broken it will be broken what will be broken; obviously, not the humans what will be broken? The rule, so that is an indication that whatever variable you are choosing for the rule and whichever quantifier you are using to govern over this rules will go up to here to cover this it which will be broken

once more there is a reference here because you are using pronoun and the noun that you are referring to use by using this it is a rule. So, it is obvious it, should be obvious that whatever quantifier you are using and with the quantifier with whatever variable you are using to refer to the rules will cover within its scope - from here up to here, why up to here? Because otherwise this reference will remain free you want cover even this reference, this it is nothing but the rule which will be broken get it.

So, unlike the previous statement which was a truth functional compound it was a horseshoe statement this one is going we are fully quantified statement that is one thing, but we also need to know that we have two kinds of subject on references. We are referring to rules and we are also referring to humans and then there are this properties also that you know if there are human, so being a human is going to be attributed predicated to one group and being a rule and being broken will be attributed to a different kind of a group. So, predications simultaneously will happen (Refer Time: 19:07).

Now, what are the quantity terms? Obviously, we are going to need because we are having two separate groups of subject term, we are going to need two separate quantifiers fine. But what are the quantifier terms and what kind of quantifier are going to need now here we see there is anything, if anything is a rule. So, again you need to ask how many things have to be rule, if anything are rule and here is a rather strange kind of quantity reference if there are humans - if there are humans how many humans there have to be for the rule to be broken and the answer is if and if there is at least one human rule will be broken. If anything is a rule what you say everything, take any x if it is a rule or take whichever x if it is a rule or we want to say there exist at least one x such that if it is rule it will broken the answer is that we have a universal reference here.

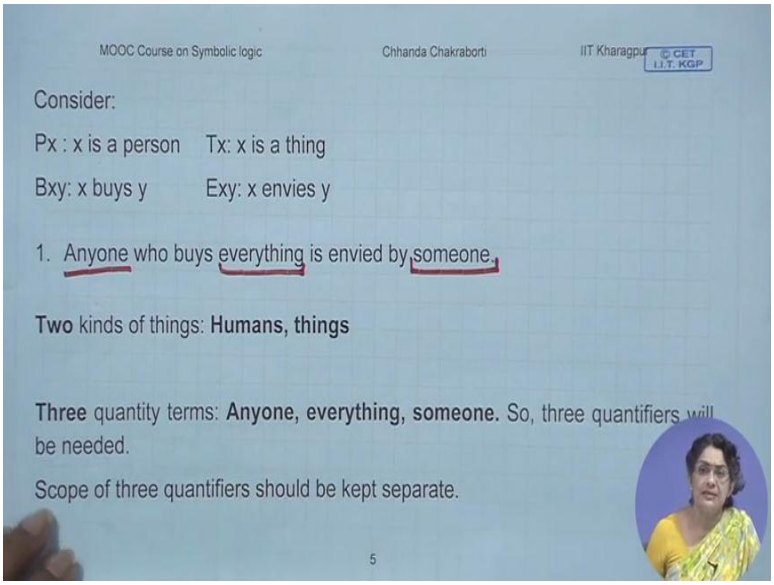
If there is even one; it is not a question about there is at least one x such that its rule it is about take anything if that happens to be rule if there are humans it will be broken that is the meaning of the sentence. So, the paraphrase, the paraphrase is something like this for any x I am choosing x, if x is a rule if there are humans if there exist at least one y such that y is human then x will be broken.

So, same x will be broken in between you have if there exist at least one y such that y is a human notice that here is; if there is a then within the then there are if then. So, we need to capture all of this, but we will start here for any x let us do that, for any x if x is a rule then this is the main (Refer Time: 21:39) that, but it is within the quantifier scope. So, the scope of this quantifier the one that is ruling over the rules will start from here it will go up to here. So, take any x or every x if x is rule then, if there exist at least one y such that why is human then x will be broken, what are we looking at? We are looking at a multiple quantifier's situation.

But this multiplier for multiple quantifier situations is such that one quantifier is within the scope of another quantifier, embedded, did you can see that? The scope of this for all x starts here and goes up to here. In between you have an existential y and its scope starts from here it ends with this y, remember that the variable that occurs with the quantifier is covered by the default and the variable adjacent to it is covered unless you have some other kind of brackets going on. Now here the only to y occurrence are these. So, there is no question about extending this scope of it until then. So, it is just starts here it end here, right here, that is a scope of this quantifier. The rest of it is within the scope of this for all x, as you can see the closing brackets sort of ensures that, this bracket it starts here, this bracket is starts here.

This is why we said again and again that you need to use a fresh variable keep the scope completely clear, especially when we have this embedded kind of multiple quantifiers situations is it alright have you understood that. So, this is one kind of a situation the previous one was another kind of a situation where I was trying to explain that you can handle it. Provided the paraphrase is correct and that you take care of indicating this scope also very very clearly. Let us try this one more.

(Refer Slide Time: 24:03)



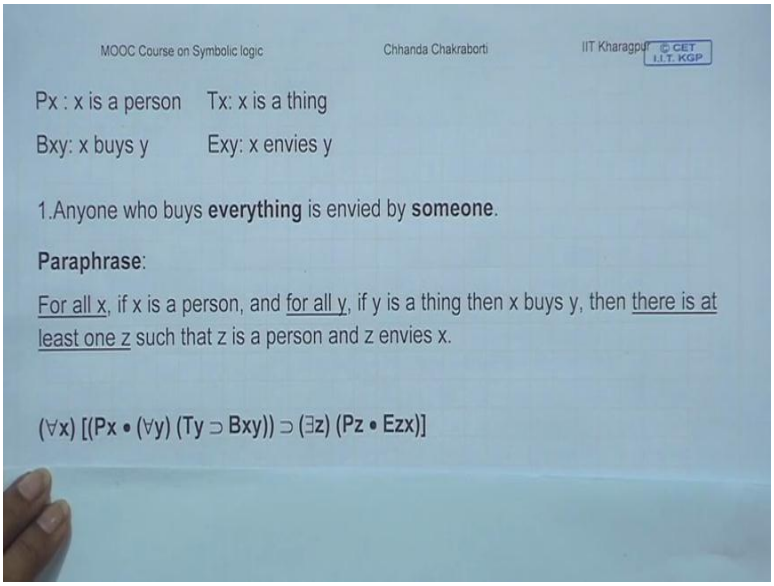
I am not mentioning the universe discourse which means that it is unrestricted. So, we are talking about this. And sometimes by looking at the translation key also you can make out how many quantifiers you are going to need. This is Px x is a person Bxy x buys Yt x, x is a thing Exy x envies y; how many kinds of things we are talking about? Here is a person kind of thing and the thing kind of things. So, at least two kinds of things we are talking about therefore, we are going to require two quantifiers in this sentence.

Let us first take a look into the sentence itself - anyone who buys everything is envies by someone; obviously, what we can call it consumer jealousy right. Somebody who has so much that he buys or she buys everything will be end with buy someone. So, once more anyone who buys everything is envied by someone. So, the first of all let us take a look into any one reference is to the thing or to the persons, two persons. So, this is going to be your looking at group then we are taking about some persons, but we have used the term anyone, so we are going to require a quantifier there - who buys everything, so everything now there you are, that is another kind of implicit quantity reference, everything. Envied by someone that is another kind of a quantity reference.

So, we have two groups - people and things within this people we have somebody who buys or people who buy everything and people who envies those people. So, within that therefore, within the people we are going to require two quantifiers because envies somebody. So, that is going to happen and notice that these are all two place predicates. So, let us open it up, first is as I said two kinds of things, but within the humans we have people who buy everything and people who end with those who buy everything. So, there we are going to need another quantifier and the quantity terms are all indicated.

Now, paraphrase, the three quantity terms are also identified anyone everything someone fine now quantifiers. The point is that the scope of these quantifiers needs to be kept separate; we are now going to do anything that sort of messes it up to see the quantifier, the scope.

(Refer Slide Time: 27:12)



So, here is the paraphrase once more, anyone means what anybody as everybody; everybody who buys how many things everything, everything has to be universal everything. Envied by someone means at least one person envies that. If you have got the hang of it you have grassed it paraphrase is going to look like this for all x, if x is a person then for all y if y is a thing that x buys y, then there is at least one z such that z is a person and z envies x, see whether I am correct or not. So, for all persons, all persons

buys all things then there is at least one person who envies the buyer this is how we are going to do that and the translation if you recall now, now we are just going to plug it in for all x, if x is a person right the property predication has to happen and for all y if y is a thing then x buys y then this whole thing then there is at least one z such that z is a person and z envies x, we will you see that how it happen.

So, this is the kind of multiple quantifiers' situation which is more advanced kind of predications where we need more than one quantifier and I have shown you in this module how to use them by keeping these scopes distinct from each other. More to follow and we are about to end this lessons on first order predicate logic soon, but see you again in the next module.

Thank you very much.