Symbolic Logic
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Lecture - 38
Scope of a Quantifier
Free and Bound Variables
Difference Between Quantified Propositions and Truth-Functional Compounds in Predicate Logic

Hello. We are back with Module number 38. And we are going to talk about little bit more about the quantifiers.

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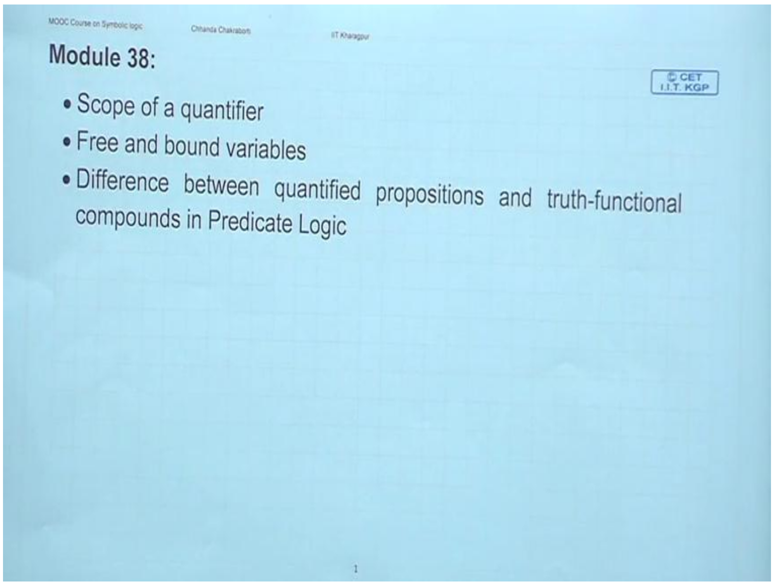

Quantifiers have been introduced to you in the earlier module symbolically and how use them in translations and so on, but we have not spoken about the scope of a quantifier. What is that and how to utilize that knowledge and so on will talk about it. But scope of a quantifier will help us to identify also an important thing namely the free and bound variables.

So, we will use the idea of the scope of quantifier to define free and bound variables. And then we also will utilize that scope of quantifier too talk about the difference between quantified propositions and truth functional compounds in predicate logic. We have both kinds in present in predicate logic, so some of you might be confused why we are calling this proposition quantified, we why we are calling this propositional truth functional compound. That confusion should be cleared, so will talk about that also. This is what we are going discuss today in our module number 38.

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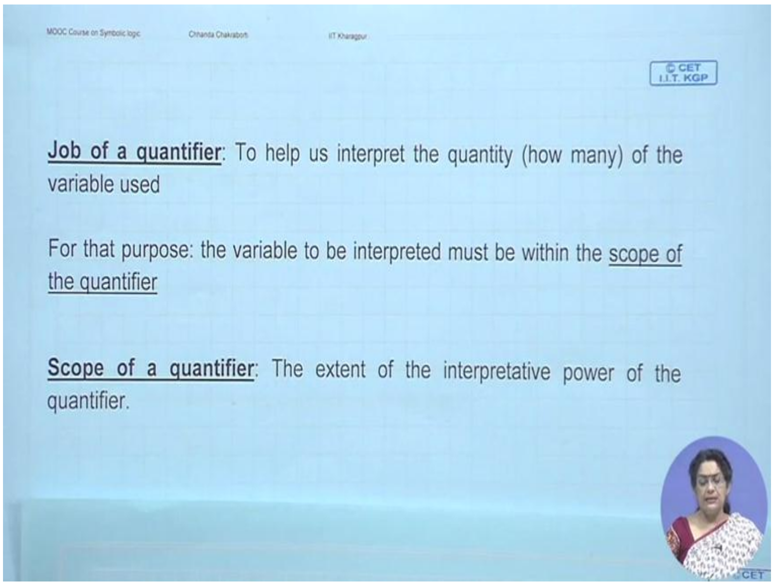

See the job of a quantifier as I have told you is to interpret how many of the variable used. So, you are talking quantity of the variable used. But for that purpose the variable has to be the variable that using has to be within the scope of the quantifier. The interpretation of the variable in terms of quantity can happen only if it lies within the scope of the quantifier. But what the scope mean here? What does it mean to say the quantifier scope is this? Well, scope here means the extent of the interpretative power of quantifier. How far can it interpret the quantifier? This is the idea and will try to, this our initial understanding of water scope is the limit or the extent of the power of the quantifier to interpret the variables. This is what we mean by a scope of a quantifier.

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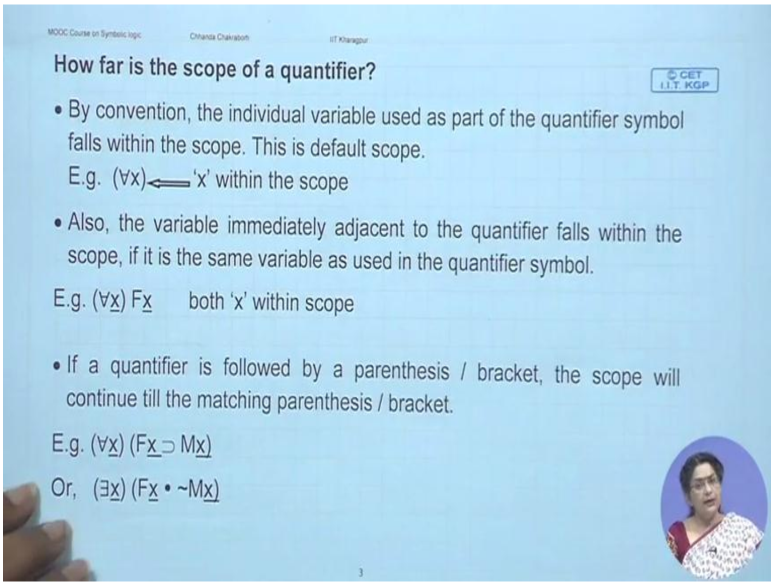

Now, comes the (Refer Time: 03:03). How far does the scope of quantifier go, how will be we know that this is where the scope of the quantifier is and this is where its not. So, let us talk about that. Now slowly we will build it up that by convention you know I have told you that quantifier comes with variable that is part of a quantifier symbol. Remember by convention the individual variable that was using as part of the quantifiers symbol is already within the scope of the quantifier. That is the default for example, if you are using for all x this is how you use quantifier right within the bracket. So, there is a variable present here namely this x and that x by default within scopes this quantifier. So, that automatically is within the scope of the quantifier this is the convention.

But then, there is also you are using other variables after that, only this cannot be full sentence so there other variables. Remember that the variable that is immediately adjacent to the quantifier symbol also lies within the scope of the quantifier. If it is the same variable that is used in the quantifier symbol. For example, if you have this for all x f x. Now here you have two occurrences of the x which is variable. Notice that already I said the first occurrence is by default within the scope of the quantifier and now I am adding to that that the variable occurring immediately adjacent to the quantifier symbol is also within the scope of quantifier which covers the second occurrence of the x. So, that is also within the scope. So, first this x is already within and then because it is immediately adjacent to the quantifier symbol this x is occurrence is also within the scope of the quantifier.

Further, if a quantifier is followed by a parenthesis and if that is the case then the variable that are occurring between the opening and the closing parenthesis matching with the quantifier symbol will be all within the scope of the quantifier. Once more, after this see up to here is default after that if you have a parenthesis coming up and same variables are also used then until the bracket is closed all of that will be assumed as within the scope of the same quantifier. For example take a look, there are three occurrences here the first two as you know where already covered or already within the scope of this quantifier. In order to bring this third occurrence within the scope of the quantifier what we have done is to use this parenthesis get it. So, because now this is the convention to read that this covers this x this x and as well as this x until the matching parenthesis is (Refer Time: 05:36) get it.

So, this is something to learn. Or for example take a look at this existential quantifier. Whatever I have said about the universal quantifier applies to the existential quantifier as well. This x by default is part of quantifier and is within the scope this is being adjacent and this is because there is this parenthesis all round it. And now you probably understand why we have been using this parenthesis all along with the quantifier expression. Mainly to indicate that the scope of the quantifier is from here to here, so here in this case scope of the quantifier is from here all the way to here, from here all the way to here, this is what way established. This is to understand how the scope of a quantifier works in our system.

Now, I am going to use this idea of scope of a quantifier to then further talk about free and bound variables.

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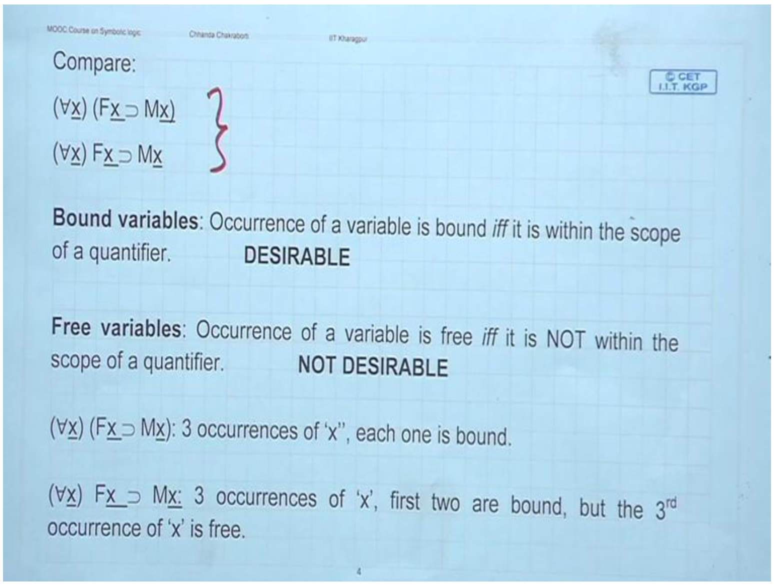

Consider for example this, these two sentences; take a closer look so that you can see what is there in the first sentence and what is there in the second sentence. Now I am going to define what a bound variable is. The occurrence of a variable that is within the scope of a quantifier is called Bound. And it goes other way; if the occurrence of a variable is within the scope of a quantifier then the occurrence of the variable is called Bound, both. This is what is desirable, that every occurrence of a variable is bound in this kind of sense that it is line within the scope of some quantifier or other; get it, this is what is desirable.

On the other hand, there is something called free variables. A free variable is the occurrence of a variable that is not bound in this sense that is, the occurrence of a variable which does not lie in the scope of a quantifier; that is what a free variable is. Remember in our first order predicate logic system it is not a desirable kind of situation, free variables and not really desirable. Why, because these open statements we interpret them. So, variable occurrence is need to interpret also and the quantifier helps us with quantity understanding there are other things also. So, remember free variables are not desirable once what we need is the bound variables situation in every properly formatted sentence of first order predicate logic.

Now, let us go back to the original pair that one we started this page with. If you look at the first sentence then we have three occurrences of the variable x. And as explained already in the previous page every occurrence of this variable x is bound. Now you know what bound variables are. So, I will say every occurrence of this x is bound, because it lies within the scope of this quantifier. Now compare that with situation we have in the second sentence. Again we have three occurrences of the variable x. The first two please note are bound because one is part of quantifier the second one is the adjacent but look at this third occurrence of x. There is no indication that this one lies within this scope of this quantifier. So, this third occurrence is free. And this is the sentence that we cannot allows syntactically as a properly formed a well formed sentence and we do not know how to interpret read this also. So, this is not a desirable situation did you understand that.

So, free occurrence is free variables have to be avoided at all cost. Over goal will be to have sentences where the occurrence of the variable is bound by some quantifier or other.

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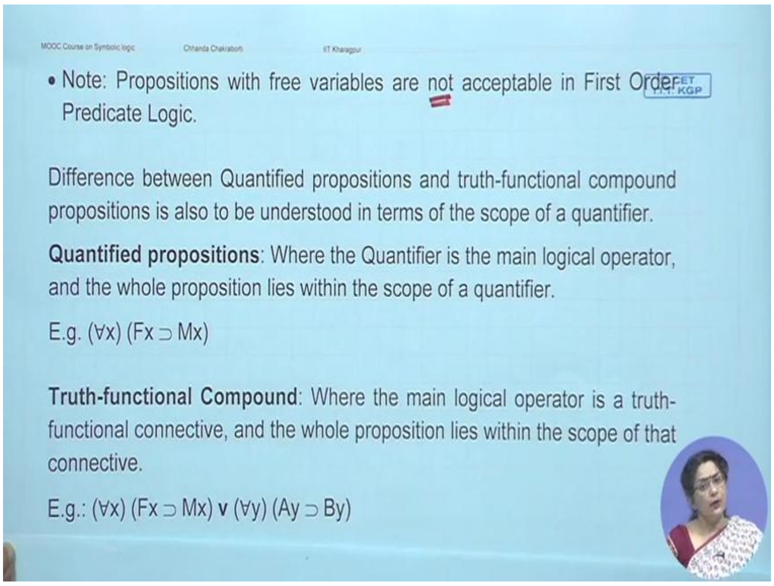

So, as said free variables are not really welcome here. You may say the freedom of expression, freedom of speech etcetera desirable but free variables is suddenly not desirable in this case in the way I explain. Now I will explain the difference between truth functional compound and quantifier statements also in terms of scope of the quantifier.

So here it comes, see when we quantified propositions we mean those propositions where quantifier is the main logical operator. Now how do we know it quantifier if the main logical operator, when the quantifier has the scope over the entire statement. Once more quantifier statements are those statements where a quantifier has the entire sentence within its scope; that is what we call quantified statements. So, entire statement has to be within the scope of a quantifier.

For example, take a look at this one every occurrence of this is bound by this quantifier and this is what we mean by saying that the entire sentence is within the scope of this quantifier, this is a legitimate quantified statement, whereas the truth functional compounds are those where the main logical operator is a truth functional connective. So, the entire sentence is going to be within the scope of that connective not under quantifier. So, the proposition is going to look different, is to be interpreted also be differently and it function also will be different.

Take a look for a example here, what you find here and here is that we have quantified statements here quantifier statements here, but the whole sentence is a truth functional compound. How do I know? Well, take a look this is your wedge the same v or wedge that was a truth functional connective and it follows the truth able of the wedge the either or except the disjuncts are all quantified statements. There is vast difference between this kind of sentence and this kind of sentence. Here you do not apply the truth table you cannot, but here this is bound to follow the truth table of the wedge. And this is what we will call the truth functional compound; this is what we call the quantified statement.

So, what we have done today in terms of scope is enormous in the sense that we can now operationalize, we can use this to understand other concepts such as distinction between truth functional compound and quantifier statement as well as free and bound variables.

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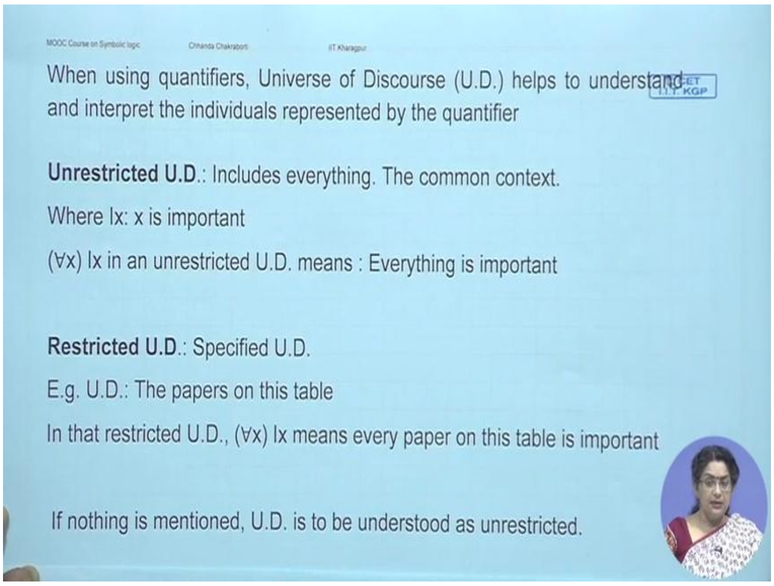

There is one more thing that we need to know and that is universe of discourse. See the quantifier helps to understand how many and the universe of discourse helps to understand what are we talking about, what kind of things are we talking about. For interpretation along with the quantifier the universe of discourse is very very important. Let me show you universe of discourse can be unrestricted as in open. If left open an unrestricted includes everything. Everything that possibly you can include with that which is what we call the common context. When we start a convention our context is open. It is assumed to be common to the convergence between them they know what is normal and what is common and so on.

So unrestricted U.D is sort of like that, you include everything that is you intend to be included. For example, let me show you how it works where i x is the predicate letter for x is important, then in that unrestricted U.D where there is no bar for all i x means everything in that U.D is important. This is what you are saying; everything is important. Now if we restrict it we can artificially restrict the U.D saying that I am only talking about this, I am only talking about certain things that you can do also. If you specify the U.D then you will see the interpretation of the sentence changes. So, let us try that with the same sentence for all x i x, but here we are going to restrict the U.D to the papers on this table which table this table the one that we are I am using right now and there are lot of papers here.

So, in this I restricted U.D where we are talking about the papers on this stable for all x i x would mean what; every paper on this table is important. See how this interpretation changed, I did not change the sentence, we did not change the quantifier, what did we change the context. The universe discourse is nothing but the context. So, it depends on the contextual meaning also as it showed so restricting the U.D change is the meaning, unrestricted U.D changes the meaning and so on so forth.

Now the default is convention is that if nothing is mentioned when you are asked in the translation or anywhere when there is no special mention of the U.D you have to understand talking about an unrestricted U.D. If it is restricted artificially that will be mentioned. We are talking about this specific U.D not all, not everything. So, keep that in mind.

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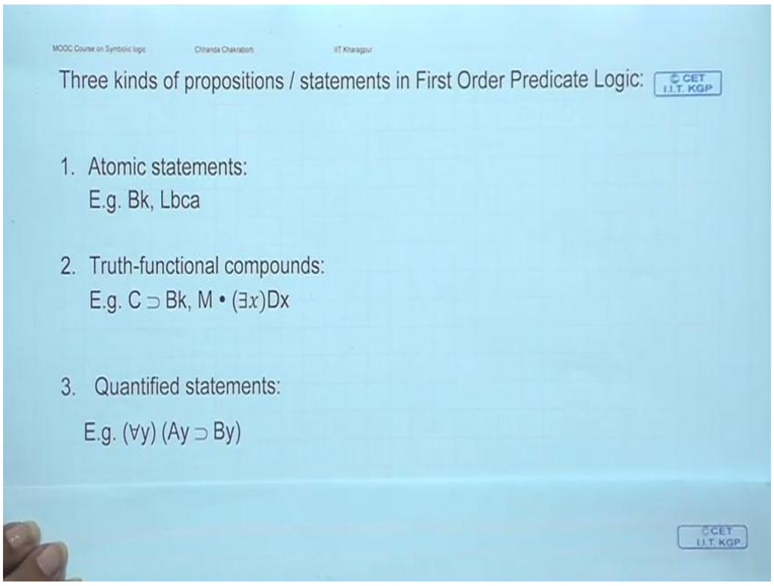

So, let us sum it up then what we have learned is that there are three kinds of statements that we allow in our first our order predicate logic. The first one are the atomic statements. You know where we have the constants appearing with predicate symbols and so on and they look like this. So you have the predicate letter followed by constant, the predicate letter followed by number of lined or ordered sequence of constant are variables we have seen that. This is one kind of situation.

Then we also have the truth functional compounds and we know how to define them, how to understand them also. And they may look like this. Please note that even in the truth functional compound you can have the presence of a quantifier, right that is (Refer Time: 18:56) bared out that is not eliminated. But the nature of sentence remains truth functional compound. So you might have things like this, you also might have this see here it is a conjunction between m and there exist at least one x such that x is d x. So, that is a truth functional compound for reason that I have given already in terms of scope.

And then there are full pledged quantifier’s statements, for example like this. So, all three are legitimate (Refer Time: 19:31) of this world of first order predicate logic we have some what must at the syntax of first order predicate logic. So, we will end it here today and will come back with more in the next module.

Thank you very much bye-bye.