Symbolic Logic
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Lecture – 33
Understanding Logical Relations in Standard Form Categorical Propositions
Traditional Square of Opposition
Immediate Inferences Based on Traditional Square of Opposition

Hello. Welcome to the module 33 of Symbolic Logic course and now embark on understanding this categorical logic or Aristotle logic.

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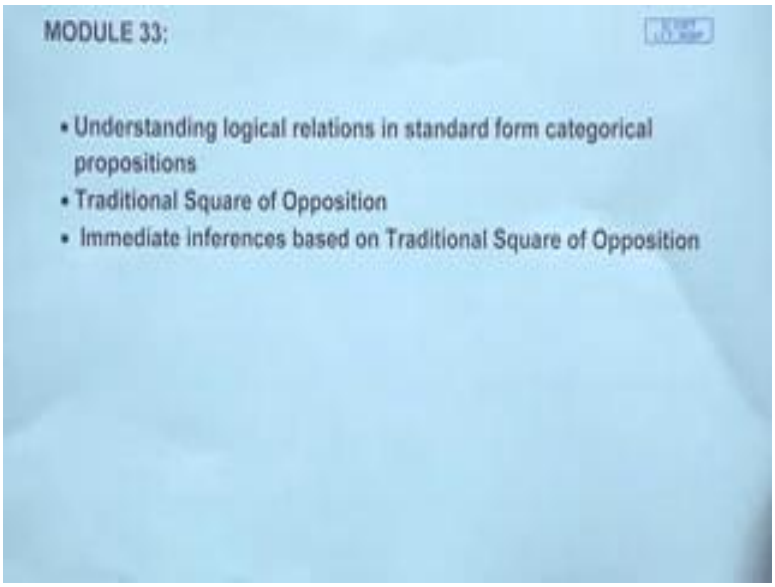

So we have a talking about these standard form categorical propositions for in the last module, but time has come now to look at them more closely. We have just barely got an introduced to them. So now, it is time to understand, what is going on inside them and we will also learn about something call the traditional square of opposition. Which holds a very important place in Aristotle's logic and then based on this traditional square of opposition I will show you how immediate inferences can be done in the Aristotelian system. So this is what we will learn in the module 33 today.

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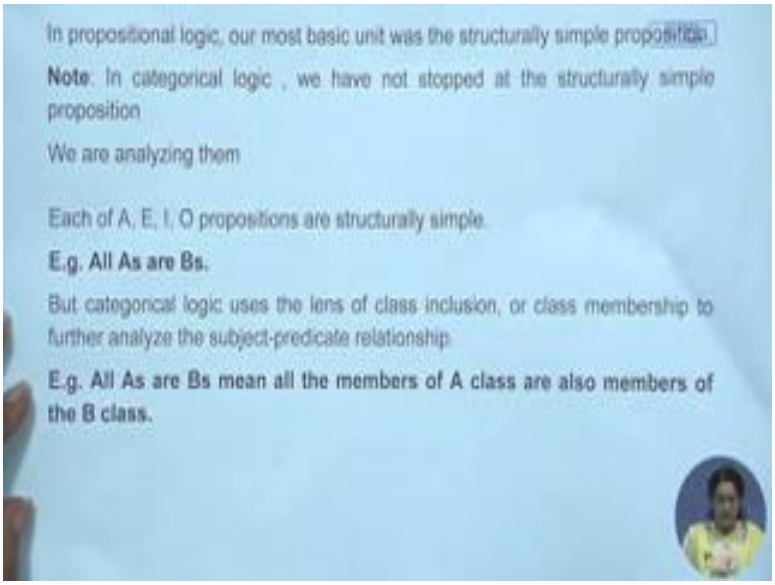

See first we need to go back to the rational, the reason why we left propositional logic. I have already mentioned this, but still nonetheless less it is better to bring it back again, that remember the in proposition logic our basic unit or ultimate unit was the structurally simple proposition right. Now in categorical logic we have gone beyond that. Beyond in the sense that we have gone inside the structurally simple proposition, now what we are doing, we are analyzing them, using what using the class inclusion lens, class relationship lens, either there is a relationship or there is none. Somehow we are using this to probe into the structurally simple propositions.

So otherwise what seem to be completely inaccessible earlier for example, if you had all As are Bs standing as a structurally simple propositions say in your propositional logic time, then this is it you would be simply assigning it a capital letter that is about it, but what we are doing we are using the lens of class inclusion, class membership to look inside this kind of propositions and what we have found that what they do when we are looking through this class inclusion that we see a distinct relationship. What is that relationship? We are being seemed that all of A is included in B. What does that mean? It means that if A is a class, then every member of the class A is included in the class B right, so all members of a class are also members of the b class.

Now that is information that we are getting through this kind of opening up of the categorical proposition. So we have gone inside this structurally simple proposition to open them up and further analyze them to find out what is the logical information given inside them.

Now that we have started so and we have used the Venn diagram and other things to see those relationships, capture those relationships, but now here comes something more. In Aristotle's logic itself, there is a mechanism to conduct immediate inferences from each other, provided when the subject term and the predicate terms are identical.

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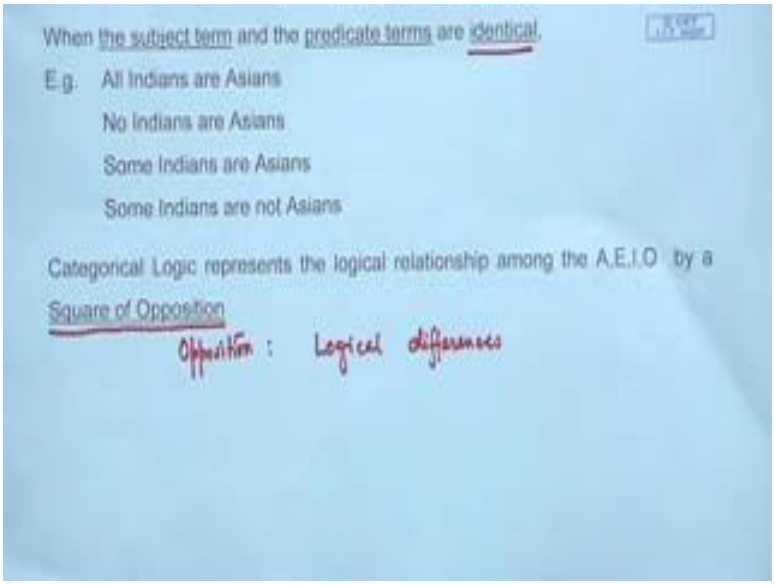

So if you line up A E I O proposition stand in standard form, when their subject term and predicate terms match right. So when they have exactly the same subject and predicate term, you can line them up in a certain sort of way. I have taken some examples here all Indians are Asians, no Indians are Asians, some Indians are Asians, some Indians are not Asians, this is A E I O.

When you have such four sums, then categorical logic represents the internal relationships among these in terms of a square of opposition. I will show you in a second square is indeed a square, it is a figure and the opposition here would mean the logical

differences. So basically what will it allows us to enjoy is the logical differences among these propositions, when they have the same subject term and the predicate term. So let us take a look into this square of opposition. I told you that this is a traditional component in Aristotle's logic. So we have preserved it as is from Aristotle's times. So take a look.

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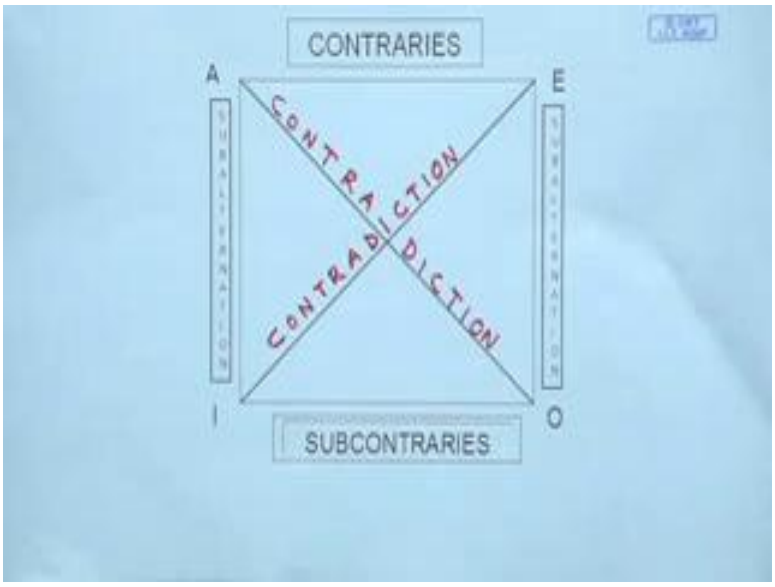

So here is what is known as the square of opposition, the traditional square of opposition. For your sake I mean this does not look like a square, perhaps it look more like a rectangle, but given that also, I mean you can see what we have done is to line up this A, E on top I and O at the bottom alright. So remember this A E I Os have exactly matching subject terms and predicate terms. When that happens the square of opposition tells us that these are the kind of logical differences or oppositions you can see them in them. I will explain that later.

Let us know first get acquainted with the name of the relationship that they have. So between A and E there is a relationship of contrary. Contrary C O N T R A Y and the pairs are called the contraries. So between A and E contrary relationship holds. Look at the bottom of the square, there is I and there is O, the relationship between them is sub

contrary. The pair is called the sub contraries alright. So this is contrary relationship, this is sub contrary relationship.

Now let us take a look at this side of the square you have A and I, E and O. The relationship that flows on the sides is called Sub Alternation alright. So this is on between these two pairs and then inside as you can see between A and O and E and I the relationship is called Contradiction ok.

So, this is the picture, this is what we would call the Traditional Square of Opposition. I have taken a actual examples to show so that we understand this better. This is suppose, instead of schematic A E I O. We put actual categorical propositions here, may be it might help you to understand it better.

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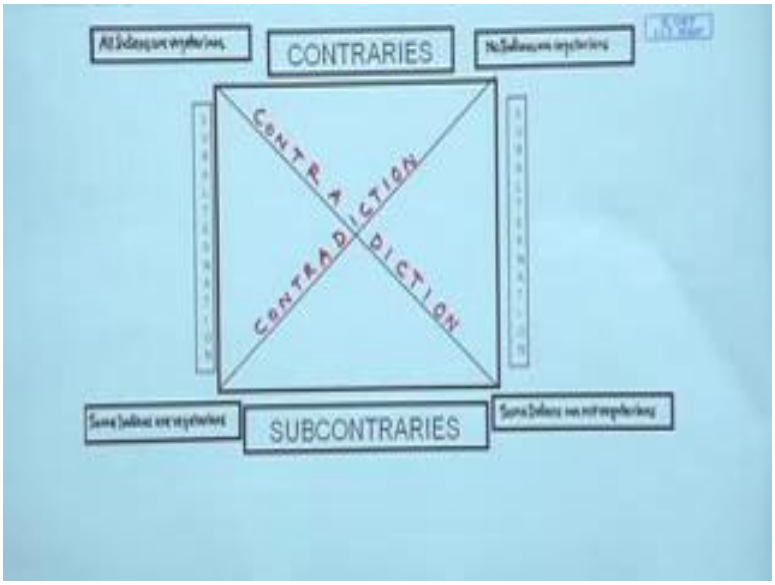

So here for example, all Indians are vegetarians, no Indians are vegetarians, some Indians are vegetarians, and some Indians are not vegetarians. You can choose your own A E I O and from this square of opposition to understand how this relations work, but keep this picture in front of you and in other words what I am going to say know. So I am going to now explain what each of this relationships are, but you need to remember

which two pairs we are talking about and how they actually fit into this description alright. So we will refer back to this picture, but let me just move it here a little.

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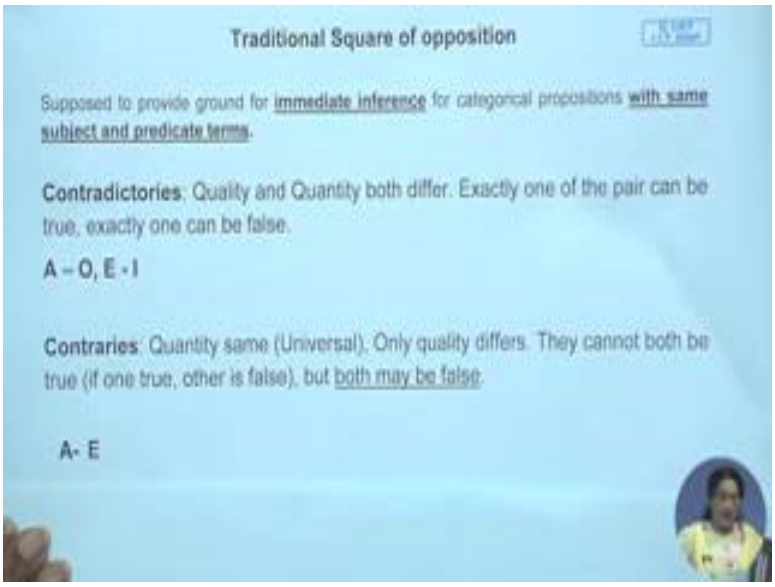

So what we are talking about is that, on the basis of the traditional square of opposition which is this, we can form some immediate inferences. So given one premise you can jump into a conclusion alright. Immediate means there is no other proposition in between, in between what in between the premise and the conclusion. So directly you can go immediately you can make some inference provided you have this kind of line up of A E I O with same subject and predicate terms.

The first relationship which is between A, O and E, I is called contradictory. Between as you can see this is A, E, sorry A, O and E, I right. Between this you can see the quality differs, if one is affirmative, the other one is negative right and the quantity also different if one is universal the other one is particular. So A and O quality, quantity differ. E and I quality, quantity differs and that is the nature of this relationship for contradictories.

What follows from that exactly one of them can be true at one time and exactly one can be false, that means. So if you know one of them is true you can immediately infer what, the other one must be false right or if you have the information that one of them is false

between these two pairs, you know the corresponding contradictory will have to be true, get it. So this is going to be the basis of some immediate inferences. We will see examples but I hope you got it. So we are talking about the A, O and E, I and these two pairs are known as the contradictories in this kind of sense. It only means quantity and quality both differ.

Then comes the contraries, let us take a look who are the contraries. The contraries if you remember on top of the square of the opposition, we had A and E. Let us bring the picture back. So these are the contraries. You have the A and the E as contraries. The - I and E are I and O are sub contraries will come there later, but first the contraries. Notice that among the contraries, the quantity is the same between A and E both are universals right. So quantity is the same, what is different is the quality. If one is affirmative the other one is negative, but notice that their relationship is different from the contradiction because they both cannot be true. That matches with the contraries, exactly one of them can be true, both of them cannot be true at the same time. So if one of them is true you know the other one must be false, but unlike the contradictories both may be false alright.

So if you know that some Indians are vegetarians, then its neither true that all Indians are vegetarian nor it is true no Indians are vegetarians is not it, both may be false, which you do not have in case of the contradictories. So both may be false and this is the nature of the contraries quantity same, quality different, but both may be false, though both may not be true at the same time. this is your contraries.

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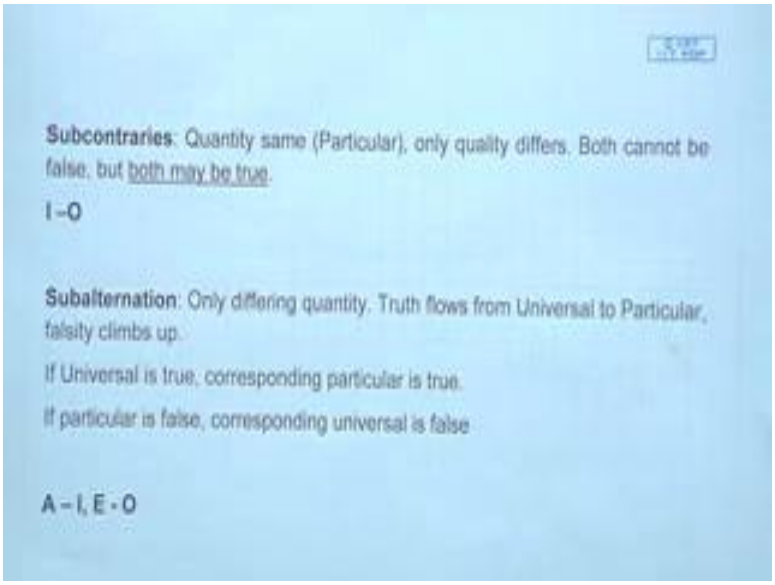

Let us get a look into the other-other relationships. So here is what we would call the Sub Contraries. Sub Contraries remember between I and O and here also what happens the quantity is the same. Quantity is the same as in both are particular. Only the quality is different. Now here though it seems like a version of contrary, there is a difference between the contrary and the sub contraries, What is that? Both cannot be false. Remember in sub contrary both could be false, but sub contraries because both are particular, you cannot have in Aristotelian system that both are false, that is you cannot have say some Indians are vegetarians is false, some Indians are not vegetarian that is also false. That the Aristotelian scheme at least will not allow, so both cannot be false though both may be true. It may be true that some Indians are vegetarians; it may be also true that some Indians are not vegetarians true.

So this is the peculiar nature of sub contraries. So what happens is that that when you know of this pair one of them is false you can easily assume the other one must be true right, but if you know one of them is true, nothing follows right. It is not necessary the other one is true you just know that there is possibility the other one might be true. So from just by knowing that one of the - I and O pair is true, nothing concrete follows about the other-other the corollary. You cannot infer anything definite about the other.

Then comes what is known as the side relationships. Remember sub alternation and we are talking about the A, I and E and the corresponding O. So there is a relationship. You have the same A and it is corresponding I, you have the E and its corresponding O and the relationship is called Sub Alternation. what happens here the only difference is there in quantity right.

So between A and I you know that they are both affirmative, quality no difference. E and I no difference in quality both are negative, but one is universal the other one is particular. So quantity differences are there is sub alternation. When that happens, the truth we say flow from the universal down. That is if you know that the universal is true, you can assume, or infer that the corresponding particular must be true. So whether it is A whether it is E, you know the corresponding particular must be true.

On the other hand the falsity climbs up, meaning what that if you know the particular is false, the corresponding universal must be false right. So if you know that the I is false for example, that it is false that some Indians are vegetarians. You can infer from it must be false also that all Indians are vegetarians. So this is the nature of sub alternation. So the pairs are A E I O and as I said that they differ in quantity and truth flows down and falsity climbs up if you remember that. So once more we are going to visit square of opposition and then use this relationship to form some immediate inferences out of this.

So contraries between A, E sub contraries, sub alternation on the sides and diagonally it present is the contradiction relationship and I have explained to you how this work, how the pairs are connected and what kind of truth value inferences you can make based on this relations that what we are going to try this immediate inferences from this.

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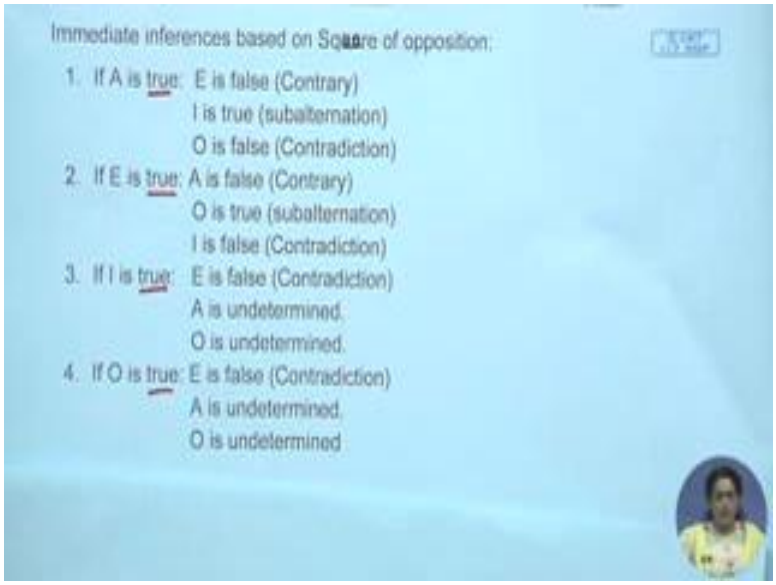

So sit down with a piece of paper and try to solve as we go along. That suppose I tell you, you choose your own A, I, E, O, but with exactly matching subject and predicate terms for you to understand this. Now suppose I tell you that A is true, what can you tell me about the truth value of the corresponding E, I and O. Did you understand the question? If I tell you that the your A is true, what can you tell me about the truth values of corresponding E, I and O based on what you know about the square of opposition.

So immediately I can see that some of you will say that o has to be false. Why? Because we know that O is false by contradiction and once more remember contrary both cannot be true. So if A is true the corresponding E will have to be false. Remember sub alternation truth flows down. So if A is true the corresponding I has to be true now see you have just figured out that they have made some inference about them and you have determined the truth values, did you do any truth table, you do any truth trees, did you just do some calculation to do that, no; what you did was utilize, what you know about the square of operation right. So similarly you can go on talk about what if the E is true and then you know that similarly this will be the truth values of the corresponding A,I and O.

Suppose I tell you that I is true, what happens to the corresponding A, E, O easiest to that probably is that E is false based on the contradiction that is nature of the contradiction, but if I is true, please note that the you do not know anything about surely, you do not know anything of necessity, what is going to be the truth value of the corresponding A. It is not necessary that the A will be true right, but that does not mean a is necessarily false. So the best answer here is the value of A is undermined. Which is you do not know we cannot be sure.

Similarly remember I is true both in sub contrary, both may be true, not must be true right. So you do not know whether O is true, you do not know that, but if you do not know that does not mean O is false does it. So it does not, therefore the best answer here is that O is will be undermined. So same thing will happen when you say O is true. As you can see what we have generated are many immediate inferences about the truth values of the corresponding one. So if this is with true what happens with when one of them is true.

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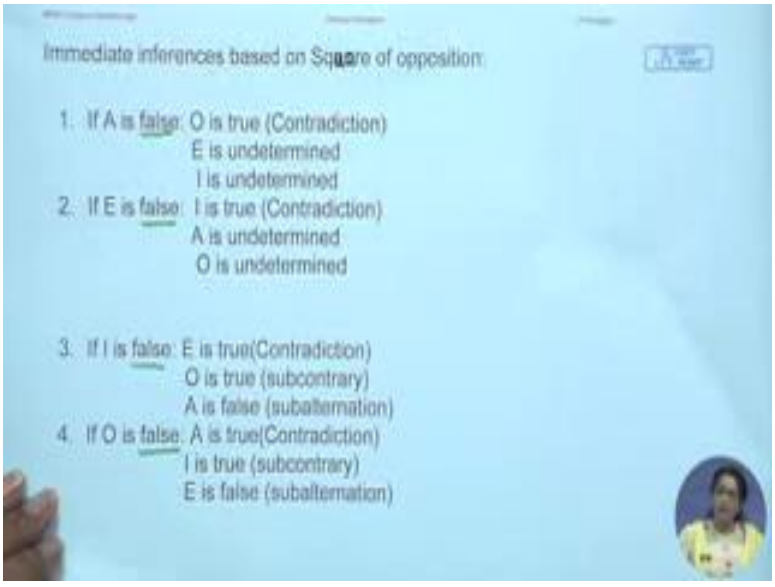

Similar thing we can do with falsity also. Suppose I tell you A is false. Then what happens to the O, E and E, I. If A is false you know immediately O has to be true. Why because of contradiction, but if A is false, you do we know what the truth value of E

would be no, because both may be false. May be does not mean must be. So you do not know that e must be false, you also do not know that e must be true. So the best answer here is E is undermined. Similarly you know that the universal is false. What happens to the corresponding particulars? Sub alternation says we do not know.

A is false does not mean the corresponding I will be false, it does not mean that it would be true either. So you go with this I is undermined, same thing follows with E is false. I suggest that you sit down with us, with this lecture and try to think this is why I said write on a piece of paper your own set of A E I O to understand this immediate inferences and work it inside your mind its very intuitive, but at the same time you need to realize the truth of what we are saying here right, so you need some practice. What if I tell you that I is false, I is false means E is true, that is for sure by contradiction, but if I is false then you know O has to be true sub contrary because both cannot be false at the same time. So I must be O must be true. If I is false then we know A is false how by sub alternation falsity climbs up and same result you can see also almost similar result in the with the case of falsity of false.

What we just did, as I was saying is immediate inferences based on square of opposition. So square of opposition we have learnt and as I told you it has a pivotal place in Aristotle's logic. An as you saw this becomes the back bone for making immediate inferences about other related and relevant categorical propositions.

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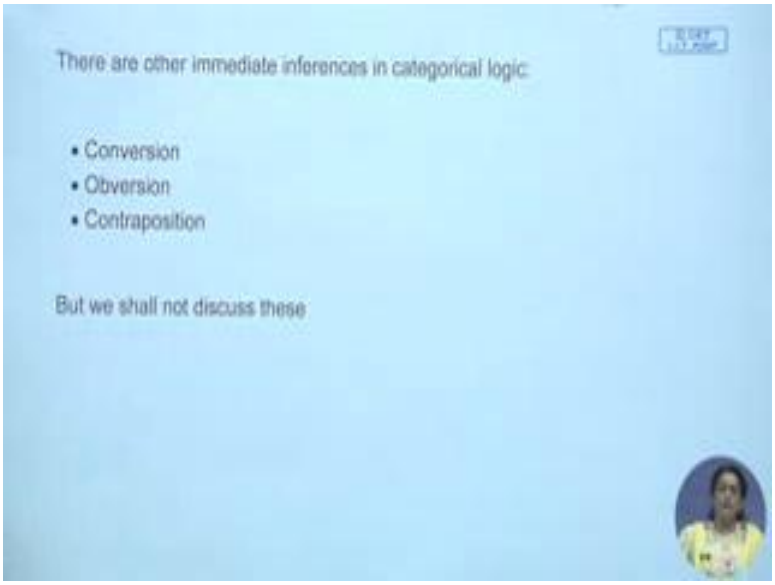

Now this does not mean that this is the only kind of immediate inferences available in categorical logic. There are 3 other kinds, there is Conversion, there is Obversion, there is Contraposition and each one is a method for immediate inferences. So given a premise you can do these operations on them to get a result, that would be a valid conclusion and where they will tell you where it is valid where it is not and so on. But we shall not discuss this here, because of paucity of time and because it is an online course, where we have to maintain other constraints. So we are not going to discuss any of this, but if you are interested you can take a look into this, but what I thought today is how to do immediate inferences based on square of opposition and with the knowledge of the square of opposition in fact you can jump into many such inferences.

So this here we are going to stop today and there will be more in the next module, but I hope this is so far is alright. Keep up the good work of practicing on what we have discussed and drawing this square of opposition to have a feel of this alright.

Thank you very much.