Symbolic Logic
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Lecture – 28
Conditional Proof
Preliminary Version
The Format
The Procedure

Hello and welcome back to this module number 28 and we are on the topic of Limited Scope Assumption Procedures, which have already learnt the indirect proof.

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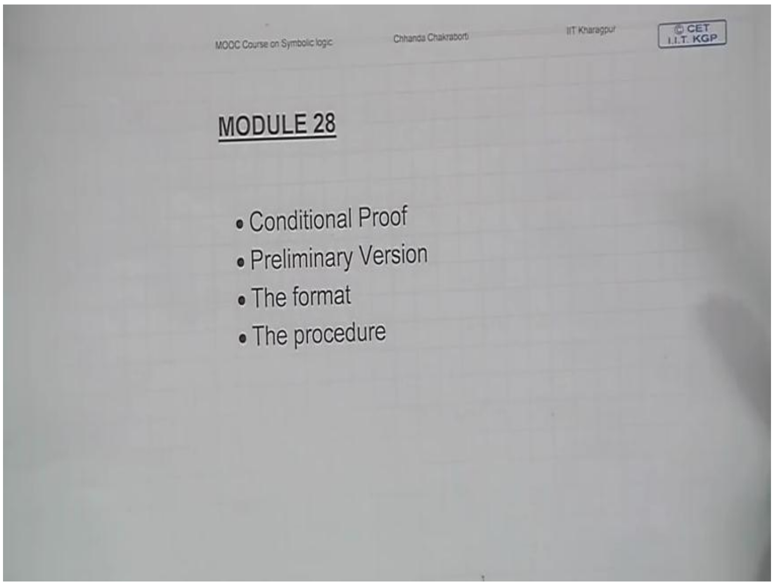

And today we are going to look into this Conditional Proof which is the other limited scope assumption proof procedure that we are going to learn. Except that I will mention that the conditional proof for the CP has more than one version and today we are going to only look into the preliminary version which is the elementary and there will be an advanced version or the strengthen version that will take up in the next module.

So currently we are because we are getting (Refer Time: 01:06) with proof procedures. So will start with the elementity one first. An obviously we need to know if you are learning the new procedure, then the format as well as the procedure and so on. So this is will going to the content of our module on number28. Before we proceed to a conditional proof, let me make one point clear about indirect proof.

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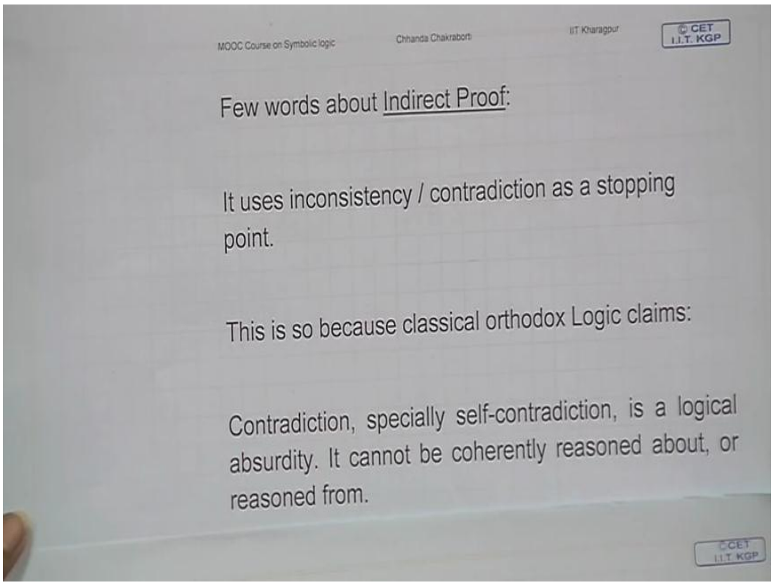

See I simply said that it sort of stop at an absurdity, where we are talking the absurdity as the self contradiction, something that is very oblivious and of the nature somewhere the proposition and its negation are both be placed as truth. So it uses this IP procedure uses inconsistency or self contradiction as a stopping point. Beyond this the argumentation does not proceed. So that sort of (Refer Time: 02:20) this self contradiction in consistency as a termination point for logical reason. Why is that? Why do we have to stop when we have encountered have self contradiction and answer is because that is what the assumption of classical orthodox logic is. It claims that you know a self contradiction is a rather abominable, unacceptable and logical undesirability. So, once you reach a self contradiction nothing can be coherently reason about or reason from. So that is what this whole think is about, that there is this law of contradiction, the law of non contradiction in orthodox logic.

This is the reason whenever you are in the domino classical orthodox logic, this encountering inconsistency of self contradiction will put you to stop put you to a stop, but notice that may not be always the case, but will come there let us try to understand what is this all about.

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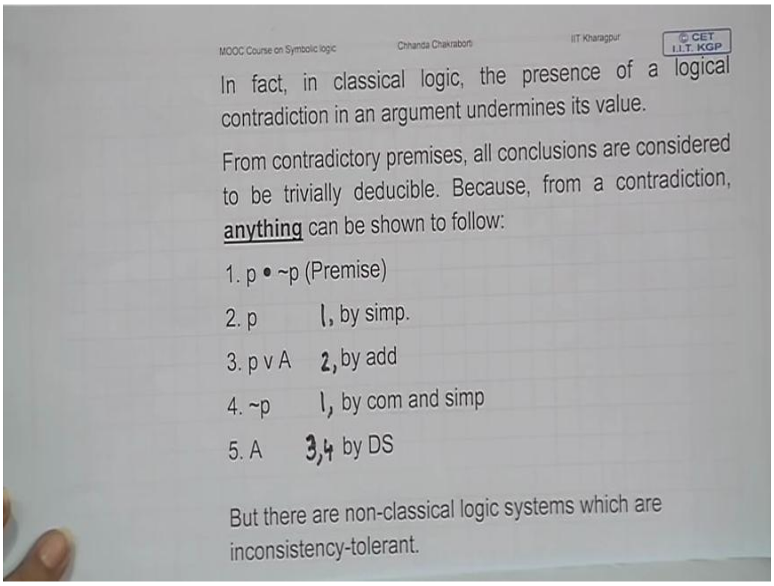

Mainly what happens in that in the classical logic, the presence of a logical contradiction in an argument, in away completely undermines its value. It makes it practically trivial and useless and. in fact, in logic formal logic, classical logic it is said that if you have the premises as contradiction, self contradictory, then all conclusions that are derived from that is to be considered trivially deducible or in other words from a contradiction anything follows and that claim we can show it in case you you cannot cross grasp it conceptually, I will show you the how this proof works like this for example, suppose you have the premise like so p dot not p, which is the patent self contradiction from that we can simplify p and then we can add A, where A is any conclusion of your choice, anything it can be a very complex proposition also just by addition. See the validity is preserved so far and then we can take not p out from A by commuting and then simplifying. So I have combined this state with your permission on line 4. On line 1 we switch the position of p and not p and take not p out fine.

But then you put 3 and 4 together and apply DS what will you get A. So in a way from contradictory set of premise you can show the derivation of any conclusion of your choice. So from a contradiction anything follows, which is why it is not really desirable that you premises have inconsistency or your premises should not have any logical self contradiction contained in them, that is if you are in the domino classical logic.

If you leave the domino of classical logic and go into some of the neurologic non classical logic there I must tell you there are systems which are inconsistency tolerant. That is they can absorb even self contradictions and still the logic system functions. So what logic since are we talking about, I mean I just going to mentions some names without explaining them, but you are welcome to look it up or if you want interested you can, you can for the study. Para consistence logic is one of them (Refer Time: 06:30) for example, is a system that believes that all contradictions need not be false you know.

So there are non classical logical systems, but that is not where we are at this classical orthodox logical system level where the self contradiction is a stopping point all right. I just thought I will mention this because you have learnt indirect proof and you if you are ask by somebody, but why do you treat that indirect proof that, if I reach the self contradiction why do have to do. So I thought I will give you some theoretical answer behind that.

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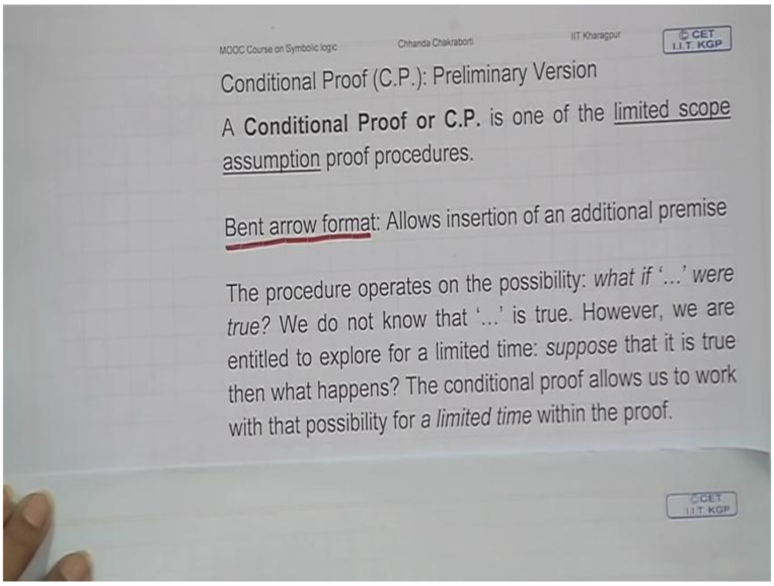

Let us now come to our today's topic, that is your Conditional Proof, but we are going to only look into the preliminary version right as I said elementary version. What it is? I have already said it is the Limited Scope Assumption Proof. So everything that I have said earlier about the limited scope assumption proof still holds for the conditional proof. For example, the bent arrow format that we have learnt in case of IP or indirect proofs still applies. The arrow head is going to point that beginning of an assumption and the movement you reach your objective, you are ready to close the assumption or discharge the assumption that is where the assumption must be closed and you know how to do that the bent arrow format by now you know. Now what is this conditional proof all about, well it operates on the possibility of what if.

So this is not about absurdity, this is simply saying what if this also where the case remember I mean in our common argumentation technique, we often say for the arguments sake let us assume this is true then that does not mean that it is true. All you are saying that let suppose for the time being that this is also true and then what. So that is exactly what this conditional proof is all about. It allows us to add an assumption for limited time and explore what happens, when that assumption holds. So for a limited time within the proof you are allowed to add an assumption and see what follows from that.

Having said that, now you will say that what am I going to assume, but before that let me this show you let us take a look into format.

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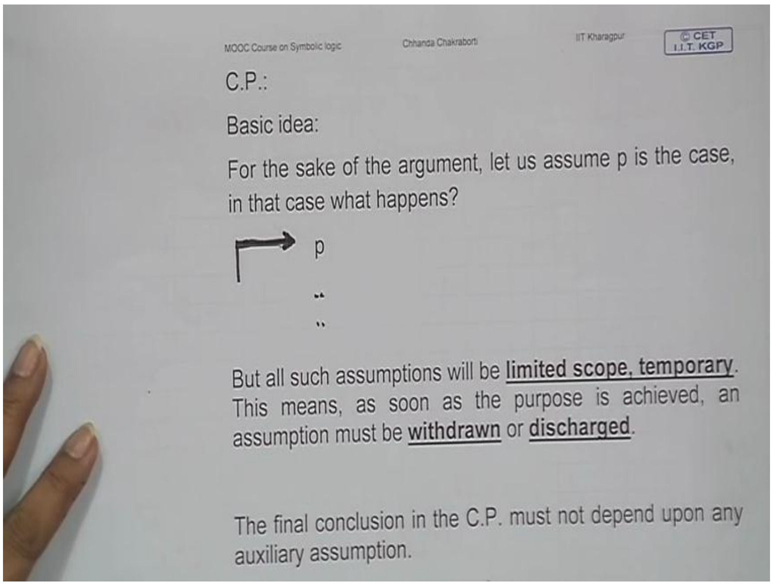

So, as I said you are saying let us assume for example p is the case, the movement you say p is the case the bent arrow technical take over and this is the kind of indication that you need to give and then within a unlimited period and as soon as you reach your target there is reason why you said let us assume p, once you have reach the target what the target is I will explained later, but once you have reach the target as before you need to discharged or withdrawn close the assumption and that is where the lined assumption will close also. So the bent arrow will closed down. Final line in a conditional proof will not depend upon the any of the limited scope assumption that we you are going to make.

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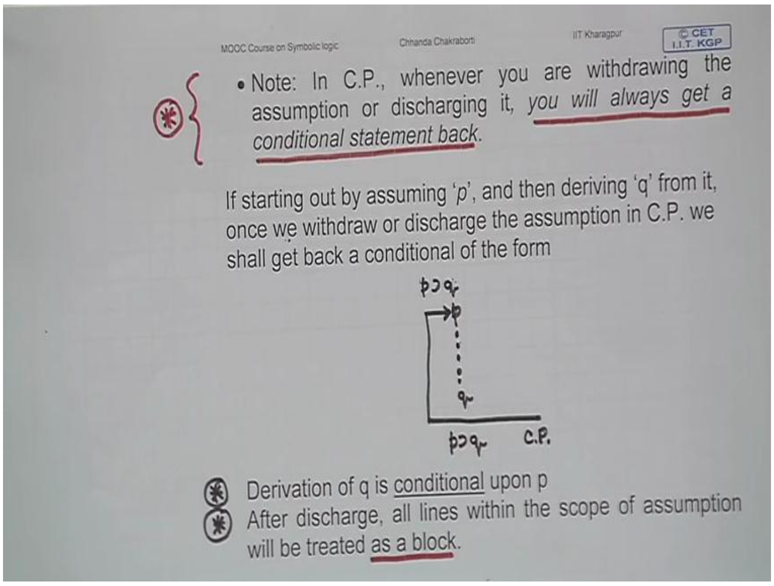

The main and the most important thing in conditional proof is this, that in conditional proof whenever you are withdrawing an assumption or discharging the assumption, you are going to get back a conditional statement. Conditional statement does not if p then q type of statement. So if you said that let us assume the p and then you derived q from it and then you said I have reached my goal q, I no longer need this p. So you are going to withdraw or discharge the assumption p. The movements you do that what are going to get back in CP you are going to get back a statement like this p horseshoe q. In a way it should make sense you have said that if p then sort of q is derivable. So, that is what you deserved in a proof. This line is going to appear in your proof and you better know what to do with this line in your proof, otherwise this is point wise exercise one after other.

So once more the different between IP and SP is this, that in CP you are saying that let assume this for some time and you have a certain target in your mind, once you reach the target you say I do not need it, but when you close the assumption when you discharge the assumption, what happens; inside the proof a conditional statement will appear where the antecedent is going to be your assumption and the consequent is going to be the line that you have derived from it based on it. So you are saying that q is conditional upon the assumption of p and the bent arrow will work, like this, this is where the arrow head will point at and then through the lines, this is where you are 19 rules are going to be operational and you have derived q and then you said I do not need any more fine then you close it the movement you close it, CP will ask you to at this 19 to your proof if p then q.

So derivation of q this is your discloser, that derivation of q is conditional upon the assumption p and once more I have to remind you just like in previous IP, everything that is contend within this, where the arrow heads start and the retains is a block. It is treated as an assumption block. An assumption block will not be accessible individually. Which means whatever lines are here, once you are outside of this block you cannot access or you cannot refer to that, you cannot use them in the proof anymore remember that. So this is once and for all is closed.

So remember in a proof if you are going to need any of these lines later on, you need to have them available to you or get everything done here. So, that final you no longer need this. So this will require some practice on your part to C what is it that I need so but in general my point is that this is the closed block and treat it like that. So in justification also it will treat as a block reference.

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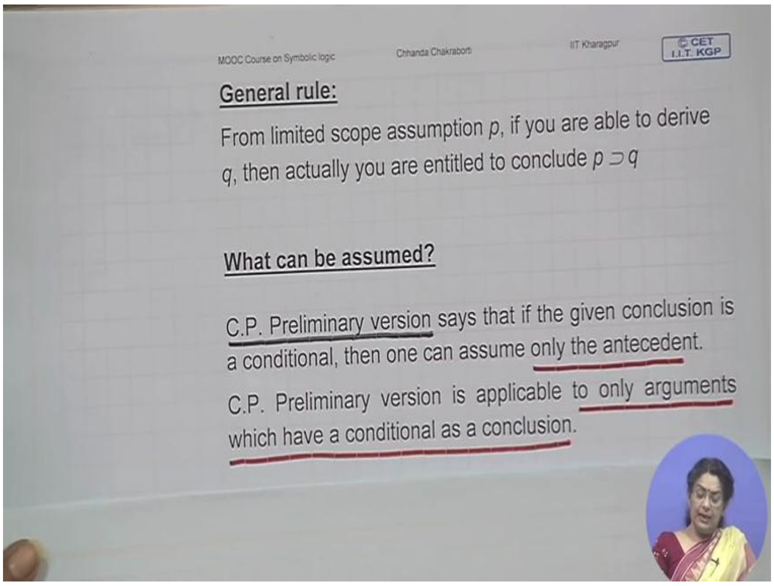

Now, the general rule therefore, for conditional proof is likes so that if on the basis of p you are able to derive q, then you are entitled to conclude if p then q, that is how the conditional proof works. Now comes the question, but. So, far I understand we have said assume p assume p, but what exactly is p? What can be assumed in CP? Then the answer that you are going to get, in the preliminary version will be different from the advance of the strengthen version, but we are now learning only the elementary of the preliminary version of the CP and that preliminary version says this. That given, that you are working with an argument, where the conclusion is a conditional statement given that, then what you can assume is only the antecedent of that conclusion. So once more if you happen to be lucky to have an argument, where the conclusion is something of the form if p then q. Then the preliminary version of CP allows assuming only p and solving for q get it. So that would be the target for you, but this is the only kind of assumption that the preliminary version of CP will allow you to add.

So, in a way therefore, the preliminary version has a certain limitation of application. It apply is to only to only those arguments, which have conclusions which are conditionals. So if your conclusion is of the type p wedge q or p and q, then this version of CP will not even be applicable right and you have found the answer what can be assume the answer in this version is the antecedent of the conclusion, where the conclusion happens to be a conditional. Have I made myself clear in this? Good. So this is how this will work.

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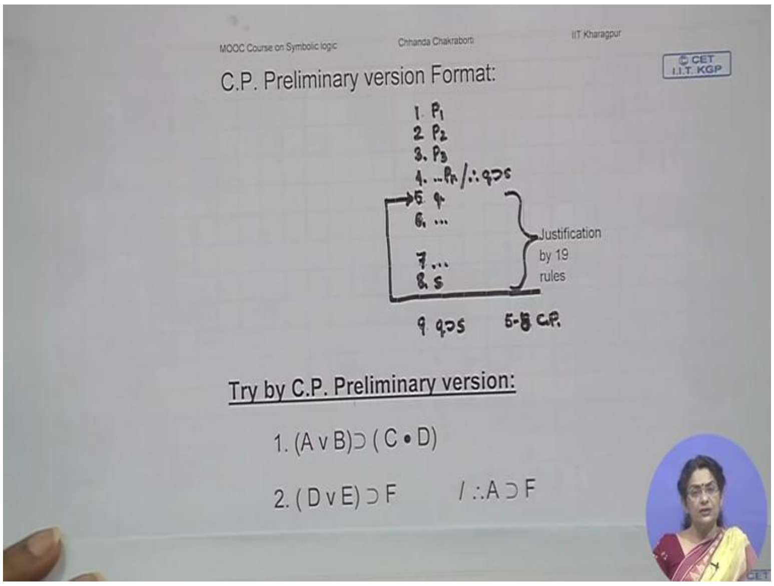

And the format you know like as in the case of IP will be somewhat similar, but you need to certain get you to the difference with IP also. So here is you are proof, here is your original argument, which happens to have a conclusion which is q horseshoe s. These are all your premises is given. The conclusion is q horseshoe s. What preliminary version of CP will allow you is to add q further as your limited scope assumption. Immediately the bent allow will take over.

From this what is your target? What are you going to show? The answer is out all these what will show is that s follows. So we are saying given all this premises and q let us suppose that q is true s follows. So that is what we will solve for s and this where your knowledge of nineteen rules is going to really again come out and help. Once you have reach this with this 19 rules, what you would say that I have reach my goal and no longer need this then you close it like and what did I say the movement, you close the assumption when you are going to get the back what, a conditional of the form q horseshoe s get it.

So the assumptions horseshoe the line that you have derived. This line is going to be show up in your proof. Fortunately this is what you wanted also derived right, this is your conclusion, how do you justify? We justify it as a block. So 5 through 8, sorry this is 8, 5 through 8. So, this is 5, this is 8, 5 through 8 and by CP. So no need to have CP appear, here this is just an indication that you are starting a limited scope assumption procedure. This is where you disclose the strategy or the proof procedure right this we have seen.

So I think we have said enough now it is time to apply it and see whether you have learnt, so can we try by conditional proof preliminary version this little argument. See it calls for the preliminary version of conditional proof because the conclusion is actually a conditional A horseshoe F. So you have premise 1, premise 2, what will be a premise 3, premise 3 will be A. Let us assume that A is true and what will you solve for? You will solve for F right and once you have reached if you can close it, like so and then you will get back A horseshoe F. Will you do that and I have the worked out solution as always with me I will show it to you in a second, but these try it out your own with the knowledge of 19 rules and see whether you can derived F from this in this using this proof procedure.

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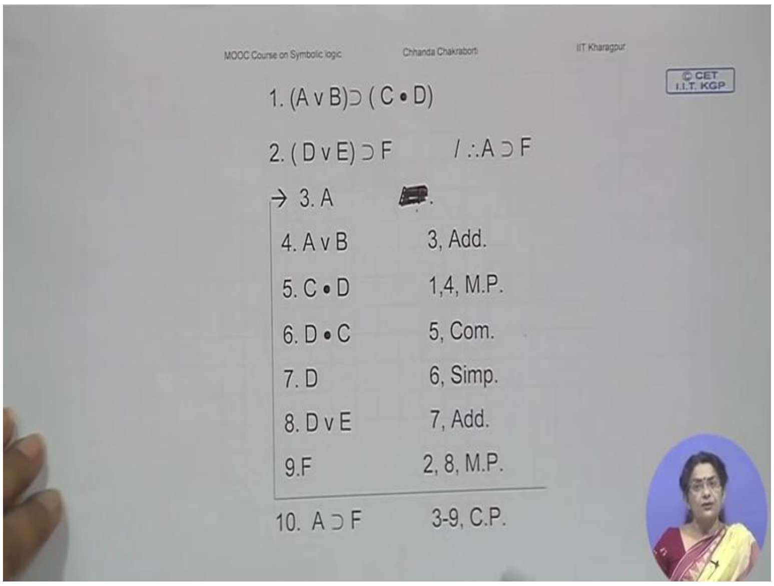

So, this where I show you the worked out solution; in case you have tried it out you can work it with me or even check it as it goes along, but this is where you start right. This is A and this is where the assumption arrow head starts. Then I do what? Well this is your planning is necessary. What you are trying to solve for is F here is F. So, if you can get D wedge E somehow then your job is done. How do you get D wedge E. Well here is D, somehow if you get A wedge B then C dot D and you know there is rule calls simplification. That will allow you to get the D out of C dot D. So plugging it altogether we work out the problem like so 2AV add B by addition and then we plug it in with one and four we get no respondents. So this is what we derived C dot D and this we commute because we want the D out by simplification and once we have the D we just add E to it D wedge E, so that we can have F out. Have I reached my target? The answer is yes. Once I reach the target what do I do? I close the assumption. So this is like that and do I get back you know what you get back namely A horseshoe F.

How did you obtain it 3 to 9, see these are all rules that you already knew and which you have been using so far. So there is nothing new about it this procedure is the only thing that you have to learn and look at the way the justification goes that something new, so 3 through 9 that is the block, that is an assumption block and we close it and by CP we mention the proof procedure right here all right. So this is not too bad this is our a preliminary version of conditional proof, we are going to look into the strengthen version later, but try to absorb the idea of the conditional proof and you will see that you once you get into this, try other proofs that you have tried earlier which have conditionals as conclusions with this preliminary version of CP.

Hence you will see that the proofs are become much easier. So with that we close this module. We will see you again in the next module.

Thank you, bye.