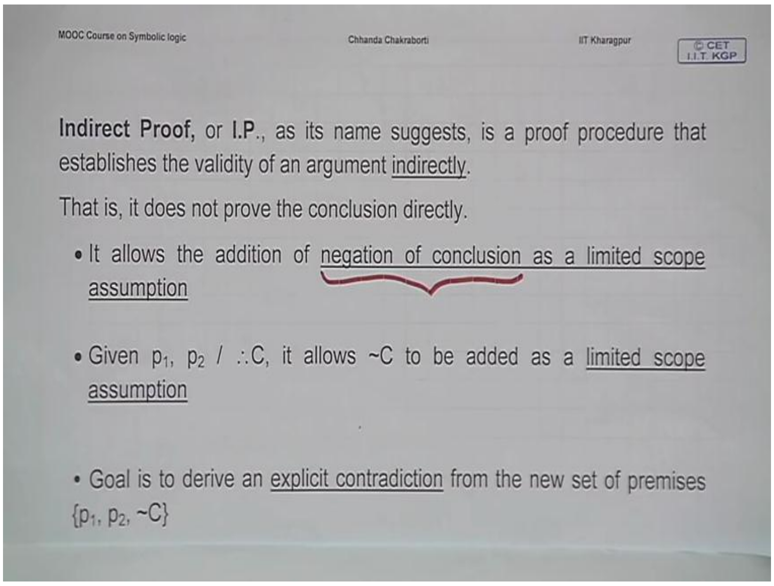
Symbolic Logic
Prof. Chhanda Chakraborti
Department of Humanities and Social Sciences
Indian Institute of Technology, Kharagpur

Lecture – 27
Indirect Proof
The Format
The Execution of the Indirect Proof

Hello and welcome back to our module 27 of this NOC course in symbolic logic. This module 27 we are looking into Limited Scope Assumption Proof and we are going to learn about the indirect proof. We are going look into what it is and what the format is, how to do the proof and so on. In this also you we are going to make use of the 19 rules that you have learned. So those days so that if you have gathered an expertise over the 19 rules, that is going to come very healthy for even in this Limited Scope Assumption Proofs. So without further I do latest proceed to understand what this indirect proof is all about.

(Refer Slide Time: 01:13)



This indirect proof or IP as we are going to call it, it is proof based procedure that work indirectly. That is the name that is why the name is like that. So what it does not do is to proof the conclusion directly. See in a formal derivation what do you do, you start from the premises and then you slowly derived the conclusion directly, directly means that the conclusion is what you actually derived out of the premises is not it with the 19 rules.

That is not what is going to happen in indirect proof, rather it is going to go roundabout way to proof the conclusion. What is that roundabout way? Will that is what we are going to learn today.

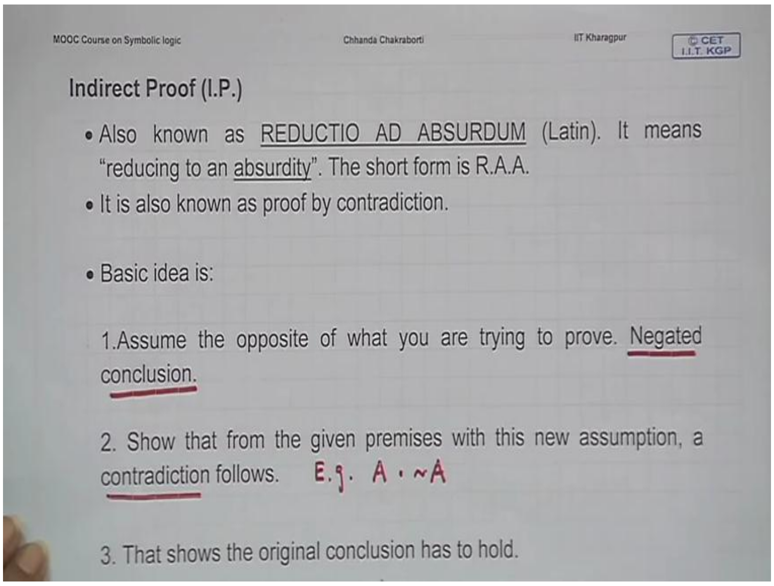
First off all note that as a limited scope assumption proof it will allow you to add a premise, at an assumption as a premise right, limited scope assumption I have already explain in the previous module that the procedure are going to allow you to insert an assumption in your premise base. So there would be some given premises and then there will be some added premise that you are going to insert to the premise base. Here in IP what will be permissible to add is negation of the conclusion given. So if you have the argument in front of you, instead of starting with the premise and then deriving that directly to the conclusion, What you do? You take the premises, you look at the conclusion, and then, you add the negation of the conclusion with the premises. Just like we did it in the truth trees remember. So that would be how this indirect proof would work.

You have the premises with that you add the negation of the conclusion also as a limited scope of assumption fine and then so if you have for example, schematically speaking, here is an argument were you have p1, p2 as premises and C is your conclusion given right. If it were a formal derivation you would be starting with p1, p2 and then try to solve for C. That is not the IP way, what will IP allows you? to do 2 add tilde C. C is your conclusion. So it will allow you to add negation of that C as a limited scope assumption right and then what you will say, but why are we doing this, what I am supposed to do that. Well the target is, that from this new set you have a new set now p1, p2 and tilde C, from this new set you try to derive with the 19 rules, that you have already done explicit contradiction, logical contradiction that is your target and the movement remember the limited scope assumptions are really limited in their scope. So you have a purpose in mind why you are assuming them, the movement you have reach the objective you draw you have to drop the assumption.

So, in this case the goal is this explicit contradiction. Once you have arrived at explicit contradiction your job is to drop the original assumption. What is the assumption? Namely tilde C right that was the beginning assumption, you drop it and then you claim therefore C has to follow from the premises get it. So this is how indirectly in round of way the proof procedure proves why C has to follow from the given premise set.

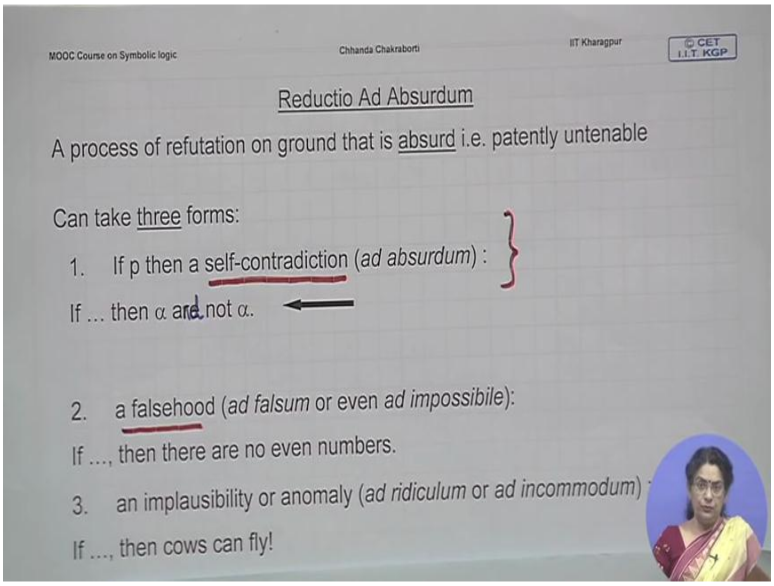
So let see whether how much we understood this or not. Now let me add few more details to this, so that our knowledge about IP is somewhat complete.

(Refer Slide Time: 05:31)



You may have encountered indirect proof another form, when in school where you will do geometric for example, euclidean geometry for example, you may have been thought, the theorems can be proven by the Reductio method. REDUCTIO AD ABSURDUM that is the Latin word - REDUCTIO AD ABSURDUM, which means reducing something to an absurdity and the short form for this kind of proof, is called R.A.A and that is precisely what IP is, it is a reduction to an absurdity. You may have said in geometric classes, let us assume that the theorem is not the case. That it does not follow from the (Refer Time: 06:26), then let us see what happens. If you have done that kind of proof, then this is what same thing will be also tried out by the IP method. I have use the word absurdity remember I said the IP other name for it is reducing it to an absurdity, REDUCTIO AD ABSURDUM. What is an absurdity? How do we show an absurdity and the medieval logicians gave us three kind of answers. So, we will take a look into that a little bit.

(Refer Slide Time: 07:07)



See what they said is that this REDUCTIO AD ABSURDUM procedure is a process of refutation, is a process of logical refutation where the ground is that what you have shown is something absurd. Absurd in the sense that it is patently, openly, untenable unexpectable, but what counts as absurdly, when we can say this is absurd so that the medieval logicians have explained that there is it can take three forms. One of them is very much obvious, which is a self contradiction. So you can go like this that if p happens, then something absolutely self contradictory follows. So if we then alpha and not alpha both followed right. If this is the case alpha and not alpha both follows then obviously, p cannot be the case.

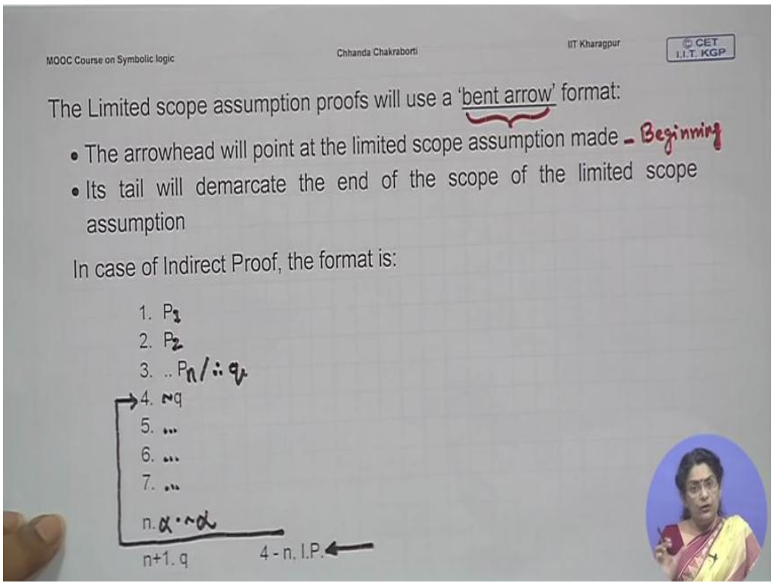
This is one kind of absurdity and this is called Self Contradiction which is one of the highest forms of absurdity, logical absurdity you can say. There can be milder versions of absurdity also for example, which is not self contradiction, but something that is widely known to be false. Clearly false for example, if p happens then there are no even numbers. If p is taken to be assume to be true then something as obliviously false as this statement follows that there are no even numbers. That is not exactly a self contradiction of this kind of form that is falls right. So, this is also an absurdity according to the medieval logicians.

The third kind of absurdity is Not a Self Contradiction, not even a patent falsehood, but an implausibility or something that is anomalous, with the given information, the current information that we have for example, if I said that if p happens then cows will start to fly, cows flying in this world is an implausibility or completely in anomaly with whatever we know about cows about our atmosphere about gravitation and so on and so for right. So this is sort of Ridiculous level ad Rediculum. So what you have shown is reducing it to a ridiculous nature. So all three these types were known by them as absurdity, reducing to this to any of this would count as a REDUCTIO AD ABSURDUM proof, but for our purpose we are going to only look into this, one namely that self patent self contradiction will be explicitly shown as derivable and that is what we will call the Reductio proof.

So, the basic idea would be that you know we are going to try out the Reductio and our goal would be to show an explicit contradiction to follow from it that would be the level of absurdity where we would say now we were reaches the target of the proof. So where do you start the answer is by assuming the negation of the conclusion, which happens to be the opposite of what your trying to proof right the negated conclusion. How do you go about well we start with the new premise base, namely are given premises plus the negated conclusion and then what we try to do with the 19 rules is that a self contradiction follows for example, if you can derived from this new set A and not A that is a clear an open an explicit contradiction.

And then you are claim should be that is the indirect proof, that why the original conclusion has to hold because if you negate it if that does not hold then a patent contradiction follows get it. So this how the indirect proofs sort of moves. So once you have understood it conceptually the working out of the proof is not at all problems.

(Refer Slide Time: 11:35)



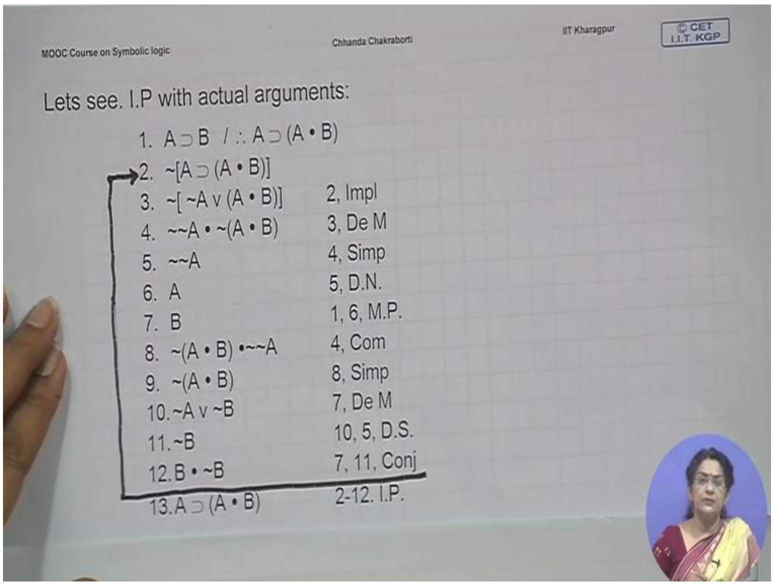
Let us talk about the format now and all limited scope of assumption proofs will follow a bent arrow format, arrow that is bent. How we will show you in second, what it will do is that you are going to start an arrow where the scope of the assumption starts and remember what I said that by the fault the assumption lies within its own scope. So wherever you are making the assumption that is where the scope of the assumption also starts right because the assumption is included with in its scope. So you are head of the arrow is going to point at the assumption itself, that is the beginning of the scope of the assumption and the till will demarcate where the scope of the limited scope assumption ends, I will show you where for example, you here you are goal is what goal is to derived an explicit contradiction. So, when you have reached that contradiction line you know that you no longer need this assumption that would be the end of the scope for this assumption.

So the format would be, now you can see the visually what I was trying to say here is like this. Suppose you have a proof like this, were 1 2 3 these are these are all your premises given premises p1, p2 up to pn and you are conclusion is q right. So, this is where you standing you have the q. What is your job? If you are doing the indirect proof the starting point would be not q remember negated conclusion and this is where the arrow head is also immediately will show up, note that there is no justification given why not because this is an indication that you are starting a limited scope assumption procedure write here. So, that is not q (Refer Time: 13:34) what is our goal with this new set we are going to derive an explicit competition. So let us see that let us assuming that we are doing that with our 19 rule, we are trying to work out and finally, we come to something like alpha and not alpha that is your explicit contradiction right. So, this is where you understand that my job done we no longer need this not q and the way to indicate that in a proof is to close the assumption scope. See so your arrowhead starts here its goes with like this and it goes under this. What does it tell you? That, this whole thing is an assumption block all right and this assumption block you refer to it also as a block, you close it. When you close it, remember this individual lines are no longer accessible to you cannot refer for example, to 6 anymore which is within the assumption block. So, it becomes like a closed box, an assumption block.

So the all of this is now packed inside and you have put a closed to that and then in IP what you do? You repeat the conclusion that is originally given this is your claim that from this said this as to follows.

Why because if it is not then this kind of contradiction follows, but you have shown that already. Please note how we are justifying this last line, 4 through n can you see the hyphen, there is no comma between 4 and n, there is a hyphen 4 through n, were the beginning is an line 4 the end is on line n right. So we refer to this whole thing as a block, 4 hyphen n. That block remember is no longer accessible, but we refer it not to separate line, but as a block. Here comes the comma the separated comma from the line numbers and the here is the first time that you said the procedure declaration is IP. So this is telling everybody the look we derive this by IP method. Let us take an actual example and maybe you can do this on your own also.

(Refer Slide Time: 16:01)



Let us take the examples that I showed you earlier that when we say that really there is no proof of this in nineteen rules. So now let us test whether with IP addition can be now show that there is exist proof in our system. So here is the argument A horseshoe B and A horseshoe A dot B. we know that its valid I have already in the earlier module, I have explained why this must be so.

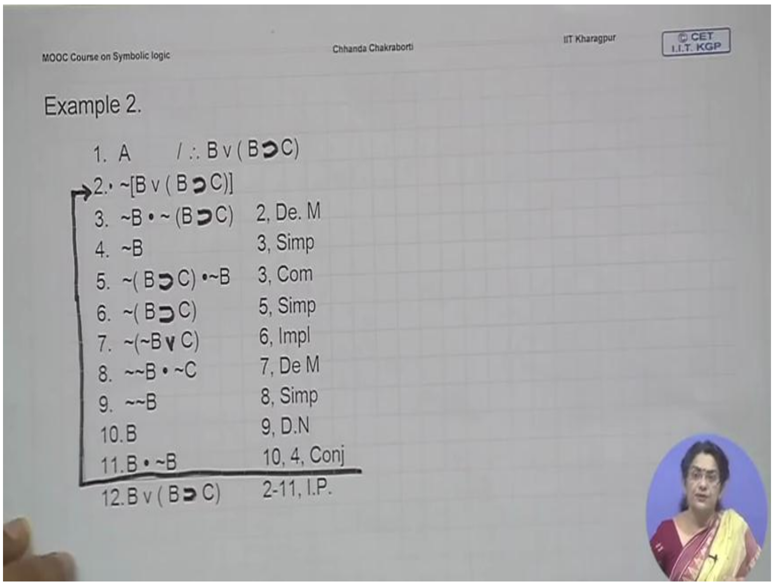
Now, we are going to adopt the IP method, therefore our starting point will be what the negation of the given conclusion? now you have one you have two what is your job the target is to derived any explicit contradiction, it could be A and not A could be B and not B all right. So that is the target. Now just simply use the 19 rules to derive at that. Look at this two make a plan how you can derive from this something of the kind that will show A and not A, B and not B something of that. Once you have reached that you close the assumption and restate this has to follow this is our target.

So, there is nothing extraordinary about that as I will show you how I have derived you can follow the proof you want like. So here comes b and not b all of these types are just to get here. All this rule are known to you these an nothing but your 19 rules. So if you have learnt to do the proof earlier that will come 19 even here as I will try to say. This is 12, the line where we have reached B dot not B.

Now, what I should do, I should close the assumption right, there I no longer did this negated conclusion and then line number thirteen would be the given conclusion and once more what is the justification from 2 through 12, 2 hyphen 12 can you say that 2 hyphen 12 IP. So by IP and with all this line has proven why this must be valid conclusion gets it.

Let try another example, the one again that sound it like rather strange and we said that there cannot be any proof with the just with the nineteen rules.

(Refer Slide Time: 18:31)



So let us try with the IP, whether now we can show why this argument has to be this has to be a true conclusion, valid conclusion. This is the A and we have B wedge B horseshoe C. IP will tell you what that you can assume this fine and now you have 1 and 2, what is your goal to derive a self contradiction of some kind. So, again your 19 rules will come really helpful, if you know it and here is how I derived Band not B. It has a accident, but this is what is derivable from this. You can see in other proofs there might be other kind of contradictions also, but try to get this kind of literals. So do not go for complex for example, even if you have B horseshoe C and not B horseshoe C that is not desirable. You try to get it into simple component like this so B and not b and all this rules are familiar to you we have really gone through this.

So with that then here comes your final absorption this has to follow 2 through 11 by I p get it. So this is our introduction this was our way to do your indirect proof and that has been the contend of this module, where we have looked into, you have just learned how to do one procedure in limited scope assumption and there is more in the next module, but in the mean time try to grasp this indirect proof how to do it if you know the format the rest is as usual the 19 rules right.

So that should not be a problem. Even if there is problem we are already there always there. So fill free to try out a little bit, thank you, that is all for this module.

Thank you.