Symbolic Logic
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Lecture – 26
Completeness What It Is
Completeness: As a Desirable Property for a Formal Logic System
Is Propositional Logic Complete

Hello, Are you ready for our lesson? So, we are in module 26 today to cover this formal derivation system that we have learned and then looking to some interesting property in it. See in the previous module, I have explained to you this nineteen rules how to do the formal derivation have to that right. So and we have done some examples together.

But what today we are going discussed is an interesting property called completeness. What is property is we will explain it also, but I will try to establish that completeness is a desirable property for a formal logic system and then we will look into whether the propositional logic system, the proof system that we have learned in the last one or two modules is that complete in this sense. So that is on our agenda for this module on number 26.

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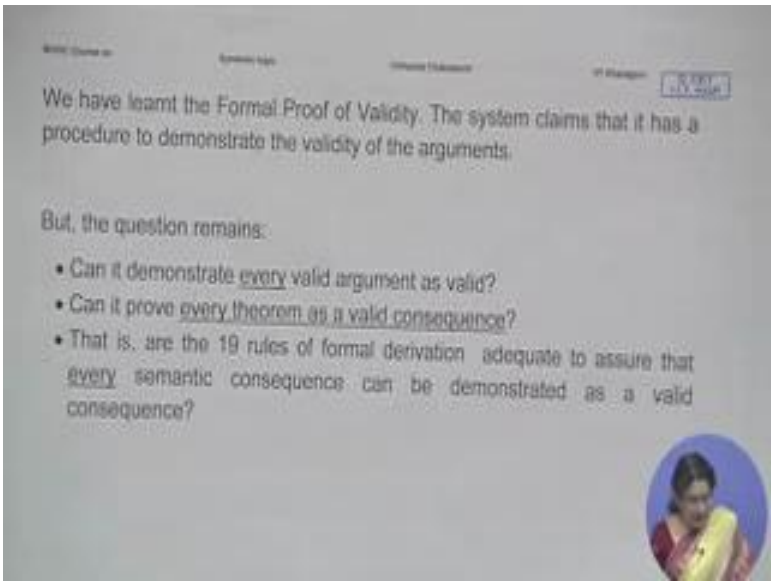

See we have learnt this formal proof of validity and which means that the system has made a claim, that we have the rules, the mechanisms, to demonstrate the validity of deductive arguments right. Now that is not the only thing that we have concerned at this point. Given an argument we know innovate that this system has our mechanism to show how the conclusion can be derived from the premises, but there is a further question here, if you pick a random deductive argument and suppose its valid also can the proof system or the derivation system is our guarantee, that it can demonstrate the validity of that argument. Did you understand what I just said? I said that we know that we have the rules that we have all the mechanisms and we know how to apply the rules.

So we know innovate that given an argument, which is deductively valid will try to prove that, prove that the conclusion follows validly from the premises, that there is an answer the question that we are asking next. What we are asking next? Is that can we guaranteed that any valid argument which is deductively valid our system is equipped to show demonstrate its validity. So for every valid argument, can be guarantee that in our system there is a proof. So every conclusion that follows validly that we can call as a theorem in our in our system is it prove for it, can we prove every theorem to be a valid consequence. In a way what we are asking is that we have. So many rules in our rule base the 19 rules of inference and of equivalents to replacement, at the adequate to this guarantee that, every semantic consequence can be demonstrated as a valid consequence.

If you still have not got it, then will try to build upon this concept, but this is not just a question that here is an argument, show me the proof, it is deeper than that, what we are asking that if you succeed in this case, in a given arguments case, that here is the conclusion and here is the proof by that, can we then make sure that if I give you any valid argument deductive argument of course, that you will be able to demonstrate its validity in your system any, any is the operative point of point here.

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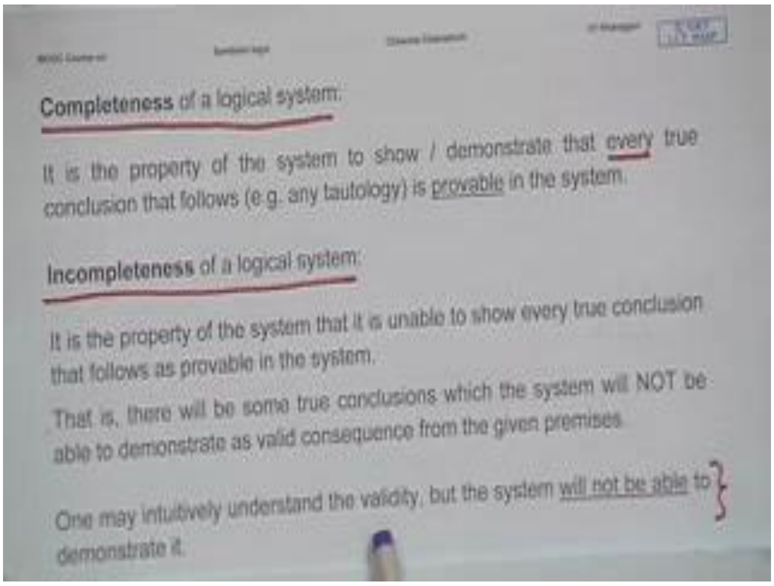

So, any given argument is there a proof for it in the system one are we discussing this as I told you that there is this property called completeness.

Now completeness is a curious property of a logical system, by which the system is able to show as an demonstrate that every true conclusion that is expressible in it including the tautology, every true conclusion is provable in the system. So if there is something that is expressible in the system as true as a true conclusion, there exists a proof in the system for it that is the quality that we will call completeness. The system is complete in this sense. What would be incompleteness of a logical system when I just opposite of that property, where the system is unable there is an inability in the system to show every true conclusion that follows as provable in the system. So it cannot show every true conclusion as provable conclusions get it. So in that case what will happen, that there many some conclusions that you know true and that they follow validly will see some examples soon.

But they are valid and you know that they are true intuitively that is clear, but what is not available is proof or a derivation how they can be derived from the given premises and that is something property that we would be call incompleteness of a logical system. So this failure to demonstrate even one you know that something is valid, you know this gap that is what incompleteness is. Now if I leave this in front of view and ask you which one is desirable in a logical system? You know the system which claims that it has a system proof, that it has a formal derivation system. Which one is desirable obviously you will

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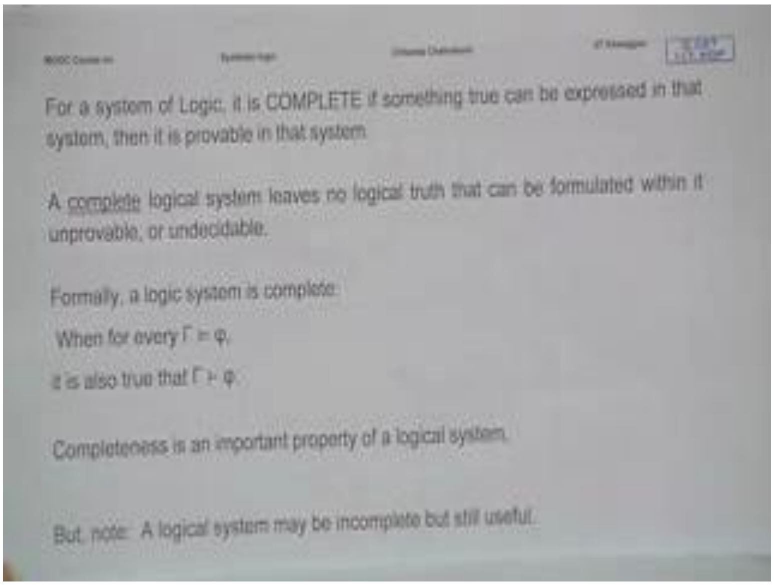

say completeness. So the proof system has to be complete, comprehensive enough to cover every true conclusion that follows is in it whereas incompleteness is somewhat questionable property.

Now this is our understanding of the completeness and incompleteness, but I am going to put add some more new answers in to this concepts. So if it is a complete system in logic, then something true that is expressible in that is provable, this is what we said. Expressible as in a syntactically expressible, you can see that it has to be a valid consequence, but provability is means that you have to show by process, the actual step by step process the theory exist a derivation for it. What happens in a complete logic system? Is that it leaves no logical truth within it unprovable or undecidable and that is a really applaudable quality in a logical system I mean it something (Refer Time: 08:26) that some no logical truth is unprovable or undecidable in the system, that is what completeness ensures. So that symbolically we can say that you know for any gamma set of statements, if alpha happens to be the semantic consequence then its also true that there is a proof for it, there is a derivation that shows how alpha powers from that set of premises.

So completeness, I am trying to emphasis upon is an extremely important property for a logic system having said that I have that there may be I mean incomplete logic systems which is still useful. Useful as in the sense that it will work for you in certain cases quite well, but the problem is that you do not when it is going to stop functioning. The incomplete logical system what will it do, it will continue to give you proofs for some arguments. What you do not know is the next one, next valid argument that you have been hand whether your system is actually able to show that next time. So that guarantee will not be there in the incompleteness. So in that way it is useful, but it is a limited useful where as what is preferred by all means is the complete logical system. So, completeness is the very important property.

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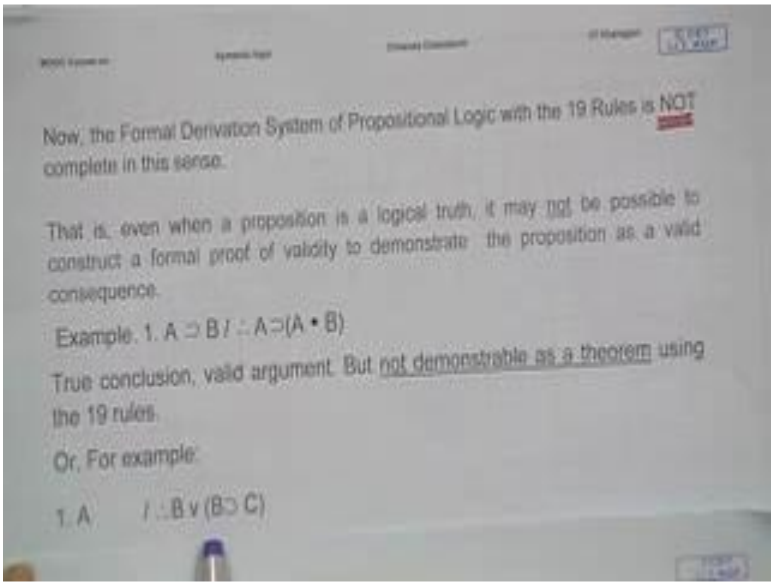

Lets now come back to our formal derivation system the one that we have learnt and that we have trying to master. Now with the 19 rules, let me just make this point very very clear that what you have learnt with the 19 rules. It is not complete in this sense that is, though the number of the rules is quite high you have 19 have them, but they cannot demonstrate the validity of every single true conclusion that follows in the system, you will not believe this, but I will try to demonstrate that. I am not going to the entire mathematical proof of it, but I will try to serve conceptually explain it to you. So what we happen and I am going to show you with examples is that, a proposition may be a logical truth, but with the 19 rules you may not be able to construct the formal proof of derivation for it in the system.

For example, look at this it is a very simple argument, the premises if A then B or A horseshoe B, the conclusion says if A then of course A and B. Now intuitively you should understand provide this has to be valid. Those of you who want to try out more; you why do not you do the truth develop this to see for yourself why this has to be valid. I am going to explain only conceptually. So, there everybody sort of gets the point. See what we are saying that a horseshoe B is true, if A then B fine, now from A remember A follows; from any given proposition itself will follow right. So from A will follow if A then A that much we know for sure. As per B that is already stated in the premise, if a then B and that is true right, given A B follows. So, when we captured that that given A, A and B follows it cannot be falls. That is my conceptual explanation to you for those who wanted do further you can do the truth able it is you can do the truth retest which I have talk you earlier anyway to see why if the premises true, the conclusion cannot be falls, which means this has to be a valid argument, but unfortunately there is no proof of it possible with the 19 rules.

The 19 rules haves they are limitations right. I mean they work in a certain way, they do not work in a other ways, with those you cannot demonstrate higher from this you can come here unfortunately, you can try, you can try you also take my word for it you can try, but let me also ensure you that you will not be able to do that. Because the rules as such that they cannot after a certain point you cannot jump to a state, which will land you here right exactly. So you may try as I said, I am in try implication try de morgan anything that you want exportation distribution anything that you one all the rules, but if there is no proof with the 19 rules, with the said nineteen rules.

Here is another example this one. Now if you did not understand might point here and your still racking pen, why we cannot have proof of it is, how is it possible that we cannot have a proof, see this premises A from this what B or B horseshoe C see it and my claim is that with the 19 rules from this premise, you cannot have a step by step formal derivation that will land you here. There is simply you know mechanism, there error. Again you do not have to take my word, but you try it and you will see conceptually I will give my explanation, why this is valid try it yourself and you will find that B wedge B horseshoe C is a tautology. So it is always true, if it is always true it does not matter what value A is if, A is true when I good, if A is falls even then it does not matter the argument would be valid right. Why? Because the conclusion is always true.

Therefore, this is a valid consequence in a trivial sense it is a valid consequence and remember if it is a logical truth that is expressible in our system, if our system was complete there should have been a proof when it right, but unfortunately you will not be able to show it. Now what is the solution, so what we have just said is that the formal derivation system, as its stands with the 19 rules it is not complete. Then what I mean are we going to leave it at that or just leave with that complaint that it is not complete, no what we are going to do is to solve a work on it.

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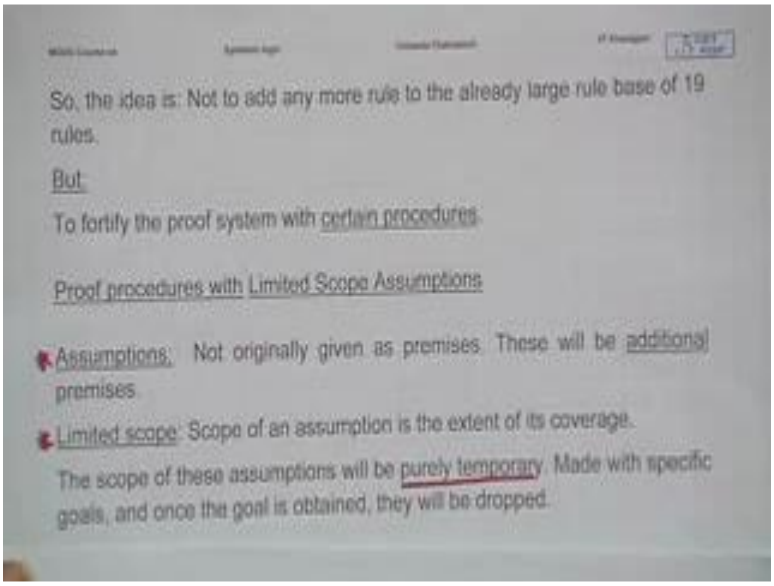

So what we give is not to add more rules because we already have 19 rules, even then we have found that the rule basis not complete it does not give us that guarantee. So we are not going to add any more rules to it, instead what we are going to do is to further strengthened the system which some procedures, some additional procedures will be added. These will be proof procedures, which will call proof procedures with limited scope assumptions.

Now when I say assumptions what we mean? Is not exactly the given premises, so assumptions on not originally given, originally given or the premises; the specifications that you start with. Assumptions are you are saying; let us assume this to be true. So there are not part of the premises, but can be added to the premise base. Listen what I said. They can be added. So they are additional premises, which should assume to be true. That kind of assumptions addition these procedures will allow.

Second I said Limited scope, what is it mean? Well the every assumption has a scope meaning how for does it covered, the extent of its coverage. So the scope of an assumption remember and assumption falls by default within its own scope it covers itself plus it covers some more. So, the scope of an assumption is where it starts and where it ends. What we are telling with in these procedures would be limited scope assumptions meaning there will be temporary, purely temporary with some limited scope so that, it does not go all the way from the beginning of the proof till the end. So the last line of the proof will be free of all assumptions and the temporary in the sent that you are starting the assumptions with some goals in mind. As soon as that goal is reached the assumption will be dropped or discharged.

So limited scope assumptions, we are going to see this in our next modules. I am just going to introduce the name

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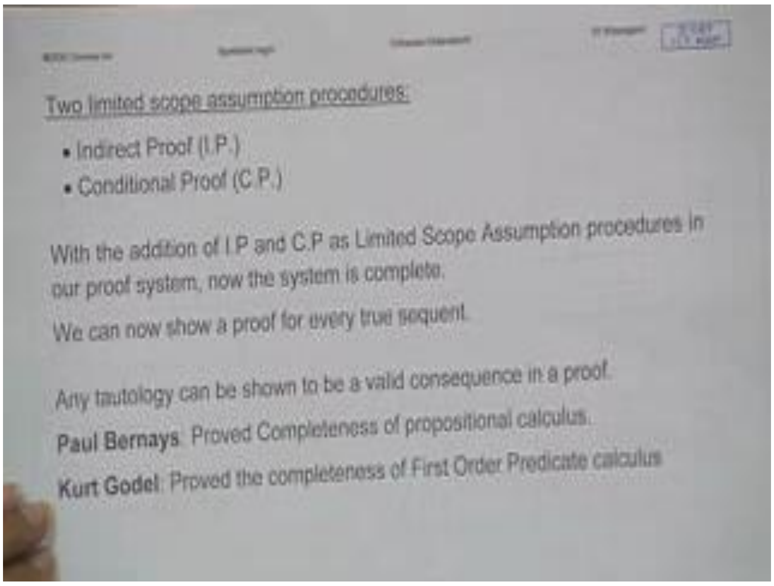

So that you know what is coming up. We will look to two limited scope assumptions in the next modules, one is called the Indirect Proof which will call IP and the other one is called Conditional Proof or CP. You were going to refer to it has CP. So and you will see after we have discussed this and after you shown you that bit the addition of these two proof procedures our proof system will then stand as complete. Then given any kind of true conclusion will be ready to come with a proof. So this is what will be achieved and if you give as a any kind of tautology, any true conclusion that follows will be able to provide a proof for it.

Now so for I am said it and I have not really gone in to the in two cases of the theoretical proof or showing you the mathematical proof, but I will refer to you like this. That if you are interested, you can look up Paul Bernays. He is the one who proved the completeness of propositional calculus. So do not take my word for it, there exist approve mathematical prove that shows why after this insertions this additional proof procedures the propositional calculus will be complete and Kurt Godel some of view may have harden in Godel. So Godel prove the completeness of the fits order predicate calculus. We have not gone to first order predicate calculus, we have not gone to first order predicate calculus here we are here at the propositional and logic label and this is the person who actually proved why and how propositional we calculus us now complete.

So, with that I am going to end this module and in the next module as I said we will pick these Limited Scope Assumption procedures and explain it to you, all right, so looking forward to see in you again in the next module.

Thank you very much.