Symbolic Logic
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Lecture – 25
Proofs with All Rules

Hello, we are going to do today the module 25 with you. This is module 25 of the symbolic logic force; where we are learning how to do the formal proof of validity and we have been exposed all right to the nineteen rules so for and we also well learning how to do the formal and profile validity. By now I hope you have learnt to rules well enough. So that reference to the rules we if need we can always bring the rules to show you, but now by now we should be familiar enough with the rules, so that we can proceed without any further explanation about them. Let us consider an example here, here is the proof, here is the argument, where you have this one premises C horseshoe A dot not B and this is second, this is still the C horseshoes A and this is A dot not B or A, how to go about this and how to construct the proof.

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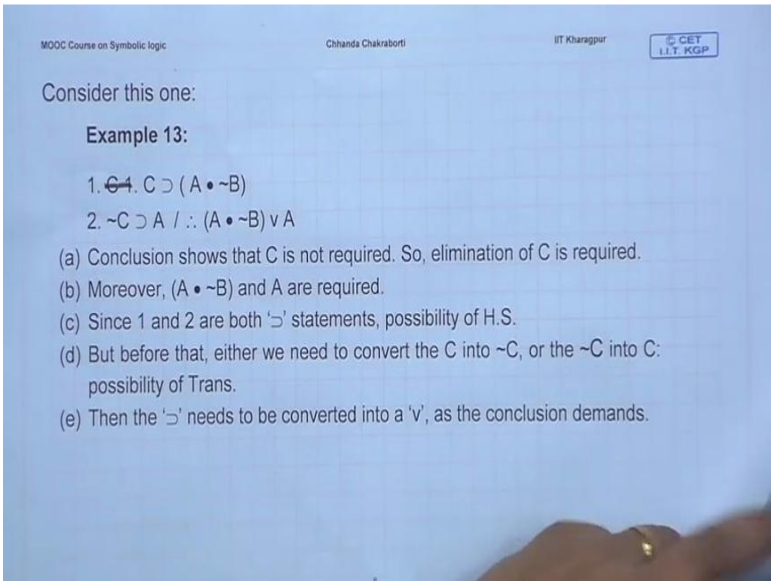

So as you can see that there is no place of C in here. So somehow we have to eliminate the C correct. So that we get this, because we have A, and we also have A dot not B in the premises. Only thing is how do I get the see out the elimination of C and we want to keep the A not A dot not B and A in the place right. Since 1 and 2 both are horseshoe statements this opens up certain possibility of horseshoe application rules and it seems like probably there is A way to do the H.S is here so that we can keep only the A and A dot not B, but how that you have to stop thinking. Somewhere you remember H.S rules that we need to have an exact match in order to use the H.S rules. This is the H.S rules.

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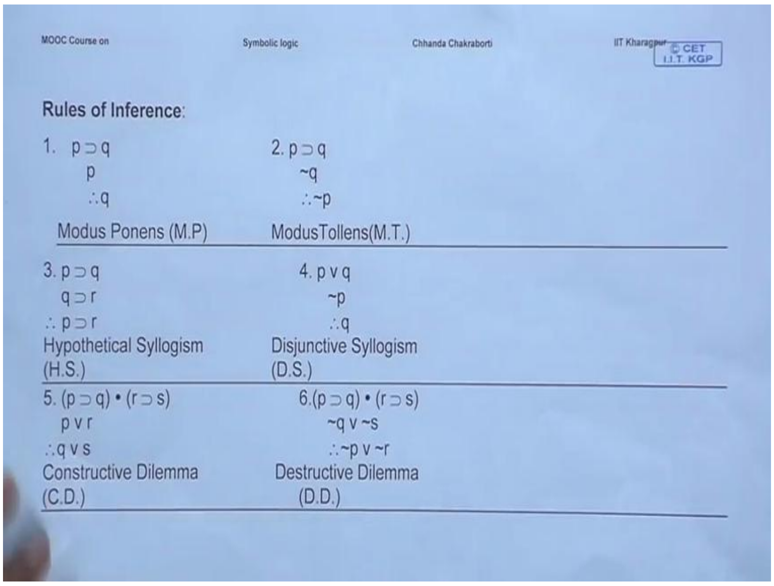

It shows that you have to have the q in the strategic positions. So the q is in these 2. premises should match, meaning what the consequent of the first conditional must exactly match with the antecedent of the second condition. So that is required. Now if you look into this two statements here, 1 and 2 you have see here and you have tilde C and one of them is in the antecedent position, the other one is also in the antecedent position.

So how can we use the H.S for example, so there is A, some prince storming is required in order to change that and then we have this wedge situation we need to have a wedge in place. All these are clues for you to start the proof answer of work on the constructed formal pro profile validity. We have A solution already, but I suggest that you tried on your own and not just copy the problem that is worked out, because of will ideas to get you started get you started to think and to actually work on to proof.

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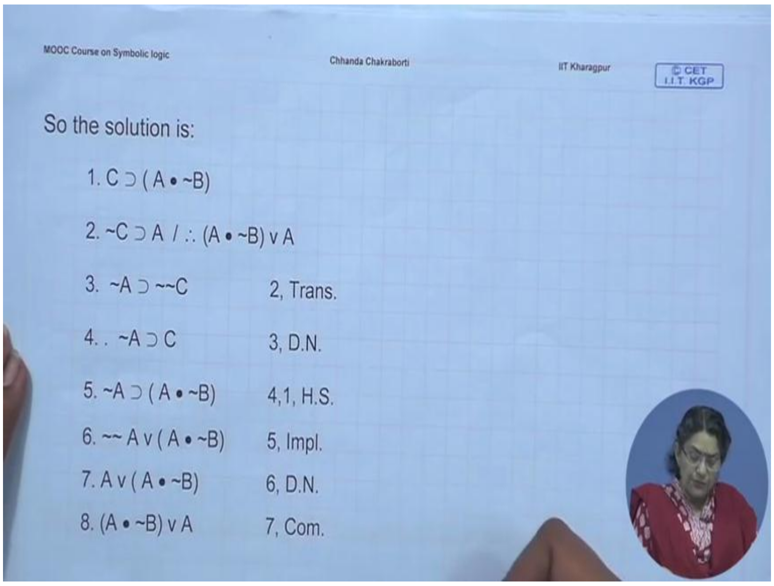

This is how I have done it. Non necessary exactly how you how to do it, but this should set your thinking into motion. When I did is to do the trans on line number 2, line number 2 is not see horseshoe A right. Now I have the trans gives me not A horseshoe not not C. The question is why I need to do the trans on line number 2 that should be clear to you otherwise there is this is not A random proof.

I know why I needed to do that because my plan is to some get this C has exactly matching with this C. So that I can then apply H.S to get this A and A dot not be somehow out. So this is what may the motive is behind my using the trans.

So now is we have done the double negation and this is what we have. When you have that you can see that this is an exact match and we can apply the h s rule here and that is what I did, so 4 and 1 because I wanted to get rid of the C; 4, 1 gives me not A horseshoe A dot not B. Sounds very close to what we have here right because the C is now eliminate - still some work is related left. So now, this horseshoe has to be change in to A this junction and you know which will gives you that is your impulse rule, which is applied to this horseshoe and we get this. Are we now at target? No, not quite because the positioning of this to disjuncts is not exactly like this. So we have to do this step also to get rid of the double negation.

Even in here you cannot stop the proof because it is not exactly the conclusion that you want to derive. This is where your. So, this is the complete proof of this argument, this is how you show that from these 2 premises this conclusion follows validly in this way. get it. So we have to know as you can see there is a combination of rules of inference, along with the rules have equivalents, which is very very normal very usual to do in A formal proof validity, but that is of the point, the point is to know which to apply when and which some already preplanned motive so that you know why you are doing what you doing.

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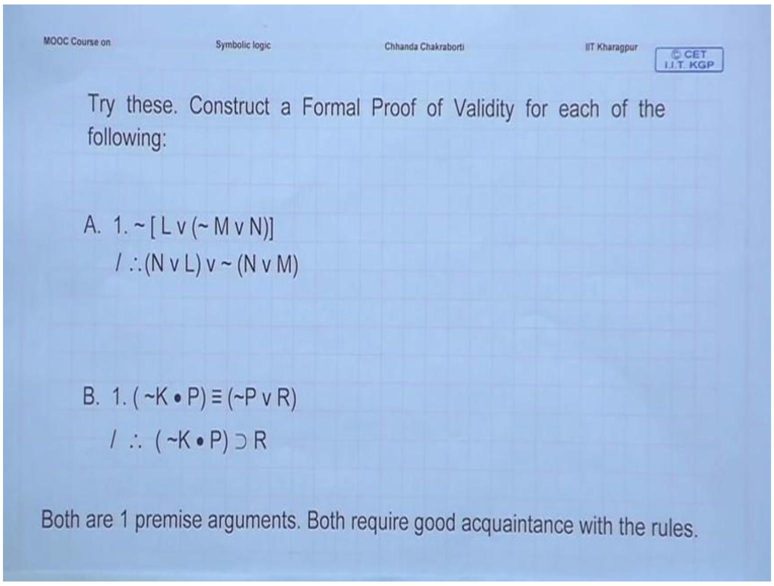

Lest see other problems and this size suggest that you do not even if I explain it here you try to do it on your own so that you have feel of the problem. So here is one problem. I suggest that you take this down and before going any further into this module, where the solutions are presented you try to work it. If you if you do not succeeded in one attempt no problem try it again, but do not give up that is the point. So this is like small puzzle that you have to be art it in or it to solve it this was my first problem and this is second problem. So these 2 I suggest again that, take it down on a piece of paper and try to solve them. You have all the rule set you disportion, you have all the time to think about it and have a plan before we go for it.

So at this point I have suggest do not look into the module, solve it first and you come back to the module to see how it has been solved by me.

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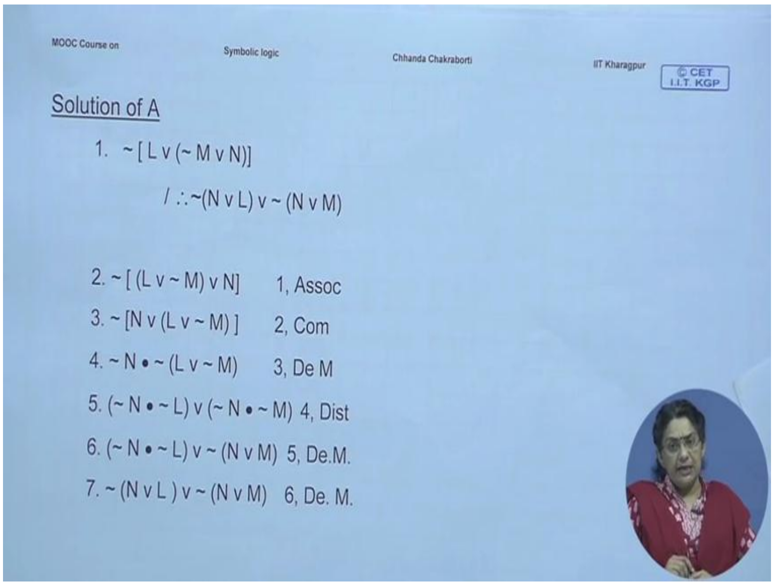

Both of these are one premise arguments there is only one premise here and both required good grass over the rules. The first one the solution of the first problem, I have worked it in this way. See this is the conclusion. N it is a negation of N or L or negation of N or M. If you look into the premise, you all your components had there, all we need is some how to approach it and they are all in as a wedge. Here also all your connectives are in wedge. So somewhere we are talking about redistribution of these. So, that this becomes in to this

So, there is hope here, but you need to work with the statements A little. How I proceeded is that I regroup them and there is A reason by I regrouped in this way. So first let me explain that the regrouping is like so that first it was inside take a look it negations sign is outside, but this l was the primary disjuncted and not M or N was the other one. What I did it was to group them in this way. So L wedge not M is one disjuncted or N is another this junked this is what association allows the regroup right, provided you have all wedge or all dots remember that. Question is Why? Why did I choose to use association? The answer realize in the fact there I see that M is distributed with L and N and we need to have N in A position to distributed over L and M and this gives me that sort of possibility, it leaves open that possibility.

So you need to know distribution rule well enough to see this possibility clear and then association is just one step to prepare other statement to the application of distribution let me show you. Then I got the position of N as the first disjuncted right, but there is still at tilde in front fine. So De Morgan will give me at conjunction. So not N dot not L wedge not M fine this is what we have so for and then we bring in distribution. We have here what am i doing I am distributing this not N with L and inside not M and with distributed groups like this. Then De Morgan on that only on this part gives me this. This is untouched, one more De Morgan will give me this and this is your target conclusion take a look, I leave with in front of you for you YouTube's are right.

The point is that you need this, but in order to get here you need to anticipate these moves a little bit. So the whole thing is to prepare the proposition in such a way that legitimately you can derive this line. Is a little bit like playing chess where you have to anticipate the moves, at least 4 or 5 steps ahead you need to think a little, you need to arrange response in such A way that you get that move open.

So similarly in the proof of validity you need to think earlier little bit the plan has to be done so that the steps come through one by one, when you learned that the target line. This was our first problem.

The second problem the solution is some over like this. You see this is the premise and this is the conclusion. Somewhere again all your components are there, but not in the way you want. So we have to process, we have to somehow replace, to get this thing out of here.

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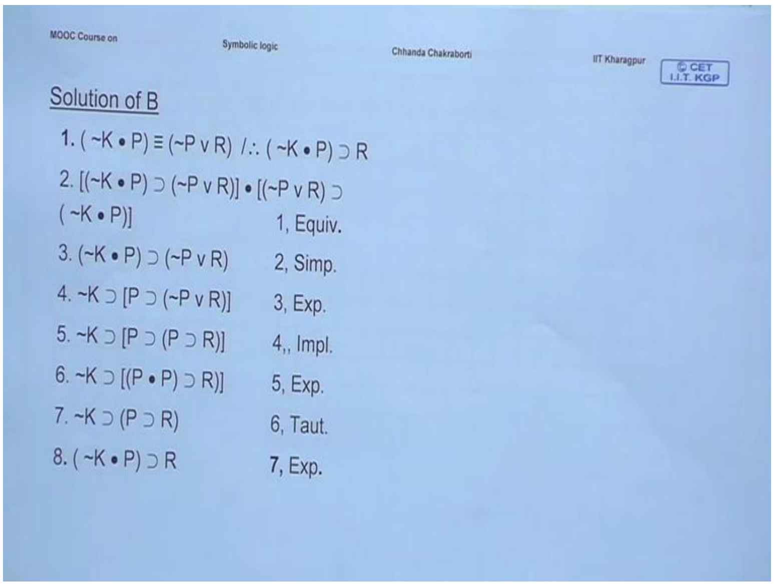

First of all this is the triple bar and we do not have place for the triple bar anywhere here, but we need is a horseshoe fine, that should give you some clue. That we are going to break open this triple bar in the horseshoes and then we may have to work on the statements to get rid of the re down then for example, there is P here, there is not P we have to somehow get read of all that to get them in to this kind of primed for. So here is the way I have approach the problem, this is my application of material equivalents rule on line number 1. There is only one premise. So, the we have to start here and what you get is a conjunction of 2 horseshoe.

If you look in to this, the horseshoe here is not K dot P horseshoe not P or R and then this is not P or R horseshoe not K and P right, this is what P dot V horseshoe q and q horseshoe P. The point is this looks long and are deserved relevant for our proof. The answer is no. We can do very well with only in the first conjunct the first horseshoe that in itself is enough. So there is A possibility of using the simplification, what you have done is to reduce this triple bar in to horseshoe first of all. So lot of extra, extraneous material has been simply chopped out of that. Then we do this and here again I have to make sure that you know this rule, the exportation rule, which is a rule of equivalents the rule of equivalents among this exportation is allows you, to work with the horseshoe statement in A certain serve away.

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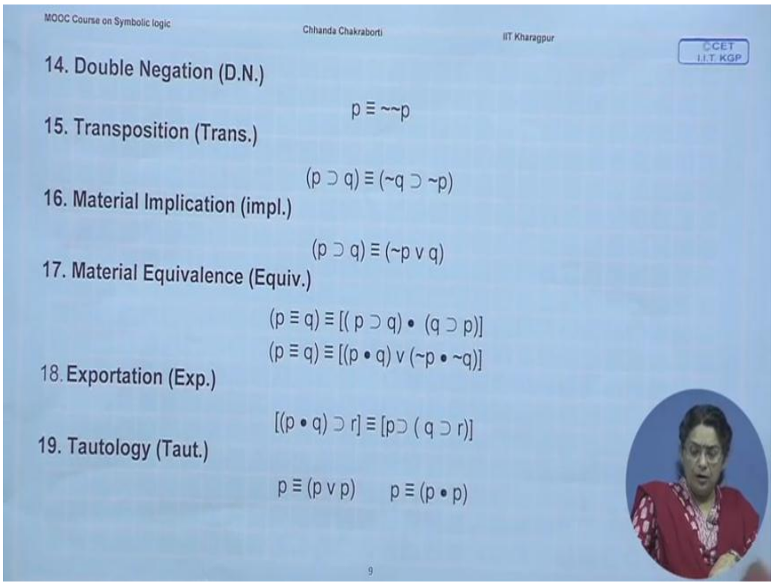

So let us remind ourselves. What does it say? The exportation rule saying that if you have P dot q then r right, then you can expand it as p horseshoe q horseshoe r or if you have a p horseshoe q horseshoe r situation like this, you can rewrite it has p dot q horseshoe r. You can compact make it into compact.

So if that is that, then look at this sentence very well and you need to you understand that what we have here is p dot q horseshoe r. This whole thing is your r. So what we can do now is to re write it like so by exportation. This is P not K horseshoe P horseshoe this is your R. This region will come provided you understand the rule exportation very well right. So that flexibility is something that you need to work on and understand. So this is what exportation will alloy you. This is expansion of what is being said here, but there is also an agenda behind it. Why are we applying exportation here, that should become clear it you as we go on.

See the reason is that in tilde P wedge R, I have seen that this is nothing, but P horseshoe R. So if you can get another P here, I say the possibility of collapsing this two P is in to one by using the rule called tautology let us see. So let me just in I am I head of that. So let us see first of all I have do done exportation, then this is your implication and this is what we can now do by exportation on this part, this is your P this is your Q this is your R.

So P horseshoe P horseshoe R by exportation would become P dot P horseshoe R correct. So I am applying the rule of equivalents only into the part where I want to and this is the exportation after applying exportation. This is what you going a get. Now you see what you are trying to get at, the tilde case right where we wanted to the P was too many times. So, we needed to work on that the R is also in the right position. So this would give you an opportunity to apply which true the tautology which says P dot P is equivalent to P right. This is the rule that we are referring to this is tautology and this rule is what I am applying here. So once we have that not K horseshoe P horseshoe R, you can see that this matter of just one state to reach that by another application of exportation. So here you want not K dot P horseshoe R.

The lesson from this is that no rule is unimportant. So you the lesson known, are the unfamiliar rules are the once that you may have to use a lot to serve of get grasp on this and rules are in rules of inference have their place, but also the rules of equivalents are also equally important. This was our way to do the formal profile for validity, but let us said whether you have enough command over the rule base by now.

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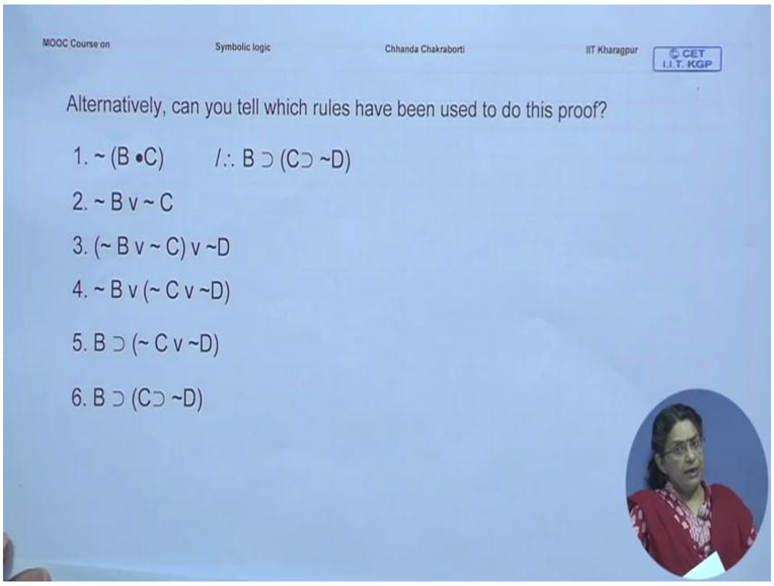

So alternatively we can ask you even this, that here is A worked out formal profile validity, take A look at this and try to tell by which rule this lines have been generated, can you do that. So with this is your original argument. Here is the worked out proof. Just try to fill up the gap and say by which rule we got this line for example this. That would an exercise for you to also check back, whether how much of the rules you have grasp when, whether you know what they do.

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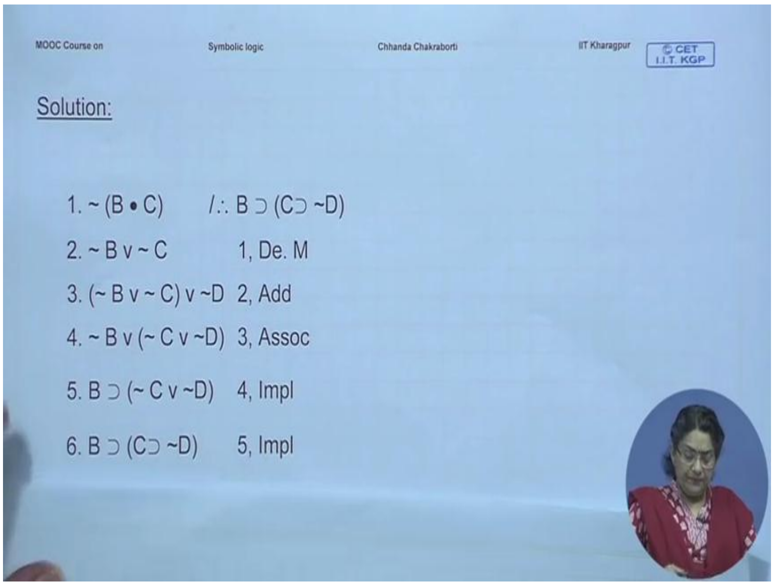

And this again I say just that you instead of looking into the remaining part of the module, you stop and work it out and then check the result with the worked out solution. So here is the solution which comes next. See this line was given and from this the next line followed by the rule De Morgan. De Morgan does that. This tilde and then within bracket the dot becomes tilde B wedge not C De Morgan.

The next one where then tilde D appears is nothing, but adding the tilde D on to this line. So 2 addition will give you this right. Here comes the application of association, this is how we got that and then this is your implication by which we have change this wedge in to the horseshoe. Still not done there is a part that we want to work on. So this you obtained by applying implication again on that part. The whole point of this exercise is to see whether you can recognized, the results of rule applications in each case and which rule is being applied.

So that it opens your eye and also you see you find out realized that this is how the rules can work out and may be it will give you some idea about how to do the proofs is also.

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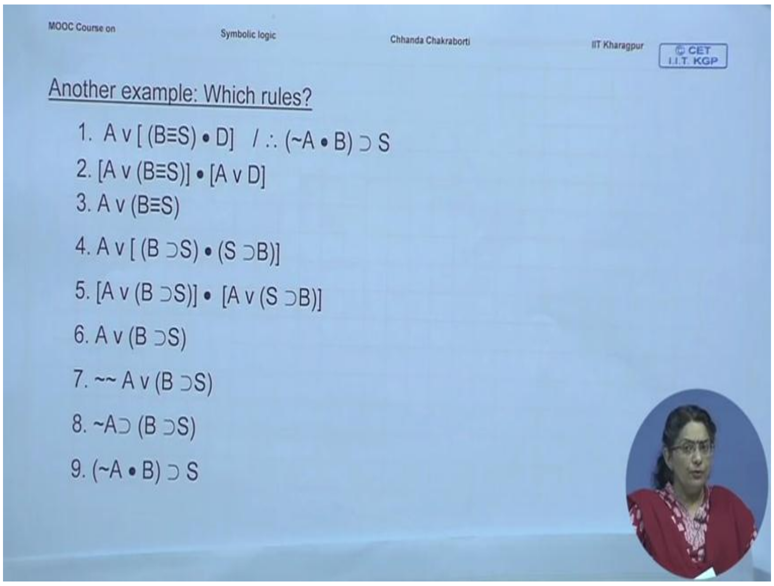

So here is another example, again I will ask you which rules. So this is a problem that is worked out, here is the premise, one premise and here is the conclusion. The every line there has been derived. The question is how is it derived? So take a look in to this worked out example and try to fill out the justification part on your own. The rule and the line number in each case try to fill this out and that would be another way to know what is it that the rules to and how can you generate a proof like this.

So this is the problem for you to work on, but I will finish the module by showing you the worked out result, which you can compare with once you finish your own work.

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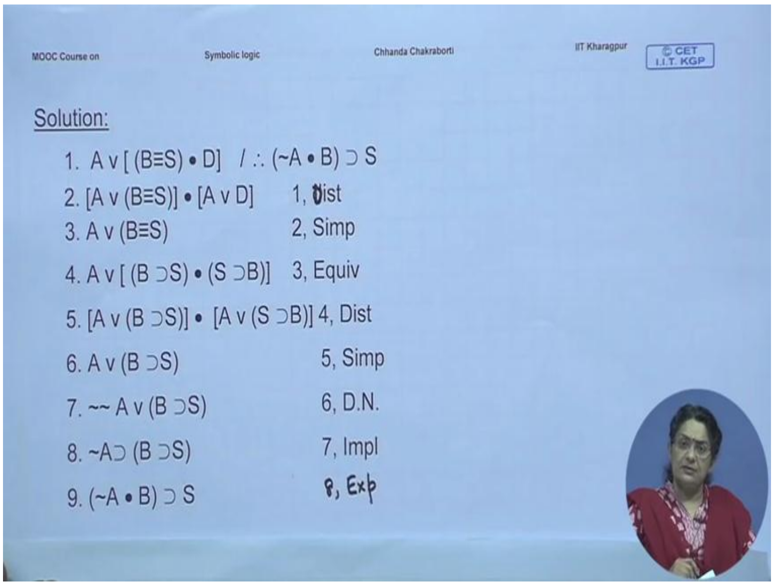

This was the original one given the argument itself. This is the line that we obtained from there. How the answer is what happen this that the A has been distributed. So A wedge B triple bar S is 1, then A wedge D is the other conjunct. What was inside became the outside connective. So 1 distribution then comes this where A wedge D has been chopped out. What you derived by simplification rule is A wedge B triple bar S. This is line 3 and application of equivalents, where this triple bar has been broken down into the conjunction of 2 conditionals. This is 4 distributions, where, what are you distributing again A over what first B horseshoe S and then is horseshoe B. So look at and the dot becomes the main connective. Which was inside will become the outside connected, that is how distribution words.

Once you have obtained that this part is being removed, eliminated by what again the rule is simplifications. So now we have only this is your final target, but we are getting close. This is double negation of course, requires no explanation and this is implication. Is this step necessary to come here from for this line? Yes, at this point yes. From here we need to have A double negation to get the implication of this for. Once you have that you can probably see now that we are absolutely there except this one step which is your exportation. By exportation we are joining this to antecedents and this is your whole antecedent here is the s correct. So this is where we can see how the proof can be worked out what rules we are applying and so on and so for.

Now, this is where I am going to n this module, because I think all that you needed has been already touched upon. The remaining part is your practice and for the practice to see are you can get a proof going. Without the plan and without the knowledge of the rule basics going to be rather problematic, but I know that the rules are like that and probably by now you have understood and master them A lot. So I do not think there is going to be any problem for doing the formal profile validity.

So with that good luck and all the best for doing this formal profile validity or derivations and this is where I am going to end this module.

Thank you very much.