Symbolic Logic

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Lecture – 24
Understanding the Rules
Strategic Tips for Doing Formal Derivation of Validity
Proof with All Rules

Hello, we are into module 24 of Symbolic Logic NOC course and we were working on the formal profile validity. So, we will continue to do that.

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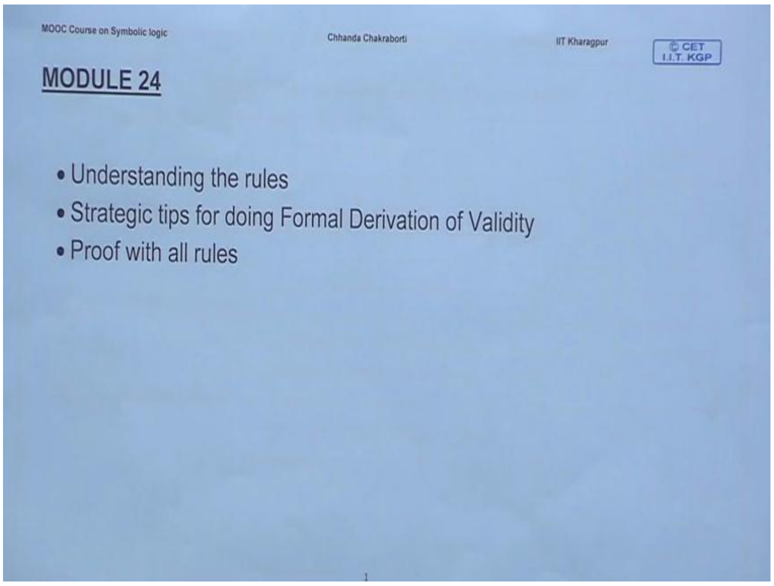

Last time we were talking about the equivalence rules, so we will continue understanding them a little bit. And then there will be some work related tips if that is possible some ideas that is, how to go ahead with the formal derivation of validity and then we will combine all the rules as in - the rules of inference as well as the equivalence rules to do the proofs. So, once more, we take a look in to the rules where replacement because we were just beginning to look at them.

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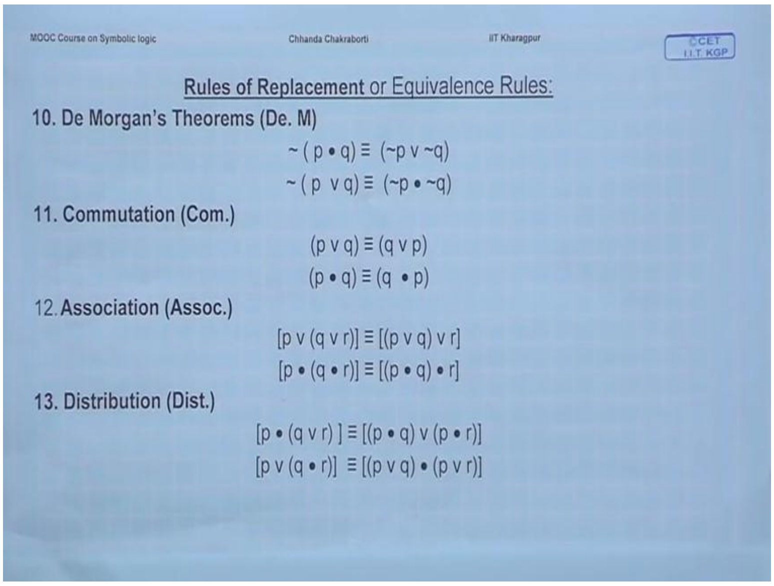

So, this four were introduced in the last module. See all of them are about dot and wedges. So, what we are looking at is how to change the connective from one to the other, in a way in a proof that could be very useful and also important to remember. For example, see here in the distribution what happened conjunction became this junction and this junction became conjunction and sometimes you know that there are rules which apply specifically to conjunction. For example, you want to get rid of the P wedge r right and you know the there is a rule of inference called simplification which works only on the conjunction. So, there is benefit of looking closely in to this equivalence rules I suggest that you also pay attention to the name that these are actually rules of replacement.

So, in a way you are not deriving anything what you are doing? You are replacing one expression with same meaning and same truth value expression that is what you are doing. So, that is how the De Morgan's theorems for example has to be read, this is how the commutation has to be read when you have a wedge And you can swap the position when you have a drought you can swap the position, when you have association you can regroup them without disturbing truth value though the emphasis changes in the association as I have shown you - P wedge q wedge r, where P is on the first emphasis whereas here the P wedge q is the emphasis and then r is the other this junk. So, this is association and this is your distribution.

Since we have to done these rules earlier in the earlier module we will proceed to no more. There will be 10 rules of replacement or equivalence rules on top of your 9 rules of inference.

So, all together in your data base or in your rule base there will be 19 rules of derivation and you will see that each one has it is own place and each one has it is advantage to know. So, pay attention to each one of them as we go along. So, we have covered so far De Morgan's theorems which we will call De. M, we have commutation which you will call in short come and we have association which we will call Assoc and this is distribution, distribution will refer to us Dist. Apart from this there are few others which we also have to sort of look into.

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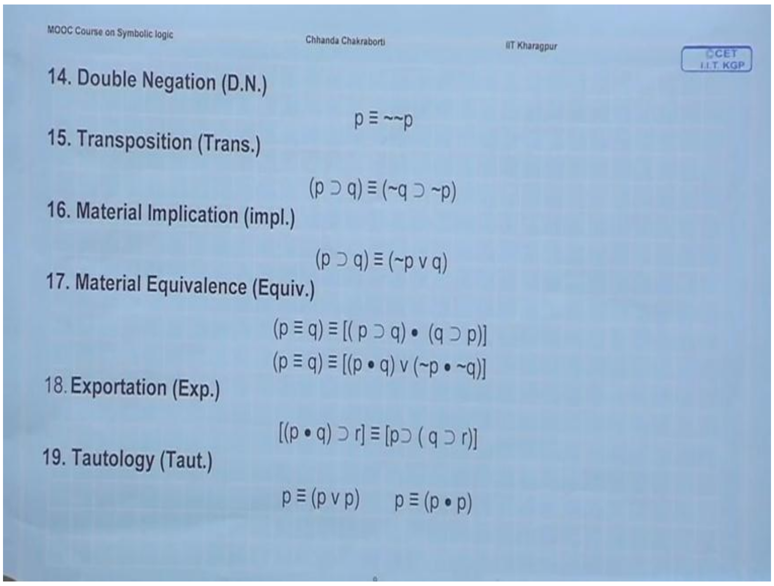

So, here are the remaining four. The first one is double negation, intuitively obvious and very well known all we are saying is that given an expression you can replace it with the negation of negation of that and similarly when you have an expression like this tilde tilde P you can rewrite it as p.

So, that is rule without the rule remember you do not have the permission to replace that. So, the system which misses this double negation rules for example, cannot do what you can so easily do though you may intuitively know, but since you are in a formal process un less there is a rule that specifically says that you can do this it can (Refer Time: 04:39). This is on the other hand transposition rule, transposition rule what you are doing? You are saying if P then q and that is replaceable by if not q then not P right. So, in a way we are restating what is expression now, now if you those of you who are conceptually would like to understand it then what we are saying here is as equivalently that is why? Because here we said that P is sufficient condition and q is necessary condition. So, if q does not happen then P does not happen this is the exactly what we have said.

Similarly when we have an expression like this you can rewrite it has P horseshoe q. So, there you are this is transposition rule which allows you to swap the position and attached to the components. This is on the other hand the material implication rule which follows from the very truth developed horseshoe, you already worked with it in the truth tree and I have mentioned it during you truth table time also. So, this is nothing but saying that if P is horseshoe can be recognized this junction in this where. So, if P then q is equivalent to saying either not P or q.

Similarly, this junction is equivalent to saying that there is a horseshoe here. Depending upon what your proof needs whether it needs this junction or whether it needs a horseshoe you are going to use this. This is material equivalence rules, they are also equivalence rules and again if you recall the truth cable of triple bar you will understand why this has to hold. What I am saying here that P triple bar q is equivalent to if P then q and if q then p, we have already established that when I explain to you what if and only if means this is exactly what we referred to that it is a conjunction of two condition. This is the reason that triple bar is called by conditional. So, that is captured in this rule.

This on the other hand follows from the very truth table of triple bar, P triple bar q is equivalent to either P and q or not P and not q this rule you have used and understood when we were doing the truth tree. So, this is these two together are going to be referred to as material equivalence and then comes exportation just like exporting it out. So, here is what you are saying is that if both P and q then r then it follows that if P then q then r or you go come the other way if P then if q then r which means that if P and q then r. So, these are interchangeable or replaceable by each and the name of the rule is exportation. The names you have to sort of remember and the rules also you need to remember, we go back to your list again and again to see the name as well as what exactly it is says.

This is the tautology rule, last one which gives you these two tautologies. So, P can be replaced either by P wedge P or P can be replaced by P dot p, similarly P dot P can be reduced p, P wedge P can be reduced to p. Please note that these two are the only two tautologies that this derivation system will allow no other tautology will be applied here. So, let us look back in to our rule list and once more learned the abbreviations because we there is no need to refer to the rules by the whole name, but it is important that you know the rules exactly and as they are. So, double negation will be referred to as D.N, transposition is as Trans, material implication as impl, material equivalence as Equiv, exportation as Exp, and tautology as Taut. And this on top of your other four rules that we have covered already that is De Morgan's commutation, assoc and distribution.

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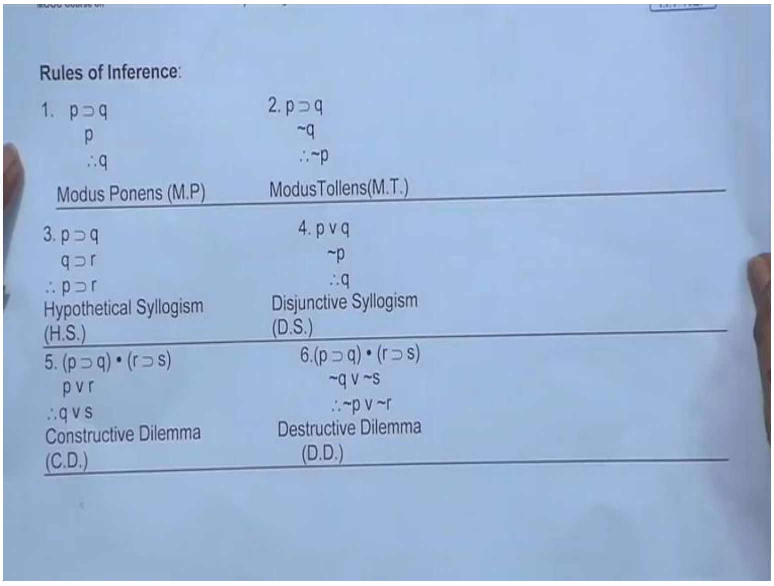

So, let us look in to this totally of these rules how many rules we have learned and what is the difference let me remind you once more. There are rules of inference; the first nine that you learned these are rules of inference. So, one directional and they have this requirement that you can derive this triple bar, triple dots actually referred to that you are deriving conclusions, so that is why they are rules of inference.

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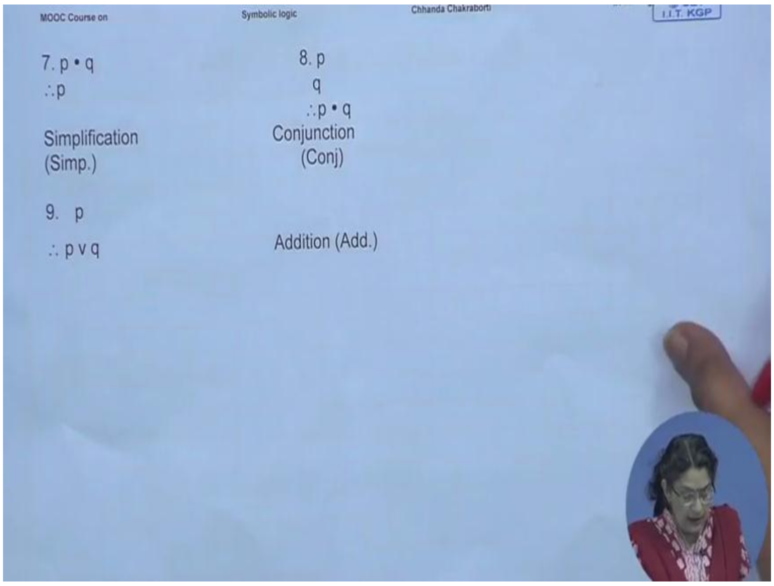

So, these 9 were our first 9 rules of inference which we are going to use and then we have the rules of replacement. So, how many rules of replacement? We have 10, these 4 plus this 6, so all together we have 19 rules at our dispersion. And as I have already mentioned couple of times, but still it may help you to go through this that the rules of equivalence or replacement rules are actually by directional you it comes you can use them any which we are want to.

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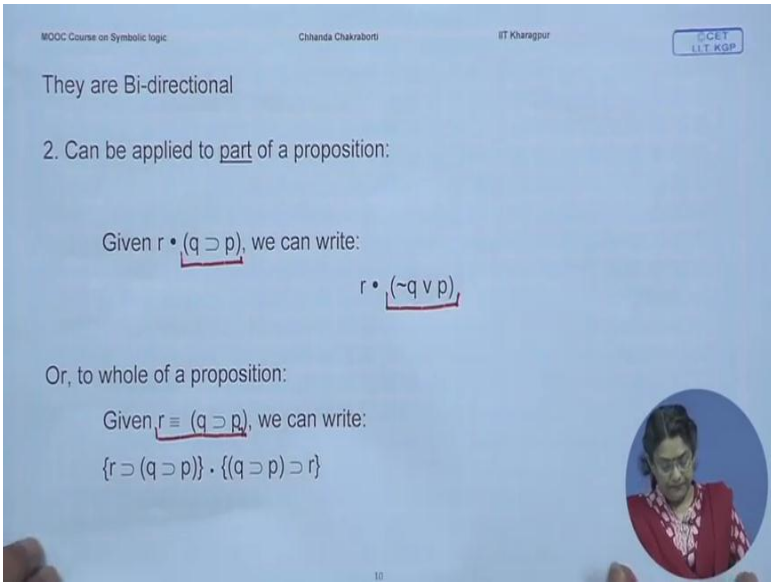

Moreover, they can be applied to part of a statement. Remember the rules of inference can be applied only to a standalone proposition as a whole, but this rules have replacement can be applied to a part of a proposition as well as to the whole of a proposition, so here you are. This is for example, suppose you are giving r dot q horseshoe p, you can rewrite it as r dot not q or P what did we do, what we did is to apply the impl rule where? Here, this is the part that we have replaced it that with. Remember the main connective is here is dot without touching it you can still replace the parts of it. Similarly this does not mean that the rules of replacement only apply to part, but you can also apply to the whole of a statement so there is no bar. For example, r triple bar q horseshoe P here it q triple bar is the main connective and you can apply the material equivalence rule to this triple bar to get this kind of horseshoe.

So, it is a conjunction of two horseshoes r horseshoe q horseshoe P and q horseshoe P horseshoe r right. So, here we are applying it to the whole sentence and here we are applying it to the bar.

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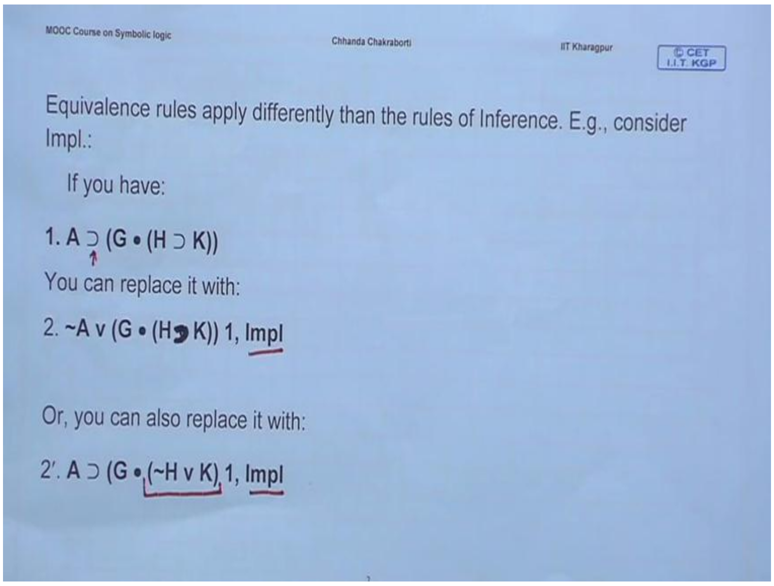

Now, same thing, same point, but let me illustrate that again further bit another example. Suppose you have a horseshoe G dot H horseshoe k, now you can do two things here depending upon what you need in the proof you can work this rule, so if you want to replace this horseshoe for some reason for with a this junction then this is where you are going to apply, here you are and it nothing changes here, but this changes in to till the a wedge And the justification is by 1 impl.

But there are two horseshoes here one is here, one is here. So, you can apply it here in the main connective or you can do it in this part without changing this horseshoe you can selectively use it on this horseshoe. The rule here impl is a rule of equivalence and that is why you could do this, but you cannot do this with a rule of inference. So, you need to know which one is a rule of inference and which one is a rule of equivalence because there are things that you can do with rules of equivalence which is not permissible if you are handling a rule of inference.

So, let us come to now, how to do the proof, how to do the derivations we have seen some examples already, but now we are going to slowly move on to doing more and as you have more rules now, so the proofs are going to be somewhat not so elementary and so on.

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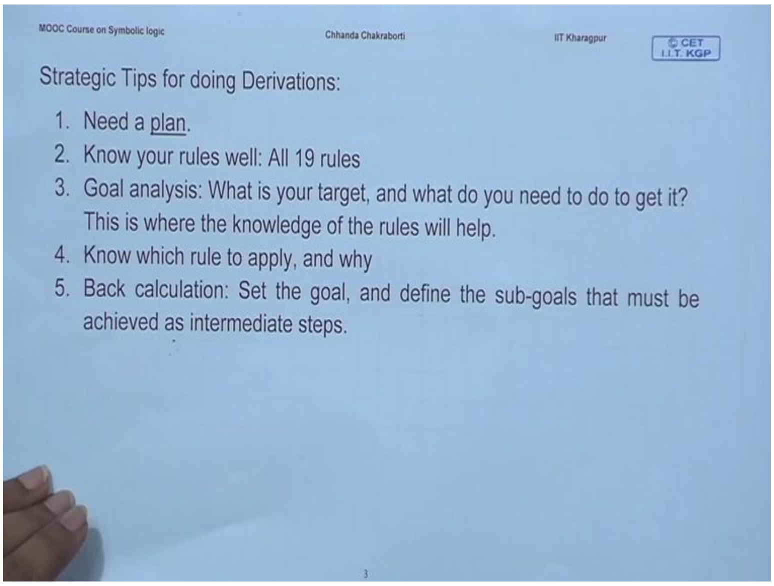

So, first of all what is that that you need to know how to prepare for doing derivations, I have already told you that you need a plan, there is nothing (Refer Time: 13:45) or nothing random it is not mindless like doing a truth table that I start somehow and I automatically it will be finished. In proofs it is like a little puzzle, you know the dancer is there, but dancer may not be very obvious or explicitly presents. So, which is why you need to think a little and have a game plan before you can event start the proof. So, first is applying, how do you formulate a plan? As I have told you that you do a rough work before you will get a head and do a rough work on this piece paper to think, how to get at the conclusion.

In order to have a plan it is mandatory essential that you know your rules all the rules. So, you have total 19 rules and it is important that you know them because otherwise you do not know which inferences are possible and which ones are not and which rule can help you in which way. So, this is why you need to practice a little with the rules to see how far you can go with these rules. Now the other thinking is that when you are looking and making a plan the rules will be there to give you the leverage, to give you the strategic advantage, but what is also needed is some set of goal analysis. So, ask yourself what is that you want, what do you want to derive and in order to get that what else do you need to get in the proof. So, is that way you know this is my ultimate goal, but before that I have to meet some of the sub goals otherwise I cannot reach my target.

The target obviously, ultimate target is the conclusion that you are going to establish, of the given argument to conclusion, but in order to get there you need to probably arrive at some of the sub goals. So, this is where the knowledge of the rules is going to immensely make a difference because many of the steps if you do not know the rules you might think that this is not even possible, whereas there are very efficient ways to get at them provided you know the rules. And when you are applying the rules know which one you are applying and why, just like in the truth tree case I always said that you should go with an open mind and open eye. So, you should know why you are picking this rule out of the rule base and not the other; obviously, there is a relevance to what you see that you are trying to do in the proof, so that why is going to be crucial why for you to keep in mind.

And then some people do back calculation also for example, I mean just like we did in the last examples in the last module that you know we were working back. So, if the goal is like this and in order to get at the goal what is that I need to have and then in order to get that what else intermediaries' steps to we need to taken so and so forth. These are tips, but what is required is that you do some practice you actually try to solves some problems with the rules, it is not that you have to memorize the rules because the more over use them it will automatically stay in your memory, but what is need is that you pay rather close attention to the rules the name as well as how they move, what do they allow, what are the legitimate conclusions that they allow.

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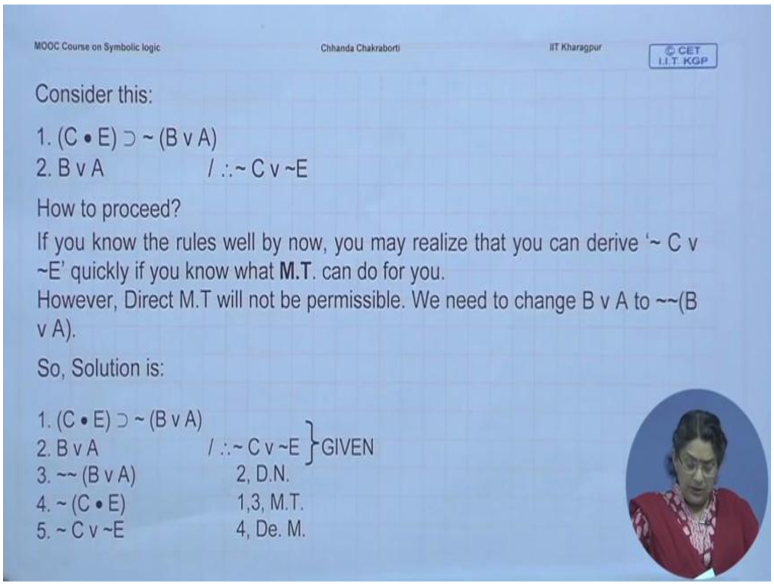

Now, consider this for example, let us see whether we can proceed in this way. Suppose you have a proof like this, now the claim is that this is a valid conclusion your job is to show how to get here. Now the premises are two and you can see that not C wedge not E is not obviously present in this, but clearly there is a way to derived that that is the claim; now you have to explicate that, so how to go about that and this is where the planning is necessary, what do you need to do? Now one thing you need probably have noticed; however, pay attention do the premises, so one thing you must have noticed is that you have B wedge he here whereas you have not B wedge A here.

So, in a way that should start some sort of thinking on your part that what exactly is to be done here. Second is that this not C wedge not E and then you look in to this first premise and you have see dot E you should immediately connect these two that there is the relationship here. Namely, that if I negate this I will probably get that. So, somewhere we need to work this out. Now main thing is that to have a plan, there are modern one ways you can go around, but here for example, you see that there is a possibility of doing more (Refer Time: 19:04), how B wedge A is actually the negation of the negation of B wedge A, right. So, because we can rewrite this immediately with the double negation rule as negation negation B wedge A, and that would give you the negation of C dot E, correct.

So, that unless you know what (Refer Time: 19:26) rule is can do for you how would you in think in this way, that is would I meant that you need to do this. Now directly we cannot apply more resonance here, I mean though it may intuitively seem to you that B wedge A is the negation of the negation, but in the formal logic pardons you need to converted it into what is known as double negation of B wedge A, so that we will be requiring. Once that is done then the rest is very easy. So, if you have finish that thinking then doing the proof is not really a problem this is what is given and then we start adding lines. So, first line as we said is to convert this line two in to double negation stay and the justification appears here.

Every time you are adding a line just as in the truth tree in formal profile validity you have to justify and that will show up on the right hand side. This is then the (Refer Time: 20:30) result, we applied one and three (Refer Time: 20:34) and we got this thing as a negation out. Why this is important? Because now you know that we want to go here and we can do this by which rule? We can do it by the De Morgan rule. So, if you do not know what De Morgan does for you then you cannot see this state very well, but if you do then it is a matter of just second to come from step 4 to step 5. So, this is how we have established that this has to be valid.

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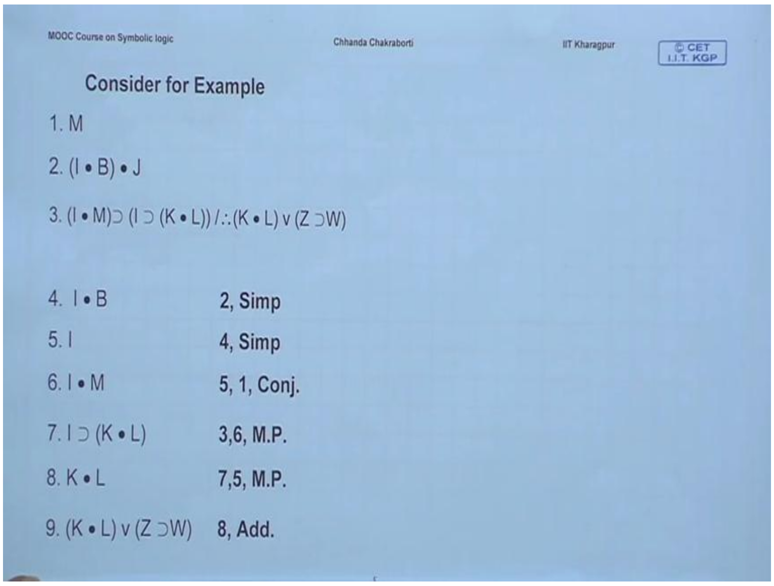

Here is another example, which has three premises and this is the conclusion. Take a good look at the premises, form a plan, a strategy and then the solution is worked out already I will explain it, but why do not you just take a look and try to have a plan how to go about so that we have this as a result.

Again, I need to remind you that you have to know the rules to see what you can achieve, any idea? Let us see that, see this is your target, now z horseshoe w is no were here, correct. So, somewhere we have to bring that in. On the other hand what is there in the proof is K dot L which is present in line number three, correct. Now, therefore, what we need is somehow if we can have I then we can have this provided we already have this part derived from line number 3. So, your sub goal is somehow to get I dot M and there is a possibility that we can see in lines one and two to get I dot M, can you see it? Here is M and here is some where I, right. So, that should tell you which rules to apply, correct.

So, if you have figured that out then the matter is very simple namely we will just go like this. This is my line 4, what did I do we just took line two and we chopped the j out remember j is now not even relevant for this, so better to simply and then further simplification why? Because all I need is I dot M, I see M and here is I, stand alone I; how did I derived that? From four, by further simplification on that line; now the job is to join this and the rule that allows you to do that is called conjunction. So, have to use that, 5 and 1 gives you I and M. The order is important have always said you wanted I dot M, now M appears first I appears here and lined five that is not the point when you are joining them you are joining in this kind of sequence you are joining 5 with 1. So, 5 is I and 1 is M and we put them together this is 5 1 conjunction.

Once you have I dot M it is a matter of just taking out I dot K dot L, I horseshoe K dot L and you have I. So, you can get K dot L without any further problem, but is this the conclusion? Not yet right, but you have this. So, how can I have this? Now take a look at the main connective that you need and if you know your rules very well then you see the possibility of using rule from the rule of inference which one is that - that is the rule of addition. So, this is your P to which you have added q, can I do that? Of course, what is being said in the rule is that if the statement is true then you can add with it anything you want with disjunction. If this is true there is no way this can be fall this is what addition rule guarantees.

So, this is how we have derived this problem, does that make sense. So, in a way you know if you do not know what the addition rule can do for you, you cannot anticipate the line in our mind. So, that is what I mean, I mean you need to think along with the rules and arrange your thinking as far as the rules that allow you to do let us take one other example and then will be finished, here we are.

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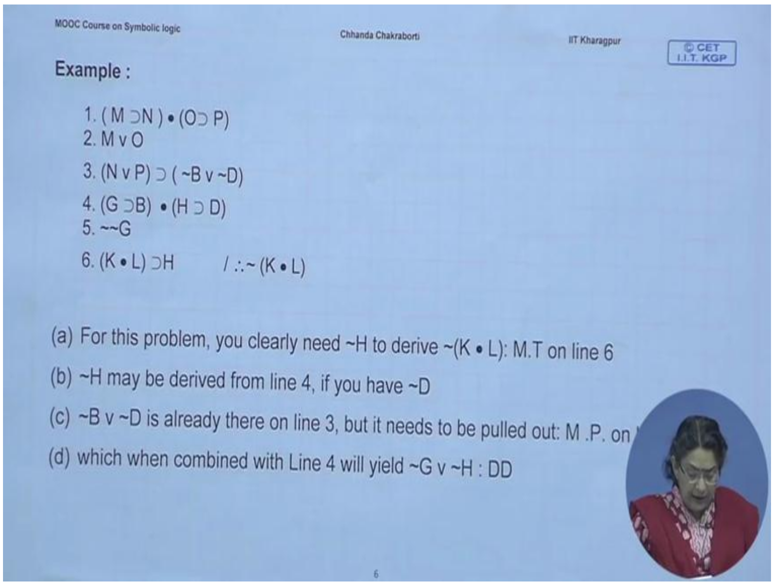

So, here is argument and this has about 6 premises and here is the conclusion, right. Now this K dot L negation of that is right here. So, unless you have not H there is no way you can take it out. So, your main idea is first of all to get this, but in order to get this we need not H. So, how from here we can get not H - take a look in to the premises very very carefully.

Here is a sort of a giveaway that you can get N or P very easily from 1 and 2, except you need to know the rule which allows you to do that, right. Once you have N or P you can get not B or not D out and then be if you can plug that N and two line number 4, it will give you not G or not H providing you know which rule to used and you how not not G. Once you have used not not G on not H not G or not H we are going to have not H. So, if you put it all together all those thinking together then here comes what will known as the whole proof take a look.

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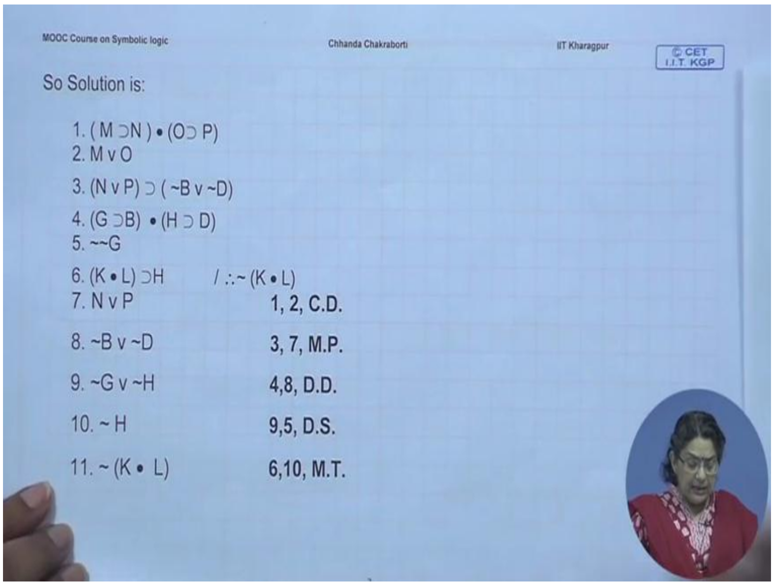

What we have said we can put it in to this way, what did we do? We used 1 and 2 C D this is the rule, unless you know the rule there is no way you can use it, this is no responance on 3 and 7, and here is the D.D destructive dilemma. If you do not know what destructive dilemma does, you do not anticipate this line that you can use; we are using it on 4 and 8 to get line number 9.

Once you are here and you have line number 5, you can apply them together and you can get not H. Why do we need not H? To get that not K dot L and that is my goal. So, this should give you an idea about how to go about with the proofs, you cannot effort not to know the rules so that first has to be sort of settle that you have to have a grasp over them. Then take a very detailed attention to the given premises because the whole game here is that there is an underline claim that within the premises somehow the conclusion is contained. All you need to do is to explicate it, derive it and then by step by step you will write there. So, there is a guarantee that the conclusion is there somewhere, but you need to find just like in a puzzle the ways to connect the premises with the rules so that you can arrived at the conclusion.

So, give yourself some practice, I think you will succeed absolutely without any problem. So, this is where I am going to end this module.

Thank you very much.